MATHEMATICS

CLASS - 9

BOARD OF SECONDARY EDUCATION, RAJASTHAN
AJMER

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MATHEMATICS

Class - IX

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MATHEMATICS

Class - IX

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PREFACE

This book has been written in accordance with the new syllabus for class IX prescribed by the Board of Secondary Education, Rajasthan, Ajmer. In presenting the book the basic object of the syllabus has been fully kept in mind and an attempt has been made to acquaint the students with the contribution of Indian Mathematician towards the development of scientific traditions. The contribution of Indian Mathematicians have been mentioned at appropriate places. Every effort has been made to present the subject in simple and lucid manner Important principal have been explained in detail.

In the interest of the students sufficient number the illustrative examples have been given. At the end of each chapter a summary of the chapter is given in the form of important points, which will help the students in revision. In each chapter objective, short and essay type questions have been given in sufficient number in the miscellaneous exercise.

We hope the book will be useful to students. Students, teachers and reviewers are requested to send their comments, suggestions and to point out any shortcoming in the book, so that the desired improvement in the book can be made in the subsequent edition.

Authors

SYLLABUS

MATHEMATICS

Class-IX

Time- 3.15 hours **Subject code- 09**

Max. Marks-100

S.N.	Name of Unit	Name of Chapter	Marks	Total Marks
1.	Vedic Mathematics	1. Vedic Mathematics	08	08
2.	Number System	1. Number System	06	06
3.	Algebra	1. Polynomials	10	20
		2. Linear Equations in Two Variable	10	
4.	Geometry	1. Introduction with geometry and Angle	02	26
		Plane Geometry line and angle		
		2. Rectilinear Figures	06	
		3. Congruence and Inequalities of Triangles	04	
		4. Construction of Triangle	04	
		5. Quadrilateral	06	
		6. Area of triangle and quadrilateral	04	
5.	Mensuration	1. Area of plane figures	07	15
		2. Surface area and volume of cube and cuboid	08	
6.	Trigonometry	1. Angle and their measurement	03	10
		2. Trigonometric ratios of acute angle	07	
7.	Statistics	1. Statistics	10	10
8.	Road Safety Education	1. Road Safety Education	05	05

Details of the Syllabus

Marks

Unit 1 Vedic Mathematics **8**

1. Fundamental concept of Vedic Mathematics (Part-I)

The meaning and applications of the sutras Ekadhikena Purvena, Ekanyunena Purvena, Vinculum (Negative) number, base, sub-base, deviation, The meaning & application of Sutra Nikhilam. Navtah Charamam dastah, Navanka and Ekadashanka methods of checking the results for addition, subtraction and multiplication operations.

Unit 2 Number system **6**

Review of Rational numbers on Number line, Irrational number, Real number and their Decimal expansions, representation of irrational number on the number line, successive magnification, Geometrical representation of a real number, operation on real numbers, laws of exponent for real numbers

Unit 3 Algebra **20**

1. Polynomials 10

Definition of a polynomial with one variable, its coefficient, constant polynomials, zero polynomial, linear polynomial, zeros of a polynomial, remainder, theorem, factorization of polynomials, Algebraic identities :

$$(x+y)^2 = x^2 + 2xy + y^2, \ (x-y)^2 = x^2 - 2xy + y^2$$

$$x^2 - y^2 = (x+y)(x-y), \ (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$$

$$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

2. Linear Equation in two variables 10

Introduction, linear equation in two variables, Rectangular co-ordinate system, graph of linear equation of two variable. Algebraic methods of solving simultaneous linear equations :- (i) Method of Elimination (by substitution, by equating the co efficients) (ii) cross-multiplication method, condition for solvability, applications of linear equations in two variables.

Unit 4 Geometry **26**

1. Introduction of Plane Geometry, Line and Angle 02

Fundamental concepts, theorems and geometric construction, geometric symbols, Angle and its measurements, transversal line and parallel line, basic constructions. Construction draw a bisector of a given line segment, draw a bisector of a given angle, construction of various angle $60°$, $120°$, $30°$, $90°$, $45°$, $135°$ with the help of ruler and compass, draw an equivalent angle of an angle on a point of a line, draw the various angles of any measurement with the help of ruler and compass. Draw a perpendicular from a point which is out side the line on any line. Draw a perpendicular at any point of the line.

2. Rectilinear Figures 6

Triangle and its angle, classification of triangle, rectilinear figures.

3. Congruence and Inequalities of Triangle 4

Theorem- Angle-Side-Angle some properties of triangle, some other concepts for the congruence of triangle, Side-Side-Side-rule, RHS-rule, inequalities of triangle perpendicular distance of a line from a external point.

4. Construction of Triangles 4

Construction of triangles when three sides are given, two sides and angle between them is given, two angles and one side is given. Construction a right-angled triangles when two sides and an angle opposite to one side is given, some difficult constructions of triangles.

5. Quadrilateral 6

Types of quadrilateral, properties of parallelogram, mid point theorem, construction of quadrilaterals when four sides and a diagonal are given, three sides and two angles between them are given, two consecutive side and angle between them and other two angles are given, construction of parallelogram and trapezium.

6. Area of triangles and Quadrilaterals 4

Introduction, area, figures made on same base and between pair of same parallel lines.

- Parallelograms made on same base and between same parallel lines are equal.
- Area of triangle between same base and same parallel lines are equal.
- If the area of two triangle are equal and one side of a triangle is equal to one side of other triangle, then their corresponding altitudes are equal.
- Baudhayan theorem - In a right angled triangle, square made on hypotenuse, is equal to the sum of the squares made on other two sides.

Converse of Baudhayan theorem- In a triangle, if square of a longest side is equal to the sum of the squares of other two sides, angle opposite to this side is a right angle.

Unit 5 Mensuration 15

1. Area of plane figures 7

Introduction, area of triangle, Heron's formula, area of an isosceles triangle, area of a equilateral triangle, area of right angled triangle, area of quadrilateral, area of parallelogram, to find the area of different quadrilateral (cyclic quadrilateral, rhombus, trapezium) and their uses.

2. Surface area and Volume of cube and cuboid 8

Introduction, cube, cuboid, diagonal of cube and cuboid. Surface area and volume of cube and cuboid.

Unit 6 Trigonometry 10

1. Angle and their measurement 3

Trigonometry, positive and negative distance, angle, positive and negative angles, angles of any magnitude and their measurement, sexagesimal system, centesimal system, circular system . Value of π, value of 1 radian.

2. Trigonometric ratios of Acute Angles 7

Right-angled triangle, trigonometric ratios of acute angle, relation between trigonometric ratios, trigonometric identities.

Unit 7 Statistics 10

1. Statistics 10

Introduction, Primary data, Secondary data, presentation of data, graphical representation of data, bargraph, histograms (According to change in base and height), frequency polygon, measures of central tendency. Mean, mode, median.

Unit 8 Road Safety Education 5

Percentage (objective, content, activity) circle (objective, content, activity), statistics (objective, content, activity) quadrilaterals (objective, content, activity), road signs, probability (objective), data.

Prescribed book :

Mathematics, Board of Secondary Education Rajasthan, Ajmer

INDEX

1

Vedic Mathematics

1.01 Introduction

Shankaracharya of Puri Swami Bharati Krishna ji Tirtha was the research scholar and inspiration in the field of Vedic Mathematics. He meditated hard for about eight years in the Sringeri math. After attaining the highest state of perfect siddhi he realized internally the Aphorisms and Mathematical Sutras described in ancient Indian literature Vedas, Brahamans', Sahintas', Vedangas etc. and reconstructed them in Sanskrit the divine language. Vedic Mathematics profounded by Swami ji is based on these Mathematical sixteen Sutras and thirteen Upsutras. These Sutras are very easy, effective, useful and correspond one to many interpretations in general. Problems of any branch of Mathematics can be solved by these Sutras easily.

1.02 Usefulness of Vedic Mathematics

By these Vedic mathematical Sutras calculations become easy and short. Less time is consumed. Students feel less mental stress. By Sutra's based methods the results or answers of the questions can be verified, hence giving the student more confidence. By these formulae the possibility of committing error by students become negligible.

By these Sutras more interest in mathematics is developed in students consequently they attain excellent achievement in subject mathematics. They can mentally solve the hard exercises. That is by Vedic Mathematics is called mental mathematics in the mathematical world. According to Swami ji the capacity and speed of calculations of students increases to five fold by the practice of Vedic Mathematics. By its practice there is an unbelievable development in their intellect and intelligence. Neocortex (mind) of the Vedic Mathematic students evolves very fast.

List of Sutras and Subsutras

Sutras	Subsutras
1. Ekadhikena Purvena (also a corollary)	1. Anurupyena
2. Nikhilam Navata-charamam Dasatah	2. Sisyate Sesamjnah
3. Urdhva-tiryagbhyam	3. Adyamadyenantya-mantyena
4. Paravartya Yojayet	4. Kevalaih Saptakam Gunyat
5. Sunyma Samyasamuchaye	5. Vestanam
6. (Anurupye) Sunyamanyat	6. Yavadunam Tavadunam
7. Sankalana-vyavakalanabhyam (also a corollary)	7. Yavadunam Tavadunikrtya Varganca Yojayet
8. Puranapuranabhyam	8. Antyayoradaskae'pi
9. Chalana-Kalanabhyam	9. Antyayoreva
10. Yavadunam	10. Samuccayagunitha
11. Vyastisamastih	11. Lopanasthapanabhyam
12. Sesanyankena Caramena	12. Vilokanam
13. Sopantyadvayamantyam	13. Gunitasamuccayah Samuccayagunitah
14. Ekanyunena Purvena	
15. Gunitasamuccayah	
16. Gunakasamuccayah	

Meaning and Applications of special Sutras

1.03 Sutra Ekadhikeana Purvena :

(a) Meaning : Sutra is composed of two words 'Ekadhika' and 'Purva', its meaning being "By one more than the previous one". Doing Ekadhika on a number means adding one to that number or marking Ekadhika dot $(\cdot)$ on its unit digit i.e..

Ekadhika of $1\dot{2} = 12 + 1 = 13$

In case if a digit of a number is marked the Ekadhika dot, its new value is to be found out e.g. Ekadhika of digit 3 of a number 1534

yields the new number $= 15\dot{3}4 = 1544$

Purvena (Purva) means 'By the previous one'. In case of digits of a number it points out the previous digit and in case of two numbers it points out the previous number e.g. In a number 685 the previous digit of five is 8 and in a multiplication 62 $\times$ 99 the previous number is 62. All the above operations can be mentally done.

(b) Applications :

(i) Addition : Sutra based method is applicable in all types of questions of addition.

Method : Write the numbers of a question columnwise. Start adding from top unit digit. At a digit where the sum equals to ten or more than ten. mark an Ekadhika dot on its previous digit. Start adding again with the unit digit of that sum. Repeat the process. Write the end sum at answer's place. Add other columns in the same way. The method is clarified by the following examples.

Example 1. Add the following :

$$\begin{array}{cccccc}
 & 3 & 7 & 9 & 9 & 5 \downarrow \\
\dot{0} & \dot{6} & \dot{8} & \dot{9} & \dot{8} & 6 \\
 & \dot{7} & \dot{5} & \dot{4} & 3 & 8 \\
\dot{0} & \dot{5} & \dot{8} & 9 & \dot{0} & 9 \\
\hline
2 & 4 & 1 & 3 & 2 & 8 \\
\hline
\end{array}$$

Steps

(i) Column, I $5 + 6 = 11$

(ii) Mark Ekadhika dot (.) on 8

(iii) Start adding again with 1

(iv) 1+8=9, Again 9+9=18

(v) Mark Ekadhika dot on 8

(vi) Wrike 8 below at the answer's place

(vii) Add other columns in the same way.

Example 2. Add the following:

km.			m.		
	2	8	0	8	4
	$\dot{3}$	2	$\dot{3}$	6	5
$\dot{0}$	6	$\dot{5}$	7	$\dot{2}$	5
	$\dot{3}$	8	$\dot{2}$	5	0
1	6	4	4	2	4

Steps

(i) Three columns in a metre.

(ii) No columns should remain vacant hence write 084 for 84.

(iii) Now add columnwise by the above method.

(ii) Subtraction :

In Vedic Mathematics subtraction can be performed by four or five methods. The best method is based on Sutra Ekadhikena Purvena and Param mitra unka (the best friend). The two digits or numbers are said to be the best friend of each other if their sum is equal to ten i.e. 2 is the best friend of 8, 6 the best friend of 4 and 0 the best friend of $\dot{9}$.

Method : When lower digit does not substract from the upper digit. Then add the param mitr unka of lower digit to the upper digit and write the sum in place of answer. Also write the mark of Ekadhikena on the previous digit of lower digit. By the repeatation the process we get the remainder. If upper digit is greater than or equal to the lower digit, then there is no need to add the param mitra unka. Substract by normal process.

The method is clarified by the following examples

Example 3. Subtract

$$\begin{array}{r} 5\ 7\ 6\ 2\ 5 \\ -\ \dot{2}\ \dot{9}\ \dot{8}\ 4\ 3 \\ \hline 2\ 7\ 7\ 8\ 2 \\ \hline \end{array}$$

Steps

(i) $5 - 3 = 2$ write it on the answer's place

(ii) 4 is not subtracted form 2. So the best friend of 4 is 6 is to be added to 2. Write the sum = 8 at answer's place.

(iii) Also mark an Ekadhikena dot at digit 8 prior to digit 4.

(iv) $\dot{8}$ = 9 is not subtracted from 6 so add the best friend of 9 i.e. 1 to 6 i.e. 1+6=7 Write it on aswers' place,

(v) Mark Ekadhika dot on 9.

(vi) Similarly 0+7=7 and 5–$\dot{2}$=2 will give the answer.

Example 4. Subtract

Steps

(i) It time unit columnwise bases are different.

(ii) In unit column of second and minute Base = 10

hour	min.	sec.
2 4	1 2	1 5
$\dot{0}$ 6	$\dot{2}$ 4	3 0
1 7	4 7	4 5

(iii) In tens's place column of second and minute, Base = 6

(iv) In Hour's column Base = 10.

(v) In all columns best friend of a digit is calculated by these bases.

Note : In Decimal Number System the base is taken as 10.

(iii) Multiplication :

In Vedic Mathematics for multiplication there are variance methods based on Sutrats. Sutra Ekadhikena Purvena based method is universal and effective. Some of its special sub methods are very easy and attractive. Here with let us define some new item . The unit digit of a number is known as Charamam digit and all other digits of a number are called Nikhilam digits. e.g. In a number 723 the Charamam digit is 3 and all the digits 7, 2, are called Nikhilam digits.

By the sutra 'Ekadhikena Purvena' when the sum of the Charamam digits is equal to 10 or power of 10 and the remainder Nikhilam digits are equal to each other, we can find the multiplication of two numbers easily.

Method :

(1) There are two sides of product one is L.H.S. and another is R.H.S.

(2) Write the product of charamam or last digits in L.H.S.

(3) In R.H.S. write the product of nikhilam digit and its Ekadhikena.

(4) In the R.H.S. keep the number of digits double of the number of zeroes in the sum of charamam digit i.e. two digits on R.H.S. if sum = 10.

(5) If number of digit in R.H.S. is less or more then we adjus the digit.

The method is explained by the following examples :

Example 5. Multiply : (Sum = 10)

Steps

83×87

$= 8 \times 9 / 3 \times 7$

$= 7221$

(i) Sum of Charamam digits 3 + 7 = 10.

(ii) remainder Nikhilam digits are equal = 8.

(iii) Two digits inR.H.S. = 21

Example 6. Multiply : (Sum = 100)

$$\begin{aligned} & 586\times 514 \\ &= 5\times 6 \;/\; 86\times 14 \\ &= 30 \;/\; 1204 \\ &= 301204 \end{aligned}$$

Steps

(i) Sum of last two digits $= 86+14=100$

(ii) remainder Nikhilam digits are equal = 5

(iii) Four digits in R.H.S. = 1204

Example 7. Multiply : (Sum = 1000).

$$\begin{aligned} & 3993\times 3007 \\ &= 3\times 4 \;/\; 993\times 007 \\ &= 12 \;/\; 006951 \\ &= 12006951 \end{aligned}$$

Steps

(i) Sum of last three digits $= 993+007=1000$

(ii) Six digits in R.H.S. = 006951

(add two zeros before 6 by adjustment)

Example 8. Multiply : (Sum = 1)

$$\begin{aligned} & 9\frac{5}{11}\times 9\frac{6}{11} \\ &= 9\times 10 \;/\; \frac{5}{11}\times\frac{6}{11} \\ &= 90\frac{30}{121} \end{aligned}$$

Steps

(i) Sum of fractios $=\frac{5}{11}+\frac{6}{11}=1$

(ii) remaider nikhilam digits equal to each other =9

Example 9. Multiply : (Sum = 1)

$$\begin{aligned} & 11\cdot 7\times 11\cdot 3 \\ &= 11\times 12 \;/\; \cdot 7\times\cdot 3 \\ &= 132\cdot 21 \end{aligned}$$

Steps

(i) Sum of decimal fractions $= \cdot 7+\cdot 3=1$

(ii) remainder Nikhilam digits equal to each other = 11.

Note: By Sutra based method all questions can be solved orally. Their products can be written in a line.

1.04 Sutra Ekanyunena Purvena :

(a) Meaning : Sutra is composed of two words - 'Eka nyunena' and 'Purvena'. Its meaning is "By one less than the previous one". Place a dot beneath the unit digit of a number which is to be less by one. This dot is known as Eka nyunena mark e.g. Eka nyunena of 7 in 57 = $5\underset{\cdot}{7}$ = 57 – 1 = 56.

Like the previous Sutra Ekadhikena Purvena in this sutra we can find the value of new number by substract one from any digit of the number. e.g. In a number 124. We get a new number 024 by taking the nyunena of digit 1. i.e. new number = $\underset{\cdot}{1}24$ = 024 = 24.

(b) Applications:

(i) Subtraction (Sutra Ekanyunena Purvena + Param mitra unka)

By this sutra every question of subtraction can be solved easily.

Method:

When the lower digit is greater than the upper number digit add the best friend of lower number digit and keep this sum digit at the answer's place. Also place an Eka nyunena dot beneath the digit which is prior to the upper number digit. The repetition of this process will yield the remainder result. The method is clearified by the following examples :

Example 10. Subtract :

$$
\begin{array}{cccc}
 & \underset{\cdot}{5} & \underset{\cdot}{6} & 0 \\
- & 3 & 7 & 4 \\
\hline
 & 1 & 8 & 6 \\
\hline
\end{array}
$$

Steps

(i) The whole process is same as Ekadhikena Purvena method.

(ii) Difference is only one that in this process we place an Ekanyunena dot beneath the digit which is prior to the upper digit instead of Ekadhikena mark.

Example 11. Subtract:

$$
\begin{array}{ccccccc}
\multicolumn{3}{c}{\text{kg.}} & & \multicolumn{3}{c}{\text{g.}} \\
\underset{\cdot}{1} & \underset{\cdot}{2} & \underset{\cdot}{5} & & 0 & \underset{\cdot}{9} & 5 \\
 & 7 & 8 & & 2 & 2 & 8 \\
\hline
0 & 4 & 6 & & 8 & 6 & 7 \\
\hline
\end{array}
$$

Steps

(i) 95 g is written as 095.

(ii) 5 + 2 (best freind of 8) = 7, write 7 at the answer's place. Also mark Eka nyunena dot beneath the digit 9.

(iii) 8 - 2 = 6

(iv) 0 + 8 (best freind of 2) = 8, write 8 at the answer's place. Also mark Eka nyunena dot beneath the digit 5.

(v) 4 + 2 (best freind of 8) = 6, write 6 at the answer's place. Also mark Eka nyunena dot beneath the digit 2.

(vi) 1 + 3 (best freind of 7) = 4, write 4 at the answer's place. Also mark Eka nyunena dot beneath the digit 1.

(viii) 0

(ii) Multiplication :

By Sutra Eka nyunena Purvena multiplication of two numbers can be easily performed if one of these numbers consists of digit nine only. For convenience the number having every digit 9 will be termed as multiplier and the other number as multiplicand.

Method :

There are two sides of the product L.H.S. = multiplicand –1

R.H.S. = multiplier – L.H.S.

Hence Multiplier × Multiplicand = Multiplicand –1 / Multiplier – L.H.S.

Three situations arieses in this multiplication :

(1) No. of digits of multiplier = No. of digits of multiplicand

(2) No. of digit of multiplier > No. of digits of multiplicand

(3) No. of digit of multiplier < No. of digits of multiplicand

I Situation :

(No. of digits of multiplier = No. of digits of multiplicand)

Let us see the following examples :

1. 8×9

L.H.S. $= 8 - 1 = 7$

R.H.S. $= 9 - 7 = 2$

$\therefore$ $8 \times 9 = 8 - 1 \ / \ 9 - 7 = 72$

2. 8567×9999

$= 8567 - 1 \ / \ 9999 - 8566$

$= 85661433$

II Situation :

(No. of digits of multiplier > No. of digits of multiplicand)

Let us see the following examples :

3. 68×999

$= 068 \times 999$

$= 067 \ / \ 999 - 067$

$= 67932$

4. 4523×999999

$= 004523 \times 999999$

$= 004522/995477$

$= 4522995477$

Note: (1) Greater the no. of digits of multiplier so many 9 digits appear in the middle of the answer e.g. see question no. 3 and 4.

(2) Sum of respective digits of L.H.S. and R.H.S. of the product will always be equal to 9, i.e. first digit of LHS + first digit of RHS = 9.

III Situation :

(No. of digits of multiplier < No. of digits of multiplicand)

The method is explained by the following examples:

5. 43×9

$= 42/9 - 42$

$= 429 \; - 42 \; = 387$

6. 512×99

$= 511/99 - 511$

$= 51199 \; - 511 \; = 50688$

Exercise 1.1

By Sutra Ekadhikena Purvena add the following:

1.
$$\begin{array}{r} 98765 \\ 63217 \\ 89522 \\ 60543 \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{r} 89789 \\ 97686 \\ 76978 \\ 86798 \\ \hline \\ \hline \end{array}$$

3.
$$\begin{array}{rr} \text{kg.} & \text{g.} \\ 178 & 45 \\ 246 & 725 \\ 569 & 188 \\ 45 & 894 \\ \hline \\ \hline \end{array}$$

4.
$$\begin{array}{rrr} \text{km.} & \text{m.} & \text{cm.} \\ 25 & 510 & 36 \\ 47 & 85 & 52 \\ 18 & 123 & 75 \\ 53 & 805 & 28 \\ \hline \\ \hline \end{array}$$

By vedic method subtract the following:

5.
$$\begin{array}{r} 746 \\ -389 \\ \hline \\ \hline \end{array}$$

6.
$$\begin{array}{r} 4032 \\ -3543 \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{r} 6007 \\ -1852 \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{r} 8317 \\ -6454 \\ \hline \\ \hline \end{array}$$

By Sutra Ekadhikena Purvena multiply the following.

9.	42×48	10.	103×107
11.	294×206	12.	413×487

Multiply with the help of Sutra Ekanyunena Purvena

13.	54×99	14.	214×999
15.	47×999	16.	342×99999
17.	73×9	18.	467×99

By Vedic method multiply the following:

19.	$15\frac{5}{7} \times 15\frac{2}{7}$	20.	$24\frac{10}{13} \times 24\frac{3}{13}$
21.	$4 \cdot 5 \times 4 \cdot 5$	22.	$9 \cdot 85 \times 9 \cdot 15$

1.05 Vinculum (Negative) Numbers :

Concept of Vinculum operation is the contribution of Vedic mathematics. By Vinculum operation calculations become easy, short and sometimes oral. By this process numbers of big digits like (6,7,8,9) can be converted into numbers of small digits like (0,1,2,3,4,5). These days Vinculum operation has been adopted by computer science also. Digits with bars $\overline{2}$, $\overline{4}$ etc. are called Vinculum digits. Their respective values are –2and –4. This bar (line) is called Vinculum line or Vinculum mark. Positive and Vinculum digits can be placed together to represent a number. e.g. $2\ \overline{3}$ or $\overline{2}\ \overline{4}$. The number $1\ \overline{2}\ \overline{4}$ is spoken as one Vinculum two Vinculum four. It will not be called one Vinculum twenty four.

1.06 Base, Sub Base, Deviation :

Base : Here Base means a Number Base. Any number greater than 1 can be a base. In Vedic mathematics to make the calculation easy and convenient bases are usually chosen as 10 or 100 or any power of 10. In metric system of numbers base is always taken as 10.

Sub-Base : Sub-Base is the multiple of base. Mostly this number always ends at zero. If Base = 10, then Sub Base .10 × x, x being a whole number, if Base = 100, then Sub Base =100 × x, x being a whole number. In case of Sub Base calculations become easy if Sub-Base are used in place of base but necessary adjustment is to be done, in the left hand side of the answer. The reasons will be explained in the coming examples.

Deviation : When Base or Sub-Base is subtracted from the given number the remainder is called the deviation.

Deviation = Number – Base

or Deviation = Number – Sub-Base

If the number is greater than the Base or Sub-Base, the deviation is positive. If number is smaller than the Base or Sub-base, the deviation is negative. The deviation consists so many digits as there are the number of zero to the Base.

Deviation of 18 w.r.t. Base 10 = +8 (Number of Zero in the Base is one)

and Deviation of 94 w.r.t. Base 100 = –06. (Number of Zero in the Base is two)

1.07 Sutra Nikhilam Navtah Charamam dastah

(a) Meaning : Sutra Nikhilam means "All from nine and last from ten". In ancient Indian Mathematics digit nine is supposed to be very creative and an apex number. The number 10 is taken to be the whole number but here the sutra indicates simply the subtraction process.Applications like Vinculum, Subtraction, Multiplication, Squaring, Cubing, Divisions etc. are based on this sutra.

(b) Applications:

(i) To convert ordinary number into Vinculum number

(Sutra Ekadhikena Purvena + Sutra Nikhilam)

When an ordinary number consists of digit 5 or more than 5, it can be converted into Vinuclum number by Nikhilam method.

Method: (1) Subtract Charmam (unit) digit from 10.

(2) Subtract the remainder digits from the number 9.

(3) Draw Vinculum line on each digit so obtained.

(4) Mark the Ekadhikena dot on the previous digit 0 or less than 5 of the remainder.

The method is explained by the following examples:

Example 12 : Convert the following ordinary number into Vinculum number:

1.
$$\begin{aligned} & 8\ 9\ 8 \\ &= 0\ 8\ 9\ 8 \\ &= \dot{0}\ \bar{1}\ 0\ \bar{2} \\ &= 1\ \bar{1}\ 0\ \bar{2} \end{aligned}$$

Steps

(i) Subtract unit digit 8 from 10.

(ii) Subtract remaining digits 9 and 8 from 9.

(iii) Draw Vinculum Bar on 2 and 1.

(iv) Place an Ekadhikena dot on 0 prior to $\bar{1}$

2.
$$\begin{aligned} & 1\ 8\ 4\ 6\ 9 \\ &= \dot{1}\ \bar{2}\ \dot{4}\ \bar{3}\ \bar{1} \\ &= 2\ \bar{2}\ 5\ \bar{3}\ \bar{1} \end{aligned}$$

Steps

(i) Subtract 9 from 10 and 6 from 9 respectively.

(ii) Draw Vinculum bar on 1 and 3.

(iii) Place Ekadhikena dot on 4.

(iv) Restart the process from digit 8.

Note : (1) When in between the bigger digit of a normal number occures a digit equal or lessthan 5 than we repeat this process.

(2) No vinculum bar is drawn on zero.

(ii) To convert Vinculum number into ordinary number

(Sutra Ekanyunena Purvena + Sutra Nikhilam)

Method: (1) Subtract the positive value of Charmam digit from 10.

(2) Subtract the positive vlaues of remaining Nikhilam digits from 9 respectively.

(3) Place an Ekanyunena dot beneath the non Vinculum digit.

(4) Repeat the process if necessary.

The method is clarified by the following examples:

Example 13 : Change the following into ordinary number:

1. $2\,\overline{4}\,\overline{3}$

$= \underset{.}{2}\,5\,7$

$= 1\,5\,7$

2. $6\,\overline{2}\,\overline{4}\,5\,\overline{3}\,\overline{2}$

$= \underset{.}{6}\,7\,6\,\underset{.}{5}\,6\,8$

$= 5\,7\,6\,4\,6\,8$

(iii) Multiplication of two numbers :

(Sutra Nikhilam – Base)

When two numbers are nearer to the base = 10 or 100 or power of 10, their multiplication can be find out easily by Sutra Nikhilam - Base.

Method: (1) Choose the base = 10 or 100 etc. nearer to the numbers.

(2) Find the deviations of the numbers with refrence to base and write them before their numbers.

(3) Divide the product place in two parts by an oblique line.

(4) Write the product of deviations on R.H.S.

(5) On L.H.S. write one number plus the deviation of the other number.

(6) Adjust the number of digits of R.H.S. according to the number of zeroes of the base.

(7) The deficiency of digit number is fulfilled by writing zero. If the product digit number is more, take it to the left hand side.

(8) If the product of deviations is negative, change it into positive by taking one or two etc. from L.H.S. Kindly note that the vlaue of one from L.H.S. is equivalent to the base value.

The method is explained by the following examples:

Example 14 : Multiply the following by Nikhilam – Base method:

1. $12 \times 14,\ \text{Base} = 10$

$$\begin{array}{ll} =12 & +2 \\ \ \ 14 & +4 \\ \hline \multicolumn{2}{l}{=14+2/2\times 4} \end{array}$$

$=168$

Steps

(i) Deviations = +2, +4

(ii) One digit in R.H.S.

2. $92 \times 87,\ \text{Base} = 100$

$$\begin{array}{ll} =92 & -08 \\ \ \ 87 & -13 \\ \hline \multicolumn{2}{l}{=92-13/(-08)(-13)} \end{array}$$

$= 79/{}_104 = 8004$

Steps

(i) Deviations $= -08, -13$

(ii) Two digits in R.H.S. so digit 1 or 104 to L.H.S.

3. $7 \times 18,\ \text{Base} = 10$

$$\begin{array}{ll} =7 & -3 \\ \ \ 18 & +8 \\ \hline \end{array}$$

$=7+8/(-3)\times 8$

$=15/-24$

$=15-3/30-24$

$=12/30-24$

$=126$

Steps

(i) Product $=15/-24$

(ii) Bring 3 from L.H.S. to R.H.S.

(iii) In R.H.S. value of 3 will be equla = 30

4. 1007×1012

$$\begin{array}{ll} =1007 & +007 \\ \ \ 1012 & +012 \\ \hline \end{array}$$

$=1012+7/084$

$=1019084$

Steps

(i) Base = 1000

(ii) d = +007, +012

(iii) Three digits in R.H.S. so write zero before 84.

(iv) Multiplication of two numbers

(Sutra Nikhilam – Sub-Base)

Sometimes it becomes difficult to multiply two big deviations. In this case concept of Sub-Base is helpful. The method is similar to the previous Sutra Nikhilam Base method except that for adjustment in L.H.S. product is multiplied by the Sub-Base digit (Upadhar Unka) and R.H.S. remains as usual. The method is explained by the following examples:

Example 15 : Multiply the following by Sutra Nikhilam – Sub-Base.

1. 32×33

$$\begin{array}{ll} =32 & +2 \\ \ \ 33 & +3 \\ \hline \end{array}$$

$= 35 \times 3/6$

$= 1056$

Steps

(i) Base = 10, Sub-Base $= 10 \times 3 = 30$.

(ii) Upadhar unka =3

(iii) deviation from Sub-Base = +2 , +3

(iv) Adjustment in L.H.S. $= 35 \times 3$

2. 54×56

$$\begin{array}{ll} =54 & +4 \\ \ \ 56 & +6 \\ \hline \end{array}$$

$= 60 \times 5/_{2}4$

$= 300/_{2}4$

$= 3024$

Steps

(i) Base =10, Sub-Base $= 10 \times 5 = 50$

(ii) Upadhar unka = 3

(iii) deviation from Sub-Base = +4, +6

(iv) L.H.S. $= 60 \times 5$

(v) Adjustment in R.H.S. will be done in the end.

3. 54×56

$$\begin{array}{ll} =54 & +4 \\ \ \ 56 & +6 \\ \hline \end{array}$$

$= 60 \times \frac{1}{2}/24$

$= 3024$

Steps

(i) Base = 100, Sub-Base $= 100 \times \frac{1}{2} = 50$

(ii) Updhar unka = ½

(iii) d =+04 and 06

(iv) Two digits in the R.H.S.

4. 206×212

$$\begin{array}{ll} =206 & +06 \\ \ \ 212 & +12 \\ \hline \end{array}$$

$= 218 \times 2/72$

$= 43672$

Steps

(i) Base = 100, Sub-Base $= 100 \times 2$

(ii) Upadhar unka = 2

(iii) d =+06, +12

(iv) Two digits in R.H.S.

(v) Multiplication of three numbers:

(Sutra Nikhilam – Base)

Multiplication process is divided into three steps :

Step I : Any number + deviations of other two numbers.

Step II : Sum of the products of two deviations.

Step III : Product of three deviations.

The method is explained by the following examples:

Example 16 : Multiply the following by Sutra Nikhilam – Base.

1. $91 \times 93 \times 96, \text{ Base } = 100$

Number	Deviation
91	-09
93	-07
96	-04

$= 93 - 09 - 04 / 36 + 28 + 63 / (-9)(-4)(-7)$

or $91 - 07 - 04$

or $96 - 09 - 07$

$= 80 / {}_{1}27 / (-252)$

$= 81 / 27 - 3 / 300 - 252$

$= 81 / 24 / 48$

$= 812448$

Steps

(i) deviation $= -09, -07, -04$

(ii) In the third part $(-09)(-07)(-04) = -252$

(iii) In the middle part $(-09)(-04) + (-07)(-04) + (-09)(-07) = 127$

(iv) 3 taken from II part to III part place value being 300

(v) $300 - 252 = 48$

2. $103 \times 105 \times 106,$

$= 106 + 03 + 05 / 15 + 30 + 18 / 90$

$= 114 / 63 / 90$

$= 1146390$

Steps

(i) Base $= 100$

(ii) deviation $= +06, +03, 05.$

(iii) rest process as above.

3. $12 \times 13 \times 15$

$= 12 + 3 + 5 / 6 + 15 + 10 / 30$

$= 20 / {}_{3}1 / {}_{3}0$

$= 2340$

Steps

(i) Base $= 10$

(ii) deviation $= 2, 3, 5.$

Note: So as many digits are to be kept in II and III parts as there are number of zeroes in the Base.

(vi) Multiplication of three numbers:

(Sutra Nikhilam – Sub-Base)

In Nikhilam Sub-Base method the first part (from the left) and middle part of the product is multiplied by (Sub-Base digit)^2 or (Upadhar Unka)^2 and (Sub Base digit) or (Upadhar Unka) respectively. This is the only difference between Nikhilam Base and Sub-Base methods.

The method is explained by the following examples:

Example 17 : Multiply the following Nikhilam Sub-Base method.

1. $21 \times 24 \times 25$

21	+1
24	+4
25	+5

Steps

(i) Base $= 10$, Sub-Base $= 10 \times 2$ =20.

(ii) Upadhar Unka= 2

(iii) deviation = +1, +4, +5

(iv) One digit each in II and III part.

$= 2^2(21+4+5)/2(4+20+5)/1\times 4\times 5$

$= 4 \times 30/2 \times 29/20$

$= 120/{}_5 8/{}_2 0$

$= 12600$

2. $502 \times 503 \times 504$

$= 5^2(502+03+04)/5(6+12+8)/2\times 3\times 4$

$= 25 \times 509/5 \times 26/24$

$= 12725/130/24$

$= 127263024$

Steps

(i) Base $= 100$,
Sub-Base = 100×5

(ii) Upadhar Unka = 5

Exercise 1.2

Change into Vinculum numbers :

1. 89
2. 878
3. 9687
4. 6578

Change into ordinary numbers :

5. $3\bar{2}1$
6. $2\bar{4}\bar{3}\bar{2}$
7. $4\bar{3}0\bar{2}$
8. $450\bar{4}\bar{9}$

Multiply the followng by Sutra Nikhilam

9. 102×107
10. 94×92
11. 72×73
12. 203×204
13. $11 \times 12 \times 13$
14. $97 \times 98 \times 99$
15. $102 \times 103 \times 104$
16. $99 \times 101 \times 103$

Methods of checking the results

There are two methods of checking the results received from any operation.

(a) Navanka method

(b) Ekadashanka Method

(a) Navanka method :

In the navanka method we find the beejanka of any number by taking base as digit 9. After subtracting 9 from the digits of a number or sum of the digits of a number remaining single digit is know as Beejanka of that number e.g. Beejanka of 947 = 2.

In the different operation the application of navanka method is explained by the following example.

Example 18 :

(1) Checking of Addition results:

	Beejanka
5 3 8 9	7
6 4 7 2	1
5 9 3 6	5
4 1 6 8	1
2 1 9 6 5	5

Steps

(i) Sum of Beejanka (row-wise)

$= 7 + 1 + 5 + 1 = 5$

(ii) Beejanka of the sum

$= 2 + 1 + 9 + 6 + 5 = 5$

Both are equal, hence answer is correct.

(2) Checking of the Subtraction results:

$$\begin{array}{r} 8\,1\,3\,4 \\ -\,5\,6\,7\,8 \\ \hline 2\,4\,5\,6 \\ \hline \end{array}$$

Steps

(i) Beejanka of the minuend $= 7 \quad = 16$

(ii) Beejanka of the subtrahend $= 8$ or -8

(iii) Beejanka of the remainder = 8

(3) Checking of the Multiplication results:

$73 \times 77 = 5621$

(i) Beejanka of the multiplicand $= 7 + 3 = 10 = 1$

(ii) Beejanka of the multiplier $= 7 + 7 = 14 = 5$

(iii) Beejanka of the multiplicand × multiplier $= 1 \times 5 = 5$

(iv) Beejanka of the product $= 5 + 6 + 2 + 1 = 5$

Beejanka of L.H.S. =Beejanka of R.H.S.

Note: 1. If two digits of any row or two digits of any column interchange their places the error cannot be spotted by Navanka method.

2. In Vedic Mathematics there are several methods to solve a question. The result can be verified by Ekadashanka Method.

(b) Ekadashanka Method or Difference Method:

In this method the difference of the sumof digits at odd places and the sum of digits at even places is called the Beejanka of the number e.g. Beejanka of 63254

$$= 4 - 5 + 2 - 3 + 6 = 4$$

The use of the Ekadashanka method for different operations is explained here with by the following examples:

(i) Checking of Addition results :

	Row-wise Beejanka
63254	$4 - 5 + 2 - 3 + 6 = 4$
54327	$7 - 2 + 3 - 4 + 5 = 9$
89325	$5 - 2 + 3 - 9 + 8 = 5$
206906	Sum = 18

Beejanka of the sum $18 = 8 - 1 = 7$

Beejanka of the sum 206906 $= 6 - 0 + 9 - 6 + 0 - 2 = 7$

Both are equal, hence answer is correct.

(ii) Checking of the Subtraction results:

	Row-wise Beejanka
7348	$8 - 4 + 3 - 7 = 0$
–5249	$9 - 4 + 2 - 5 = 2$
2099	

Beejanka of differene = (i) – (ii) = 0 – 2 = –2

Beejanka of the Remainder = 9 – 9 + 0 – 2 = – 2

Since both are equal, the answer is correct.

(iii) Checking of the Multiplication results:

$54 \times 56 = 3024$

Beejanka of 54 $= 4 - 5 = -1$

Beejanka of 56 $= 6 - 5 = +1$

Multiplicationof Beejanka $(-1) \times (+1) = -1$

Beejanka of the product $= 4 - 2 + 0 - 3$
$= -1$

Both are equal, hence answer is correct.

❑ ❑ ❑

Answers

Exercise 1.1

1.	312047	2.	351251
3.	1039 kg., 852 gm.	4.	144 Km. 524 m. 91 cm.
5.	357	6.	489
7.	4155	8.	1863
9.	2016	10.	11021
11.	60564	12.	201131
13.	5346	14.	213786
15.	46953	16.	34199658
17.	657	18.	46233
19.	$240\frac{10}{49}$	20.	$600\frac{30}{169}$
21.	$20 \cdot 25$	22.	$90 \cdot 1275$

Exercise 1.2

1.	$1\bar{1}\bar{1}$	2.	$1\bar{1}\bar{2}\bar{2}$
3.	$10\bar{3}\bar{1}\bar{3}$	4.	$1\bar{3}\bar{4}\bar{2}\bar{2}$
5.	281	6.	1568
7.	3698	8.	44951
9	10914	10.	8648
11.	5256	12.	41412
13.	1716	14.	941094
15.	1092624	16.	1029897

❑

2

Number System

2.01 Review of Rational Numbers on the Number Line

Numbers have great impotance in our daily life. We have studied about different numbers begining from natural numbers to rational numbers. Now, we review them on the number line.

(i) Natural numbers

This line increases towards the right hand side from 1 to infinity.

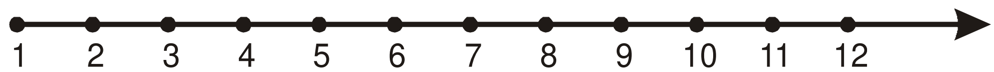


Fig. 2.01

(ii) Whole numbers

This line increases towards the right hand side from 0 to infinity.

Fig. 2.02

(iii) Integers

This line increases towards both sides starting from 0 to infinity.

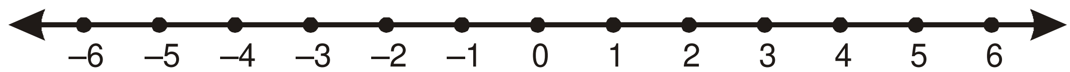


Fig. 2.03

(iv) Rational numbers

This line increases towards both sides starting from 0 to infinity.

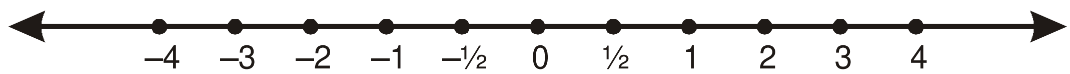


Fig. 2.04

But there are many numbers found between -1, 0 and 0, 1. We can use the concept of mean to find out the rational numbers between two rational numbers. There are infinitely rational numbers between two rational numbers.

With the course of changing time, numbers also developed. First of all, natural numbers were developed. Sum and multiplication of two natural numbers is also a natural number. The set of natural numbers is denoted by *N, i.e.,*

$$N = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$$

If we want to solve the question $x + 7 = 7$, then we get the value of x is 0. So we can not slove this equation with the help of natural numbers so the number zero is included to the set of natural numbers and this new set is known as set of whole numbers which is denoted by *W, i.e.,*

$$W = \{0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$$

If we slove of the equation $x + 15 = 6$. and found the value of x, then we need a number $x = -9$ which is not a whole number. Here, again we need to develop an another set of numbers. If we include negative numbers to the set of whole numbers, we get another set of numbers which is known as set of integers. The set of integers is denoted by *I* or *Z, i.e.,*

$$Z = \{-3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Now, we notice that some more numbers also lie between two integers which come out when integer is divided by another integer for example $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \ldots$

A number which can be represented in the form of $\frac{\mathrm{p}}{\mathrm{q}}$ is called a rational number, where p and q both are integer but $q \neq 0$ either $\frac{\mathrm{p}}{\mathrm{q}}$ is turminating or it is recurring. In a set of rational number, natural numbers, whole numbers and integers are included.

There are infinitely rational numbers between two rational numbers.

2.02 Irrational Numbers

Let us see the number line again and think whether all the numbers have been included on this number line or some number have been still remaining. Let us discuss the remaining numbers that are not rational number. Such numbers that cannot be expressed in the form of $\frac{p}{q}$, where p and q are intergers and $q \neq 0$, is called an irrational number. As we know that there are infinitely rational numbers. In the same way there are infinitely irrational numbers, for which some examples are given below :

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, 0.15150015000150000\ldots.$$

Recall that when we use the symbol "$\sqrt{\ }$", we assume that it is the square root of a positive number. So $\sqrt{25} = 5$, though 5 and -5 both are square root of 25.

The set of all rational and irrational numbers is named as the set of real numbers which is denoted by R.

2.03. Real Numbers and their Decimal Expansions

Now, we will discuss decimal expansion to distinguish between rational and irrational numbers. We will also explain how real numbers can be represented on the number line using their decimal expansions. Let us start with the rational numbers.

Let us see three examples : $\frac{3}{8}, \frac{8}{9}, \frac{6}{7}$

$$\frac{3}{8} = 0.375$$

$$\frac{8}{9} = 0.88888...$$

$$\frac{6}{7} = 0.857142857142...$$

In all of the above examples, $\frac{p}{q}, \ (q \neq 0)$ is applied on rational number, then we get the various situation after dividing p by q which are following,

I Condition : Remainder becomes zero

In the example of $\frac{3}{8}$ the remainder becomes 0 after some steps. Decimal expansion of $\frac{3}{8}$ is 0.375. Let us consider some other example like $\frac{1}{4} = 0.25, \ \frac{456}{125} = 3.648$. In these examples, the decimal expansion terminates after a finite number of steps. The expansion of such numbers is called terminating decimal expansion.

II Condition : The remainder never becomes zero but we get a repeatition of digits in the quotient.

For example $\frac{8}{9} = 0.888888...$ and $\frac{6}{7} = 0.857142857142...$ The expansion of such numbers is non-terminating recurring.

In the decimal expansion of $\frac{8}{9}$, 8 repeats. So the usual way of showing repeatiton of 8 is to write it as $0.\bar{8}$. Similarly, in decimal expansion of $\frac{6}{7}$ the block 857142 repeats, So we write $\frac{6}{7}$ in the form $0.\overline{857142}$, where the bar above the digits indicates that block of digits that repeats. Similarly, 2.67474 can also be written as $2.6\overline{74}$. From above examples, we get non-terminating (repeating) decimal expansion. In this way we found that there are only two cases of decimal expansion in rational numbers, they can either be terminating or non terminating repeating (recurring) decimals.

Example 1. Show that 2.152786 is a rational number or express 2.152786 in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : We have $2.152786 = \frac{2152786}{1000000}$, so it is a rational number.

Example 2. Show that 0.8888 . . . = $0.\overline{8}$ can be expressed as $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Solution : Let

$$x = 0.\overline{8}$$

or $\qquad x = 0.8888\ ... \qquad$ . . . (i)

Multiplying both the sides of equation 10, we get

$$10x = 10 \times (.8888....) = 8.888\ ...$$

$$10x = 8.888... \qquad \text{. . . (ii)}$$

Subtracting (i) from (ii), we have

$$9x = 8.000... = 8$$

$$\Rightarrow \qquad x = \frac{8}{9}$$

Example 3. Show that $0.\overline{47}$ = 0.474747 . . . can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : Let $\qquad x = 0.4747... \qquad$ . . . (i)

Since two digits are repeating, so multipling equation (i) by 100, we get

$$100x = 47.474747... \qquad \text{. . . (ii)}$$

Substracting (i) from equation (ii), we get

$$99x = 47.000... = 47$$

$$\therefore \qquad x = \frac{47}{99}$$

Example 4. Show that $0.12\bar{3} = 0.123333...$ **can be expressed as** $\frac{p}{q}$**, where p and q are integers and** $q \neq 0$

Solution : Let $x = 0.12333$ . . . (i)

Here, 1 and 2 do not repeat, but 3 repeats, since first two digits are non-repeating.

$$100x = 12.333 \qquad \text{. . . (ii)}$$

Again, multipling the equation (ii) by 10, we get

$$1000x = 123.333 \qquad \text{. . . (iii)}$$

Now subtracting equation (ii) form equation (iii), we get

$$900x = 111.000$$

$$x = \frac{111}{900} = \frac{37}{300}$$

So, every number with a non-terminating recurring decimal expansion can be expresses in the form of $\frac{p}{q}$, where p and q are integers and $p \neq 0$.

The decimal expansion of a rational number is either terminating or non-terminating recurring. Now, we think a number like $x = 0.150120015000150000...$ and we find that this number can not be changed in the form of $\frac{p}{q}$, (where p and q are integers and $q \neq 0$.)

So, due to specific property of number like this, it is called irrational number.

Hence, the number whose decimal expansion is non-terminating, non-recurring is called irrational number.

Infinitely many irrational numbers can be generated equivalent to x.

Decimal expansion of some irrational number $\sqrt{2}$, $\sqrt{3}$ are given here.

$$\sqrt{2} = 1.41421356237...$$

$$\sqrt{3} = 1.73205080756...$$

For over the years, mathematicians have developed various techniques to produce more and more digits in the decimal expansion of irrational numbers. For example, to find digits in the decimal expansion of $\sqrt{2}$ by the division method. From the sulbasutras (rules of chords), a mathematical book of vedic period (800 B.C.- 500 B.C.). We get the approximate value.

Similarly the history of finding the decimal expansion of π is also quite interesting

Greek's famous scientist Archimedes calculated the value of π in decimal expansion. He showed $3.140845 < \pi < 3.142857$. Aryabhatta (476-550AD), the great Indian mathematician and astronomer, found the value of π exact to four decimal places (3.1416). Using high speed computers and advanced algorithms, the value of π has been computed to many decimal places.

Example 5. Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Solution : We can easily calculate that $\frac{1}{7} = 0.142857142857... = 0.\overline{142857}$

and $\frac{2}{7} = 0.2857142857142... = 0.\overline{2857142}$

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a number which is non-terminating, non-recurring lying between them. Thus, we can find infinitely many numbers. An example of a number like this is $0.150150015000150000...$.

Exercise 2.1

1. Classify the following numbers as rational or irrational :

   (i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3797 (iv) 7.4784478 . . .
   (v) 1.101001000100001 . . .

2. Write three numbers whose decimal expansions are non-terminating non-recurring.

3. Write the following in decimal form and show the type of decimal expansion

   (i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$

   (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

4. Express the following in the form of $\frac{p}{q}$, where p and q are the integers and $q \neq 0$:

   (i) 0.3 (ii) $0.\overline{47}$ (iii) $1.\overline{27}$ (iv) $1.2\overline{35}$

5. Find three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Representation of Real Numbers on the Number Line

As we have studied in the previous section that a real number can be either a rational or an irrational number. So we can say that any real number can be represented on the number line in its unique point and every point on the number line represents a unique real number. Due to this, the number line is called the real number line. With the help of some examples we shall study the representation method of irrational numbers on the number line.

Example 6. Represent $\sqrt{2}$ on the real number line.

Solution : It can be done easily. Take a square $OABC$ with the side of unit length (See the Fig. 2.05)

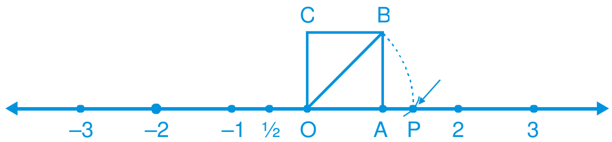


Fig. 2.05

Then using the Bodhyan theorem you can see that $OB = \sqrt{1^2 + 1^2} = \sqrt{2}$. Taking a distance of OB in *a* compass and with centre O, draw an arc on the number line that intersects it at the point P. Then point P on number line corresponds to $\sqrt{2}$.

Example 7. Represent $\sqrt{3}$ on the number line.

Solution : Let us see the Fig. 2.06.

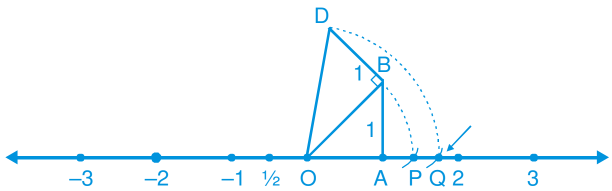


Fig. 2.06

Draw a perpendicular BD on the side OB with unit length. (As shown in the Fig. 2.06).

Then according to Bodhyan theorem $OD = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$. Taking O as centre and with the radius OD, draw an arc which intersect the number line at Q then Q corresponds to $\sqrt{3}$.

Similarly, after determining the place of $\sqrt{n-1}$ on the number line, we can easily determine the place of $\sqrt{n}$, where n is a positive integer.

Process of Successive Magnification

In the previous section, we have studied that a real number has its decimal expansion. We can represent the given number on a number line with the help of decimal expansion. Let us locate 2.05 on the number line as we see it on a sacle.

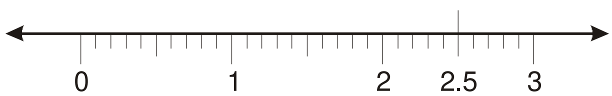


Fig. 2.07

For example, we want to determine the place of 2.775 on a number line. We have to consider the Fig 2.08 given below. We notice that 2.775 is located somewhere between 2 and 3. Divide this distance into ten equal parts. After that divide the distance between 2.7 and 2.8 in ten equal parts again. We divide the distance between 2.77 and 2.78 into ten equal parts. The point 2.775 is fifth point of this division. This process of visualisation of numbers in the number line through a magnifiying glass is called the process of successive magnification. So, we have seen that it is possible by sufficient successive magnifications to visualise the position of a real number with a terminating decimal expansion on the number line.

In the same way we can see the position of non-terminating non recurring real number on the number line with the help of this process.

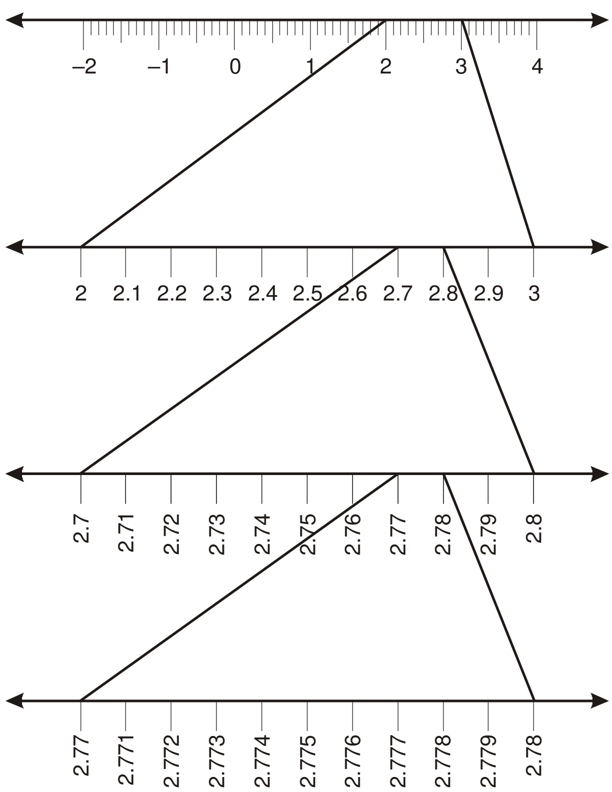


Fig. 2.08

Based on imagination of successive magnification form given in the above examples, we can again say that every real number is represented by a unique point on the number line. Further, every point on the number line represents one and only one real numer.

2.4. Geometrically Representation of a Real Number

If a is a natural number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$. The same definition can be extended for positive real numbers. Let $a > 0$ be a real number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$.

Now we shall show how the value of $\sqrt{x}$ can be found out on the number line geometrically, where x is a positive integer.

To find $\sqrt{x}$, for any positive real number x , we mark B so that $AB = x$ and as in Fig. 2.09 mark C so that $BC = 1$.

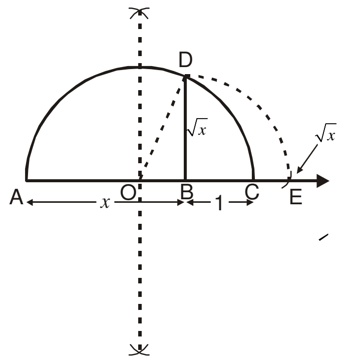


Fig. 2.09

In a right angled $\Delta\, OBD$ (Fig. 2.09), we have

So, $$OC = OD = OA = \frac{AB + BC}{2} = \frac{x+1}{2} \text{ unit}$$

$$OB = AB - OA = x - \left(\frac{x+1}{2}\right) = \frac{x-1}{2} \text{ unit}$$

By bodhyan theorem, we get

$$BD^2 = OD^2 - OB^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = \frac{4x}{4} = x$$

$\Rightarrow \qquad BD^2 = x$

$\Rightarrow \qquad BD = \sqrt{x}$

The representation of diagrametic and geometric method can be obtained by this construction which shows that $\sqrt{x}$ exists for all real numbers $x > 0$. If you want to know the position of $\sqrt{x}$ on the number line, then let us treat the line BC as the number line, with

B as zero, C as 1, and so on. Draw an arc with centre B and radius BD, which intersects the number line E (see the Fig. 2.09). Then E represents $\sqrt{x}$.

We can extend the idea of square roots to cube roots, fourth roots, fifth roots and in general n^{th} roots, where n is a positive integer. Recall your understanding of square roots and cube roots form earlier classes.

What is $\sqrt[3]{27}$? We know that $\sqrt[3]{27}=3$. Find the value of $\sqrt[5]{243}$. If $b=\sqrt[5]{243}$ then $b^5=243$, $b^5=(3)^5 \Rightarrow b=3$. So, that value of $\sqrt[5]{243}=3$. In the same way $\sqrt[n]{a}$ can be defined, where $a>0$ and n is a positive integer.

Let $a > 0$ be real number and n is a positive integer, then $\sqrt[n]{a}=b$, if $b^n=a$ where $b>0$.

Symbol "$\sqrt[n]{\ }$" is called the radical sign. $\sqrt[n]{a}$ can be expressed as $a^{1/n}$.

2.05. Operation on Real Numbers

We have studied in the previous classes, that rational numbers satisfy the commutative, associative and distributive laws for addition and multiplication. We have also studied that if we add, substract, multiply or divide (except zero) two rational numbers, we still get a rational number. That is rational numbers are closed with respect to addition, substration, multiplication and division. We also notice that irrational number also satisfy the commutative, associative, distributive laws for addition and multiplication. However the sum, difference, quotients and products of irrational are not always irrational. For example $\left(\sqrt{5}\right)-\left(\sqrt{5}\right)$, $\left(\sqrt{7}\right)\cdot\left(\sqrt{7}\right)$ and $\dfrac{\sqrt{13}}{\sqrt{13}}$ are rational numbers.

Let us see what happens when a rational number is added with an irrational number and a rational number is multiplied by an irrational number?

For an example, $\sqrt{5}$ is an irrational number, then the number $2+\sqrt{5}$ and $2\sqrt{5}$ are of what kind ? Undoubtfully these are irrational number because these numbers give the non-terminating non-recurring decimal expansion. Non-terminating non-recurring decimal expansion of irrational numbers can be understood more by the following examples.

Example 8. Check whether $7\sqrt{5}, \dfrac{7}{\sqrt{5}}, \sqrt{2}, \sqrt{2}+24,\ \pi-3$ are irrational or not.

Solution : We know that, $\sqrt{5}=2.236...,\ \sqrt{2}=1.4142...,\ \pi=3.1415$

Then $\quad 7\sqrt{5}=15.652...,\ \dfrac{7}{\sqrt{5}}=\dfrac{7\sqrt{5}}{\sqrt{5}\sqrt{5}}=\dfrac{7\sqrt{5}}{5}=3.1304...$,

$\sqrt{2}\quad 24\quad 25.4142....,\qquad 3\quad 0.1415$

All of these are non-terminating non-recurring decimals. Hence, all of these are irraitonal numbers.

Example 9. Add : $2\sqrt{3}+3\sqrt{5}$ and $\sqrt{3}-\sqrt{5}$.

Solution :
$$\left(2\sqrt{3}+3\sqrt{5}\right)+\left(\sqrt{3}-\sqrt{5}\right)$$
$$=\left(2\sqrt{3}+\sqrt{3}\right)+\left(3\sqrt{5}-\sqrt{5}\right)$$
$$=(2+1)\sqrt{3}+(3-1)\sqrt{5}$$
$$=3\sqrt{3}+2\sqrt{5}$$

Example 10. Multiply : $6\sqrt{7}$ **by** $2\sqrt{7}$ **.**

Solution :
$$6\sqrt{7}\times 2\sqrt{7}=6\times 2\times\sqrt{7}\times\sqrt{7}$$
$$=12\times 7=84$$

Example 11. Divide : $8\sqrt{15}$ **by** $2\sqrt{5}$ **.**

Solution :
$$8\sqrt{15}\div 2\sqrt{5}=\frac{8\sqrt{3\times 5}}{2\sqrt{5}}=\frac{8\sqrt{3}\sqrt{5}}{2\sqrt{5}}=4\sqrt{3}$$

We can derive the following conclusions form these examples.

(i) The sum or difference of a rational number and an irrational number is irrational.

(ii) The product or quotient of a non-zero rational number with an irrational number is irrational.

(iii) If we add, substract, multiply or divide two irrational numbers, the result may be rational or irrational.

Here, some identities related to square root, which are useful for rationalizing have been given.

Let a and b are positive real numbers, then

(i) $\sqrt{ab}=\sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

(iii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$

(iv) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}-\sqrt{d}\right)=\sqrt{ac}-\sqrt{ad}+\sqrt{bc}-\sqrt{bd}$

(v) $\left(\sqrt{a}+\sqrt{b}\right)^2=a+2\sqrt{ab}+b$

(vi) $\frac{1}{a+\sqrt{b}}=\frac{a-\sqrt{b}}{a^2-b}$

(vii) $\dfrac{1}{a+b\sqrt{x}}=\dfrac{a-b\sqrt{x}}{a^2-b^2x}$,where x is a natural number.

(viii) $\dfrac{1}{\sqrt{x}+\sqrt{y}}=\dfrac{\sqrt{x}-\sqrt{y}}{x-y}$, where x and y are natural numbers.

Above identities will be used to rationalise the denominator. When the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominatior is a rational number is called rationalising the denominator.

Example 12. Rationalise the denominator of $\dfrac{1}{\sqrt{2}}$.

Solution : We know that $\sqrt{2}\cdot\sqrt{2}=2$, which is rational number.

Multiply the numerator and denominator by $\sqrt{2}$ we have

$$\frac{1}{\sqrt{2}}=\frac{1}{2}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

In this form, it is easy to locate $\dfrac{1}{\sqrt{2}}$ on the number line. It is half way between 0 and $\sqrt{2}$.

Example 13. Rationalise the denominator of $\dfrac{1}{2+\sqrt{3}}$.

Solution : Here, the conjugate of the denominator $\left(2+\sqrt{3}\right)$ is $\left(2-\sqrt{3}\right)$. Multiply the numerator and denominator by $\left(2-\sqrt{3}\right)$, we have

$$\frac{1}{2+\sqrt{3}}=\frac{1}{\left(2+\sqrt{3}\right)}\times\frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

Example 14. Rationalise the denominator $\dfrac{5}{\sqrt{3}-\sqrt{5}}$

Solution : By applying the identity (iii).

We have $\dfrac{5}{\sqrt{3}-\sqrt{5}}=\dfrac{5}{\sqrt{3}-\sqrt{5}}\times\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\dfrac{5\left(\sqrt{3}+\sqrt{5}\right)}{3-5}=\left(-\dfrac{5}{2}\right)\left(\sqrt{3}+\sqrt{5}\right)$

Example 15. **Rationalise the denominator** $\frac{1}{7+3\sqrt{2}}$

Solution : Here, conjugate of the denominator $(7+3\sqrt{2})$ is $(7-3\sqrt{2})$. Multiply the numerator and denominator by $(7-3\sqrt{2})$, we have

$$\frac{1}{7+3\sqrt{2}} = \frac{1}{7+3\sqrt{2}} \times \left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}}\right) = \frac{7-3\sqrt{2}}{49-18} = \frac{7-3\sqrt{2}}{31}$$

Exercise 2.2

1. Classify the following numbers as rational or irraitonal :

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$

(iii) $\frac{2\sqrt{11}}{7\sqrt{11}}$ (iv) $\frac{1}{\sqrt{3}}$

(v) 5π

2. Rationalise the denominator of the following :

(i) $\frac{1}{5+3\sqrt{7}}$ (ii) $\frac{1}{\sqrt{2}+\sqrt{3}}$ (iii) $\frac{1}{\sqrt{7}-2}$

3. If $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$, then find the values of a and b. When a and b are rational numbers.

2.6. Laws of Exponent for Real Numbers

As we have studied in the previous classes about the laws of exponents. Here a, m and n are natural numbers a is called the base and m and n are the exponents.

Here a, m, n and b are natural numbers.

(i) $a^m a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$

(iii) $\frac{a^m}{a^n} = a^{m-n}, m > n$ (iv) $a^m b^m = (ab)^m$

What is $(a)^{\circ}$? Its value is 1. Therefore, $(a)^{\circ} = 1$

The value of $\frac{1}{a^n} = a^{-n}$. Now we can use these laws on negative exponents. For example :

(i) $7^2 \cdot 7^{-5} = 7^{2-5} = 7^{-3} = \frac{1}{7^3}$

(ii) $\left(5^3\right)^{-7} = 5^{-21}$

(iii) $\frac{23^{-15}}{23^7} = 23^{-15-7} = 23^{-22}$

(iv) $(6)^{-3}(7)^{-3} = (42)^{-3}$

We can extend the laws of exponents that we have studied earlier, even when base is a positive real number and the exponents are rational numbers. In the previous section, we have defined $\sqrt[n]{a}$ for real number, where $a > 0$.

$$x^n = a \Rightarrow x = a^{1/n} = \sqrt[n]{a}$$

$$4^{3/2} = \left(4^{1/2}\right)^3 = 2^3 = 8$$

$$4^{3/2} = \left(4^3\right)^{1/2} = (64)^{1/2} = \left(8^2\right)^{1/2} = 8$$

Let $a > 0$ be a real number. Let m and n be integers such that m and n have no common factors other than 1, and $n > 0$, then :

(i) $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ (ii) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$

Let $a > 0$ be a real number and p and q are rational numbers, then :

(i) $a^p \cdot a^q = a^{p+q}$ (ii) $\left(a^p\right)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{p-q}$ (iv) $a^p b^p = (ab)^p$

Simplify :

(i) $4^{2/3} \cdot 4^{1/3} = 4^{2/3+1/3} = 4^{3/3} = 4^1 = 4$ (ii) $\left(3^{1/5}\right)^8 = 3^{8/5}$

(iii) $\frac{9^{1/5}}{9^{1/3}} = 9^{1/5-1/3} = 9^{(3-5)/15} = 9^{-2/15}$

Exercise 2.3

1. Find the value of :

(i) $81^{1/2}$ (ii) $64^{1/6}$ (iii) $(125)^{1/3}$

2. Find the value of :

(i) $(4)^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$

3. Simplify :

(i) $2^{2/3} \cdot 2^{1/7}$ (ii) $\left(\frac{1}{3^3}\right)^7$

4. Find the value of x :

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

Answer Sheet

Exercise 2.1

1. (i), (iv) and (v) irrational

(ii) and

(iii) rational

2. 0.01001000100001 . . .; 0.202002000200002 . . ., 0.00300030000 . . .

3. (i) 0.36 terminating ;

(ii) $0.\overline{09}$ non-terminating recuring;

(iii) 4.125 terminating ;

(iv) $0.\overline{230769}$ non-terminating recuring

(v) $0.\overline{18}$ non-terminating recuring ;

(vi) 0.8225 terminating

4. (i) $\frac{1}{3}$ (ii) $\frac{47}{99}$ (iii) $\frac{14}{11}$ (iv) $\frac{233}{990}$

5. 0.75075007500075000075 . . .

0.767076700767000767 . . .

0.808008000800008 . . .

Exercise 2.2

1. (i) irrational ; (ii) rational ; (iii) rational ; (iv) irrational ; (v) irrational

2. (i) $-\frac{1}{38}\left(5-3\sqrt{7}\right)$; (ii) $-\left(\sqrt{2}-\sqrt{3}\right)$; (iii) $\frac{1}{3}\left(\sqrt{7}+2\right)$

3. $a=\frac{13}{7},\ b=\frac{9}{7}$

Exercise 2.3

1. (i) 9 ; (ii) 2 ; (iii) 5

2. (i) 8 ; (ii) 4 ; (iii) 8

3. (i) $2^{\frac{2}{21}}$; (ii) 3^{-21}

4. $x = 3$

3

Polynomials

3.01. Introduction

In earlier classes, we have studied various operation of algebraic expressions in which algebraic experssions are included in these classes, we have used the following algebraic indentities for factorisation.

$$(x+y)^2 = x^2 + 2xy + y^2,$$

$$(x-y)^2 = x^2 - 2xy + y^2 \text{ and}$$

$$x^2 - y^2 = (x+y)(x-y).$$

In this chapter we shall study a particular type of algebraic expressions, called *polynomials*, and the terminology related to it. We shall also study some algebraic identities to factorization of polynomials.

3.02. Polynomials

We know that a variable is denoted by the symbols x, y, z, etc. A variable can take any real value. When any constant and variables are expressed with four main operations, then the combination is called the algebraic expression. For examples, $3x$, $5x$, $-x$, $-\frac{3}{2}x$ etc., are algebraic expressions. General form of an algebraic expression is ax, where a is constant and x is variable. $3x$, $x^2 + 3x$, $x^3 + 2x^2 - 4x + 5$ etc. are algebraic expressions. In all of these the exponents of variable x is a whole number. These type of expressions are called the polynomials in one variable. In all of above examples x is a variable. A polynomial is denoted as $p(x)$, $g(x)$, $q(y)$ ..... etc.

For example

$$p(x) = 3x^2 + 4x - 5$$

$$g(x) = x^3 + 1$$

$$q(y) = y^3 + 2y - 1$$
$$s(t) = 3 - t - 2t^2 + 5t^3$$

A polynomial can have a lot of but finite number of terms

$$p(x) = a_n x^n + a_{n-1} x^{n\ 1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots a_2, a_1, a_0$ are the constants and $a_n \neq 0$. In polynomial $x^2 + 3x$; x^2 and $3x$ are terms of the polynomial. In a polynomial every term has a coefficient.

In polynomial $5x^3 - 2x^2 + x + 3$

coefficient of x^3 $= 5$, coefficient of x^2 $= -2$

coefficient of x $= 1$, coefficient of x^o $= 3$

Is 3 is a polynomial?

3, –7, 9 etc., are called **constant polynomials.**

0 is called zero polynomial.

Polynomials having only one term are called **monomials.**

For example, $3x,\ 5x^2,\ -3x^3,\ 2,\ t^2,\ y$ etc.

Polynomials having two terms are called **binomials.**

For example, $x + 2,\ x^2 - 2x,\ y^n + 2,\ t^{30} - t^3$ etc.

Similarly, polynomials having three terms are called trinomials.

For example, $p(x) = x^2 + x + 1$

$$g(x) = x - x^2 + \sqrt{3}$$
$$t(y) = y^3 + y + 3$$
$$s(t) = t^4 + t^2 - 2$$

The highest power of variable in a polynomial is called **the degree of the polynomial.**

In $p(x) = 4x^3 - 2x^2 + 8x - 21;$ the hightest power of the term $4x^3$ $= 3$.

In $q(y) = 3y^7 - 4y^6 + y + 9$ the hightest power of the term $3y^7$ $= 7$.

So, the degree of polynomials $p(x)$ and $q(y)$ are 3 and 7 respectively.

In the constant monomial $g(x) = 2$, the term 2 with highest power $= 2x^0$. So its degree = 0

Conclusion : The degree of a non-zero constant polynomial is zero.

Now analyse the following $p(x) = 5x + 4,\ g(y) = 12y,\ r(t) = 4 - 2t$ and

$s(u) = \sqrt{3} + 2u$.

Degree of all these polynomials is 1 (one).

Polynomials having degree 1 are called the linear polynomials.

Generally, a linear polynomial is expressed as :

$$p(x) = ax + b,\ a \neq 0$$

The maximum terms of a linear polynomial is two. It means a linear polynomial is either a binomial or a monomial.

Observe the following polynomial :

$p(x) = 2x^2 - 3x + 15,\quad g(x) = 5x^2 + 3$ and $g(y) = y^2 + 2y$

These polynomials have degree 2 are called the quadratic polynomials.

Generally, a quadratic polynomial is expressed as :

$$p(x) = ax^2 + bx + c,\ a \neq 0$$

A quadratic polynomial in one variable has at most three terms. It means a quadratic polynomial can be monomial, binomial or trinomial.

A polynomial of degree three is called a cubic polynomial. A cubic polynomial is expressed as $p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$ can have at most four terms. A polynomial in one varible x of degree n is an expression of the form :

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, $a_n, a_{n-1}, \ldots a_1, a_0$ are constants.

In particular if $a_0 = a_1 = a_2 = a_3 = \ldots = a_n = 0$ (all the constants are zero), we get the zero polynomial, that is denoted by 0. The degree of the zero polynomial is not defined.

Consider an algebraic expression $x + \frac{1}{x}$.

$$x + \frac{1}{x} = x + x^{-1}$$

The exponent of 2nd term of expression is -1 which is not a whole number.

$$\sqrt{x} + 5 = x^{1/2} + 5$$

The exponent of $x^{1/2}$ is $\frac{1}{2}$, which is not a whole number.

$$\sqrt[3]{y} + y^3 = y^{1/3} + y^3$$

Here, exponent of $y^{1/3}$ is $\frac{1}{3}$, which is not a whole number.

All the above expressions are not polynomials because as none of them has exponent as a whole number.

We have studied polynomials having one variables. There are polynomials having more then one variable for example, x^2+y^2+xyz, $p^2+8q^3+r^4$, t^2+s^3. These polynomials have 3, 3 and 2 variables respectively. We shall study these polynomials leter on.

Exercise 3.1

1. Which of the following expressions are polynomials in one variable. Find the number of terms also :?

   (i) $3x^2-5x+13$ (ii) $y^2+2\sqrt{3}$ (iii) $y+\frac{3}{y}$

   (iv) 3 (v) $2\sqrt{x}+\sqrt{3}\,x$ (vi) $x^{12}+y^3+t^{20}$

2. Write the coefficient of x^2 in following expressions :

   (i) $12+3x+5x^2$ (ii) $7-11x+x^3$ (iii) $\sqrt{3}x-7$ (iv) $\frac{\pi}{2}x^2+x$

3. Write an example of binomial of degree 45.
4. Write an example of monomial of degree 120.
5. Write an example of trinomial of degree 8.
6. Can you give some examples other than questions number 3, 4, and 5 have been given? If yes, then give two more examples for each.
7. Write the degree of each of the following polynomials :

   (i) $12-3x+2x^3$ (ii) $5y-\sqrt{2}$ (iii) 9 (iv) $3+4t^2$

3.03. Zeros of a Polynomial

Consider a polynomial

$$p(x)=2x^3-3x^2+4x-2$$

If we replace *x* by 2 everywhere in $p(x)$, we get

$$\begin{aligned} p(2) &= 2\times(2)^3-3\times(2)^2+4\times2-2 \\ &= 2\times8-3\times4+4\times2-2 \\ &= 16-12+8-2=10 \end{aligned}$$

So, we can say that the value of *p(x)* at $x=2$ is 10.

Similarly $p(0)=2\times(0)^3-3\times(0)^2+4\times0-2=-2$

and $p(-1)=2(-1)^3-3(-1)^2+4\times(-1)-2$

$$=2\times-1-3\times1-4\times1-2=-11$$

We can say that the value of polynomial $p(x)$ can be obtained by replacing the $x = \alpha$ in $p(\alpha)$.

Example 1. Find the value of the polynomial $p(x) = 8x^2 - 3x + 7$, at $x = -1$ and $x = 2$.

Solution : $p(x) = 8x^2 - 3x + 7$

The value of the polynomial $p(x)$ at $x = -1$ is :

$$p(-1) = 8(-1)^2 - 3(-1) + 7$$
$$= 8 + 3 + 7 = 18$$

Again, the value of the polynomial $p(x)$ at $x = 2$ is :

$$p(2) = 8(2)^2 - 3(2) + 7$$
$$= 32 - 6 + 7 = 33$$

Example 2. Find the value of the polynomial $p(x) = 2x^3 - 13x^2 + 17x + 12$, at $x = -\frac{1}{2}$.

Solution : $p(x) = 2x^3 - 13x^2 + 17x + 12$

Put $x = -\frac{1}{2}$

$$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 13\left(-\frac{1}{2}\right)^2 + 17\left(-\frac{1}{2}\right) + 12$$
$$= 2 \times \frac{-1}{8} - 13 \times \frac{1}{4} + 17 \times \frac{-1}{2} + 12$$
$$= -\frac{1}{4} - \frac{13}{4} - \frac{17}{2} + 12 = 0$$

Example 3. Find the value of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ at $x = 1$.

Solution : $p(x) = x^3 - 6x^2 + 11x - 6$

$$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6$$
$$= 1 - 6 + 11 - 6 = -12 + 12 = 0$$

In the above example since $p(1) = 0$, so we say that 1 is a zero of polynomial $p(x)$.

In general we can say that a zero of a polynomial $p(x)$ is a number "α" (alpha) such that $p(\alpha) = 0$.

What is the value of $p(1) = 0$, for polynomial $p(x) = x - 1$

$$p(1) = 1 - 1 = 0$$

We observed that the zero of the polynomial $p(x) = x - 1$ is obtained by equating it to 0 i.e., $x - 1 = 0$, which gives $x = 1$. We say $p(x) = 0$ is a polynomial equation and 1 is the root of the polynomial equation $p(x) = 0$. So we can say that 1 is the zero of the polynomial $x - 1$, or a root of the polynomial equation $x - 1 = 0$.

What is the zero of the constant polynomial 7?

It has no zero because replacing x by any number in $7x^0$ still gives us 7. In fact, a non-zero constant polynomial has no zero.

What about the zero of the zero polynomial ? By convention, every real number is a *zero of the zero polynomial.*

Examples 4. Check whether 3 and -3 are zeroes of the polynomial $p(x) = x + 3$.

Solution : $p(x) = x + 3$

$$\therefore \quad p(3) = 3 + 3 = 6$$

$$p(-3) = -3 + 3 = 0$$

Therefore, -3 is a zero of the polynomial $p(x) = x + 3$ but 3 is not a zero.

Example 5. Find a zero of the polynomial p(x) = 3x+2.

Solution : Finding a zero of $p(x)$ is the same solving the equation $p(x) = 0$

$$\therefore \quad p(x) = 3x + 2 = 0 \quad \Rightarrow 0 = 3x + 2$$

$$\Rightarrow \quad 3x = -2 \quad \Rightarrow x = \frac{-2}{3}$$

Now, if $p(x) = ax + b$, $a \neq 0$, is a linear polynomial, then we can find a zero of $p(x)$ from above examples.

It means finding a zero of the polynomial *p(x)* is to solve the polynomial equation *p(x) = 0*.

Now, $p(x) = 0 \Rightarrow ax + b = 0, \quad a \neq 0$

$$ax = -b$$

$$x = -\frac{b}{a}$$

So, $x = -\frac{b}{a}$ is the only one zero of $p(x)$ i.e., a linear polynomial has one and only one zero.

Example 6. Verify that 3 and 0 are the zeroes of the polynomial $x^2 - 3x$.

Solution : Let $p(x) = x^2 - 3x$

Then $\quad p(3) = (3)^2 - 3(3) = 9 - 9 = 0$

and $\quad p(0) = (0)^2 - 3(0) = 0 - 0 = 0$

Hence, 3 and 0 are both zero of the polynomial $x^2 - 3x$.

We can get the follwing conclusion.

1. A zero of a polynomial need not to be a 0.
2. 0 may be a zero of a polynomial.
3. Every linear polynomial has one and only one zero.
4. A polynomial may have more than one zero.

Exercise 3.2

1. Find the value of the polynomial $2x^3 - 13x^2 + 17x + 12$ at the following value of x :

   (i) $x = 2$ (ii) $x = -3$ (iii) $x = 0$ (iv) $x = -1$

2. Find the $P(2), P(1)$ and $P(0)$ for each of the following polynomials :

   (i) $p(x) = x^2 - x + 1$ (ii) $p(y) = (y+1)(y-1)$

   (iii) $p(x) = x^3$ (iv) $p(t) = 2 + t + t^2 - t^3$

3. Verify whether the followings are zeroes of the polynomial indicated against them :

   (i) $p(x) = x^2 - 1;\ x = 1,\ -1$ (ii) $p(x) = 2x + 1;\ x = -\frac{1}{2}$

   (iii) $p(x) = 4x + 5\ ;\ x = \frac{-5}{4}$ (iv) $p(x) = 3x^2;\ x = 0$

   (v) $p(x) = (x-3)(x+5);\ x = 3, -5$ (vi) $p(x) = ax + b;\ x = -\frac{b}{a}$

   (vii) $p(x) = 3x^2 - 1;\ x = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (viii) $p(x) = 3x + 2;\ x = \frac{-2}{3}$

4. Find the zeros of the following polynomials :

(i) $p(x) = x - 4$ (ii) $p(x) = 4x$

(iii) $p(x) = bx,\ b \neq 0$ (iv) $p(x) = x + 3$

(v) $p(x) = 2x - 1$ (vi) $p(x) = 3x + 7$

(vii) $p(x) = cx + d,\ c \neq 0,\ c, d$ are real numbers.

Remainder Theorem :

We know that when we divide 25 by 7, we get quotient 3 and remainder 4. Mathematically we express it as

$$25 = (3 \times 7) + 4$$

Similarly if we divide 48 by 8, we get

$$48 = (6 \times 8) + 0$$

Here, remainder is (0) and we say that 8 is s factor of 48 or 48 is a multiple of 8. In the same way we can divide a polynomial by another polynomial. In first case if divisor is monomial. For example on divide polynomial $3x^3 + 2x^2 + x$ by monomial x.

$$\left(3x^3 + 2x^2 + x\right) \div x = \frac{3x^3}{x} + \frac{2x^2}{x} + \frac{x}{x} = 3x^2 + 2x + 1,$$

Here, you have noticed that x is common to each term of $\left(3x^3 + 2x^2 + x\right)$

$3x^3 + 2x^2 + x = x\left(3x^2 + 2x + 1\right)$. We say that x and $\left(3x^2 + 2x + 1\right)$ are factors of $3x^3 + 2x^2 + x$ and $3x^3 + 2x^2 + x$ is a multiple of x as well as a multiple of $3x^2 + 2x + 1$.

Now divide $5x^2 + x + 1$ by x.

$$\left(5x^2 + x + 1\right) \div x = \left(5x^2 \div x\right) + \left(x \div x\right) + \left(1 \div x\right)$$

Here, when 1 is divided by x, we don't get a polynomial term. So, in this case we stop here, and note that 1 is the remainder. Thus, we have

$$5x^2 + x + 1 = \left[(5x + 1) \times x\right] + 1$$

Here, we get $(5x + 1)$ as a quotient and 1 as a remainder. So $(5x - 1)$ is not a factor of $5x^2 + x + 1$. Since, the remainder is not zero.

Dividend = (Divisor × Quotient) + Remainder.

In general, if $p(x)$ and $g(x)$ are two such polynomials that the degree of $p(x)$ is greater then $g(x)$ and $g(x) \neq 0$, then we get two polynomial $q(x)$ and $r(x)$.

$$p(x) = g(x) \cdot q(x) + r(x) \text{ where } r(x) = 0$$

Or the degree of $r(x)$ is smaller than that of degree of $g(x)$;

When $p(x)$ is divided by $g(x)$, we get quotient $q(x)$ and remainder $r(x)$.

Example 7. Divide p(x) by g(x); where p(x) = $7x + 5x^2 + 3$ and g(x) = $x + 1$.

Solution :

$$\begin{array}{r|l} & 5x+2 \\ \hline x+1 & 5x^2+7x+3 \\ & 5x^2+5x \\ & - \quad - \\ \hline & 2x+3 \\ & 2x+2 \\ & - \quad - \\ \hline & \quad 1 \end{array}$$

Note : We take the following steps for above division operation.

Step I : We write the dividend $7x+5x^2+3$ and divisor $x+1$ in the standard form arranging the terms in the descending order i.e. dividend as $5x^2+7x+3$ and divisor as $x+1$.

Step II : We divide the first term of the dividend by the first term of the divisor; i.e., we divide $5x^2$ by x get $5x$. This gives us the first term of the quotient.

Step III : We multiply the divisor by the first term of quotient $5x$ and subtract this product $5x^2+5x$ form the dividend. This gives us the remainder as $2x+3$.

Step IV : We take this remainder $2x+3$ as the new dividend. We repeat the step II to get 2 is the second term of the quotient.

Step V : Similarly as step III. We multiply the divisor $x+1$ by the second terms of quotient 2 and subtract the product $2x+2$ from the dividend $2x+3$. This gives us 1 as remainder.

This process continues till the remainder is 0 or the degree of the new dividend is less than the degree of the divisor. At the last stage, dividend becomes the remainder and the sum of the quotients gives us the whole quotient.

In this example divisor is a linear polynomial. Let us see the relation between the remainder and certain values of dividend.

In $p(x) = 5x^2+7x+3$, substituting –1 in place of x, we get

$$p(-1) = 5(-1)^2 + 7(-1) + 3 = 5 - 7 + 3 = 1$$

Hence the remainder obtained on dividing $p(x) = 5x^2+7x+3$ by $(x+1)$ is the same as the value of the polynomial $p(x)$ at the zero of the polynomial $(x+1)$ i.e., –1.

Let us consider some more examples.

Example 8. Divide the polynomial $2x^4 - 3x^3 + 3x + 1$ by $x + 1$.

Solution :

$$
\begin{array}{r|l}
 & 2x^3 - 5x^2 + 5x - 2 \\
\hline
 & 2x^4 - 3x^3 + 3x + 1 \\
x + 1 & 2x^4 + 2x^3 \\
 & - \quad - \\
\hline
 & -5x^3 \qquad + 3x + 1 \\
 & -5x^3 - 5x^2 \\
 & + \quad + \\
\hline
 & 5x^2 + 3x + 1 \\
 & 5x^2 + 5x \\
 & - \quad - \\
\hline
 & -2x + 1 \\
 & -2x - 2 \\
 & + \quad + \\
\hline
 & 3
\end{array}
$$

Remainder $= 3$

Here the zero of divisor $x + 1$ is -1. So on putting $x = -1$ in $p(x)$.

$$p(-1) = 2(-1)^4 - 3(-1)^3 + 3(-1) + 1$$

$$= 2 + 3 - 3 + 1 = 3$$

$=$ Remainder

Example 9. Find the remainder obtained on dividing $p(x) = x - 1$ by $x^3 - 1$.

Solution :

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
 & x^3 - 1 \\
x - 1 & x^3 \qquad - x^2 \\
 & - \qquad + \\
\hline
 & x^2 \qquad - 1 \\
 & x^2 - x \\
 & - \quad + \\
\hline
 & x - 1 \\
 & x - 1 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Now the remainder is 0.

The root of divisor $x - 1$ is $x = 1$ and $p(x) = x^3 - 1$

$\therefore \qquad p(1) = (1)^3 - 1 = 1 - 1 = 0$

So $p(1) = 0$ is equal to the remainder obtained by actual division.

In this way, it is a simple method to find the remainder obtained on dividing a polynomial by a linear polynomial. We shall now generalise this fact in the form of a theorem.

Remainder Theorem

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x-a$, then the remainder is $p(a)$.

Proof : Let $p(x)$ be any polynomial with degree greater than or equal to 1. Suppose that when $p(x)$ is divided by $x-a$, the quotient is $q(x)$ and the remainder is $r(x)$, i.e.,

$$p(x) = (x-a)q(x) + r(x)$$

Since, the degree of $x-a$ is 1 and the degree of $r(x)$ is less then degree of $(x-a)$, the degree of $r(x) = 0$. It means that $r(x)$ is a constant, say r.

So for every value of x, $r(x) = r$

Therefore $\quad p(x) = (x-a)q(x) + r$

In particular, if $x = a$, this equation gives us

$$p(a) = (a-a)q(x) + r$$

$$= 0 \times q(x) + r = r$$

$$= r$$

Hence Proved.

Example 10. Find the remainder when $x^4 - 4x^2 + x^3 + 2x + 1$ is divided by $x - 1$.

Solution : Let $p(x) = x^4 - 4x^2 + x^3 + 2x + 1$

The zero of $x-1$ is 1.

Thus, $\quad p(1) = (1)^4 - 4(1)^2 + (1)^3 + 2(1) + 1$

$$= 1 - 4 + 1 + 2 + 1 = 5 - 4 = 1$$

Thus, remainder = 1

Example 11. Verify whether the polynomial $p(x) = 4x^3 - 12x^2 + 13x - 4$ is a multiple of $g(x) = 2x - 1$.

Solution : As we know, $p(x)$ will be a multiple of $g(x)$ only when $g(x)$ divides $p(x)$ completely *i.e.*, remainder is zero.

So, $g(x) = 2x - 1 = 0$

$$x = \frac{1}{2}$$

$\therefore$ $$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 13\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{2} - 3 + \frac{13}{2} - 4 = 0$$

Hence, $g(x)$ is a factor of $p(x)$, means $p(x)$ is multiple of $g(x)$.

Exercise 3.3

1. Find the remainder, if the polynmomial $x^4 + x^3 - 3x^2 + 3x + 1$ is divided by following linear expression :

   (i) $x - 1$ (ii) $x - \frac{1}{2}$ (iii) $x + \pi$ (iv) $3 + 2x$ (v) x

2. Find the remainder, when $2x^3 + 2ax^2 - 5x + a$ is divided by $x + a$.

3. Check whether $x + 1$ is factor of $x^3 + 3x^2 + 3x + 1$ or not.

4. We get the same remainder if polynomials $x^3 + x^2 - 4x + a$ and $2x^3 + ax^2 + 3x - 3$ are divided by $x - 2$. Find the value of a.

Factorisation of Polynomials :

It is found by observing the example 11. Since, the remainder $p\left(\frac{1}{2}\right) = 0$,

therefore, $g(x) = (2x - 1)$, is factor of $p(x)$. So for a polynomial $p(x)$

$$p(x) = (2x - 1)q(x)$$

This is the particular case of the theorem that is given below.

Factor Theroem : If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number such that $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$, i.e. if $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$.

Example 12. Examine whether $x - 3$ is a factor of polynomials

$x^3 - 3x^2 + 4x - 12$ and $3x - 9$.

Solution : Given, $p(x) = x^3 - 3x^2 + 4x - 12$, $q(x) = 3x - 9$

According to factor theorem if $(x-3)$ is a factor of $p(x)$ and $q(x)$, then :

$$p(3) = q(3) = 0$$

The zero of $(x-3)$ is 3.

$$p(3) = (3)^3 - 3(3)^2 + 4(3) - 12$$

$$= 27 - 27 + 12 - 12 = 0$$

Thus, $x-3$ is a factor of $p(x)$

Similarly, ` $q(3) = 3 \times 3 - 9 = 0$

Thus, $x-3$ is a factor of $q(x)$ also.

Example 13. Find the value of a if x - 5 is a factor of the polynomial $x^3 - 3x^2 + ax - 10$.

Solution : As $x-5$ is a factor of $p(x) = x^3 - 3x^2 + ax - 10$

$\therefore$ $p(5) = 0$

Now, $p(5) = (5)^3 - 3(5)^2 + a(5) - 10 = 0$

$\Rightarrow$ $125 - 75 + 5a - 10 = 0$

$\Rightarrow$ $40 + 5a = 0$

Thus, $a = -\dfrac{40}{5} = -8$

The factor theroem is used to factorise some polynomials of degree 2 and 3. We are already familiar with the factorisation of quadratic polynomials like $ax^2 + bx + c$ where $a \neq 0$ and a, b, c are constants by splitting the middle term.

Let $ax^2 + bx + c = (px+q)(rx+s)$

$= prx^2 + (ps+qr)x + qs$

Comparing the coeffficients of both side, we get

$a = pr$

$b = ps + qr$

$c = qs$

Where b is the sum of two numbers ps and qr, whose product is

$$(ps)(qr) = (pr)(qs) = a \cdot c$$

Therefore, we can say that to factorise $ax^2 + bx + c$, we have to write b as the sum of those two numbers whose product is ac.

Example 14. Factorise $6x^2+17x+5$ by splitting the middle term and using the factor theorem.

Solution : 1. By spliting the middle term :

We have to split middle term 17 into such numbers whose sum is 17 and product is $6 \times 5 = 30$

Factors of 30 are, $1 \times 30 = 30$

$2 \times 15 = 30$

$3 \times 10 = 30$

$5 \times 6 = 30$

The sum of the pair of 2 and 15 is 17. So

$$6x^2+17x+5 = 6x^2+(2+15)x+5$$

$$= 6x^2+2x+15x+5$$

$$= 2x(3x+1)+5(3x+1)$$

$$= (3x+1)(2x+5)$$

2. By factorisation theorem

$$6x^2+17x+5 = 6\left(x^2+\frac{17}{6}x+\frac{5}{6}\right) = 6\cdot p(x)$$

Let the zeroes of $p(x)$ are a and b, then

$$6x^2+17x+5 = 6(x-a)(x-b)$$

So, $$ab = \frac{5}{6}$$

Now, possible values of a and b $= \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{5}{2}, \pm\frac{5}{3}, \pm 1$

Putting the values respectively, we have

$$p\left(\frac{1}{2}\right) = \frac{1}{4}+\frac{17}{12}+\frac{5}{6} \neq 0$$

$$p\left(-\frac{1}{3}\right) = \frac{1}{9}+\frac{17}{6}\times\frac{-1}{3}+\frac{5}{6}$$

$$= \frac{1}{9}-\frac{17}{18}+\frac{5}{6} = 0$$

So $\left(x+\frac{1}{3}\right)$ is a factor of $p(x)$

Similarly, by trial, we can find that $\left(x+\frac{5}{2}\right)$ is a factor of $p(x)$

Therefore, $$6x^2+17x+5=6\left(x+\frac{1}{3}\right)\left(x+\frac{5}{2}\right)$$

$$=6\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right)$$

$$=(3x+1)(2x+5)$$

Example 15. Factorise $x^2-7x+12$ with the help of factor theorem.

Solution : Let $p(x)=x^2-7x+12$

Now, if $p(x)=(x-a)(x-b)$, then

Here, constant term $ab=12$

So, to look for the factors of $p(x)$ we find the factors of 12.

Factors of 12 = 1, 2, 3, 4, 6

$$p(3)=(3)^2-7(3)+12=0$$

So, $(x-3)$ is factor of $p(x)$

Similarly, $$p(4)=(4)^2-7(4)+12=0$$

So, $(x-4)$ is factor of $p(x)$

Thus, $x^2-7x+12=(x-3)(x-4)$

Example 16. Using the factor theorem, factorise the polynomial $x^4+x^3-7x^2-x+6$.

Solution : Let $p(x)=x^4+x^3-7x^2-x+6$

The factors of constant term 6 = $\pm 1, \pm 2, \pm 3$ and ± 6

$$p(1)=(1)^4+(1)^3-7(1)^2-1+6=8-8=0$$

So, $(x-1)$ is factor of $p(x)$.

Similarly, $$p(-1)=(-1)^4+(-1)^3-7(-1)^2-(-1)+6=8-8=0$$

So, $(x+1)$ is also a factor of $p(x)$

$$p(2)=(2)^4+(2)^3-7(2)^2-(2)+6=30-30=0$$

Therefore, $(x-2)$ is another factor of $p(x)$

$$p(-2)=(-2)^4+(-2)^3-7(-2)^2-(-2)+6=24-36\neq 0$$

So, $(x+2)$ is not a factor of $p(x)$

$$p(-3)=(-3)^4+(-3)^3-7(-3)^2-(-3)+6=90-90=0$$

So, $(x+3)$ is a factor of $p(x)$ ।

Since, $p(x)$ is a polynomial of degree 4, therefore it can not have the factors more than 4.

$\therefore \quad p(x)=k(x-1)(x+1)(x-2)(x+3)$

$\Rightarrow \quad x^4+x^3-7x^2-x+6=k(x-1)(x+1)(x-2)(x+3) \quad . . . (1)$

put $x=0$, we get

$0+0+0-0+6=k(-1)(1)(-2)(3)$

$\Rightarrow \quad 6=6k$

$\Rightarrow \quad k=1$

Replacing 1 for k in the equation (i) we get

$x^4+x^3-7x^2-x+6=(x-1)(x+1)(x-2)(x+3)$

Exercise 3.4

1. Determine which of the following polynomials has $(x-1)$ as a factor :

(i) $x^4-2x^3-3x^2+2x+2$ (ii) $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3-3x^2+x-2$ (iv) $x^3-x^2-(2+\sqrt{3})x+\sqrt{3}$

2. Using the factor theorem, find if $g(x)$ is a factor of $p(x)$?

(i) $p(x)=3x^3-x^2-3x+1;\ g(x)=x+1$

(ii) $p(x)=2x^4-7x^3-13x^2+63x-45;\ g(x)=x-1$

(iii) $p(x)=3x^3+3x^2+3x+1;\ g(x)=x+2$

(iv) $p(x)=2x^3+x^2-2x-1;\ g(x)=2x+1$

3. Find the value of k, when $(x-5)$ is a factor of the polynomial $x^3-3x^2+kx-10$.

4. Find the value of k, when $(x-1)$ is a factor of the polynomial $2x^2+kx+\sqrt{2}$.

5. Find the value of a and b, if $(x+1)$ and $(x-1)$ are the factors of the polynomial $x^4+ax^3-3x^2+2x+b$.

6. Factorise :

(i) $3x^2+7x+2$ (ii) $4x^2-x-3$

(iii) $12x^2-7x+1$ (iv) $6x^2+5x-6$

7. Find the zeroes of the polynomials :

(i) $x^3+6x^2+11x+6$ (ii) x^3+2x^2-x-2

(iii) $x^4-2x^3-7x^2+8x+12$ (iv) x^3-2x^2-x+2

(v) x^3-3x^2-9x-5 (vi) $x^3-23x^2+142x-120$

Algebraic Identities :

In our earlier classes, we studied that an algebraic identity is an algebraic equation that is true for all values of the variables occuring in it. We have already studied the identities given below in our previous classes.

Identitiy I : $(x+y)^2=x^2+2xy+y^2$

Identitiy II : $(x-y)^2=x^2-2xy+y^2$

Identitiy III : $x^2-y^2=(x+y)(x-y)$

Identitiy IV : $(x+a)(x+b)=x^2+(a+b)x+ab$

All of above identities involved product of binomials. Let us extend the Identity I to a trimonial $x+y+z$. We shall compute $(x+y+z)^2$.

Let $x+y=t$, then

$$\therefore \quad (x+y+z)^2=(t+z)^2$$

$$=t^2+2tz+z^2 \quad \text{(Using Identity I)}$$

$$=(x+y)^2+2(x+y)z+z^2$$

Substituting the value of t, we have

$$=x^2+2xy+y^2+2xz+2yz+z^2$$

$$=x^2+y^2+z^2+2xy+2yz+2zx$$

So, we get the following identity :

Identity V : $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Example 17. Expand : $(2x+4y+3z)^2$

Solution : On compairing with identity V.

$x = 2x$, $y = 4y$, $z = 3z$

On using the Identity V.

$$(2x+4y+3z)^2 = (2x)^2+(4y)^2+(3z)^2+2(2x)(4y)+2(4y)(3z)+2(3z)(2x)$$

$$= 4x^2+16y^2+9z^2+16xy+24yz+12zx$$

Example 18. Expand : $(2a-3b-4c)^2$

Solution : Using Identity V

$$(2a-3b-4c)^2 = [2a+(-3b)+(-4c)]^2$$

$$= (2a)^2+(-3b)^2+(-4c)^2+2(2a)(-3b)+2(-3b)(-4c)+2(-4c)(2a)$$

$$= 4a^2+9b^2+16c^2-12ab+24bc-16ac$$

Example 19. Factorise : $4x^2+y^2+z^2-4xy-2yz+4xz$

Solution : $4x^2+y^2+z^2-4xy-2yz+4xz$

$$= (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)$$

$$= (2x-y+z)^2 \quad \text{(Using Identity V)}$$

$$= (2x-y+z)(2x-y+z)$$

Now, let us extend identity I to compute $(x+y)^3$.

Here, $(x+y)^3 = (x+y)(x+y)^2$

$$= (x+y)(x^2+2xy+y^2)$$

$$= x(x^2+2xy+y^2)+y(x^2+2xy+y^2)$$

$$= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3$$

$$= x^3+3x^2y+3xy^2+y^3$$

$$= x^3+y^3+3xy(x+y)$$

So, we get the following identites :

Identity VI : $(x+y)^3 = x^3+y^3+3xy(x+y)$

Replacing y by $-y$ in the above identity, we get

Identity VII : $(x-y)^3 = x^3 - y^3 - 3xy(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

Example 20. Using the identities expand the following expressions :

(i) $(4a+3b)^3$ (ii) $(3x-5y)^3$

Solution : (i) Comparing the expression with $(x+y)^3$, we find that

$$x = 4a \text{ and } y = 3b$$

$$(4a+3b)^3 = (4a)^3 + (3b)^3 + 3(4a)(3b)(4a+3b)$$

$$= 64a^3 + 27b^3 + 144a^2b + 108ab^2$$

(ii) Comparing the expression with $(x-y)^3$, we find that

$$x = 3x, \quad y = 5y$$

$$\therefore \quad (3x-5y)^3 = (3x)^3 - (5y)^3 - 3(3x)(5y)(3x-5y)$$

$$= 27x^3 - 125y^3 - 135x^2y + 225xy^2$$

Example 21. Using the suitable identity evaluate each of the following :

(i) $(102)^3$ (ii) $(998)^3$

Solution : (i) $(102)^3 = (100+2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100+2)$ (Using the Identity VI)

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(ii) $(998)^3 = (1000-2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

Example 22. Factorise : $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Solution : $8x^3 + 27y^3 + 36x^2y + 54xy^2$

$= (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2$

$= (2x+3y)^3$ (Using Identity VI)

$= (2x+3y)(2x+3y)(2x+3y)$

Let us find out an important identity :

On expanding $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, we get

$$= x\left(x^2+y^2+z^2-xy-yz-zx\right)+y\left(x^2+y^2+z^2-xy-yz-zx\right)+$$
$$z\left(x^2+y^2+z^2-xy-yz-zx\right)$$
$$= x^3+xy^2+xz^2-x^2y-xyz-x^2z+x^2y+y^3+yz^2-xy^2-y^2z-xyz$$
$$+x^2z+y^2z+z^3-xyz-yz^2-xz^2$$
$$= x^3+y^3+z^3-3xyz \text{ (By simplification)}$$

So we obtain the following identity.

Identity VIII: $x^3+y^3+z^3-3xyz=\left(x+y+z\right)\left(x^2+y^2+z^2-xy-yz-zx\right)$

Example 23. Factorise : $27x^3+y^3+z^3-9xyz$

Solution : $27x^3+y^3+z^3-9xyz=\left(3x\right)^3+\left(y\right)^3+\left(z\right)^3-3\left(3x\right)\left(y\right)\left(z\right)$

$$=\left(3x+y+z\right)\left[\left(3x\right)^2+y^2+z^2-3x\cdot y-y\cdot z-z\cdot 3x\right]$$

(Using the identity (VIII))

$$=\left(3x+y+z\right)\left(9x^2+y^2+z^2-3xy-yz-3zx\right)$$

Exercise 3.5

1. Use the suitable identities to find the product of :

(i) $\left(x+3\right)\left(x+7\right)$ (ii) $\left(x-5\right)\left(x+8\right)$ (iii) $\left(2x+7\right)\left(3x-5\right)$

(iv) $\left(5-3x\right)\left(3+2x\right)$ (v) $\left(x^2+\frac{3}{5}\right)\left(x^2-\frac{3}{5}\right)$ (vi) $\left(x+2\right)\left(x-5\right)$

2. Using the algebric identities, find the product of following :

(i) 104×109 (ii) 94×97 (iii) 103×97

3. Using the suitable identities, factorise the following.

(i) $x^2+6xy+9y^2$ (ii) x^2-4x+4 (iii) $\frac{x^2}{100}-y^2$

4. Expand the following with the help of suitable identities :

(i) $\left(2a-3b-c\right)^2$ (ii) $\left(2+x-2y\right)^2$ (iii) $\left(a+2b+4c\right)$

(iv) $\left(m+2n-5p\right)^2$ (v) $\left(3a-7b-c^2\right)^2$ (vi) $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$

5. Factorise :

(i) $9x^2+4y^2+16z^2-12xy-16yz+24xz$

(ii) $x^2+2y^2+8z^2+2\sqrt{2}xy-8yz-4\sqrt{2}xz$

6. Expand the following cubes :

(i) $(3a-2b)^3$ (ii) $(1+2x)^3$ (iii) $\left(\frac{2}{5}x+3\right)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

7. Evaluate the following using suitable identities :

(i) $(98)^3$ (ii) $(103)^3$ (iii) $(999)^3$

8. Factorise :

(i) $x^3+8y^3+6x^2y+12xy^2$ (ii) $27a^3-8b^3-54a^2b+36ab^2$

(iii) $27-125x^3-135x+225x^2$ (iv) $125x^3-64y^3-300x^2y+240xy^2$

9. Factorise :

(i) $64a^3+27b^3$ (ii) $125x^3-8y^3$

10. Verify that :

(i) $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

(ii) $27a^3+b^3+c^3=(3a+b+c)\left[9a^2+b^2+c^2-3ab-bc-3ac\right]$

11. If $x+y+z=0$, then verify that $x^3+y^3+z^3=3xyz$

12. Using the suitable identitites compute :

(i) $(30)^3+(20)^3+(-50)^3$ (ii) $(-15)^3+(28)^3+(-13)^3$

[**Hint :**Use identity if $x+y+z=0$, then $x^3+y^3+z^3=3xyz$]

Important Points

1. A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form

   $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$

   Where $a_0, a_1, a_2, \ldots$ are constant and $a_n \neq 0$

   $a_0, a_1, a_2 \ldots a_n$ are respectively the coefficients of $x^0, x, x^2, \ldots$ and n is called the degree of the polynomial.

   $a_n x^n, a_{n-1}, x^{n-1}, \ldots, a_0$ where $a_n \neq 0$ is called a term of the polynomial $p(x)$
2. A polynomial of one term is called a monomial.
3. A polynomial of two terms is called a binomial.
4. A polynomial of three terms is called a trinomial.
5. A polynomial of degree one is called a linear polynomial.
6. A polynomial of degree two is called a quadratic polynomial.
7. A polynomial of degree three is called a cubic plynomial.
8. A real number 'a' is a zero of the polynomial $p(x)$, if $p(a) = 0$.
9. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial.
10. Remainder Theorem : If $p(x)$ is any polynomial of degree greater than or equal to 1 and a is a real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$
11. Factor theorem : $(x - a)$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also if $x - a$ is a factor of $p(x)$, then $p(a) = 0$
12. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
13. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
14. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
15. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Answer

Exercise 3.1

1. (i) Polynomial, one variable.

(ii) Polynomial, one variable

(iii) and (v) are not polynomial, because each exponent is not a whole number.

(iv) Constant polynomial

(vi) Polynomial, three variables.

2. (i) 1 (ii) 0 (iii) 0 (iv) $\frac{\pi}{2}$

3. $5x^{45}+7$ (other polynomials may be possible)

4. $3x^{120}$ (other polynomials may be possible)

5. $2x^8+3x^4+5x$ (other polynomials are also possible)

6. Possible, write yourself.

7. (i) 3 (ii) 1 (iii) 0 (iv) 2

Exercise 3.2

1. (i) 10 (ii) –210 (iii) 12 (iv) –20

2. (i) 3, 1, 1 (ii) 3, 0, –1 (iii) 8, 1, 0 (iv) 0, 3, 2

4. (i) 4 (ii) 0 (iii) 0 (iv) –3

(v) $\frac{1}{2}$ (vi) $-\frac{7}{3}$ (vii) $-\frac{d}{c}$

Exercise 3.3

1. (i) 1 (ii) $\frac{31}{16}$ (iii) $\pi^4-\pi^3-3\pi^2-3\pi+1$

(iv) $-\frac{137}{16}$ (v) 1

2. 6a

3. Yes, because remainder is zero.

4. $a=-5$

Exercise 3.4

1. (i) and (iii) has a factor $(x-1)$;

(ii) and (iv) do not have a factor $(x-1)$

2. (i) Yes (ii) Yes (iii) No (iv) Yes

3. (i) $k=-8$

4. $k=-\left(2+\sqrt{2}\right)$

5. $a=-2,\ b=2$

6. (i) $(x+2)(3x+1)$; (ii) $(x-1)(4x+3)$;

(iii) $(3x-1)(4x-1)$; (iv) $(2x+3)(3x-2)$

7. (i) $-1,\ -2,\ -3$; (ii) $-2,\ -1,\ 1$; (iii) $-2,\ -1,\ 2,\ 3$;

(iv) $-1,\ 1,\ 2$; (v) $-1,\ 5$; (vi) $1,\ 10,\ 12$

Exercise 3.5

1. (i) $x^2+10x+21$ (ii) $x^2+3x-40$ (iii) $6x^2+11x-35$

(iv) $15+x-6x^2$ (v) $x^4-\dfrac{9}{25}$ (vi) $x^2-3x-10$

2. (i) 11336 (ii) 9118 (iii) 9991

3. (i) $(x+3y)(x+3y)$ (ii) $(x-2)(x-2)$ (iii) $\left(\dfrac{x}{10}+y\right)\left(\dfrac{x}{10}-y\right)$

4. (i) $4a^2\times 9b^2+c^2-12ab+6bc-4ac$

(ii) $4+x^2+4y^2+4x-4xy-8y$

(iii) $a^2+4b^2+16c^2+4ab+16bc+8ac$

(iv) $m^2+4n^2+25p^2+4mn-20np-10pm$

(v) $9a^2+49b^2+c^2-42ab+14bc-6ac$

(vi) $\dfrac{x^2}{y^2}+\dfrac{y^2}{z^2}+\dfrac{z^2}{x^2}+\dfrac{2x}{z}+\dfrac{2y}{x}+\dfrac{2z}{y}$

5. (i) $(3x-2y+4z)^2$ (ii) $\left(x+\sqrt{2}y-2\sqrt{2}z\right)^2$

6. (i) $27a^3 - 8b^3 - 54a^2b + 36ab^2$ (ii) $1 + 8x^3 + 6x + 12x^2$

(iii) $\frac{8}{125}x^3 + 27 + \frac{36}{25}x^2 + \frac{54}{5}x$ (iv) $x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

7. (i) 941192 ; (ii) 1092727; (iii) 997002999

8. (i) $(x+2y)^3$; (ii) $(3a-2b)^3$; (iii) $(3-5x)^3$; (iv) $(5x-4y)^3$

9. (i) $(4a+3b)(16a^2-12ab+9b^2)$; (ii) $(5x-2y)(25x^2+10xy+4y^2)$

12. (i) –90000; (ii) 16380

4

Linear Equations in Two Variables

4.01. Introduction

In previous classes you have studied about linear equations in one variable. The equations in which degree of variables is one are called linear equation. Some of the examples of the linear equations are

(i) $x+3=8$ (ii) $2y+10=28$

(iii) $4x-7=2x+3$ (iv) $5m=40$

The equations satified by a value of the variable, which when substuituted in place of variable in the equation i.e. the left hand side and right hand side of the equation is equal is called the solution of the equation.

We know the fact about the equations that the solution of a linear equation is not affected when :

(i) the same number is added or subtracted from both sides of an equation.

(ii) the same number is multiplied or same non zero number divide both the sides of the equation.

In general, a linear equation in one variable can be expressed in the form of $ax+b=0$, where a and b are real number. Here $a \neq 0$ and x is the variable. The solution of equation $ax+b=0$ is $x=\frac{-b}{a}$. The equaltion in one variable has a unique (one and only one) solution, that is called the root of equation .

Linear Equations in Two Variables

The equation in which there are two variables of degree one is called linear equation in two variables.

Let us understand the co-ordinate system before study of the equations in two variables.

4.02. Rectangular Co-ordinate System

We shall clear the concept of rectangular co-ordinate system before solving a linear equation graphically.

(a) Rectangular Co-ordinate System :

We have already learnt that how to represent a real number on the number line. There are many situations, in which to find a point we are need to describe its position with reference to more than one line. Sometimes a point does not fall on the number line instead it is located somewhere in the plane. So we expand the principle related to number line.

So a dot in the plane can be represented by two perpendicular lines one of them is horizontal and other one is vertical. The horizontal line is called x-axis as well as vertical line is called y-axis and they are represented as XOX' and YOY' respectively.

The intersecting point of both lines known as origin and is denoted by symbol O. Positive integers lie to the right side of origin on x-axis (towards OX) and negative integers lie to the left side (towards OX'). Similarly positive integers lie above the origin of y-axis (towards OY) and negative integers lie below the origin (towards OY')

(b) Quadrants

Two axes XX' and YY' divide the plane into four parts which are called quadrants (means one fourth part). The expansion of these quadrants is infinite. XOY, YOX', $X'OY'$ and $Y'OX$ are the first, second, third and fourth quadrants respectively.

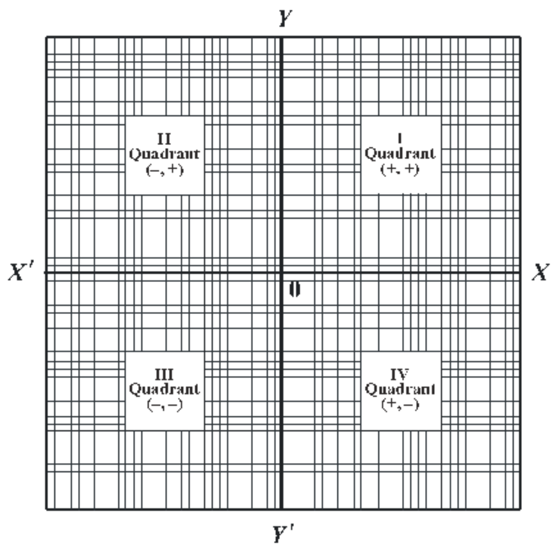


Fig. 4.01

(c) Plotting of Points

Let P be a point in the first quadrant. To reach this point we have to move 3 units towards OX and then 4 units towards OY, then this point can be expressed as $P(3,4)$. 3 is the x-coordinate of P on x-axis and 4 is the y-coordinate of P on y-axis. x-coordinate is called **abscissa** and y-coordinate is called **ordinate.** In this way, there is an abscissa x and an ordinate y for every point in the plane. These are represented by a ordered pair (x,y). Ordered pair (x,y) is called the coordinate of that point.

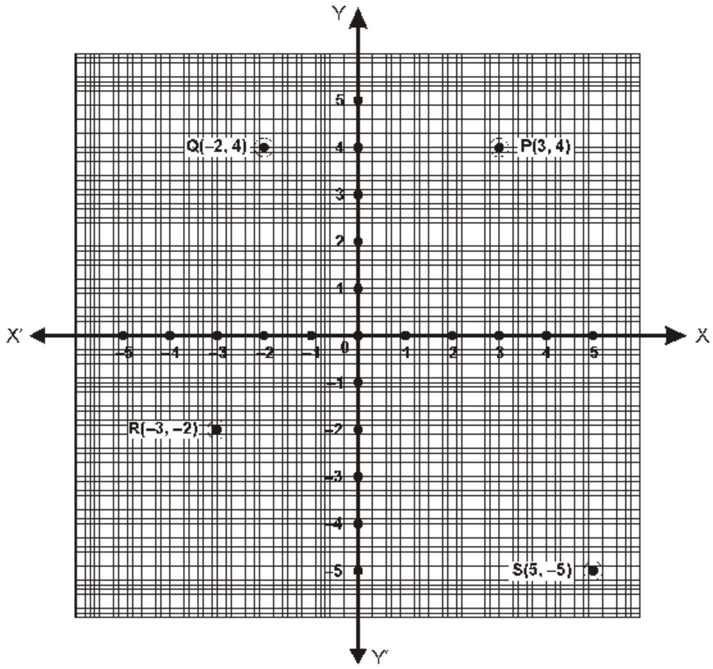


Fig. 4.02

Now consider the point $P(3,4)$. We observe that this point is above the x-axis and right to the y-axis, so its both abscissa and ordinate are positive. Therefore, the point P (3,4) lie in the first quadrant.

Similary we see that the coordinate in ordered pair form of points Q, R and S are resectively $(-2,4)$, $(-3,-2)$ and $(5,-5)$ as shown in the figure 4.02.

Again if we plote the point P (3, 4) then we move 3 unit in the direction towards OX right to O and then we move upward 4 unit parallel to OY from that point. This is the actual position of point $P(3, 4)$ in the plane.

So, to locate the point $Q(-2,4)$ we move 2 units in the direction of OX' form O and then we move upward 4 units parallel to OY. The point Q lie in the IInd quadrant.

In the same way we can plot the points $R(-3,-2)$ and $S(5,-5)$ in the plane.

Note :

1. The ordinate of each point on x-axis is zero.
2. The abscissa of each point on y-axis is zero.
3. Coordinates of origin are (0,0).

4.03. Graph of Linear equation in two variables

Take an example $x + y = 9$

The solutions of the equation are the values of variables x and y which satisfy the given equation. Let us see what values of x and y satisfy the above equation. See the following table.

x	0	1	2	3	4	5	6	7
y	9	8	7	6	5	4	3	2

These are some solutions that satisfy the equation but we can say that infinitely many values of variables x and y satisfy the solution.

We should plot the value of x variable on the X-axis and the value of variable y on Y-axis, and cordinates of x and y are written as (x,y). When all these points are joined, we get a straight line that is called the graph of the equation. Solutions of the equation are pointed on the obtained line and according to the graph every point on the line is the solution of the equation.

Construction of a line is a series of infinitely many points. So we can say that an equation in two variables has infinitely many solutions.

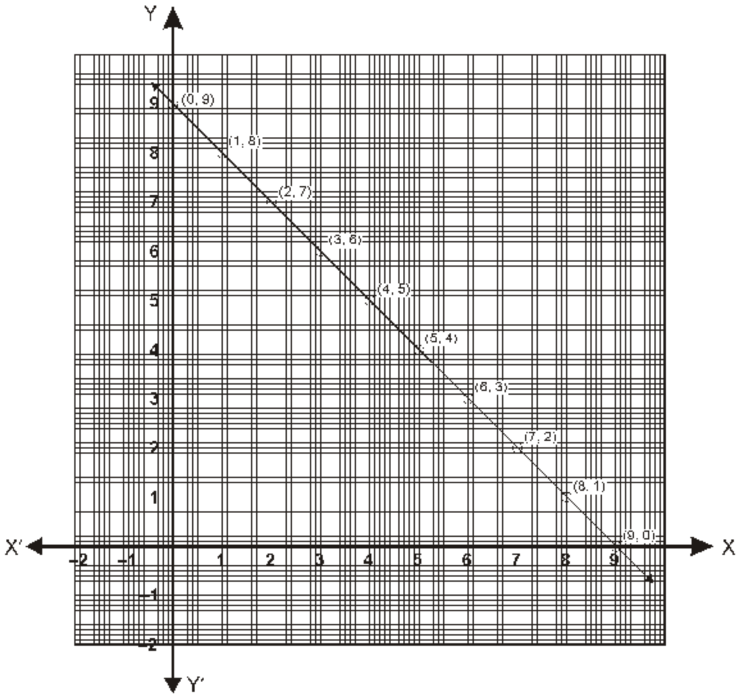


Fig. 4.03

Note :

1. The graphical representation of an equation in two variables is always a straight line.

2. Every point on the graph line gives the solution of the equation.

3. Any point, which does not lie on graph line, is not a solution of the equation. To obtain the graph of a linear equation in two variables, it is enough to plot two points corresponsing to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the correction of the graph.

Example 1. Draw the graph of equation $3x + y = 2$

Solution : Given equation is

$$3x + y = 2 \quad \Rightarrow \quad y = 2 - 3x$$

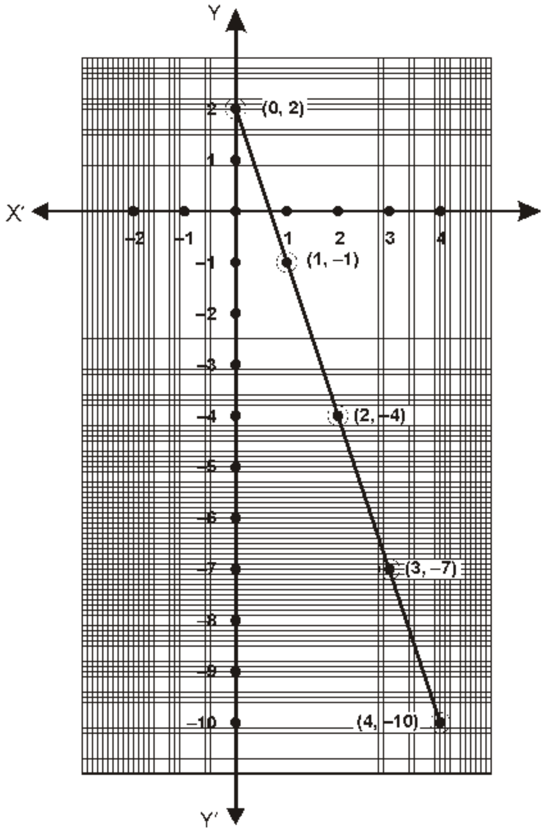


Fig. 4.04

We prepare a table as follows by writing the values of y below the corresponding values of x

x	0	1	2	3	4
y	2	-1	-4	-7	-10

From the graph of the equation, it is clear that every point on the graph line is a solution of the equation. If more than one equations in two variables are to be graphed on the same graph paper, then following conditions may be obtained :

(i) The graph line of two equations may intersect each other on a point.

(ii) Graph lines of two equations may be parallel and they never intersects each other.

(iii) Both lines may be coincident. In the first condition, the intersecting point of two lines shows the solution of both the lines. So,the coordinates of that point satisfy the two equations.

To obtain the unique solution of a linear equation in two variables, two linear equations will be required. Such linear equations in two variables are called the simultaneous equations.

The solution of the pair of linear equation by graph.

Illustrative Examples

Example 2. Solve the following equations by graphical method.

$$x + y = 3 \ ; \ 3x - 2y = 4$$

Solution : By the given equations we make the separate tables from their possible solutions.

Given, $x + y = 3$ . . . (i)

$\Rightarrow$ $x = 3 - y$

x	1	2	3
y	2	1	0

Similarly, $3x - 2y = 4$ or $x = \dfrac{4 + 2y}{3}$ . . . (ii)

x	2	4	6
y	1	4	7

Draw the graph by plotting the above points given in the tables and then by joining the line.

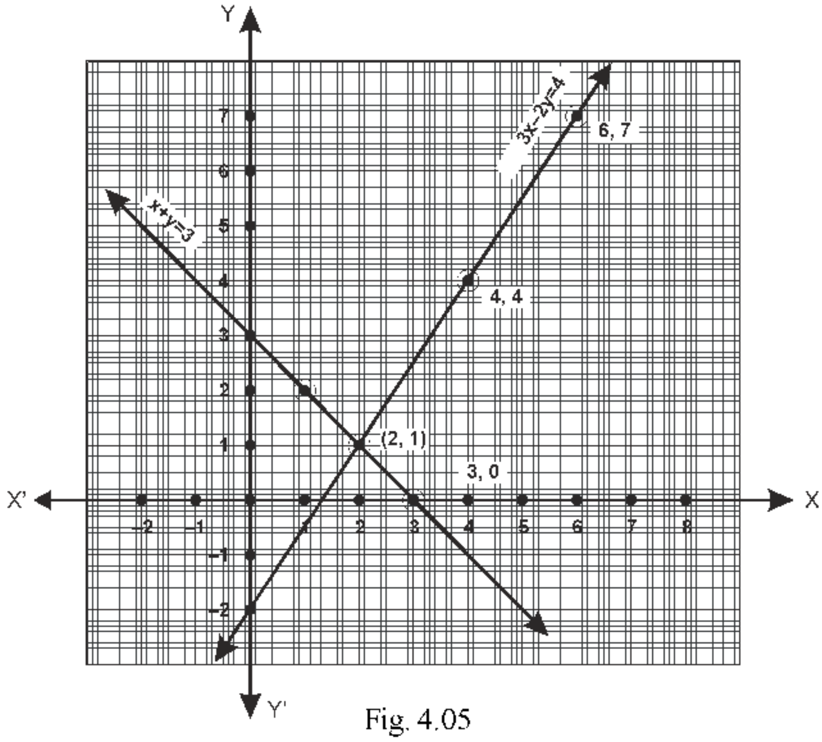


Fig. 4.05

Both the lines intersect at a point. The coordinates of point are (2, 1). Thus, the solution of the given equations is $x = 2$ and $y = 1$.

Example 3. **Solve the following pair of equations for x and y by graphical Method : $2x + 3y = 13$; $5x - 2y = 4$**

Solution : Given system of equations is :

$$2x + 3y = 13 \quad ...(1)$$

$$5x - 2y = 4 \quad ...(2)$$

Graph of the equation $2x + 3y = 13$ or $y = \frac{13 - 2x}{3}$

We have the following table for some possible solutions of the equation :

x	-1	2	5
y	5	3	1

Similarly euation $5x - 2y = 4$ or $y = \frac{5x - 4}{2}$.

x	-2	4	2
y	-7	8	3

Plotting the points $(-1, 5)$, (2, 3) and (5, 1) and drawing a line joining them, we get the graph of the equation $2x+3y=13$ as shown in Fig. 4.06. We have the following table for values of (*x*, *y*).

Plotting the points $(-2,-7),(4,8)$ on the same graph paper and drawing a line joining them, we obtain the graph of the equation $5x-2y=4$.

Clearly, the two lines intersect at point P(2, 3). Thus, $x=2,\ y=3$ is the solution of the given system.

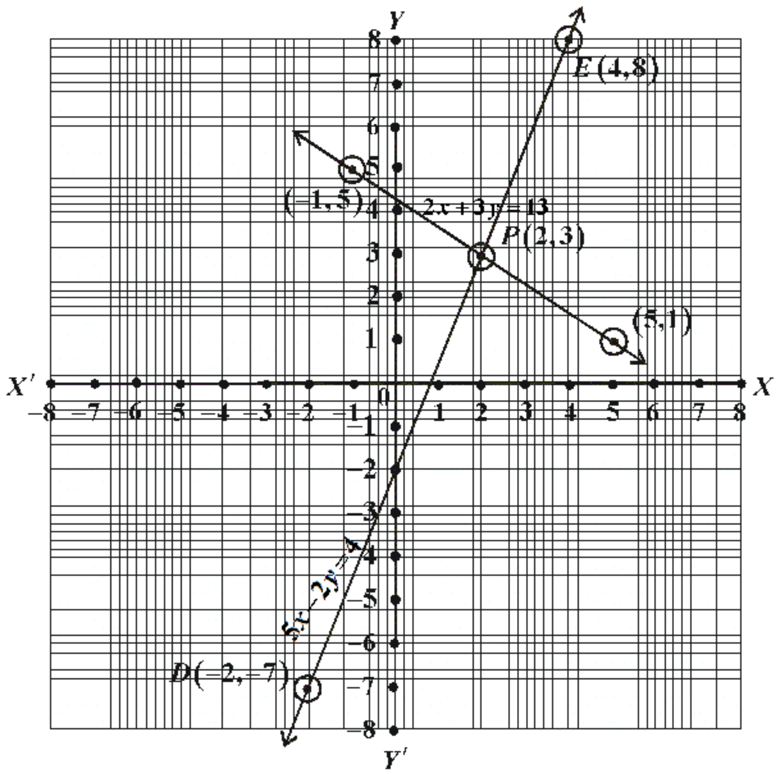


Fig. 4.06

Example 4. Solve the following equations by graphical method.

$$\mathbf{2x-6y+10=0\ ;\ 3x-9y+15=0}$$

Solution : Given system of equation is :

$$2x-6y+10=0 \qquad \dots(1)$$

$$3x-9y+15=0 \qquad \dots(2)$$

Table for equation $2x-6y+10=0$

x	1	–5	7
y	2	0	4

Table for equation $3x-9y+15=0$

x	4	–2	–8
y	3	1	–1

Now, we plot the points $(1,2),(-5,0)$ and $(7,4)$ on a graph and join them. We get a straight line AB in the form of the graph of equation $2x-6y+10=0$. Again, we plot the points $(4,3),(-2,-1)$ and $(-8,-1)$. We see that all three points lie on the line AB. So, both the lines are coincide. So, the equations have infinitely many solutions. And the solution of the equation $2x-6y+10=0$, will be the solution of the system.

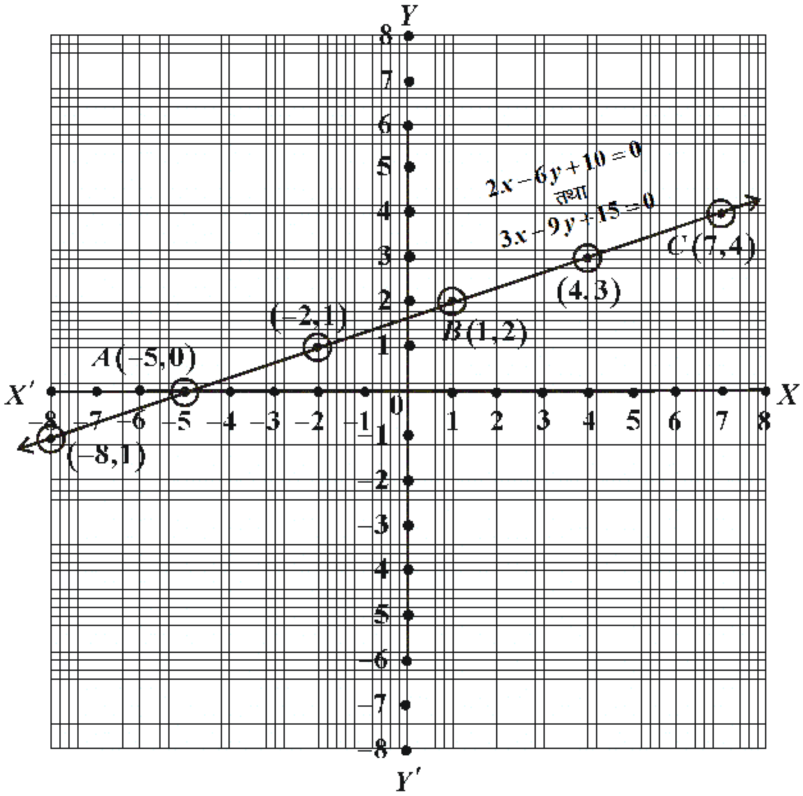


Fig. 4.07

Example 5. Solve the system of equations given below graphically. Also find the nature of system $2x+3y=12$; $2x+3y=6$.

Solution : We have two linear equations.

$$2x+3y=12 \quad \text{... (1)}$$

$$2x+3y=6 \quad \text{...(2)}$$

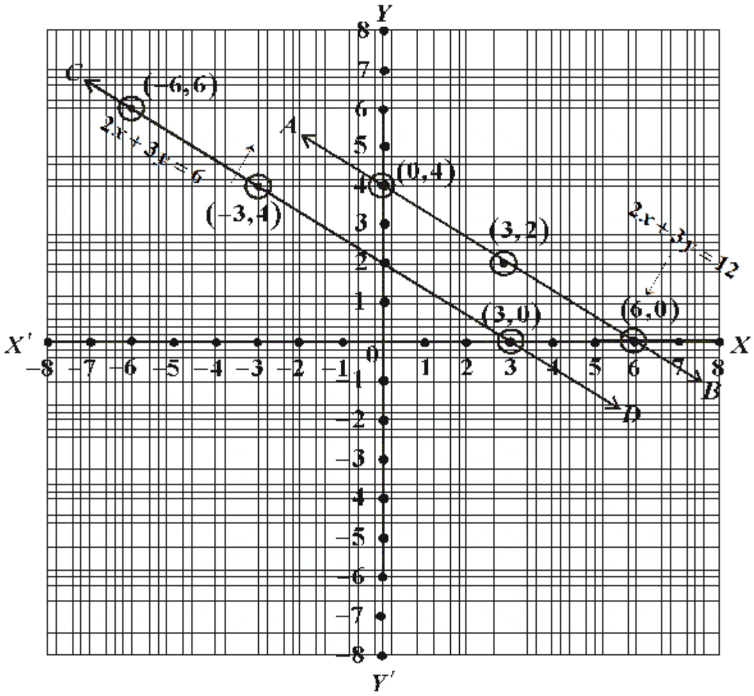


Fig.4.08

Now on plotting the points $(6,0),(3,2),(0,4)$. When we join these three points, we obtain a straight line AB. Which is a graph of equation $2x+3y-12=0$.

Again on plotting the points $(3,0),(-3,4)$ and $(-6,6)$. We get the graph of equation $2x+3y=6$ is a straight line CD by joining these points.

Now we see that the lines AB and CD which we obtained graphically are parallel it means the given system of linear equations is inconsistent. Hence, there is no solution of the given equations.

Exercise 4.1

Solve the following equations graphically.

1. $x+3y=6$
   $2x-3y=12$

2. $2x+y=6$
   $2x-y+2=0$

3. $x-2y=6$
   $3x-6y=0$

4. $x+y=4$
   $2x-3y=3$

5. $2x-3y+13=0$
   $3x-2y+12=0$

6. $3x-4y=1\,;\,-2x+\frac{8}{3}y=5$

7. $2x+\frac{y}{2}-5=0\,;\,\frac{x}{2}+y=-4$

8. $0\cdot 3x+0\cdot 4y=3\cdot 2\,;\,0\cdot 6x+0\cdot 8y=2\cdot 4$

9. $2x+3y=8\,;\,4x-\frac{3}{2}y=1$

10. $3x-y=2\,;\,6x-2y=4$

11. $3x+2y=0\,;\,2x+y=-1$

4.04. Algebraic Methods of Solving Simultaneous Linear Equations

Simultaneous equations is the pair of two linear equations in two variables. The values of both variables that satisfy both the equations are solutions of the simultaneous equations.

The solution of the given system of linear equations can be obtained by using following algebraic method.

(i) Method of elimination (by substitution)

(ii) Method of elimination (by equating the co-efficient)

(iii) Method of cross multiplication (General method)

(i) Method of Elimination (by Substitution) :

In this method, the value of a variable in an equation of simultaneous linear equation is expressed in the form of other variable of the equation. Now the value of the variable so obtained substituted in another equation of simultaneous linear equation. Consequently the second equation is equation in one variable. Solving this equation in one variable, we can easily find the value of the variable used in the equation. Then substituting this value in any of equation we obtain the value of other variable. The example given below will be helpful to understand this method.

Example 6. Solve the following system of equations by substuting method.

$$x+3y=11$$
$$4x-y=5$$

Solution : The given system of equations is :

$$x+3y=11 \qquad \ldots(1)$$
$$4x-y=5 \qquad \ldots(2)$$

From equation (1), we get

$$x=11-3y \qquad \ldots(3)$$

Substuting this value of x in (2), we get.

$$4(11-3y)-y=5$$

or $$44-12y-y=5$$
or $$44-13y=5$$
or $$13y=39$$
or $$y=\frac{39}{13}$$
$\therefore$ $$y=3$$

Putting $y=3$ in equation (1), we get :

$$x=11-3(3)$$
or $$x=11-9$$
or $$x=2$$

Hence, the solution of the given system of equations is $x=2,\ y=3$

(ii) Method of Elimilation (by Equating the Coefficient) :

In this method, one or both equations of the simultaneous equations are multiplied by such number so that the co-eficients of one variable in two equations may be equal. Now, according to the situation both the equations are added or subtracted, so that we can get an equation in one variable easily. Now this value is substituted in any of the two equations. In this way, the value of other variable is found.

Example 7. Solve the pair of equations using the method of elemination by equating the co-efficient.

$$4x+5y=31$$
$$7x-2y=22$$

Solution : The given system of equations is :

$$4x+5y=31 \qquad \ldots(1)$$
$$7x-2y=22 \qquad \ldots(2)$$

Let us eliminate y from the given equations. The coeficients of y in the given equations are 5 and 2 respectively. The LCM of 5 and 2 is 10. So, to make the coefficients of y equal, we multiply equation (1) by 2 and equation (2) by 5, we get

$$8x+10y=62 \qquad \ldots(3)$$
$$35x-10y=110 \qquad \ldots(4)$$

Adding the equations (3) and (4) we get :

$$43x=172 \quad \text{or} \quad x=\frac{172}{43}$$

$\therefore \qquad x = 4$

Substituting $x = 4$ in equation (1), we get

$$4(4) + 5y = 31 \quad \text{or} \quad 16 + 5y = 31$$

or $\quad 5y = 31 - 16 \quad \Rightarrow \quad y = \dfrac{15}{5}$

$\therefore \quad y = 3$

Hence, the solution of the given system of equations is $x = 4,\ y = 3$.

Using this method, we can solve such equations which are made of reciprocals of variables.

The method is so clear by the example given below.

Example 8. Solve the following equations :

$$\frac{20}{x} + \frac{2}{y} = 6, \ \frac{10}{x} - \frac{1}{y} = 2$$

Solution : Given , system of equations is :

$$\frac{20}{x} + \frac{2}{y} = 6 \qquad \ldots (1)$$

$$\frac{10}{x} - \frac{1}{y} = 2 \qquad \ldots (2)$$

Multiply equation (2) by 2, we get

$$\frac{20}{x} - \frac{2}{y} = 4 \qquad \ldots (3)$$

Adding equation (1) and (3), we get

$$\frac{40}{x} = 10 \quad \Rightarrow \quad x = \frac{40}{10}$$

or $\qquad x = 4$

Substituting $x = 4$ in equation (1), we get

$$\frac{20}{4} + \frac{2}{y} = 6 \quad \text{or} \quad 5 + \frac{2}{y} = 6$$

or $\quad \dfrac{2}{y} = 6 - 5 \quad$ or $\quad \dfrac{2}{y} = 1$

or $\quad y = 2$

Thus, the solution of the given system of equations is , $x = 4,\ y = 2$

Example 9. Find the solution of following system of equations :

$$5x + 6y = 3xy, \quad 10x + 9y = 5xy$$

Solution : The given system of equations :

$$5x + 6y = 3xy \qquad \text{. . . (1)}$$

$$10x + 9y = 5xy \qquad \text{. . . (2)}$$

Dividing the equation (1) and (2) both by xy, we have

$$\frac{5}{y} + \frac{6}{x} = 3 \qquad \text{. . . (3)}$$

$$\frac{10}{y} + \frac{9}{x} = 5 \qquad \text{. . . (4)}$$

Taking $\frac{1}{x} = m$ and $\frac{1}{y} = n$. The given system of equations become

$$5n + 6m = 3 \qquad \text{. . . (5)}$$

$$10n + 9m = 5 \qquad \text{. . . (6)}$$

Multiply equation (5) by 2, we get

$$10n + 12m = 6 \qquad \text{. . . (7)}$$

Substracting equation (6) from equation (7), we get

$$3m = 1 \quad \Rightarrow \quad m = \frac{1}{3}$$

Putting $m = \frac{1}{3}$ in equation (6), we get

$$10n + 9\left(\frac{1}{3}\right) = 5$$

or $\quad 10n + 3 = 5 \quad \Rightarrow \quad 10n = 5 - 3$

or $\quad 10n = 2 \quad \Rightarrow \quad n = \frac{2}{10}$

or $\quad n = \frac{1}{5}$

Now $m = \frac{1}{3} \Rightarrow \frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$

and $\quad n=\frac{1}{5} \Rightarrow \frac{1}{y}=\frac{1}{5} \Rightarrow y=5$

Thus, the solution of the equation is $x=3,\ y=5$

Exercise 4.2

Solve the following equations by the method of elimination (by substitution) :

1. $2x+3y=9$
   $3x+4y=5$
2. $x+2y=-1$
   $2x-3y=12$
3. $3x+2y=11$
   $2x+3y=4$
4. $8x+5y=9$
   $3x+2y=4$
5. $4x-5y=39$
   $2x-7y=51$
6. $5x-2y=19$
   $3x+y=18$

Solve the following equations by method of elemialtion by equating the coefficients :

7. $2x+y=13$
   $5x-3y=16$
8. $0.4x+0.3y=1.7$
   $0.7x-0.2y=0.8$
9. $\frac{x}{7}+\frac{y}{3}=5$
   $\frac{x}{2}-\frac{y}{9}=6$
10. $11x+15y=-23$
    $7x-2y=20$
11. $3x-7y+10=0$
    $y-2x=3$
12. $x+2y=\frac{3}{2}$
    $2x+y=\frac{3}{2}$

Solve the following equations :

13. $8v-3u=5uv$
    $6v-5u=-2uv$
14. $\frac{1}{2x}-\frac{1}{y}=-1$
    $\frac{1}{x}+\frac{1}{2y}=8$
15. $\frac{5}{(x+y)}-\frac{2}{(x-y)}=-1$
    $\frac{15}{(x+y)}+\frac{7}{(x-y)}=10$

Cross- Multiplication Method

Cross-multiplication is a general method to solve the simultaneous equations. Here we are going to clear this method.

Let the given system of equations be

$$a_1x + b_1y + c_1 = 0 \quad \ldots (1)$$

$$a_2x + b_2y + c_2 = 0 \quad \ldots (2)$$

Multiplying equation (1) by b_2 and (2) by b_1 respectively, we get

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \quad \ldots (3)$$

$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \quad \ldots (4)$$

Substracting equation (4) from equation (3), we get

$$\left(a_1b_2 - a_2b_1\right)x + b_2c_1 - b_1c_2 = 0$$

or $$\left(a_1b_2 - a_2b_1\right)x = b_1c_2 - b_2c_1$$

$\Rightarrow$ $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \ldots (5)$$

Similarly multiplying equation (1) by a_2 and equation (2) by a_1, we get

$$a_1b_2x + a_2b_1y + c_1a_2 = 0 \quad \ldots (6)$$

$$a_1a_2x + a_1b_2y + c_2a_1 = 0 \quad \ldots (7)$$

Substracting equation (7) form equation (6)

$$\left(a_2b_1 - a_1b_2\right)y + c_1a_2 - c_2a_1 = 0$$

or $$\left(a_2b_1 - a_1b_2\right)y = -c_1a_2 + c_2a_1$$

or $$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad \ldots (8)$$

Hence, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

The above solution of equation can be written as :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

This result is shown by the following diagram, so that we can remembering the solution of given equation easily.

$$\frac{x}{\begin{matrix} b_1 & & c_1 \\ & \times & \\ b_2 & & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & & a_1 \\ & \times & \\ c_2 & & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & & b_1 \\ & \times & \\ a_2 & & b_2 \end{matrix}}$$

In the above diagram, the direction of arrows shows the multiplication of the related numbers. First of all we multiply downwards and then subtract the multiplication upward from it.

In this method,the numbers are multiplied across, so it is called the cross multiplication method. In this system all the terms are taken to the left hand side and then right hand side become zero.

4.05. Condition for Solvability

In a pair of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, the solution of the equation depends on the ratio of corresponding coefficients.

1. First condition : if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.

2. Second condition : if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations is inconsistent, *i.e.*, it has no solution.

3. **Third condition :** if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of equations has infinitly many solutions.

Example 10. Solve the equations given below by cross multiplication method.

$$2x + 3y - 17 = 0 \ ; \ 3x - 2y - 6 = 0$$

Solution : By cross multiplication method, we get

$$\frac{x}{\begin{matrix} 3 & & -17 \\ & \times & \\ -2 & & -6 \end{matrix}} = \frac{y}{\begin{matrix} -17 & & 2 \\ & \times & \\ -6 & & 3 \end{matrix}} = \frac{1}{\begin{matrix} 2 & & 3 \\ & \times & \\ 3 & & -2 \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{(3)(-6)-(-2)(-17)} = \frac{y}{(-17)(3)-(-6)(2)} = \frac{1}{(2)(-2)-(3)(3)}$$

$$\Rightarrow \quad \frac{x}{-18-34} = \frac{y}{-51+12} = \frac{1}{-4-9}$$

$$\Rightarrow \quad \frac{x}{-52} = \frac{y}{-39} = \frac{1}{-13}$$

$$\Rightarrow \quad x = \frac{-52}{-13} \text{ and } y = \frac{-39}{-13}$$

$$\Rightarrow \quad x = 4 \text{ and } y = 3$$

Hence, the solution of the equations is $x = 4, \ y = 3$

Example 11. Check the consistency of the equations given below. If they are consistent, solve them.

$$2x + 3y = 7$$

$$6x + 9y = 15$$

Solution : The equations are

$$2x + 3y = 7$$

$$6x + 9y = 15$$

Taking all terms to the left side, we get

$$2x + 3y - 7 = 0$$

and $\quad 6x + 9y - 15 = 0$

here $\quad \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$

and $\quad \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$

also $\quad \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

Here, we see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of equations is inconsistent and there is no solution.

Exercise 4.3

Check the equations given below, if they have a unique solution, no solution or infinitely many solutions. Incase there is a unique solution, find it;

1. $\quad 2x + y = 35$ $\qquad$ 2. $\quad 2x - y = 6$

$3x+4y=65$	$x-y=2$
3. $3x+2y+25=0$	4. $x+2y+1=0$
$2x+y+10=0$	$2x-3y-12=0$

5. If the following system of equation has no solution, find the value of k.

   (i) $2x+ky=1$, $3x-5y=7$

   (ii) $kx+2y=5$, $3x+y=1$

6. Find the solution of system of equations $mx-ny=m^2+n^2$, $x+y=2m$

7. Find the value of λ if the system of equations $3x+\lambda y+1=0$, $2x+y-9=0$

   (i) has a unique solution (ii) has no solution.

4.06 Application of linear equations in two variables

With the help of a pair of simultaneous linear equations we can solve the practical problems related to our daily life. In solving such problems, we may use the following steps :

(i) We use the variable for unknown quantities involved in the given problem.

(ii) Convert the conditions which are given in the verble from of problem in equation form by using the variables.

(iii) On solving these equation by appropriate method. We get the value of variables.

Example 12. 10 students of a class participated in a essay competition. If the number of boys is 4 more than the number of girls, find the number of boys and the girls separately participated in the competition.

Solution : Let the number of boys participating in the competition be x and number of girls be y

Given that the total number of students participating in the competition is 10. So number of boys + number of girls = 10

$$\Rightarrow \qquad x+y=10$$

It is also given that the number of boys is 4 more than the number of girls

Number of boys − number of girls = 4

$$x-y=4$$

According to the situation we get a pair of equations as :

$$x+y=10 \qquad \ldots(1)$$

$$x-y=4 \qquad \ldots(2)$$

Adding equations (1) and (2) , we get

$$2x=14$$

or $\quad x = 7$

Putting the value of x in equation (1), we get

$$7 + y = 10$$

or $\quad y = 10 - 7$

or $\quad y = 3$

Hence, the number of boys are 7 and the number of girls are 3.

Example 13. The ratio of the salaries of two persons is 9 : 7 and the ratio of their expenditure is 4 : 3. If each of the two persons saves ₹ 2000 per month, find out their salaries.

Solution : Let the salary of first person be ₹ x and the salary of second person by ₹ y.

According to the problem $x : y = 9 : 7$

or $\quad \dfrac{x}{y} = \dfrac{9}{7} \quad$ or $\quad 7x = 9y$

or $\quad 7x - 9y = 0 \qquad \ldots(1)$

The two persons save ₹ 2000 per month separately.

Their monthly expenditure are ₹ $(x - 2000)$ and ₹ $(y - 2000)$ respectively.

The ratios of their expenditure is $4 : 3$

$\therefore \quad (x - 2000) : (y - 2000) = 4 : 3$

$$\frac{(x - 2000)}{(y - 2000)} = \frac{4}{3}$$

or $\quad 3(x - 2000) = 4(y - 2000)$

or $\quad 3x - 6000 = 4y - 8000$

or $\quad 3x - 4y + 2000 = 0 \qquad \ldots(2)$

From equation (1), we get $x = \dfrac{9y}{7}$

Susbtituting $x = \dfrac{9y}{7}$ in equation (2), we get

$$3\left(\frac{9y}{7}\right) - 4y + 2000 = 0$$

or $\dfrac{27y}{7} - 4y + 2000 = 0$

or $27y - 28y + 14000 = 0$ or $-y = -14000$

or $y = 14000$

Putting the value of y in equation (1), we get

$$7x - 9(14000) = 0$$

or $7x - 126000 = 0$

or $7x = 126000$ or $x = \dfrac{126000}{7}$

or $x = 18000$

Hence the salaries of two persons are ₹ 18000 and ₹ 14000 respectively.

Example 14. The sum of the digits of a two digit number is 12. If 18 is subtracted form the number, the place of two digits is interchanged. Find digits the number.

Solution : Let the one's digit and ten's digits of the number of x and y respectively.

So the number is $x + 10y$

According to the problem, the sum of the digits of this number is 12. So

$$x + y = 12 \qquad \ldots (1)$$

and $x + 10y - 18 = 10x + y$ or $10y - y + x - 10x = 18$

or $9y - 9x = 18$ or $9x - 9y = -18$

or $x - y = -2 \qquad \ldots (2)$

Adding equation (1) and (2), we have

$$2x = 10 \text{ or } x = 5$$

Putting the value of x in equation (1), we get

$$5 + y = 12$$

or $y = 12 - 5$

or $y = 7$

Thus, the number $10y + x = 10 \times 7 + 5 = 75$

Exercise 4.4

Solve the problems given below :

1. In a two-digit number, the one's digit is 3 times the ten's digit. If 10 is added to the 2 times of the number, its digits interchange their places in the new number. Find the number.
2. The perimeter of a rectangle is 56 cm. The ratio of length and its breadth is 4 : 3. Find the length and breadth of the rectangle.
3. The ratio of two numbers is 3 : 4. If 5 is substracted from each of the number, the ratio becomes 5 : 7. Find the numbers.
4. The age of a father is 5 years more than the age of 6 times of his son's age. After 7 years the age of father will be 3 more than the age of 3 times of his son's age. Find their present ages.
5. Ram said to Shyam, "If you give me ₹ 100, my money will be doubled to your money." Then Shyam said to Ram, "If you give me ₹ 10, my money will be 6 times to yours. Find how many rupees does each of them have?
6. The cost of 4 chairs and 3 tables is ₹ 2100 and the cost of 5 chairs and 2 tables is ₹ 1750. Find the cost of one chair and one table separately.
7. If 3 times of a larger number is divided by a smaller number the quotient is 4 and remainder is 3 and when 7 times of the smaller number is divided by the larger number, then quotient is 5 and remainder is 1. Find the two numbers.
8. A two-digit number is 4 times the sum of its digits and 2 times the product of it's digits. Find the number.
9. If one is added to the numerator and the denominator separately of a fraction, then the facrction becomes $\frac{4}{5}$ and if 5 is substracted form both numerator and denominator, then the fraction becomes $\frac{1}{2}$. Find the fraction.
10. 5 years ago Geeta's age was 3 times of Kamla's age. After 10 years Geeta's age will be 2 times of Kamla's age. Find their present ages.
11. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, he takes 4 hours. But,if he travels 130 km by train and the rest by car, he takes 18 minutes longer, Find the speed of the train and car.

Important Points

1. A linear equation in two variables is in the form of $ax+by+c=0$, where a, b, and c are real numbers and $a \neq 0, b \neq 0$.
2. A linear equation in two variables has infinitely many solutions.
3. The nature of a pair of two linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given as :

   (a) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are consistent and has a unique solution.

   (b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent with infinitely many solutions.

   (c) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equation is inconsistent and have no solution.
4. If x and y are digits at one's and ten's places respectively in a number, then the number will be $10y+x$.

Miscellaneous Exercise-4

Choose the correct answer (Questions 1 to 10)

1. Find the value of x, if $y = 2x - 3$ and $y = 5$.

   (a) 1 (b) 2 (c) 3 (d) 4

2. The pair of solution that satisfy the equation $2x + y = 6$ is :

   (a) (1, 2) (b) (2, 1) (c) (2, 2) (d) (1, 1)

3. If $\frac{4}{x} + 5y = 7$ and $x = -\frac{4}{3}$, then the value of y is :

   (a) $\frac{37}{15}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

4. If $\frac{3}{x} + 4y = 5$ and $y = 1$, then the value of x is :

   (a) 3 (b) $\frac{1}{3}$ (c) -3 (d) $-\frac{1}{3}$

5. If $x = 1$, then find the value of y in the equation $\frac{4}{x} + \frac{3}{y} = 5$:

   (a) 1 (b) $\frac{1}{3}$ (c) 3 (d) -3

6. If one's digit and ten's digit of a number are x and y respectively, then the number will be :

   (a) $10x+y$ (b) $10y+x$ (c) $x+y$ (d) xy

7. The son's age is one third of his mother's age. If the present age of the mother is x years, then after 12 years son's age will be :

   (a) $\frac{x}{3}+12$ (b) $\frac{x+12}{3}$ (c) $x+4$ (d) $\frac{x}{3}-12$

8. The coordinate of a point at the x- axis is :

(a) $(2,3)$ (b) $(2,0)$ (c) $(0,2)$ (d) $(2,2)$

9. The coordinates of origin are :

(a) $(0,0)$ (b) $(0,1)$ (c) $(1,0)$ (d) $(1,1)$

10. In which quadrant does the point $(3,-4)$ lie ?

(a) First (b) Second (c) Third (d) Fourth

11. In equation $5y-3x-10=0$, express y in terms of x , Find the point on the graph. Where the line represented by $5y-3x-10=0$ cuts y-axis.

12. Taking the values of x form -2 to 2 prepare a table of the equation $y=2x+1$. Also draw the graph of the equation.

13. Solve the simultaneous equations given below :

$0\cdot 5x+0\cdot 6y=2\cdot 3;\ 0\cdot 2x+0\cdot 7y=2\cdot 3$

14. Solve the system of the equations :

$$2x+3y=9\,;\,3x+4y=5$$

15. Solve the simultaneous equations

$$\frac{1}{2x}-\frac{1}{y}=-1\,;\,\frac{1}{x}+\frac{1}{2y}=8\,;\ \text{where } x\neq 0,\, y\neq 0$$

16. There are two numbers such that if 7 is added to the smaller number then sum becomes doubel to the larger number and if 4 is added to the larger number it becomes three times of the smaller one. Find the two numbers.

17. Numerator of a fraction is 4 less than it's denomilator. If 2 is subtracted form the numerator and 1 is added to the denominator, then the denomilator becomes 8 times of the numerator. Find the fraction.

18. The cost of 5 books and 7 pens is ₹ 79 and the cost of 7 books and 5 pens is ₹ 77. Find the cost of 1 book and 2 pens.

19. When a two digit number is multipled by 9, it becomes 2 times of the number that is obtained by interchanging the digits. If the difference of the digits is 7, find the number.

20. In a triangle ABC, $\angle A=x^\circ, \angle B=3x^\circ$ and $\angle C=y^\circ$. Now, if $5x^\circ-3y^\circ+30^\circ=0$, prove that given triangle is a right angled triangle.

21. Solve the following equations graphically :

(a) $x+y=4\,;\ x=y$ (b) $x+y=3\,;\ 2x+5y=12$

(c) $2x-3y-6=0\,;\ 2x+y+10=0$ (d) $2x+y-3=0\,;\ 2x-3y-7=0$

22. Solve the system of equations $2x-y=1; x+2y=8$ graphically and find the co-ordinates of the points where corresponding lines intersect y-axis.

Answer

Exercise 4.1

(1) $x = 6,\ y = 0$ (2) $x = 1,\ y = 4$ (3) No solution (4) $x = 3,\ y = 1$

(5) $x = -2,\ y = 3$ (6) inconsistent, no solution (7) $(4,\ -6)$

(8) inconsistent, no solution (9) $(1,\ 2)$

(10) coincide, infinitely many solutions (11) $(-2,\ 3)$

Exercise 4.2

(1) $x = -21,\ y = 17$ (2) $x = 3,\ y = -2$ (3) $x = 5,\ y = -2$ (4) $x = -2,\ y = 5$

(5) $x = 1,\ y = -7$ (6) $x = 5,\ y = 3$ (7) $x = 5,\ y = 3$ (8) $x = 2,\ y = 3$

(9) $x = 14,\ y = 9$ (10) $x = 2,\ y = -3$ (11) $x = -1,\ y = 1$ (12) $x = \frac{1}{2},\ y = \frac{1}{2}$

(13) $u = \frac{22}{31},\ v = \frac{11}{23}$ (14) $x = \frac{1}{6},\ y = \frac{1}{4}$ (15) $x = 3,\ y = 2$

Exercise 4.3

(1) $x = 15,\ y = 5$ (2) $x = 4,\ y = 2$ (3) $x = 5,\ y = -20$ (4) $x = 3,\ y = -2$

(5) (i) $k = -\frac{10}{3}$; (ii) $k = 6$ (6) $x = (m+n),\ y = m - n$

(7) (i) For unique solution $\lambda \neq \frac{3}{2}$ (ii) For no solution $\lambda = \frac{3}{2}$

Exercise 4.4

(1) 26 (2) length 16 cm, width 12 cm (3) 30 and 40

(4) age of father is 29 years and age of his son is 4 years.

(5) Ram's money is ₹ 40 and Shyam's money is ₹ 170.

(6) Cost of chair ₹ 150 and table is ₹ 500

(7) The larger number is 25 and smaller number is 18. (8) 36 (9) 7 / 9

(10) the age of Geeta is 50 years, age of Kamla is 20 years.

(11) Train 100 km/h and car 80 km/h.

Miscellaneous Exercise 4

1. (D) 2. (C) 3. (B) 4. (A) 5. (C) 6. (A) 7. (A)

8. (B) 9. (A) 10. (D) 11. $y = \frac{3x+10}{5}$, $(0, 2)$

12.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

13. $x = 1, y = 3$ 14. $x = -21, y = 17$

15. $x = \frac{1}{6}, y = \frac{1}{4}$ 16. 5,3 17. $\frac{3}{7}$ 18. ₹ 20 19. 18

21. (a) $x = 2, y = 2$ (b) $x = 1, y = 2$ (c) $x = -3, y = -4$ (d) $x = 2, y = -1$

22. $x = 2, y = 3$ first line meets the y-axis at point (0, -1) and second line meets the y-axis at point (0, 4).

5

Introduction to Plane Geometry, Line and Angle

5.01 Historical Introduction

Harappa and Mohan-jo-dro (now in Pakistan), Kalibanga (Rajasthan) and Lothal (Gujrat) testify clearly that a rich civilization flourished in a large track of land during the period extending from 2500 B.C. to 1750 B.C. in ancient India. The relics of this civilization prove that the people of this period had special knowledge of Geometry and Geometrical formations. On the basis of this knowledge they constructed buildings, roads, circles, arcs where mensuration is of great importance. Babylonian has also fomulated formulae for finding out the area of various linear shapes which are available in Rhind Papyrus (1650 B.C.)

5.02 History of Indian Geometry

India has been the birth place of Geometry too. The foundation of Geometry had been laid in the shulav age or in the age of Vedang and Astronomy. During this period it was known by various names, such as Shulav mathematics, Shulav Science, Rajju Mathematics, Rajju-Figures. Rajju being synonym of shulav it began to be called Rajju Mathematics which later on changed to Geometry.

Similarly such terms as Rajju Mathematics, Kshetra Samas, Kshetra behaviour, Mensuration, Geometry and Boomiti were used for the work of measuring fields. Since ancient times altras for the performance of yagya used to be built. Geometry was the basis of their construction.

It is said in astronomy that :-

"Veda hi yajanartham bhi pravaritah"

That is , the Vedas also have been used in the work of performing yagya. According to the need of various yajanans (yagya). Different types of altars were required to be built. For this, various shape such as rectangular arcs, rectangular, triangular etc. were developed while constructing an altar care was taken that the areas of all they altars must be equal to the area of standard altar. Hence for this knowledge of geometrical formation such as forming a square on straight line, converting it into circle equal to the area of the square drawing a circle around the square, drawing a circle within the square and doubling the area of the circle etc. was very essential.

If we think a bit seriously, we come to know that two words are very important in Geometry. They are Rajju (rope) and measurement. Hence the science or mathematics which was developed with the help of Shulav, began to be called Shulav science and Shulav mathematics. Indian mathematicians contributed a lot in this field. They formulated formulae for forming various shapes which were known by their names such as Baudhayan Shulav Formula. Apastamb Shulav formula, Katyayan Shulav formula, Manav Shulav formula, Mestrayan Shulav formula, Varah Shulav formula, Baudhav Shulv formula etc.

Among the main achievements of Shulav period "Sumkon Tribhuz ka Prameya" (theorem of right angle tringle), that is a square formed on hypotenuse is equel to the sum of the squares formed on remaining two arms. This theorem was widely used in India many centuries before Pythagoras (580 B.C.).

Baudhayan Therem (800 B.C.)

"Deergh chatur srisya khasnya Rajjuh Parshavmani

Tiryankmani yatprithambhute kurutastadubhayam karoti"

The sum of the areas of squares formed on the perpendicular line and the base line of a rectangle is equal to the area of the square formed on the diagonal. It is worth noting that Pythagoras (580 B.C.) established this theorem about 300 years after 'Baudhayan'. Hence it is proper to call this theorem as Baudhayan theorem.

Among Indian Geometricians are Bhraham Gupta (598 A.D.) who found out the area of cyclic quadrilateral in terms of its perimeter and rams, Arya Bhat (476 A.D.) who found out the area of equilateral tringle, volume of pyramid and value of π and Bhaskar-II (1114 A.D.) who proved the Baudhayan theorem, by split method.

Later on the Greek mathematicians (300 B.C.) systematized this knowledge by providing its facts through inductive reasoning and published it in the book titled 'Elements". These days we study Geometry in this way.

5.03 Basic Concepts

Geometry is studied by taking some basic concepts as basis. These basic concepts are understood by examples and experiences. For these, no proofs are given. There are three basic concepts in the study of Geometry which are very important. These are (1) Point (2) Line and (3) Surface. Now we shall study these with the help of some examples.

(1) Point : A minute sign made by a fine pointed pencil, the corner of the black board are the examples of a point. If the point of the pencil is fine, it will make a fine point. Generally the points are shown by the capital letter of English alphabet that is A, B, C, D etc.

(2) Line : If we fold a piece 3 of paper, a line is formed on the surface of the piece of paper. Line is also regarded as a concepts in the same way as a point is regarded. In the figure 5.01, the line AB is shown by AB. The line can also be shown by small letters of English alphabet such as l, m, n.

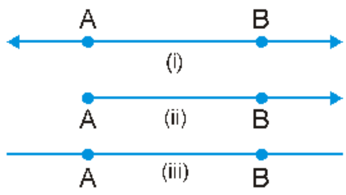


Fig. 5.01

(3) Plane : Floor, roof, wall, table are similar examples of plane. Although there are some dimensions of these examples, yet the geometrical planes extend endlessly in all the directions. A plane is extended in length and width. is has no thickness.

(4) Postulates : In Geometry there are some concepts which are regrarded as true without any proof and on the basis of which other geometrical facts are proved. such facts are called 'Postulates'. Some of main postulates are-

(i) There are infinite points on a line.

(ii) A line can be extended as much as we desire.

(iii) Infinite number of lines can be drawn from a point.

(iv) One and only one straight line can be drawn through two points.

(v) One and only one line can be drawn from a point paralled to a line.

(vi) All the right angles are equal.

(vii) Like supplementary and complementary angles are equal to each other.

(viii) A line segment can be bisected at one point only.

(ix) An angle can be bisected by one line only.

5.04 Inductive and Deductive Reasoning

Various rules which are established by various examples. Empirical findings are called inductive reasoning. Such findings are always true.

A special method of reasoning wherein the rules are proved with the help of evidences is known as deductive reasoning.

Theorem (Prameya) and Construction (Nirmeya) :

(1) Theorem : The rules which are verified by the inductive reasoning are called theorem. In geometry following steps are adopted in proving a theorem.

Corollary : On proving the theorem some results are drawn, which are understood easily. Such results are called “Corollary”.

Constructions : The geometrical forms formed by using geometric rules are called 'Construction'.

5.05 Geometric Symbols

The terms used in geometry are written in he form of symbols. Symbols of some words are given in the following table :

S.No.	Words	Symbol	S.No.	Words	Symbol
1.	Because	$\because$	8.	Right angle	∟
2.	Therefore	$\therefore$	9.	Perpendicular	$\perp$
3.	Greaterthan	$>$	10.	Triangle	$\triangle$
4.	Lessthan	$<$	11.	Parallel	$\parallel$
5.	Congruent	$\cong$	12.	Circle	$\bigcirc$
6.	Similar	$\sim$	13.	Arc	$\frown$
7.	Angle	$\angle$	14.	Not equal to	$\neq$

5.06 Angle and its Measurements

Angle : Any two rays whose starting point is same make an angle. In fig 5.02, starting from O and two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are starting. In this figure angle at point O is known as angle. According to figure, one ray is at point A and second ray is at point B, then this angle can be expressed as $\angle AOB$ or $\angle BOA$,OA and OB are sides of or Common point O is known as vertex of angle. Sometimes for convenience an angle is denoted by numbers or words as given is fig. 5.03 and 5.04.

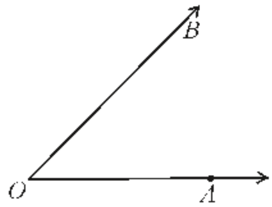


Fig. 5.02

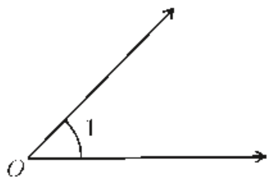


Fig. 5.03

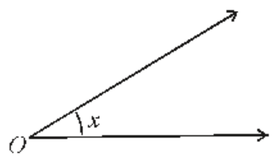


Fig. 5.04

Measurement of Angle : Suppose a revolving line with respect to point O is in OA position reaches in OB position as shown in fig. 5.05, then this revolving position is known as $\angle AOB$ and quantity of this angle is known as its measurement.

If a line OA by completing a round around O and comes in its original position, then measurement of such angle is distrubuted is 360 equal parts and it is denoted by 360^0 (degree) so similiarly 1 Part =1 degree=1^0 and 360 part = 360^0.

In rotation form 1 degree; 1 minute and 1 seconds represents 1^0, 1 and 1" respectively.

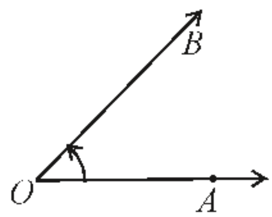


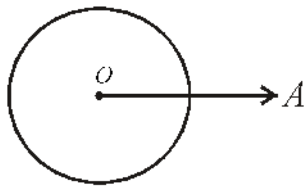
Fig. 5.05

If every degree is divided into 60 equal parts, then such each part is known as one kala (1 minute), if one minute is divided into 60 equal parts, then each part is known as vikala (second).

$1^0 = 60$ (minutes) $= 60$ kala

1 kala (minuts) $= 60$ vikala

In symbolic form of one degree, one minute and one second can be expressed 1^0, $1'$ and $1''$ For measurement of angle use of a protractor, which have 0^0 to 180^0 scale.



(1 round $= 360°$)

Fig. 5.06

Vertically Opposite Angles

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays. In Fig. 5.07 lines AB and CD intersects each other at point O and makes angle $\angle AOC$ and $\angle BOD$, $\angle AOD$, $\angle BOC$ which are vertically opposite angle.

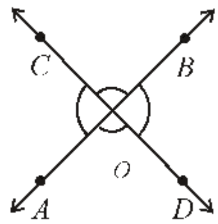


Fig. 5.07

Angle around a point

The sides of vertically angles are opposite rays.

Angle on around of a point : if various rays are starting from a point and angles obtained in this manner are know as angles around a point. As we studied that in this measurement an angle made around a point by revolving ray is $360°$.

Here, sum of angle around point O is $360°$.

or, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

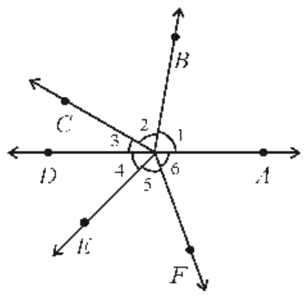


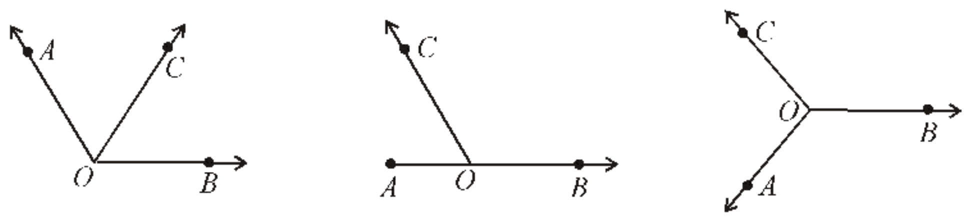
Fig. 5.08

5.07 A linear Pair of Angles

In Fig. 5.09, carefully observe the pair of angles.

Every pair of angles ($\angle AOC$ and $\angle BOC$) are adjacent angles.

In Fig. 5.09 (ii), a pair of angle is such that their total is 180^0 such angle is known as linear angle combination. It is clear that linear angle combination of adjacent angles are supplementary.



(i) (ii) (iii)

Fig. 5.09

Defination :

If the non-common arms of two adjacent angles form a line, then these angles are called linear pair of angles.

Linear Pair Axiom

Theorem 5.1 : If two lines intersect each other, then the vertically opposite angles are equal to each other.

Given : Lines AB and CD intersect each other at point O.

To Prove : Vertically opposite angles

$\angle AOC = \angle DOB$ and $\angle AOD = \angle BOC$

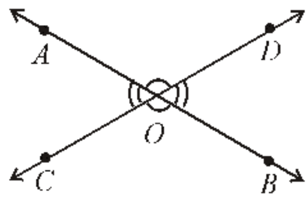


Fig. 5.10

Proof:

$\because$ Ray OD stands on line AB.

$\therefore \quad \angle AOD + \angle DOB = 180°$ (By linear pair Axiom) ....(i)

Similiary, $\quad \angle AOC + \angle AOD = 180°$ ......(ii)

From equation (i) and (ii), we get

$$\angle AOD + \angle DOB = \angle AOC + \angle AOD$$

$$\Rightarrow \quad \angle AOC = \angle DOB$$

Similarly, we can prove that :

$$\angle AOD = \angle BOC$$

Hence proved.

Cordallary 1 : If two or more than two lines intersect each other on a point, then the sum of the angles on intersecting point is $360°$.

Cordallary 2 : Bisector of vertically opposite angles are in a line.

Illustrative Examples

Example 1. In Fig. 5.11, $\angle 1$ and $\angle 2$ are linear pair $\angle 2 - \angle 1 = 18°$. If then find out the value of $\angle 1$ and $\angle 2$.

Sol. : $\angle 2 + \angle 1 = 180°$ (Linear pair axiom) ....(1)

Given, $\quad \angle 2 - \angle 1 = 18°$ ....(2)

On adding equation (1) and (2), we get

$$\angle 2 = \frac{198}{2} = 99° \quad \text{...........(3)}$$

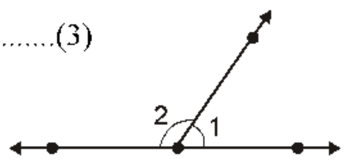


Fig. 5.11

Substitution $\angle 2 = 99°$ in equation (1), we get

$$99° + \angle 1 = 180°$$

$$\angle 1 = 180° - 99° = 81°$$

Thus, $\angle 1 = 81^\circ$ and $\angle 2 = 99^\circ$

Example 2. In Fig. 5.12, find the value of $\angle AOB, \angle BOC, \angle COD$ and $\angle DOE$ where $\angle AOE = 100^\circ$.

Sol : We know that the sum of the angles at a point is 360^0

$\therefore \quad y + 2y + 4y + 6y + 100^\circ = 360^\circ$

$\Rightarrow \quad 13y = 260^\circ$

$\Rightarrow \quad y = \dfrac{260}{13} = 20^\circ$

$\angle AOB = y = 20^\circ,$

$\angle BOC = 2y = 40^\circ,$

$\angle COD = 4y = 80^\circ$

$\angle DOE = 6y = 120^\circ$

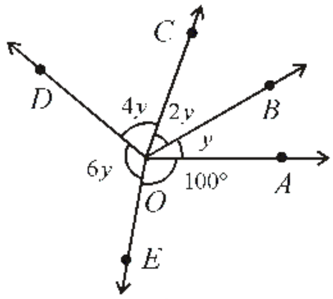


Fig. 5.12

Example 3. In Fig. 5.13, find the value of $\angle x, \angle y$ and $\angle z$.

Sol : Here, $\angle y = 140^\circ$ (Vertically opposite angles)

and $\angle x + 140^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \quad \angle x = 40^\circ$

Also $\quad \angle x = \angle z$ (Verticalliy opposite angle)

$\Rightarrow \quad \angle z = 40^\circ$

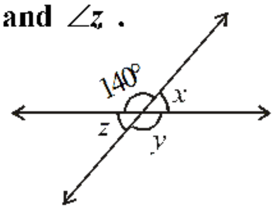


Fig. 5.13

Example 3. In Fig. 5.14, line AB and line OP meet at point O. line OD and OE are the bisectors of angle $\angle BOP$ and $\angle POA$, then find the neeasure of $\angle EOD$.

Sol : Let $\angle BOP = x$, and $\angle POA = y$

From figure,

$\angle x + \angle y = 180^\circ$ (Linear pair) .....(i)

From figure

$\angle x = 2\angle 1, \angle y = 2\angle 2$ ....(ii)

$\therefore$ From equation (i) and (ii), we get

$2\angle 1 + 2\angle 2 = 180^\circ$ (Linear pair)

$\Rightarrow \quad 2(\angle 1 + \angle 2) = 180^\circ$

$\Rightarrow \quad \angle 1 + \angle 2 = 90^\circ$

Thus $\quad \angle EOD = 90^\circ$

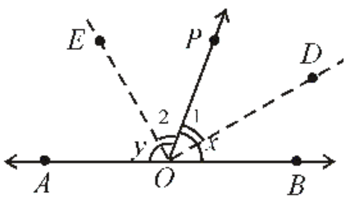


Fig. 5.14

Example 5. An angle is half of its supplementary angle, then find out the value of each angle.

Sol : Let one of the angle is x, then the value of its supplement is angle = $\dfrac{(180 - x)}{2}$

We know that sum of supplimentry angles is 180^0.

So $\quad x = \dfrac{180 - x}{2}$

$2x = 180 - x$

$3x = 180^0$

$\therefore \quad x = 60^0$

Hence, the angles are 120° and 60° .

Exercise 5.1

1. If angles $(2x+4)$ and $(x-1)$ form a linear pair, then find out the measure of the angles.

2. In Fig. 5.15 :

(i) Find the measure of $\angle BOD$

(ii) Find the measure of $\angle AOD$

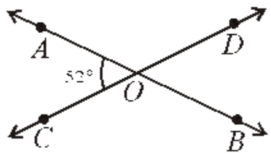


Fig. 5.15

(iii) Which pair of angles have vertically opposite angles?

(iv) Find the adjacent supplementary angles of $\angle AOC$.

3. In the given figure 5.16. if $\angle PQR = \angle PRQ$, then prove $\angle PQS = \angle PRT$.

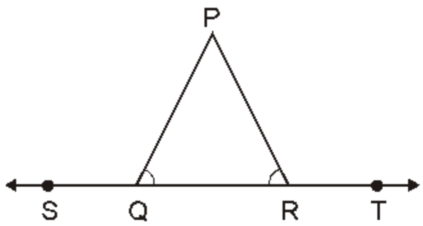


Fig. 5.16

4. In fig. 5.17, OP, OQ OR and OS are four rays, then prove that :

$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$

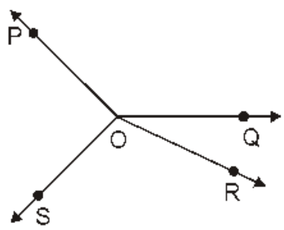


Fig. 5.17

5. In Fig. 5.18 $\angle x + \angle y = \angle p + \angle q$, then prove that AOB is a straight line.

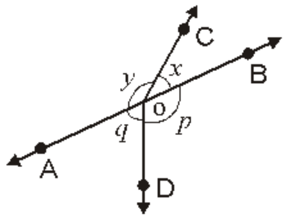
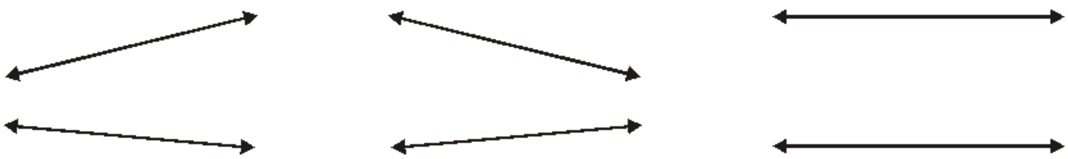


Fig. 5.18

5.08 Intersecting lines and Parallel Lines

If you are asked to draw pairs of two straight lines, then lines will be definately shown as in Fig. 5.19.



(i) (ii) (iii)

Fig. 5.19

Now, measure the distance between the lines in two different places. What do you observe?

No doubt you will see that in Fig. 5.19 (i) and (ii), the distance between the pair of lines is not equal at every point, i.e. if we extend these lines in forward or backward direction, then these lines will interest each other. Hence, these lines are intersecting lines. In Fig. 5.19 (iii), the difference between the lines is equal at every point, if we extend the lines in forward or backward direction up to infinite, then they will not intersect each other, Hence, there are parallel lines.

Transversal : If the group of two or more lines are insected by a line at different points, then it is known as transversal as shown in Fig. 5.20, line l, ntersect lines m and n at points P and Q respectively . So line l is a transversal for lines m and n .Do you observe that four angles at points P and Q. Yes, at point P, four angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ and at point Q four angles $\angle 5$, $\angle 6$, $\angle 7$, $\angle 8$ are formed.

Here $\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$ are exterior angles and $\angle 2$, $\angle 3$, $\angle 5$ and $\angle 8$ are interior angles.

Remember you have studied the numbering of angles made by a transversal. Let us remind them again.

(a) Corresponding angle :

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$

(iii) $\angle 3$ and $\angle 7$ (iv) $\angle 4$ and $\angle 8$

(b) Alternate interior angles :

(i) $\angle 2$ and $\angle 8$ (ii) $\angle 3$ and $\angle 5$

(c) Alternate exterior angles:

(i) $\angle 1$ and $\angle 7$ (ii) $\angle 4$ and $\angle 6$

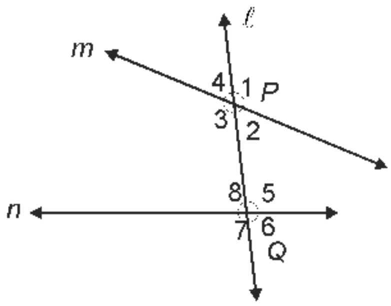


Fig. 5.20

(d) Interior angles on the same side of the transversal:

(i) $\angle 2$ and $\angle 5$ (ii) $\angle 3$ and $\angle 8$

When a transversal line intersect two or more parallel lines, then :

(i) Corresponding angles are equal

(ii) Alternate angles are equal

(iii) Interior angles are on the same side of a transversal line are supplementary and Opposite all of the statements are also true. If a transversal line cuts two lines and corresponding angles are same then the lines are perallel.

Theorem 5.2. ***If a transversal intersect two or more parallel lines, then alternate interior angles are equal to each other.***

Given : *l* and *m* are two parallel lines and a transversal intersect them at points *G* and *H*. $(\angle 2, \angle 1)$ and $(\angle 3, \angle 4)$ are pair of alternate angles.

To Prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof

$\angle 2 = \angle 6$ (Vertically opposite angles) ... (i)

$\angle 1 = \angle 6$ (Corresponding angles) ... (ii)

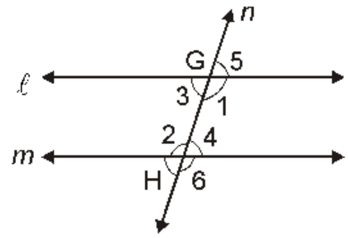


Fig. 5.21

From equation (1) and (2), we get

$\Rightarrow$ $\angle 1 = \angle 2$

Similarly $\angle 4 = \angle 5$ (Corresponding angles) ... (iii)

and $\angle 3 = \angle 5$ (Vertically opposite angles) ... (iv)

From equation (iii) and (iv) , we get

$\Rightarrow$ $\angle 3 = \angle 4$ **Hence proved**

Throrem 5.3 **(Converse of Theorm 5.2)**

If a transversal intersects two lines and alternate interior angles are equal, then the lines are paralles to each other.

Given : l and m are two lines which is interected by a transversal n at points G and H, then alternate angles are $\angle 2 = \angle 8$ and $\angle 3 = \angle 5$.

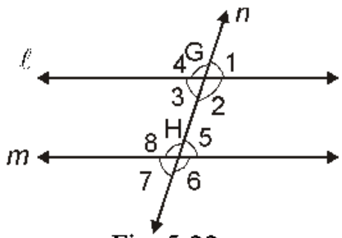


Fig. 5.22

To Prove : $\ell \parallel m$

Proof :

$\angle 1 = \angle 3$ (Alternate interior angles) ....(1)

$\angle 3 = \angle 5$ (Given) ....(2)

From equation (1) and (2), we get $\angle 1 = \angle 5$ (But these are corresponding angles)

So, $\ell \parallel m$ **Hence Proved.**

Theorem 5.4. ***If a transversal intersect two parallel lines, then the sum interior angles is equal to two right angles or 180°.***

Given : ℓ and m two parallel lines which are intersected by a transversal n at point G and H and angles at G and H are $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ are formed.

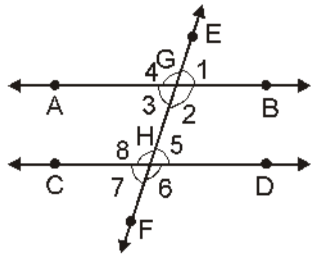


Fig. 5.23

To Prove : $\angle 2 + \angle 5 = 180°$ and $\angle 3 + \angle 8 = 180°$

Proof $\angle 1 + \angle 2 = 180°$ (Linear pair) ...(i)

$\angle 1 = \angle 5$ (Corresponding angles) ... (ii)

From equation (i) and (ii), we get

$\angle 2 + \angle 5 = 180°$

Similarly, $\angle 3 + \angle 4 = 180°$ (linear pair) ... (iii)

and $\angle 4 = \angle 8$ (Corresponding angles) ... (iv)

From equations (iii) and (iv), we get

$\angle 3 + \angle 8 = 180°$ **Hence proved**

Theorem 5.5 (Converse of Theorem 5.4)

If a transversal intersect two lines, and the sum of angles of the same side of the transversal is two right angle, then lines are parallel.

Given : ℓ and m are two lines which is intersected by a transversal n at points G and H, and formed angles $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ then alternate angles are such that : $\angle 2 + \angle 5 = 180°$ and $\angle 3 + \angle 8 = 180°$

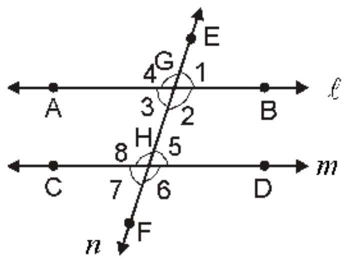


Fig. 5.24

To prove : $\ell \parallel m$

Proof :

$\angle 1 + \angle 2 = 180°$ (Linear pair axiom) ...(1)

$\angle 2 + \angle 5 = 180°$ (Given) ...(2)

From equation (1) and (2), we get

$\angle 1 = \angle 5$

But these are corresponding angles.

Thus, $\ell \parallel m$

Theorem 5.6 : ***If two lines are parallel to third line, then all three lines will be parallel to each other,i.e., if $\ell \parallel n$ and $m \parallel n$, then $\ell \parallel m$***

To prove : $\ell \parallel m$

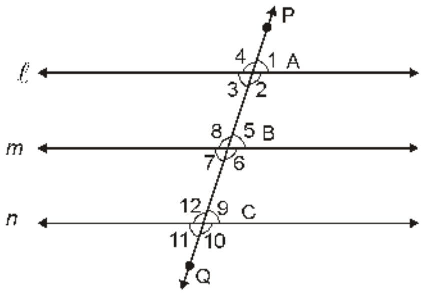


Fig. 5.25

Proof : Since, $l \parallel n$

$\Rightarrow$ $\angle 1 = \angle 9$

(Corresponding angles) . . . (1)

$m \parallel n$ and PQ is transversal

$\angle 5 = \angle 9$

(Corresponding angles) . . . (2)

From equations (1) and (2)

$\angle 1 = \angle 5$ . . . (3)

Since, $\angle 1$ and $\angle 5$ are corresponding angles, therefore by corresponding angles property $\ell \parallel m$.

Hence Proved

Example 6. In Fig. 5.26 lines $\ell \parallel m$ and a transverasal n is intersects them. If $\angle 1 = 55°$, then find out the remaining angles.

Sol : Since, $\angle 3 = \angle 1$ (Vertically opposite angles)

$\angle 1 = 55°$ (Given)

Therefore, $\angle 3 = 55°$

$\angle 1 + \angle 4 = 180°$ (Linear Pair axiom)

$\Rightarrow$ $55° + \angle 4 = 180° \Rightarrow \angle 4 = 125°$

Since, $\angle 2 = \angle 4$ (Vertically opposite angles)

$\Rightarrow$ $\angle 2 = 125°$

Since, $\angle 1 = \angle 6$ (Corresponding angles)

$\angle 1 = 55°$

Since, $\angle 4 = \angle 8$ (Corresponding angles)

$\angle 4 = 125°$

Since, $\angle 7 = \angle 6$ (Vertically opposite angles)

$\Rightarrow$ $\angle 6 = \angle 7 = 55°$

Since, $\angle 5 = \angle 8$ (Corresponding angles)

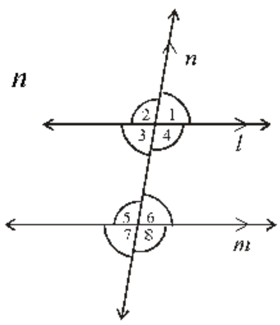


Fig. 5.26

$\Rightarrow \qquad \angle 6 = 55^\circ$

$\angle 5 = \angle 8$

and $\qquad \angle 8 = 125^\circ \qquad \therefore \qquad \angle 5 = 125^\circ$

Example 7. In Fig. 5.27, $\ell \parallel m$, then find out the equal angles and also write the reason.

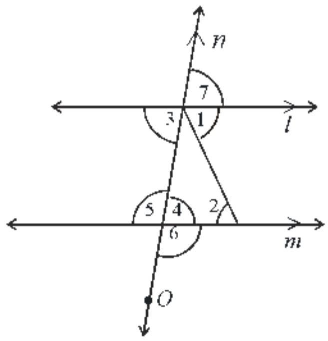


Fig.5.27

Sol : Here $\angle 1 = \angle 2$ (Alternate angle)

$\angle 3 = \angle 4$ (Alternate angle)

$\angle 5 = \angle 6$ (Vertically opposite angle)

$\angle 3 = \angle 7$ (Vertically opposite angle)

$\angle 4 = \angle 7$ (Corresponding angle)

Example 8. In Fig. 5.28, *m* and *n* two plane mirror which are perpendicular to each other. Show that incident ray *CA* is parallel to reflected ray *BD*.

Sol : Let us consider that perpendicular of *A* and *B* meet at point *P* because both the glasses are perpendicular.

$\therefore \qquad BP \parallel OA$ and $AP \parallel OB$

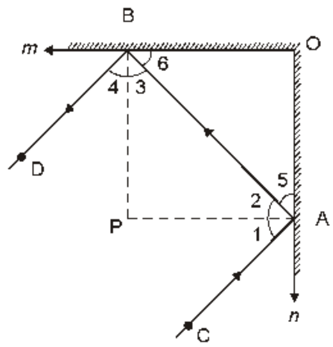


Fig.5.28

$\because \quad BP \parallel OA$ and BA is a transveral.

$\therefore \qquad \angle 3 = \angle 5$ (Alternate angles) ...(1)

and $PA \perp OA \Rightarrow \angle PAO = 90^\circ$

$\angle PAO = \angle 2 + \angle 5 = 90^\circ$ ...(2)

From equation (1) and (2), we get

$\angle 2 + \angle 3 = 90^\circ$ ...(3)

$\because \quad \angle 1 = \angle 2$ and $\angle 4 = \angle 3$ (angle of Incidence = angle Reflection) ...(4)

From equation (3) and (4), we get

$\angle 1 + \angle 4 = 90^\circ$ ...(5)

Adding equations (3) and (5), we get

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$\Rightarrow \angle CAB + \angle DBA = 180^\circ$ (Interior angles on the same side of the transversal)

Thus, $CA \parallel BD$

Example 9. In Fig. 5.29, BA || ED and BC || EF. Show that : $\angle ABC = \angle DEF$

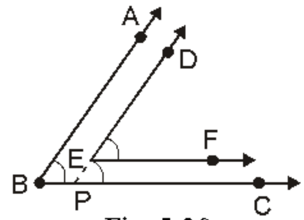


Fig. 5.29

Sol : Extend DE in such away that it will meet BC at point P.

$\because$ $BC \parallel EF$ (Given)

If we consider DP as transversal, then

$\angle DEF = \angle DPC$ (Corresponding angles) . . . (1)

$AB \parallel DE$ (Given)

then $AB \parallel DP$ (DE is extended uo to P) and consider BC as a transversal.

$\angle ABC = \angle DPC$ (Corresponding angles) . . . (2)

From equation (1) and (2), we get

$$\angle ABC = \angle DEF$$

Exercise 5.2

1. In Fig. 5.30, lines AB, CD and EF are parallel to each other. Find the values of $\angle x, \angle y, \angle z$ and $\angle p$

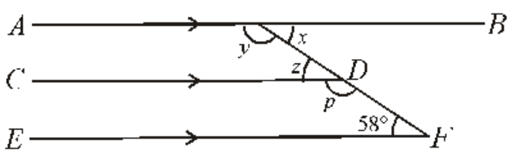


Fig. 5.30

2. In Fig. 5.31, $AB \parallel EF$ find the values of $\angle x$ and $\angle y$.

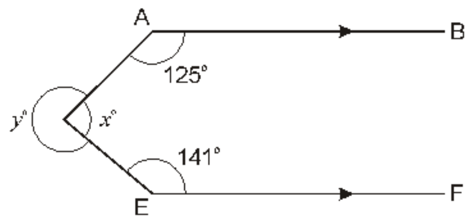


Fig. 5.31

3. In Fig. 5.32, if $\ell \parallel m$, then find the angles equivalent to $\angle 1$.

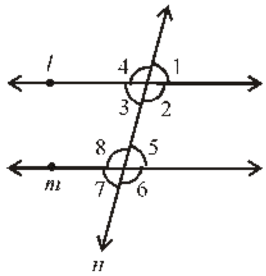


Fig. 5.32

4. In Fig. 5.33, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$, then show that m and n are parallel.

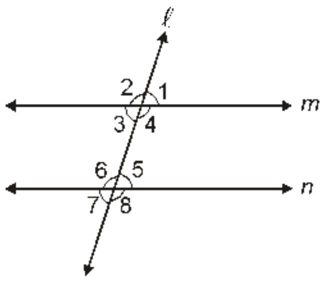


Fig. 5.33

5. AP and BQ are two bisectors of alternate angles of two parallel lines ℓ and m and its transversal n. Show that $AP \parallel BQ$.

6. In Fig. 5.34, $BA \parallel ED$, then show : $\angle ABC + \angle DEF = 180^\circ$.

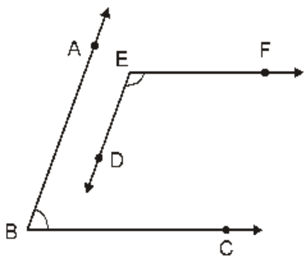


Fig. 5.34

7. In Fig. 5.35, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$, then

find out the value of $\angle APB$.

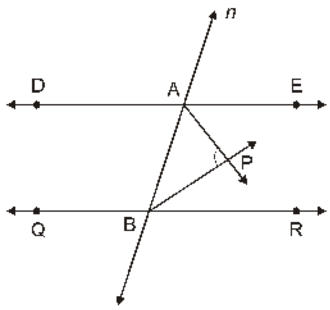


Fig. 5.35

8. If two straight lines are perpendicular to two parallel lines, then show that these straight lines are parallel to each other.

9. In Fig. 5.36, $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, then find the value of x.

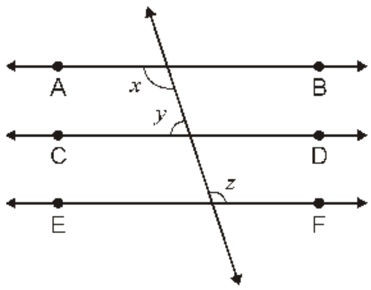


Fig. 5.36

10. In Fig. 5.37, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strickes the mirror PQ to B , the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB\ |\ CD$.

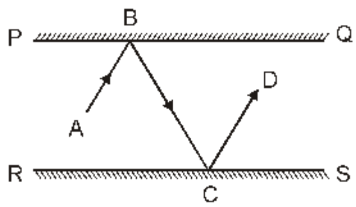


Fig. 5.37

11. In Fig. 5.38, if $PQ \parallel RS$, $\angle MXQ = 135^\circ$ and $\angle MYR = 40^\circ$, then find $\angle XMY$.

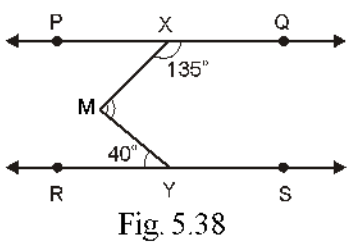


Fig. 5.38

5.9. Basic Constructions

While proving the theorem or solving the questions related to theorem which figures,shapes are made they have very less accuracy but geometric construction with the help of geometric tools like rural, protector, set squares and compasses can be made more accurate. During drawing following points should be remember.

1. Whatever geometric is to be made, first we have to draw the rough diagram with the dark pencil.
2. Steps of drawing should be explained in words.

Few Simple Construction :

You have studied some geometric construction in previous classes such as to draw a line of desired length and to draw an angle of desired measure. You have also studied various geometric facts in pervious chapter in this book. On the basis of previous chapter's knowledge and facts we can draw few geometric construction.

Geometric construction 1

Bisection of a given line segment i.e., bisection of line segment AB.

Construction : Draw a line segment AB of desired length. Taking A and B as a centre and take arc of more than half radius on the both side of line segment AB which intersect each other at point P and Q. Now, join PQ where it will intersect the line AB marked that point PQ where it will intersect the line AB maked that point O. Point O is the point of bisection of line AB as shown in Fig. 5.39.

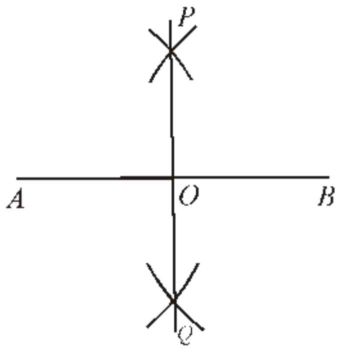


Fig. 5.39

Base of Construction : The point which have equal distance from the two given points is perpendicular bisector of the line joining them. PQ is the perpendicular bisector of line segment AB.

Geometric Construction 2

Bisection of any given angle say $\angle BAC$

Construction : Taking point A as centre of given angle $\angle BAC$ and with any radius draw an arc, which interest the side AB and AC at point P and Q. Now, taking P and Q as a centre and radius more than half of PQ, draw two arcs which intersect each other at point O. Now, join AO which bisect $\angle BAC$.

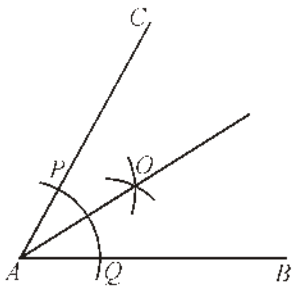


Fig. 5.40

Base of Construction : The points which have equal distance from lines AC and AB is bisecter of $\angle CAB$. Now join AO. The line between the angles is known as bisector of $\angle BAC$.

Geometric Construction 3

Draw an angle of 60° with ruler and compass.

Draw an angle of $60°$ on line segment AB at point O.

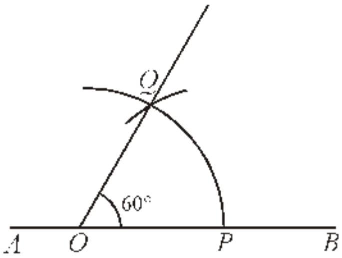


Fig. 5.41

Construction : Taking O as centre draw an arc of any radius on line AB which intersect AB at point P. Now taking P as centre draw an arc with same radius which will a intersect the arc at point Q, join OQ. Thus, $\angle BOQ = 60°$ is the required angle as shown in Fig. 5.41.

Base of Construction : Equilateral triangle ΔPOQ which have every angle $60°$.

Geometric Construction 4

Draw an angle of 120° with the help of ruler and compass.

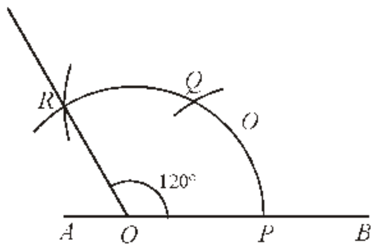


Fig. 5.42

Draw an angle of $120°$ on line AB at point O .

Construction : Taking O as a centre draw an arc of any radius which will intersect AB at point P. Now, taking P as centre draw an arc of same radius as two times (twice) which will intersect the arc on points Q and R respectively. Now, join OR. So, $\angle BOR = 120°$, as shown in Fig. 5.42.

Geometric Construction 5

Construction of various angles

(A) *Construction of angle* 30° :

$$\left[\because 30° = \frac{60°}{2}\right]$$

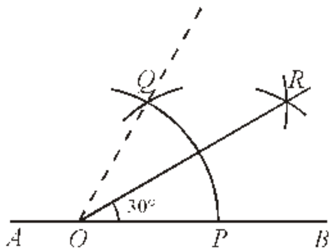


Fig. 5.43

Construction : With the help of ruler and compass draw an angle BOQ of 60° according to compass and geometric construction no. 2. Draw the bisector of $\angle BOQ$.

Hence, $\angle BOR = 30°$ (Fig. 5.43).

(A) *Construction of* 90° *angle :*

First Method :

$$\left[\because 90° = 60° + \frac{60°}{2}\right]$$

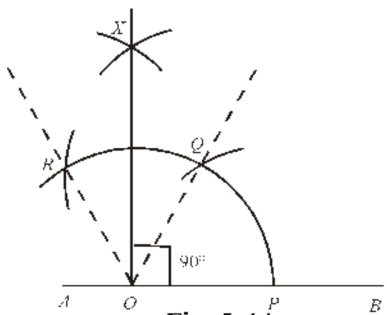


Fig. 5.44

Construction : With the help of ruler and compass draw $60°$ and $120°$ take two arcs and mark then Q and R points. On the basis of compass and geometric construction 2. Draw the bisector $\angle QOR$ and draw OX similar $\angle BOX = 90°$ (fig. 5.44)

Second Method :

$$\left[\because 90° = \frac{180°}{2}\right]$$

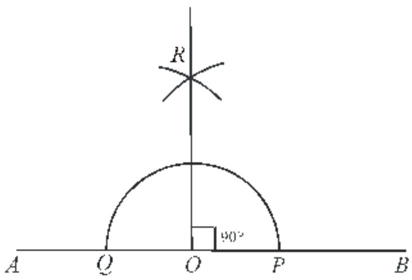


Fig. 5.45

Construction : Angle on straight line is $BOA = 180°$ according to compass and geometric construction and bisector OR.

$\therefore$ $\angle BOR = 90°$ (Fig. 5.45)

(C) Construction of 45°

$$\left[\because 45° = \frac{90°}{2}\right]$$

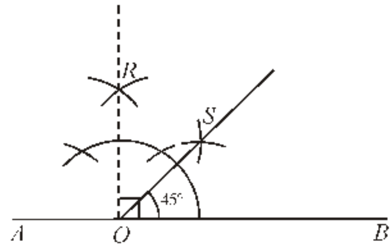


Fig. 5.46

Construction : Draw a $90°$ angle with the help of ruler and compass $\angle BOR = 90°$. According to compass and geometric construction 2, bisector of $\angle BOR$ by OS (Fig. 5.46).

So, $\angle BOS = 45°$

(D) *Construction of* 135° :

$$\left[\because 135° = 90° + \frac{90°}{2} \quad \text{or} \quad 180° - \frac{90°}{2}\right]$$

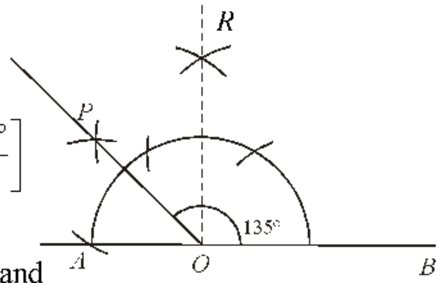


Fig. 5.47

Construction : Draw a $90°$ angle with the help of ruler and compass $\angle BOR$ according to compass and geometric construction 2. Draw the bisector of $\angle ROA$ and its bisector by OP.

So, $\angle BOP = 135°$. (Fig. 5.47)

Similarly, you can draw the following angles

1. $15° = \frac{30°}{2}$
2. $75° = 60° + 15°$
3. $105° = 90° + 15°$
4. $150° = 120° + 30°$ or $150° = 180° - \frac{60°}{2}$

$= 180° - 30°$

Geometric Construction 6

Draw an equivelant angle to an angle drawn on a point of a line.

Construction : Draw an equivalent angle equal to $\angle D$ of line segment AB at a point O. Taking D as centre and taking appropriate radius. Draw an arc which will intersect the sides at P and Q. Now, taking O as centre, draw the same arc which will intersect the line segment AB at R.

Now, taking R as a centre, draw an arc of radius PQ which will intersect the pervious arc at S. Now, join OS then $\angle BOS$ is desired angle.

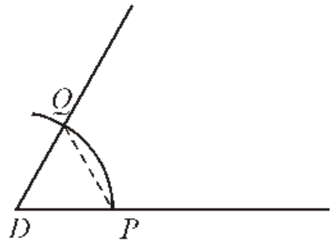


Fig. 5.48

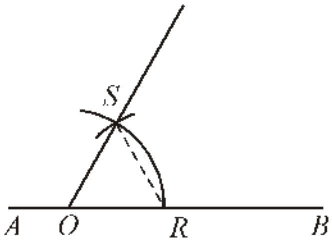


Fig. 5.49

Geometric Construction 7.

In pervious classes we have learnt to draw the various angles with the help of angle measuring instrument. Let us draw the various angles without angle mesurement instrument with the help of scale and protractor.

Step 1 : Draw a line segment AB of 6 cm as shown in Fig. 5.50.

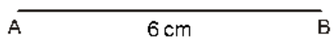


Fig. 5.50

Step 2 : Open the compass upto 6 cm and taking A as centre, draw a circle such that it passes through B. Now, taking 6 cm as radius draw equal arcs on the circle and mark the cuts as C, D, E, F and G.

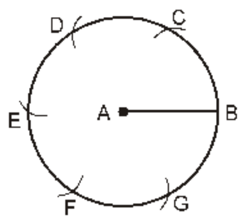


Fig. 5.51

Step 3 : Now, join AB and BC. Join in such away that it will start from where O is started and marked them 1-1 cm. Similarly , CD, DE, EF, FG and GB.

Here marked after one cm distance makes a 10° angle form each outer from point A.

if it divided into mm, then it will make 1° degree angle at distance of 1 mm form point A.

Suppose, we have to draw an angle of 40°, then we have to move up to 4th point where 40 is written. Now, join A to that point $\angle BAH = 40^{\circ}$.

Suppose, we have to draw 130° angle then we have to move upto point D and after that 130°, then $\angle BAK = 130^{\circ}$.

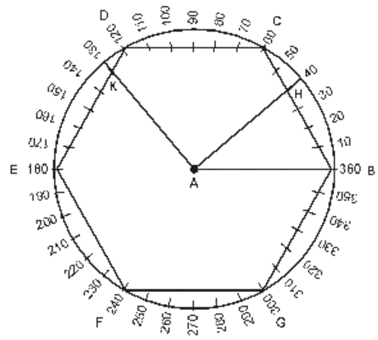


Fig. 5.52

Note : In this method, by using millimeter sacle is used then we can drawn angle 1° to 360°.

Remark : In the above method 6 cm straight line and another end where angle to be drawn. A 6 cm circle to be drawn and divide circle in 6 equal parts. (Hence, every part But not joining every point we have to join the desired line.

Geometric Construction 8

On any given line to draw a perpendicular form a point which is outside the line.

Method 1 : AB is straight line there is a point P outside the line. Draw a perpendicular.

Composition : Taking P as a centre and radius more than the distance from P to AB draw two arc which will intersect the line AB at points C and D. Taking C and D as a centre and radius more than half of CD draw two arcs below AB which will intersect each other at Q. Now join PQ. It will meet AB at point O. Thus, PO is the desired perpendicular.

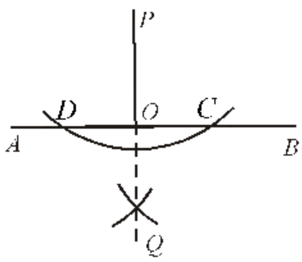


Fig. 5.53

Base of Construction : PQ is the perpendicular bisector of C and D which have equal distance.

Method 2 : Keep the one side of set square on line AB. Keep second set square in such a way so that it can move on sacle and it should be stable. Move set square so that it should move up to P. Draw line PQ to AB form point P.

Hence, PQ is the required perpendicular (Fig. 5.54).

Geometric Construction 9

Draw a perpendicular at any point O on line A B.

Construction : Let a is any point on ; line AB Taking O as centre of given line AB draw an arc which will intersect the line AB at points C and

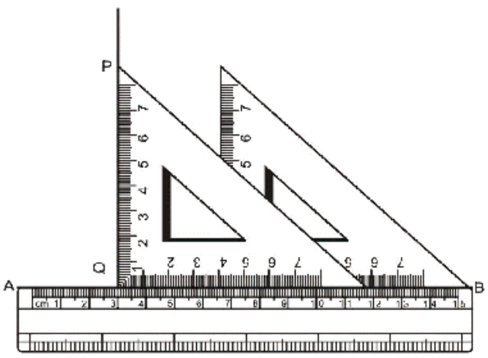


Fig. 5.54

D . Now taking C and D as centre taking more than half distance in compass draw an arc which will intetrsect each other at point P. Now join PO. Hence, PO is the required perpendicualr Fig. (5.55).

Base of Construction : If we join PC and PD, then ΔPOC and ΔPOD are congurent. Result is $\angle POC = \angle POD = 90^\circ$.

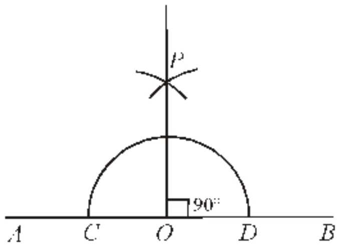


Fig. 5.55

5.10. Construction of Parallel Lines

Geometric Construction 10

From any outer point P draw a line parallel to a given line.

When two lines are intersected by a transversal, corresponding and alerante angles are equal then lines are parallel to each other.

(A) Corresponding Angle Method :

Taking a point Q on line AB and join with P. Extend QPto R. Now, at point P on QR draw $\angle DPR$ which is equal to $\angle BQR$. Hence, CD is the required line parallel to AB as shown in (Fig. 5.56).

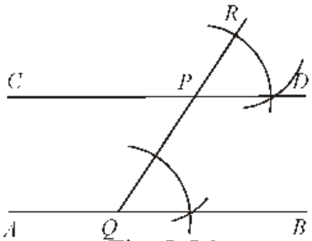


Fig. 5.56

(B) Alternate Angle Method :

Take a point Q on AB and join this with P of QP draw alternate angles $\angle CPQ = \angle BQP$. Hence, CD is the required line parallel to AB as shown in Fig. 5.57.

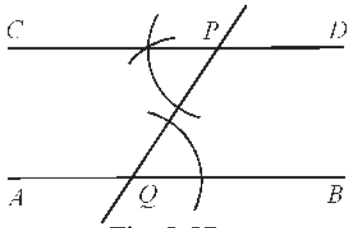


Fig. 5.57

(C) Construction of Parallel Lines with the help of Set Square :

Let AB be a straight line to which a parallel line has to be drawn from point P. Except the diagonal of set square keep other side on AB. Its another side set in such a way that its one side should be fix. Move the set square in such a way that it should be moved on P. Now, draw line PC. Hence, PC is the required parallel line (Fig. 5.58).

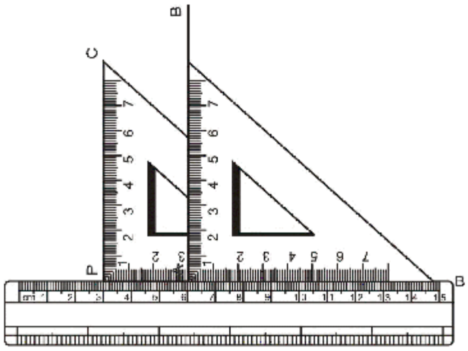


Fig. 5.58

Exercise 5.3

1. Draw a line segment $AB = 10$ cm. Draw its bisector and verify your answer.
2. Draw an angle of $120°$. Draw bisector of this angle. Measure both the angles and verify your answer.
3. Draw an angle of $40°$ with the protractor also draw an angle equal to this angle equal to this angle with ruler and compass.
4. Draw a line of 6 cm. Draw a perpendicular to it from an outer point P.
5. Draw $\angle ABC = 120°$. Draw a line parallel to BC from A.
6. Draw a line segment of 9 cm. Divide this in three equal parts with ruler and compass.
7. Draw a line segment of 10 cm. With the help of ruler and compass divide it into 3 : 2. Measure their length.
8. Draw a line segment of 6 cm. Divide it into 1 : 2 : 3 with the help of ruler and compass.
9. Draw the following angles with ruler and compass $45°$, $75°$, $105°$, $150°$.
10. Without using protractor draw the following angles :

    (i) $12°$ (ii) $20°$ (iii) $80°$ (iv) $100°$

    (v) $155°$ (vi) $218°$ (vii) $307°$ (viii) $127°$

    Verify these angles with protractor.

❑

Important Points

1. When two rays are originated form a point, then it makes a shape of angle. Common point of rays is known as vertex and rays are known as sides of angle.
2. If one line is standing on another line, then they make a right angle.
3. If a revolving ray takes a complete round, then it makes angles equal to four right angles, *i.e.*, $360°$.
4. An acute angle has a magnitude between $0°$ and $90°$.
5. An obtuse angle has a magnitude between $90°$ and $180°$.
6. Measurement of straight angle on a plane is equal to two right angles.
7. A reflex angle has a magnitude of more than two right angles but less than four right angles.
8 .If the sum of two angles is equal to $90°$, then these angles are known as complementary angles.
9. If the sum of two angles is equal to $180°$, then these angles are known as supplementary angles.
10. If in two vertex and one side is common and other side is opposite to common side then angles are known as adjacent angle.
11. If the measurment of two adjacent angles is $180°$, then these angles are known as linear pair.
12. Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays and they are equal.
13. It two parallel lines are intersected by a trasnversal, then :
    (i) Corresponding angles are equal,
    (ii) Alternate angles are equal,
    (iii) Sum of interior angles of one side is equal to $180°$.
14. If two lines are intersectd by a transversal and the angles made by them are such that
    (i) Corresponding angles are equal, or
    (ii) Alternate angles are equal, or
    (iii) Sum of interior angle on one side is equal to $180°$, then the lines are parallel to each other.

Miscellaneous Exercise-5

1. In fig. 5.59 If $AB |\ CD || EF,\ PQ \quad RS,\ \angle RQD = 25^o$ and $\angle CQP = 60^o$, then the value of $\angle QRS$ is equal to :

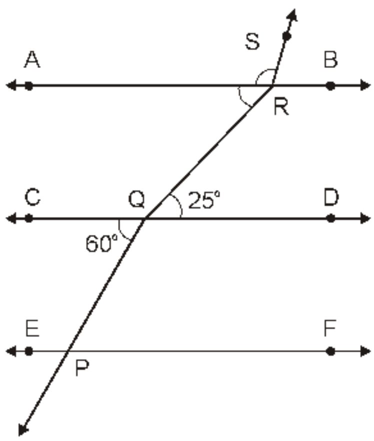


Fig. 5.59

(A) 85° (B) 135° (C) 145° (C) 110°

2. In Fig. 5.60., POQ is a straight line. The value of x is :

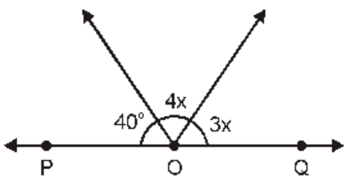


Fig. 5.60

(A) 20° (B) 25° (C) 30° (C) 35°

3. Fig. 5.61, $OP \quad RS,\ \angle OPQ = 110^o$ and $\angle QRS = 130^o$, then $\angle PQR$ is equal to :

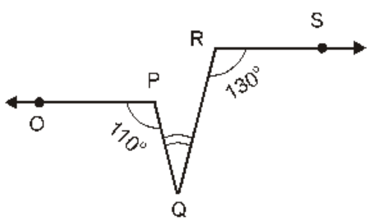


Fig. 5.61

(A) 40° (B) 50° (C) 60° (C) 70°

4. In Fig. 5.62, reflex angle $\angle AOB$ is equal to :

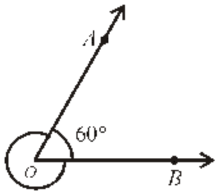


Fig. 5.62

(A) 60° (B) 120° (C) 300° (C) 360°

5. In Fig. 5.63, two straight lines AB and CD are intersecting each other at point O . Angles so formed are marked Fig. 5.63.

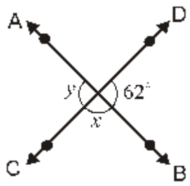


Fig. 5.63

Here, value of $\angle x - \angle y$ is :

(A) 56° (B) 118° (C) 62° (D) 180°

6. In Fig. 5.64., which pair of angles is not a pair of corresponding angles :

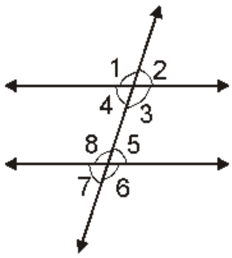


Fig. 5.64

(A) $\angle 1, \angle 5$ (B) $\angle 2, \angle 6$ (C) $\angle 3, \angle 7$ (D) $\angle 3, \angle 5$

7. In fig. 5.65, if *l* and *m* are two parallel lines which are intersected by transversal line at points G and H and the angles so formed are shown.

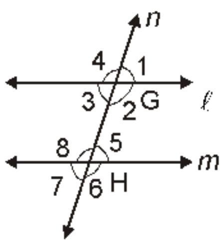


Fig. 5.65

If $\angle 1$ is an acute angle, then which statement is false :

(A) $\angle 1+\angle 2=180°$ (B) $\angle 2+\angle 5=180°$

(C) $\angle 3+\angle 8=180°$ (D) $\angle 2+\angle 6=180°$

8. Find the value of x in Fig. 5.66.

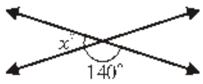


Fig. 5.66

9. In Fig. 5.67, $AB \parallel CD$. Find the value of $\angle x$ and $\angle y$.

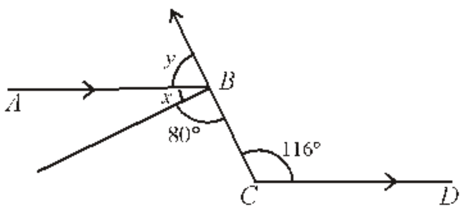


Fig. 5.67

10. In Fig. 5.68, lines *l* and *m* are parallel. Find the value of $\angle x$.

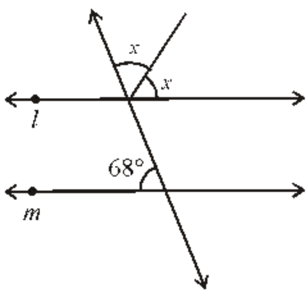


Fig. 5.68

11. In Fig. 5.69, which lines are parallel and why ?

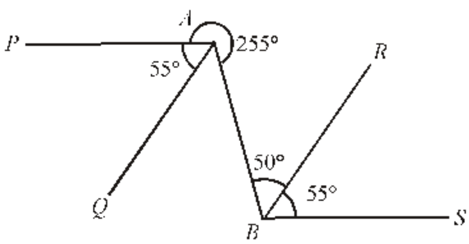


Fig. 5.69

12. In Fig. 5.70, $AC \parallel PQ$ and $AB \parallel RS$ then find the value of $\angle y$. Also, write the statements you have used.

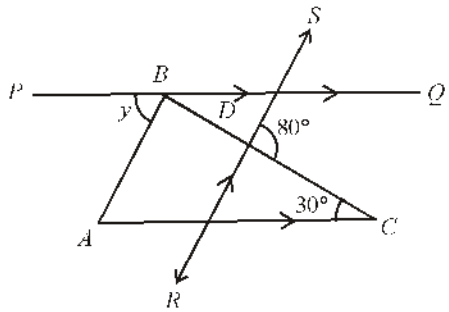


Fig. 5.70

13. In Fig. 5.71, $AB \parallel CD$ and $PQ \parallel EF$, then find the value of $\angle x$.

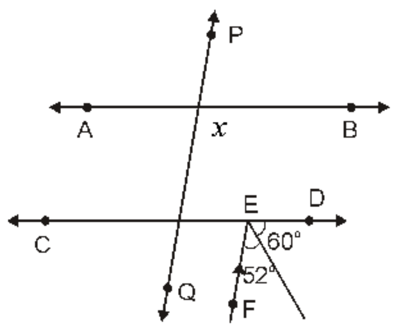


Fig. 5.71

14. In Fig. 5.72, out of lines *l, m, n, p, q* and *r* which lines are parallel and why.

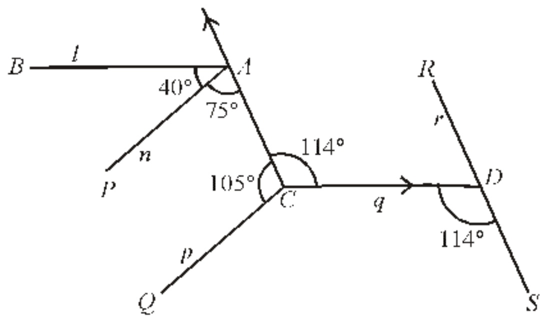


Fig. 5.72

15. In Fig. 5.73, two straight lines are intersecting each other. If $\angle 1 + \angle 2 + \angle 3 = 230^\circ$ then find the values of $\angle 1$ and $\angle 4$.

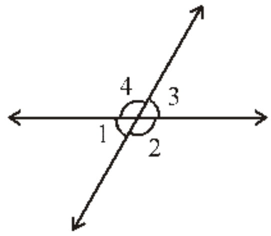


Fig. 5.73

16. In Fig. 5.74, PQ and QR are two mirrors which are joined at Q at an angle of 30° to each other. If incident ray AB is parallel to mirror RC, then find the values of $\angle BCQ$, $\angle CBQ$ and $\angle BDC$.

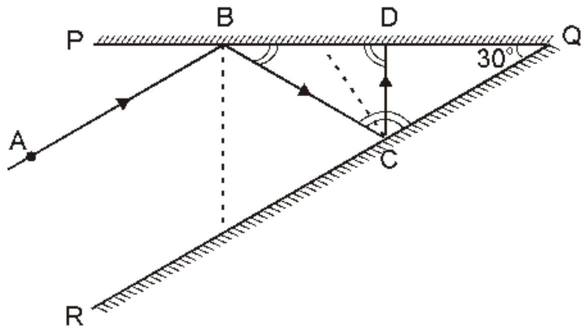


Fig. 5.74

Answer

Exercise 5.1

1. 122° and 58°

2. (i) 52° (ii) 128° (iii) $\angle AOC, \angle BOD$ and $\angle AOD, \angle BOC$

(iv) $\angle AOD$ and $\angle BOC$

Exercise 5.2

1. $\angle x = 58^\circ,\ \angle y = 122^\circ,\ \angle z = 58^\circ,\ \angle p = 122^\circ$

2. $\angle x = 94^\circ,\ \angle y = 266^\circ$

3. $\angle 3,\ \angle 5,\ \angle 7$

7. 90°

9. 126°

11. 85°

Miscellaneous Exercise - 5

1. (C) **2.** (A) **3.** (C) **4.** (C)

5. (A) **6.** (D) **7.** (B)

8. 40°

9. $x = 36,\ y = 64$

10. $x = 56^\circ$

11. $AQ \parallel BR,\ AP \parallel BS$

12. $y = 110^\circ$

13. 112°

14. $m \parallel r,\ n \parallel p$

15. $\angle 1 = 50,\ \angle 4 = 113^\circ$

16. $\angle BCQ = 120^\circ,\ \angle CBQ = 30^\circ,\ \angle BDC = 90^\circ$

6

RECTILINEAR FIGURES

6.01 Triangle and its Angles :

We have sutdied about angles, made by straight lines at a point. In this chapter, we shall study about a plane figure formed by more than two lines. If we take three non-collinear points in a plane and join them by taking two points at one time, then we will get three line segments. Thus, the figure so formed bounded by three line segments is called a tirangle.

"A plane figure bounded by three line segments by joining three non-collinear points in a plane is called a triangle."

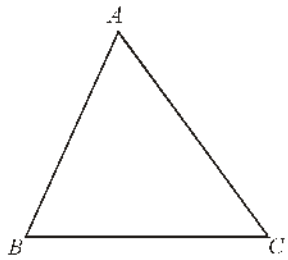


Fig. 6.01

Fig. 6.01 three non-collinear points A, B and C are joined. Figure ABC, so formed by joining line segments AB, BC and CA, is called triangle. Symbol 'Δ' is used in place of the word 'triangle'. So, triangle ABC will be denoted by ΔABC. These three points which makes a triangle are called vertices of triangle. Three line segments of triangle are called its **sides**. Angles formed at the vertices of a triangle by three line segments are called **angles** of triangle.

From Fig.6.02, it is clear that in ΔABC :

(i) points A, B and C are its vertices.

(ii) line segments AB, BC and CA are its sides.

(iii) $\angle CAB$, $(or\ \angle A)$, $\angle ABC$ $(or\ \angle B)$ $\angle BCA$ $(or\ \angle C)$

are the angles of the triangle If the sides of ΔABC are extended in order, then angle between extended side and adjacent side is called an **exterior angle** of triangle.

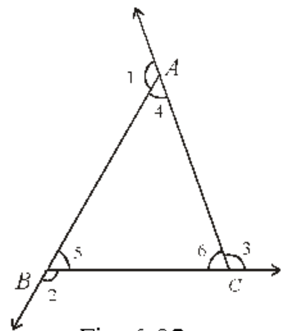


Fig. 6.02

In figure 6.2 $\angle 1$, $\angle 2$, $\angle 3$ are exterior angles of triangle ABC. $\angle 4$, $\angle 5$, $\angle 6$ are interior angles of triangle ABC.

Triangles can be classified on the basis of their sides or angles.

6.02 Classification of Triangles on the Basis of Sides

(i) Scalene Triangle : A triangle having all the three sides of different measure is called scalene triangle. In Fig. 6.03, ΔABC is a scalene triangle.

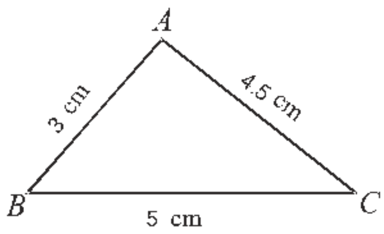


Fig. 6.03

(ii) Isosceles Triangle : If two sides of a triangle are of equal measure, then it is called an isosceles triangle. In Fig. 6.04, ΔPQR is an isosceles triangle in which $PQ = PR$.

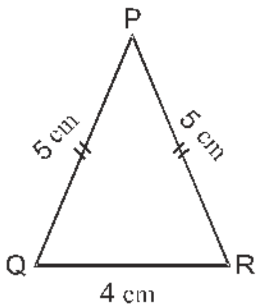


Fig. 6.04

(iii) Equilateral Triangle : A triangle, whose all sides are equal is called an equilateral triangle. In Fig. 6.05, ΔABC is an equilateral triangle in which $AB = BC = CA$.

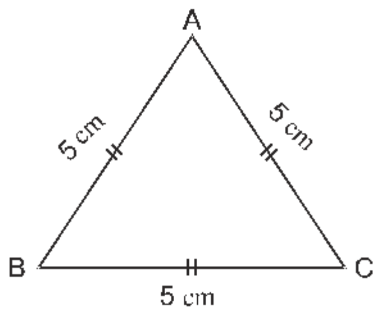


Fig. 6.05

6.03 Classification of Triangle on the basis of Angles :

(i) Acute-angled Triangle : A triangle, whose each angle is acute, is called an acute-angled triangle. In Fig. 6.06, ΔPQR is an acute-angled triangle, since $\angle P$, $\angle Q$ and $\angle R$ are acute angles.

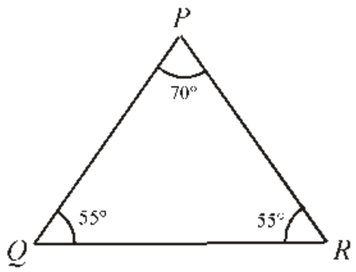


Fig. 6.06

(ii) Right-angled Triangle : A triangle with one angle a right angle is called a right-angled triangle. In Fig. 6.07, ΔABC is a right-angled triangle, since $\angle B = 90^\circ$.

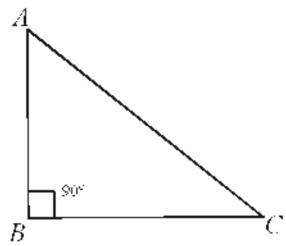


Fig. 6.07

(iii) **Obtuse-angled Triangle :** A triangle with one angle an obtuse angle, is known as an obtuse-angled triangle. In Fig. 6.08, ΔPQR is an obtuse-angled triangle, since her $\angle PQR = 120^\circ$.

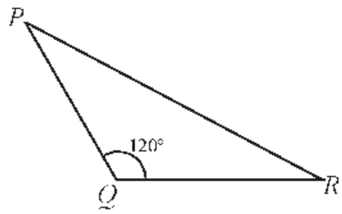


Fig. 6.08

The sum of three angles of a triangle is 180° proof of this geometrical fact is following.

Theorem 6.1. ***The sum of the three angle of a triangle is equal to two right angles.***

Given : A triangle ABC and its angles namely $\angle 1$, $\angle 2$ and $\angle 3$.

To prove $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Construction : Through A, draw a line parallel to BC.

Proof : $\because$ $BC \parallel \ell$

$\angle 2 = \angle 5$ (Alternate angles) ...(1)

and $\angle 3 = \angle 4$ (Alternate angles) ....(2)

On adding equations (1) and (2), we get

$\angle 2 + \angle 3 = \angle 5 + \angle 4$ ...(3)

Adding to both sides of equation (3), we get

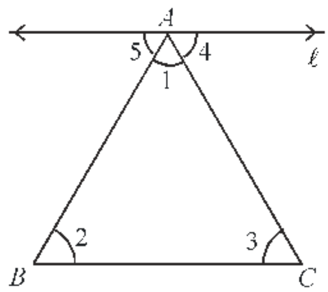


Fig. 6.09

$$\angle 1+\angle 2+\angle 3=\angle 1+\angle 5+\angle 4 \qquad ......(4) \qquad ...(iv)$$

(Sum of angles at a point on a line is 180°)

$$\angle 1+\angle 5+\angle 4=180^\circ \qquad ......(5)$$

From (4) and (5),

Thus $\angle 1+\angle 2+\angle 3=180^\circ$ Hence Proved

Corollary 1. If a side of a triangle is produced the exterior angle so formed is equal to the sum of the two interor opposite angles.

Sum of the three interior angles of a triangle is 180°

In Fig. 6.10. $\angle 1+\angle 2+\angle 3=180^\circ \qquad ...(i)$

By linear pair property

$$\angle 3+\angle 4=180^\circ \qquad ...(ii)$$

From equationas (i) and (ii), we get

$$\angle 1+\angle 2+\angle 3=\angle 3+\angle 4$$

Thus. $\angle 1+\angle 2=\angle 4$

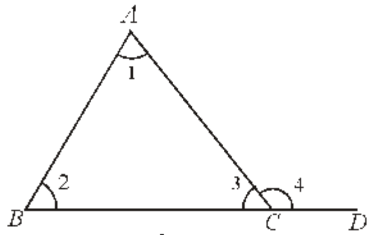


Fig. 6.10

Corollary 2. An exterior angle of a triangle is greater then either of the interior opposite angles.

In Fig. 6.10, $\angle 4=\angle 1+\angle 2$ (From Corollary 1)

$\Rightarrow \qquad \angle 4>\angle 1$

and $\qquad \angle 4>\angle 2$

Corollary 3. In a right-angled triangle, right angle is the greatest angle,

$\because$ Sum of the three angles of a triangle $=180^\circ$

$\therefore$ Sum of one right angle + Sum of two other angles $=180^\circ$

$\Rightarrow$ 90^0+ Sum of other two angles $=180^\circ$

$\therefore$ Sum of other two angles $=90^\circ$

$\Rightarrow$ Remaining two angles are acute angles.

Thus, Right angle is greater than remaining two acute angles.

Note: In each triangle, at least two angles are acute angles.

Corollary 4. Sum of the four angles of a quadrilateral is equal to four right angles.

In Fig. 6.11, ABCD is a quadrilateral having four angles $\angle A, \angle B, \angle C$ and $\angle D$. Line AC divides quadrilateral into two triangles.

In ΔABC $\angle 1+\angle 2+\angle 3=180^\circ$ ....(1)

and in ΔADC , $\angle 4+\angle 5+\angle 6=180^\circ$ ....(2)

From equations (i) and (ii), we get

$$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6=360^\circ$$

$\Rightarrow \quad (\angle 1+\angle 4)+\angle 2+(\angle 3+\angle 5)+\angle 6=360^\circ$

Thus, $\quad \angle A+\angle B+\angle C+\angle D=360^\circ$

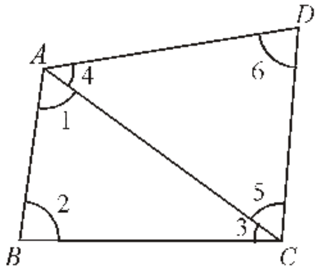


Fig. 6.11

Illustrative Example

Example 1 : In Fig. 6.12, one angle of ΔABC is **40°**. If the difference between remaining two angles is **30°** then find them.

Solution : Let $\angle x$ and $\angle y$ are remaining two angles of ΔABC.

$\therefore \quad \angle x+\angle y+40^\circ=180^\circ$

$\Rightarrow \quad \angle x+\angle y=140^\circ \quad$ ......(i)

$\Rightarrow \quad \angle x-\angle y=30^\circ$ (Given) ......(ii)

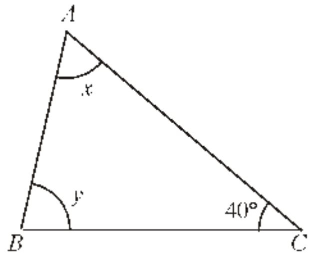


Fig. 6.12

On adding (i) and (ii), we get

$$\angle x+\angle y+\angle x-\angle y=140^\circ+30^\circ$$

$\Rightarrow \quad 2\angle x=170^\circ$

$\Rightarrow \quad \angle x=85^\circ$

and from (i) $\quad \angle y=140^\circ-\angle x$

$\quad =140^\circ-85^\circ=55^\circ$

Thus, the required angles are $\angle x=85^\circ$ and $\angle y=55^\circ$.

Example 2. From Fig. 6.13, find $\angle RPQ, \angle QRP$ and $\angle PQR$.

Solution : From Fig. 6.13

$$\angle x+\angle x=126^\circ$$

$\Rightarrow \quad 2\angle x=126^\circ$

$\Rightarrow \quad \angle x=63^\circ$

and $\quad \angle RPQ=63^\circ$

and $\quad \angle PQR=63^\circ$

now $\quad \angle y+\angle 126^\circ=180^\circ$ (Linear pair)

$\therefore \quad \angle y=180^\circ-126^\circ=54^\circ$

or $\quad \angle QRP=54^\circ$

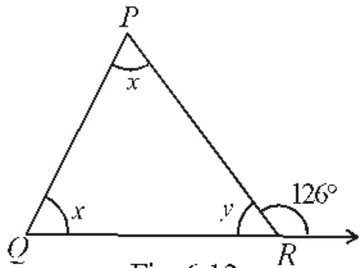


Fig. 6.13

Example 3. In Fig. 6.14, Find $\angle x, \angle y$ and $\angle ACD$ Here, $BA \parallel CE$.

Solution : Here, $\angle x=42^\circ$ (Alternate angles)

Now $\angle ACD = \angle x + 66°$

$= 42° + 66°$

$= 108°$

$\angle y + \angle ACD = 180°$ ... (i)

or $\angle y + 108° = 180°$

$\Rightarrow$ $\angle y = 180° - 108°$

$\Rightarrow$ $\angle y = 72°$

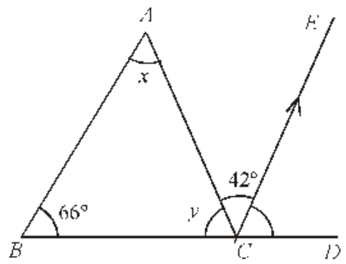


Fig. 6.14

Example 4. **If in a ΔABC, bisectors of angle B and C intersect each other at point 'O', then show that $\angle BOC = 90° + \frac{1}{2}\angle A$.**

Solution : Draw ΔABC as shown in Fig. 6.15 and then draw BO and CO, the bisectors of $\angle B$ and $\angle C$.

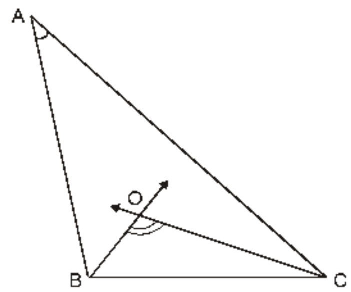


Fig. 6.15

$\angle A + \angle ABC + \angle ACB = 180°$

(Sum of three angles of a Δ is $180°$)

or $\frac{1}{2}\angle A + \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = \frac{1}{2} \times 180°$

or $\frac{1}{2}\angle A + \angle OBC + \angle OCB = 90°$ ....(1)

Given BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively.

$\because$ $\angle BOC + \angle OBC + \angle OCB = 180°$ (the dangles of ΔOBC) ....(2)

Subtracting (1) from (2), we get

$$\angle BOC + \angle OBC + \angle OCB - \frac{1}{2}\angle A - \angle OBC - \angle OCB = 180° - 90°$$

$\Rightarrow$ $\angle BOC - \frac{1}{2}\angle A = 90°$

Thus, $\angle BOC = 90° + \frac{1}{2}\angle A$.

Example 5. **In Fig. 6.16, if $BE \perp AC$, $\angle EBC = 30°$ and $\angle FAC = 20°$, then find the values of $\angle x$ and $\angle y$.**

Solution : In ΔBCE,

$90° + 30° + x = 180°$

or $\quad 120^\circ + \angle x = 180^\circ$

or $\quad \angle x = 180^\circ - 120^\circ$

or $\quad \angle x = 60^\circ$

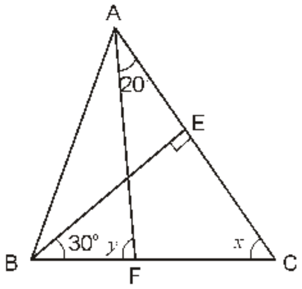


Fig. 6.16

Now, $\angle y = \angle FAC + \angle x$ (Exterior angle=Sum of interior opposite angles)

$$\angle y = 20^\circ + 60^\circ = 80^\circ$$

Exercise 6.1

1. In Fig. 6.17, find all the angles of ΔABC

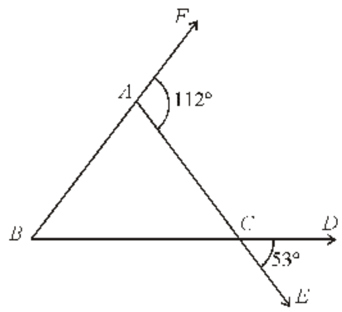


Fig. 6.17

2. In Fig. 6.18, ΔABC is an equilateral triangle. Find the values of $\angle x$, $\angle y$ and $\angle z$ from the figure.

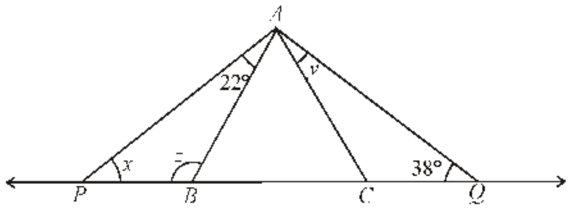


Fig. 6.18

3. In Fig. 6.19, sides AB and AC of ΔABC are produced to E and D respectively. If angle bisectors BO and CO of $\angle CBE$ and $\angle BCD$ meet each other at point O, then prove that;

$$\angle BOC = 90^\circ - \frac{\angle x}{2}$$

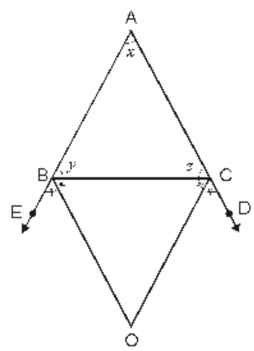


Fig. 6.19

4. In Fig. 6.20, $\angle P = 52^\circ$ and $\angle PQO = 64^\circ$, if QO and RO are the angle bisectors of $\angle PQR$ and $\angle PRQ$ respectively, then find the values of $\angle x$ and $\angle y$.

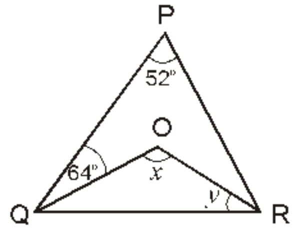


Fig. 6.20

5. In Fig. 6.21, if AB || DE, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, then find the value of DCE.

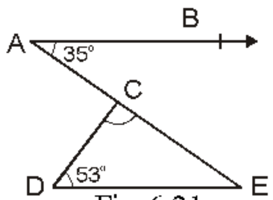


Fig. 6.21

6. In Fig. 6.22, if lines PQ and RS Intersects each other at point T, such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, then find $\angle SQT$.

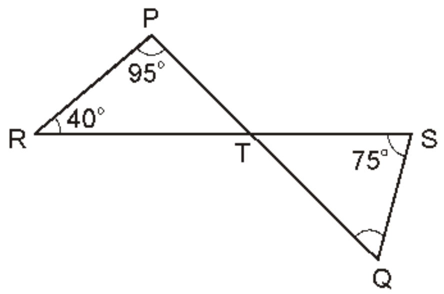


Fig. 6.22

7. In Fig. 6.23, sides QP and RQ of a triangle PQR are produced upto S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$ then find $\angle PRQ$.

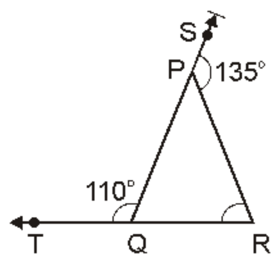


Fig. 6.23

8. In Fig. 6.24. If $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.

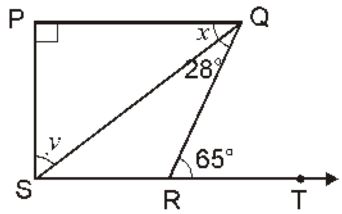


Fig. 6.24

9. In Fig. 6.25, side QR of ΔPQR is produced to point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meets at point T, then prove that : $\angle QTR = \frac{1}{2}\angle QPR$

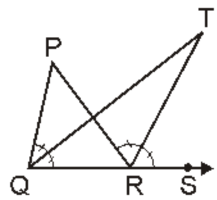


Fig. 6.25

10. In ABC, A is right angle. L is any point on BC such that $AL \perp BC$. Prove that : $\angle BAL = \angle ACB$

11. The angles of a triangle are in ratio 2:3:4. Find the all angles of the triangle.

6.04 Rectilinear Figures

A closed plane figure, bounded by at least three straight lines is called rectilinear figure.If we take $n\,(n \geq 3)$ different points on the plane such that :

(i) Line segments made by any two points out of n considered points, should not passes through any remaining point ($n-2$ points) except its own end points.

(ii) Two lines segment from a point is not collinear.

Then join these points in an order and figure so obtained is called polygon of n sides. These points are called vertices of the polygon and line segments which make polygon are called **sides of polygon**. Angles subtended by line segments at the vertices are called **interior angles** of polygon.

It is clear that in a n sided polygon :

(i) There are n vertices.

(ii) There are n sides.

(iii) There are n interior angles.

Polygons are classified on the basis of number of sides and the measure of angles.

(i) Triangle : When n=3, then rectilinear figure is called triangle.

(ii) Quadrilateral : When n=4, the rectilinear figure is called quadrilateral.

(iii) Pentagon : A polygon with 5 sides is called pentagon. In Fig.6.26, ABCDE is a pentagon, which AB, BC, CD, DE and EA are its sides and $\angle EAB, \angle ABC, \angle BCD, \angle CDE$ and $\angle DEA$ are its interior angles.

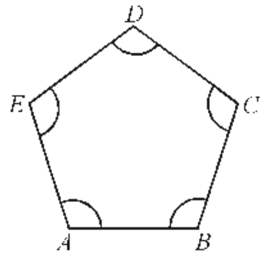


Fig. 6.26

(iv) Hexagon : A polygon with 6 sides is called hexagon. In Fig. 6.27 ABCDEF is a hexagon. Angles namely $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ are its interior angles and AB, BC, CD, DE, EF and FA are its sides.

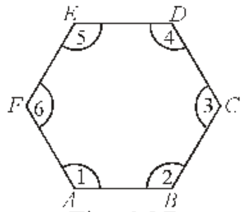


Fig. 6.27

(v) Heptagon : A polygon with 7 sides is called heptagon. In Fig. 6.28, ABCDEFG is a heptagon.

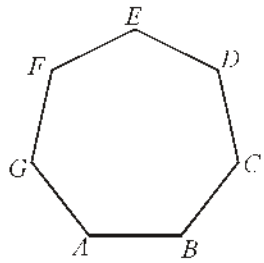


Fig. 6.28

(vi) Octagon : A polygon with 8 sides is called an octagon. In Fig. 6.29, ABCDEFGH is an octagon.

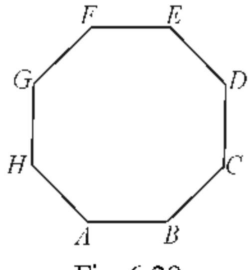


Fig. 6.29

Convex Polygon : If each angle of a polygon is less then two right angles, then the polygon is called a convex polygon. Unless otherwise stated, a polygon means a convex polygon.

Concave Polygon : if at least one angle of the polygon is greater than two right angle, it is called a concave polygon. In the given figure 6.30, ABCDEFGH is a concave polygon, since its interior angle $\angle FED$ is greater than two right angles.

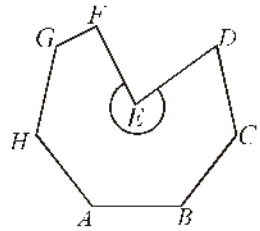


Fig. 6.30

Equilateral Polygon : A polygon with all sides equal is called an equilateral polygon. (Fig. 6.31)

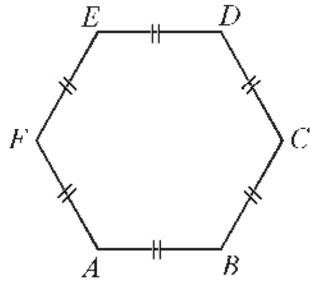


Fig. 6.31

Equiangular Polygon : A polygon with all interior angles equal is called an equiangular polygon. (Fig. 6.32)

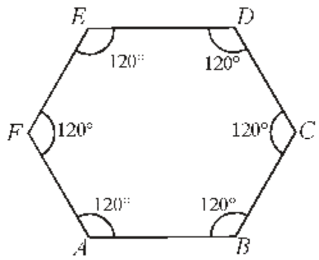


Fig. 6.32

Regular Polygon : A polygon which is equilateral and equiangular, is called regular polygon (Fig. 6.33).

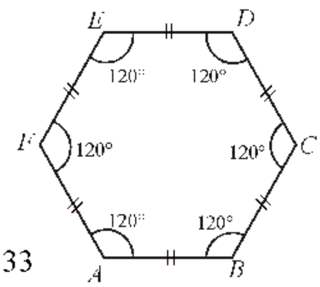


Fig. 6.33

Exterior angles of a polygon : if the sides of a polygon are produced in order (*i.e.* clockwise or anticlockwise direction) then the angles outsides the polygon, which are supplementary angles of interior angles, and called exterior angles of the polygon.

In Fig. 6.34, the sides of polygon ABCDEF is produced in same order and we obtained exterior angles as

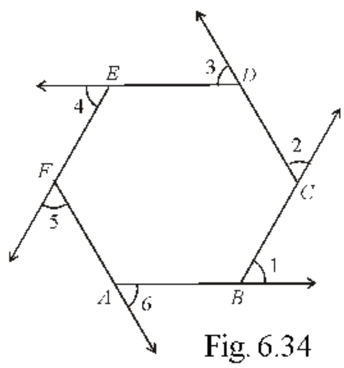


Fig. 6.34

$\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$.

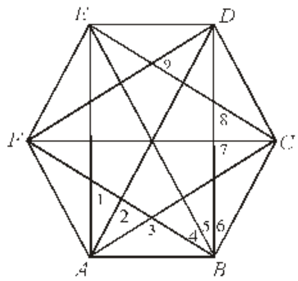


Fig. 6.35

Perimeter of a Polygon : The sum of all sides of a polygon is called its **perimeter.**

Diagonals of a Polygon : By joining the vertices of a polygon we get the straight lines (that are not the sides) are called diagonals of polygon. In Fig. 6.35, lines AC, AD and AE are diagonals from point A. Here, Total 9 diagonals

Total number of diagonals in n-sided polygon $= \dfrac{n(n-1)}{2} - n$

e.g. if $n-6$, then number of diagonals

$$= \frac{6(6-1)}{2} - 6 = \frac{6 \times 5}{2} - 6 = 15 - 6 = 9$$

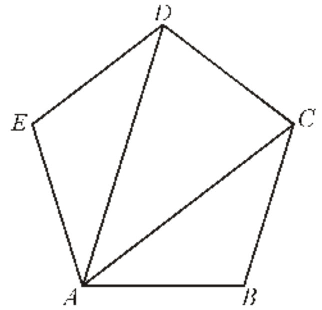


Fig. 6.36

In a n-sided polygon, the diagonals drawn from vertices make (n-2) triangles. In Fig. 6.36, diagonals AC and AD of pentagon ABCDE makes three traingles. On the basis of above facts we will establish the formula to find the sum of all the interior angles of any polygon, and on this basis we will find the sum of all extrior angles of polygon.

Theorem 6.2

(i) Sum of inteior angles of an n-sided polygon is equal to (2n-4) right angle.

(ii) Sum of exterior angles of an n-sided polygon is equal to 4 right angles.

(iii) Each interior angles of a regular polygon is equal to $\dfrac{1}{n}(2n-4)$ right angle.

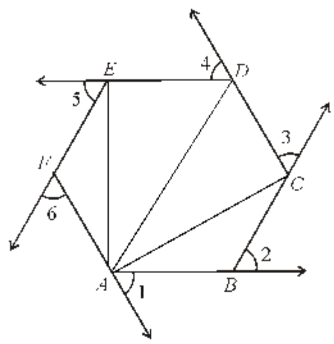


Fig. 6.37

The sides AB, BC, CD, DE, EF, FA, ... of a n sided polygon are produced in same order. In This way $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6 \cdots$ are respectively exterior angles at the vertices A, B, C, D, E, F, ...

To prove

(i) Sum of all interior angles = (2n - 4) right angles

(ii) Sum of all exterior angles = 4 right angles

(iii) Each interior angle of a regular polygon = $\dfrac{(2n-4)}{n}$ right angle

Construction : Draw diagonals AC, AD, AE, ... from the vertex A. In the way we get (n-2) triangles in total.

Proof (i) : We know diagonals of polygon from its vertices divides it into (n-2) triangles.

Sum of all interior angles of n sided polygon

=Sum of interior angles of (n-2) triangles made by the diagonals from the vertices

= (n - 2) x 2 right angle

= (2n - 4) right angle **Hence Proved**

Proof (ii) : We know that when the sides of polygon are produced, then sum of its pair of interior and exterior angles is equal to 2 right angles. Thus, there are n pairs of interior and exterior in a n-sided polygon and their sum = n x 2 right angles = 2n right angles.

Sum of n interior angles = (2n - 4) right angle.

Sum of n exterior angles = 2n right angle - (2n - 4) right angle.

= (2n - 2n + 4) right angle = 4 right angle.

Proof (iii) : If polygon is regular then its each interior angle $= \frac{1}{n}(2n-4)$ right angle

Proof : We know that the sum of all the interior angles of n-sided polygon is equal to ($2n$-4) right angles. We know that, each angle of regular polygon is same. Let each angle of regular polygon is x^0, then $nx = (2n-4)$ right angles (sum of all n angles)

$$x = \frac{1}{n}(2n-4) \text{ right angles}$$

Hence Proved.

Illustrative Examples

Example 6. In a regular polygon, number of sides is 10. Find its each interior angle.

Solution : First Method : $\because$ Each exterior angles or of n-sided regular polygon

$$= \frac{4}{n} \quad \text{(right angles)}$$

$$= \frac{4}{10} \times 90^\circ \quad (\text{Here } n = 10)$$

$$= 36^\circ$$

$\therefore$ **Each interior angle** $= 180^\circ - 36^\circ$

$= 144^\circ$

Second Method :

Each interior angle of n sided regular polygon $= \frac{(2n-4)\text{ right angle}}{n}$

$$= \frac{(2\times 10-4)\times 90^o}{10}$$

$$= \frac{16\times 90^o}{10} = 144^o$$

Example 7. Find the sum of all interior angles of a heptagon.

Solution : Sum of interior angles of n sided regular polygon $= (2n-4)$ right angle.

$\therefore$ Sum of all the interior angles of 7 sided rgular polygon.

$$= (2\times 7-4)\times 90^o = 10\times 90^o = 900^o$$

$$= 10\times 90^o = 900^o$$

Alternative method :

Number of triangles in regular polygon made by all the diagonals from a vertex of a heptagon =7-2=5.

$\because$ Sum of interior angles of a triangle $= 180^o$

$\therefore$ Sum of interior angles of 5 triangles $= 5\times 180^o = 900^o$

Example 8. If each interior angle of a regular polygon is 175^o, then find number of sides.

Solution : $\because$ 1 interior angle+1 exterior angle $= 180^o$

1 exterior angle $= 180^o - 1$ interior angle

$$= 180^o - 175^o = 5^o$$

$\because$ Sum of n exterior angles $= 360^o$ (Let the number of sides are n.)

$\therefore$ $n\times 5^o = 360^o$

Thus, $n = \dfrac{360^o}{5^o} = 72$

Example 9. Can a regular polygon be possible in which each interior angle is 115^o ? Check it.

Solution : $\because$ It is given that each interior angle of regular polygon $= 115^o$ (If it possible.)

Each exterior angle $= 180^o - 115^o = 65^o$

let the number of sides be n.

$\because$ Sum of exterior angles of a polygon =4 right angles

$\Rightarrow$ $n\times 65^o = 360^o$

$\Rightarrow$ $n = \dfrac{360^o}{65^o} = \dfrac{72}{13} = 5\dfrac{7}{13} \neq$ a whole number

Thus, a regular polygon cannot possible in which each interior angle is 115^o.

Exercise 6.2

1. If a regular polygon has 8 sides, then:

(i) find the sum of its exterior angles.

(ii) find the measure of each exterior angle.

(iii) find the sum of its interior angle.

(iv) find the measure of each interior angle.

2. If the sum of interior angles of a regular polygon is 2160^o, then what will be the number of sides of polygon? Find it.

3. Check, whether a regular polygon is possible with each interior angles 137^o.

4. Find $\angle CED$ and $\angle BDE$ in the following Fig. 6.38.

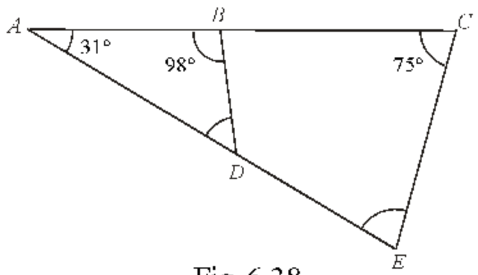


Fig. 6.38

Important Points

1. A plane figure bounded by n line segements is called n-sided polygon. In a n-sided polygon, there are n vertices, n interior angles and n sides.
2. Triangle, quadrilateral, pertagon , hextagon, heptagon and octagon are the name of the polygon according to $n = 3,4,5,6,7,8$ respectively.
3. Sum of three interior angles of a triangle is 180^o.
4. Sum of all interior angles of n-sided regular polygon is equal to (2n - 4) right angles.
5. A polygon with equal sides and equal angles is called regular polygon.
6. Each interior angles of n-sided regular polygon = $\frac{(2n-4)}{n}$ right angles.
7. Sum of all exterior angles of a polygon = 360^0
8. Each exterior angle of n-sided regular polygon = $\frac{4}{n}$ right angles.

Miscellaneous Exercise - 6

1. If two angles of a triangle are of measure $90°$ and $30°$ then the third angles is :
   (A) $90°$ (B) $30°$ (C) $60°$ (D) $120°$ [ ]
2. Three angles of triangle are in ratio 2:3:4, then the measure of its greatest angle is :
   (A) $80°$ (B) $60°$ (C) $40°$ (D) $180°$ [ ]
3. Measure of each angle of an equilateral triangle is :
   (A) $90°$ (B) $30°$ (C) $45°$ (D) $60°$ [ ]
4. Four angles of a quadrilateral are in the ratio $1:2:3:4$,then the measure of its smallest angle is :
   (A) $120°$ (B) $36°$ (C) $18°$ (D) $10°$ [ ]
5. In Fig.6.39, side BC of ΔABC is produced upto points D, If $\angle A = 55°$ and $\angle B = 60°$,then the measure of $\angle ACD$ is :
   (A) $120°$ (B) $110°$
   (C) $115°$ (D) $125°$ [ ]

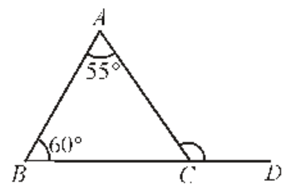


Fig.6.39

6. Sum of all interior angles of a hexagon is :
   (A) $720°$ (B) $360°$ (C) $540°$ (D) $1080°$ [ ]
7. The sum of exterior angles made by producing sides (in same order) of n-sided regular polygon :
   (A) n-right angles (B) $2n$ right angles
   (C) $(2n-4)$ right angles (D) 4 right angles [ ]
8. In a regular polygon, number of sides in n, then the measure of each interior angle is :
   (A) $\frac{360^0}{n}$ (B) $\left(\frac{2n-4}{n}\right)$ right angles
   (C) n right angles (D) $2n$ right angles [ ]
9. If one angles of a triangle is equal to the sum of other two angles, then the triangle is :
   (A) Isosceles triangle (B) Obtuse angled triangle
   (C) Equilateral triangle (D) Right angled triangle [ ]
10. The exterior angle of a triangle is $105°$ and the interior opposite angles are same. Each of the equal angle is :
    (A) $37\frac{1}{2}^{\circ}$ (B) $52\frac{1}{2}^{\circ}$ (C) $72\frac{1}{2}^{\circ}$ (D) $75°$ [ ]

11. The angles of a triangle are in the ratio 5 : 3 : 7. This triangle is :

(A) Acute angled triangle (B) Obtuse angled triangle

(C) Right angled triangle (D) Isosceles triangle [ ]

12. If one angle of a triangle is 130° , then angle between the anglular bisector of two angles of a triangle may be :

(A) 50° (B) 65° (C) 145° (D) 155° [ ]

13. In Fig. 6.40, find $\angle A$.

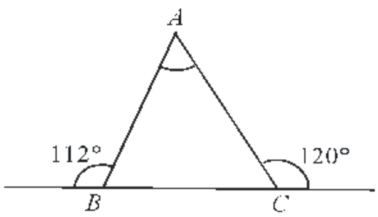


Fig. 6.40

14. In Fig. 6.41, $\angle B = 60^\circ$ and $\angle C = 40^\circ$. Find the measure of $\angle A$.

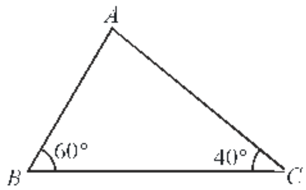


Fig. 6.41

15. In Fig. 6.42 $m \parallel QR$, then find $\angle QPR$

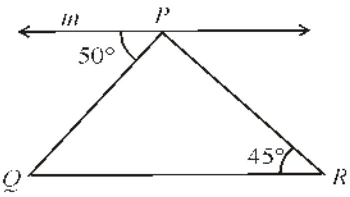


Fig. 6.42

16. In Fig. 6.43, find $\angle A$

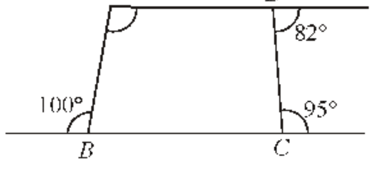


Fig. 6.43

17. Four interior angles of a pentagon are $40°, 75°, 125°$ and $135°$, then find the measure of the fifth angle.

18. If each exterior angle of a regular polygon is 45^0, then find the number of its sides.

19. The number of sides of a regular polygon is 12, then find the measure of each interior angle.

20. Sum of all interior angles of a polygon is 10 right angles. Find the number of sides.

21. Check, whether a polygon is possible if each of its interior angle is of measure $110°$.

22. If in a ΔABC, $\angle A + \angle B = \angle C$, then find the greatest angle of ΔABC.

23. Find the sum of interior angles of an octagon.

24. Find each interior angle of regular decagon.

25. $110°, 130°$ and $x°$ are the exterior angles obtained by producing the sides of a triangle in same order. Find the value of $x°$.

26. A hexagon has one interior angle of measure $165°$ and remaining each interior angle of measure $x°$, then find measure of remaining angle.

27. In Fig. 6.44, $AB \parallel DC$, Find $\angle x, \angle y$ and $\angle z$.

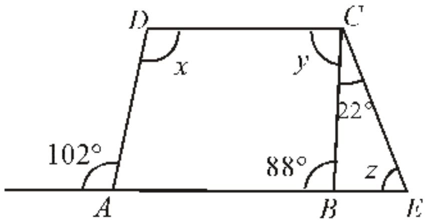


Fig. 6.44

28. In Fig. 6.45, find the value of $\angle x$ and $\angle y$, where $\angle x - \angle y = 10°$.

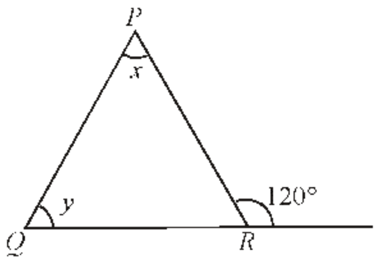


Fig. 6.45

29. In a polygon, two angles are of 90^0 each remaining each angle is of measure 150^0. Find the number of sides of this polygon.

30. In Fig. 6.46, prove that : $\angle x + \angle y = \angle A + \angle C$.

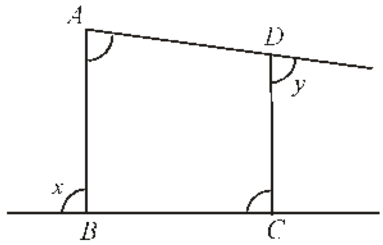


Fig. 6.46

31. In Fig. 6.47, lines BO and CO are the bisectors of $\angle B$ and $\angle C$, Find angle x

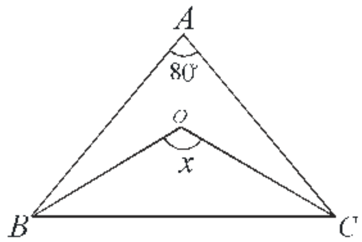


Fig. 6.47

32. In Fig 6.48, $\angle Q > \angle R$, PA is the biscetor of QPR and $PM \perp QR$. Prove that :

$$\angle APM = \frac{1}{2}(\angle Q - \angle R)$$

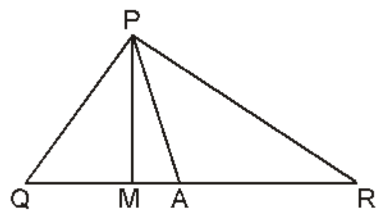


Fig. 6.48

Answers

Exercise 6.1

1. $\angle A = 68°$, $\angle B = 59°$, $\angle C = 53°$
2. $\angle x = 38°$, $\angle y = 22°$, $\angle z = 120°$

4. $\angle x = 116°$, $\angle y = 32°$
5. $92°$
6. $60°$
7. $65°$

9. $\angle x = 37°$ and $\angle y = 53°$

11. $40°$, $60°$, $80°$

Exercise 6.2

1. (i) $360°$ (ii) $45°$ (iii) $1080°$ (iv) $135°$
2. 14 3. *No* 4. $74°$

Miscellaneous Exercise 6

1. (C)	2. (A)	3. (D)	4. (B)	5. (C)	6. (A)
7. (D)	8. (B)	9. (D)	10. (B)	11. (A)	12. (D)

13. $\angle A = 52°$ 14. $80°$ 15. $85°$ 16. 97 17. 165

18. 8 19. $150°$ 20. 17 21. No 22. $\angle C$ 23. $1080°$

24. $144°$ 25. $120°$ 26. $111°$ 27. $x = 102°$, $y = 92°$, $z = 66°$

28. $x = 65°$, $y = 55°$ 29. 6 30. (B) 31. $140°$

❑

7

Congruence and Inequalities of Triangles

7.1 Introduction

Earlier, we have studied about triangles and their properties. In this chapter, we will study about the congruence rules of triangles and some other properties of triangles and inequalities in triangles.

7.2 Congruence of triangles

You have ever made several copies of your photograph of same size from a photographer. Similarly, you have seen bangles of same size in the wrist of your mother and seen postal stamps with same photo. Such figures are identical. If you choose any two such figures out of these and placed upon each other, then they exactly coincide.

Do you know, in geometry these figures are known by which name? These are called congruent figures. Congruent mean identically equal, i.e., figures with same shape and same size.

Thus, two triangles are congruent if and only if one of them can be made to uperpose on the other, so as to cover it exactly.

Axiom : (1)

If two sides and one included angle of a triangle are equal to the corresponding two sides and included angle of the other triangle, then the two triangles are congruent. (SAS rule of congruence)

Theorem 7.1. Angle-Side-Angle Rule (ASA Rule)

If two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the othe triangle, then the triangles are congruent.

Given : ABC and DEF are two triangles, in which $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BC = EF$

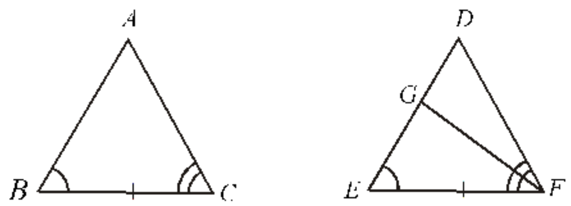


Fig. 7.01

To prove : $\Delta ABC \cong \Delta DEF$

Proof : Here, comparing the length of sides AB and DE of ΔABC and ΔDEF, following three conditions are possible :

(i) $AB = DE$ (ii) $AB < DE$ (iii) $AB > DE$

Condition (i) : If $AB = DE$, then in ΔABC and ΔDEF

$AB = DE$ (Say)

$\angle ABC = \angle DEF$ (Given)

$BC = EF$ (Given)

Thus, ΔABC and ΔDEF are congruent by Side-Angle Side rule.

i.e., $\Delta ABC \cong \Delta DEF$

Condition (ii) : When $AB < DE$, then take a point G on side DE such that $AB = GE$ and join GF (Fig. 7.01)

For ΔABC and ΔGEF

$AB = GE$ (Say)

$BC = EF$ (Given)

$\angle ABC = \angle GEF$ (Given) $[\because \angle GEF = \angle DEF]$

i.e., by SAS rule $\Delta ABC \cong \Delta GEF$

Thus, $\angle ACB = \angle GFE$ ...(1)

and $\angle ACB = \angle DFE$ (Given) ...(2)

From (1) and (2) $\angle GFE = \angle DFE$ is impossible, unless GF and DF do not coincide. It means points G and D coincide.

$\therefore$ $AB = DE$

Thus, by SAS rule,

$\Delta ABC \cong \Delta DEF$.

Condition (iii) : When $AB > DE$ then according to Fig. 7.02 take a point G on side AB of ΔABC such that $BG = ED$

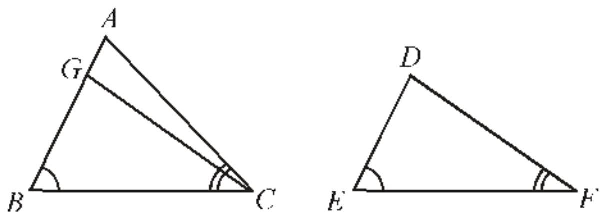


Fig. 7.02

Here, according to condition (ii), we can prove that point G will coincide point A i.e., $AB = DE$ and by Side-Angle-Side rule, $\Delta\, ABC \cong \Delta\, DEF$.

Thus, in all three condition, $\Delta\, ABC \cong \Delta\, DEF$. Hence proved.

Note : We know that the sum of three interior angles of a triangle is $180°$. Therefore, when two angles of a triangle are equal to two angles of another triangle, then their third angles will automatically by same. We will prove the following corollary on the basis of this law.

Corollary : Angle-Angle-Side Rule (AAS Rule)

If two angles and one side of one triangle are equal to the corresponding two angles and one side of the other triangle, then the two triangles are congruent.

Given : In $\Delta\, ABC$ and $\Delta\, DEF$ $\angle B = \angle E; \angle A = \angle D$ and side BC = side EF

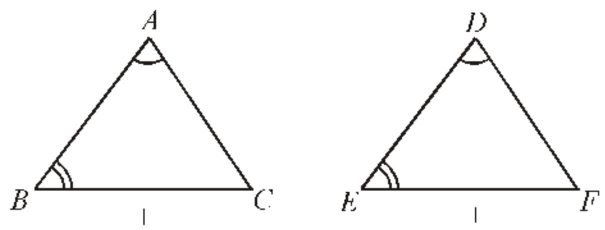


Fig. 7.03

To prove : $\Delta\, ABC \cong \Delta\, DEF$

Proof: We know that the sum of three interior angles of a triangle is $180°$.

$$\angle A + \angle B + \angle C = 180° \qquad \ldots(1)$$

$$\angle D + \angle E + \angle F = 180° \qquad \ldots(2)$$

From (1) and (2) $\angle A + \angle B + \angle C = \angle D + \angle E + \angle F \quad \ldots(3)$

Given that $\angle B = \angle E$ $\angle A = \angle D$

Thus, $\angle C = \angle F$ [From (3)] $\ldots(4)$

Now, in $\Delta\, ABC$ and $\Delta\, DEF$

$\angle B = \angle E$ (Given)

$BC = EF$ (given)

$\angle C = \angle F$ [From (4)]

By Angle-Side-Angle rule $\Delta\, ABC \cong \Delta\, DEF$. Hence proved

Illustrative Examples

Example 1. In Fig. 7.04, C is the mid-point of AB , $\angle BCD = \angle ACE$ and $\angle DAB = \angle EBA$ then show that :

(i) $\Delta DAC \cong \Delta EBC$

(ii) $DA = EB.$

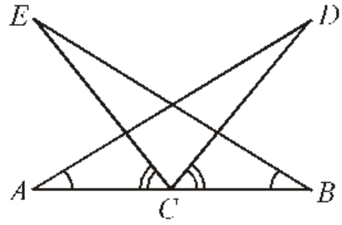


Fig. 7.04

Sol : Given : In Fig. 7.04,

$AC = BC$, $\angle DAB = \angle EBA$ and $\angle BCD = \angle ACE$

To prove : (i) $\Delta DAC \cong \Delta EBC$ (ii) $DA = EB.$

Proof : It is given that C, is the mid-point of side AB.

So, $AC = BC$ ... (1)

and $\angle BCD = \angle ACE$ (Given) ... (2)

Adding $\angle DCE$ both sides,

$$\angle BCD + \angle DCE = \angle ACE + \angle DCE$$

or $\angle ECB = \angle DCA$ ... (3)

Now, in ΔDAC and ΔEBC

$\angle DAC = \angle EBC$ (Given that)

$AC = BC$ [From (1)]

$\angle DCA = \angle ECB$ [From (3)]

By Angle-Side-Angle rule,

$$\Delta DAC \cong \Delta EBC$$

By the property of congruence, corresponding sides of two triangles are same.

Thus, $DA = EB$ Hence proved.

Example 2. In fig. 7.05, in a quadrilateral $ABCD$ $BC = AD$ and $\angle ADC = \angle BCD$ then prove that :

(i) $AC = BD$ (ii) $\angle ACD = \angle CDB.$

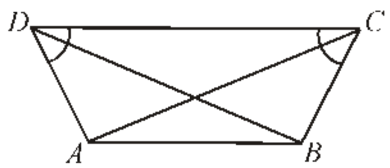


Fig. 7.05

Sol : According to Fig. 7.05, it is given that :

$BC = AD$ and $\angle ADC = \angle BCD$

So, in ΔADC and ΔBCD

$AD = BC$ (Given)

$CD = CD$ (Common side)

$\angle ADC = \angle BCD$ (Given)

So, by Side-Angle-Side rule, $\Delta ADC \cong \Delta BCD$

Since, corresponding sides and corresponding angles of congruent triangles are same.

Therefore, $AC = BD$ and $\angle ACD = \angle CDB.$ or $\angle ACD = \angle CDB.$ Hence proved.

Example 3. AB **is a line segment and line** ℓ **is its perpendicular bisector. If** P **is any point on** ℓ **, then show that** P **is equidistant from points** A **and** B

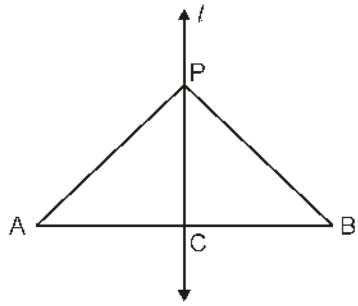


Fig. 7.06

Sol. : AB is a line segment and line ℓ passes through the mid-point C of AB (see fig. 7.06). We have to show that $PA = PB$. For this, think about ΔPCA and ΔPCB . It is given that

$AC = BC$ (C is the mid-point of AB)

$\angle PCA = \angle PCB = 90°$ (Given)

$PC = PC$ (Common)

So, $\Delta PCA \cong \Delta PCB$ (SAS Rule)

Therefore, $PA = PB$ (Corresponding sides of congruent triangles) Hence proved

Example 4. In Fig. 7.07, $AE = EC$ **and** $DE = BE$ **then show that :**

(i) $\Delta AED \cong \Delta CEB$ **(ii)** $\angle A = \angle C$.

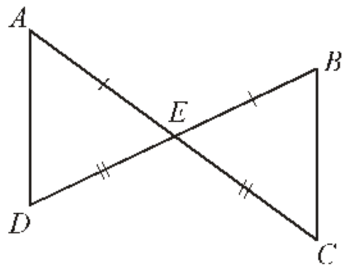


Fig. 7.07

Sol. : According Fig. 7.07, it is given that

$AE = EC$

$DE = BE$ ...(1)

Now, for ΔAED and ΔBEC

$AE = EC$ (given)

$\angle AED = \angle CEB$ (vertically opposite angles)

$DE = EB$ (given)

Therefore, by Side-Angle-Side rule $\Delta AED \cong \Delta CEB$ and their corresponding angles $\angle A = \angle C$ $\angle D = \angle B$ Hence proved

Example 5. In Fig. 7.08, $AD = BC$ **and** $BD = CA$ **then show that :**

(i) $\angle ADB = \angle BCA$

(ii) $\angle DAB = \angle CBA$.

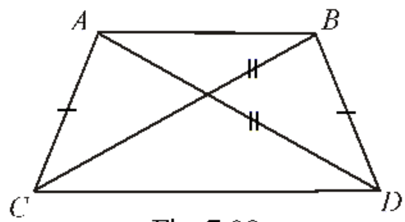


Fig. 7.08

Sol. : In Fig. 7.08 $AD = BC$ and $BD = CA$

So, in ΔABD and ΔABC

$$\left.\begin{array}{l} AD = BC \\ BD = CA \end{array}\right\} \quad \text{(Given)}$$

$AB = AB$ (Common)

So, by Side-Side-Side rule

$\Delta ABD \cong \Delta ABC$

So, corresponding angles

(i) $\angle ADB = \angle BCA$ (ii) $\angle DAB = \angle CBA$ Hence proved

Exercise 7.1

1. In ABC and PQR $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to the side AB of ΔABC , so that two triangles become congruent? Given reason to your answer.
2. In triangles ABC and PQR $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side BC of ΔABC ,so that two triangles are congruent? Give reason to your answer.
3. If two sides and one angle of a triangle is equal to the two sides and one angle of other triangle, then two triangles should be congruent. Is this statement true? Why?
4. If two angles and one side of a triangle is equal to the two angles and one side of other triangle then triangles sure should be congruent. Is this statement true? Why?
5. It is given that $\Delta ABC \cong \Delta RPQ$. Is $BC = QR$ true? Why?
6. If $\Delta PQR \cong \Delta EDF$, then is this true $PR = EF$? Given reason your answer.
7. In fig. 7.09, diagonal AC of quadrilateral $ABCD$, is the bisector of $\angle A$ and $\angle C$? Prove that : $AB = AD$ and $CB = CD$.

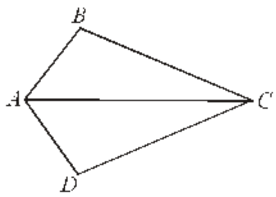


Fig. 7.09

8. In Fig. 7.10, in quadrilateral $ADBC$, $\angle ABC = \angle ABD$ and $BC = BD$,then prove that $\Delta ABC \cong \Delta ABD$.

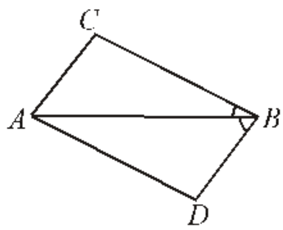


Fig. 7.10

9. According to Fig. 7.11 $AB \parallel DC$ and $AD \parallel BC$ then prove that : $\Delta ADB \cong \Delta CBD$.

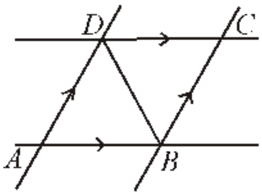


Fig. 7.11

10. In Fig. 7.12 , if $AB \parallel DC$ and E is the mid-point of side AC , then prove that E is the mid-point of side BD

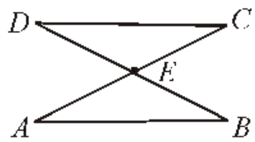


Fig. 7.12

7.3 Special Properties of Triangle

You have already studied about two conditions of congruence of triangles. Now, we will use thier results to prove the theorems related to an isosceles triangle and remaining theorems of congruence of triangles.

7.4 Isosceles Triangle

A triangle with two equal sides is called an isosceles triangle.

Theorem 7.3 : If two sides of a triangle are equal, then their opposite angles are also equal.

or

In an isosceles triangle, angles opposite to equal sides are equal.

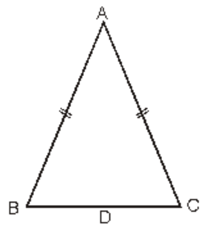


Fig. 7.13

Given : $\Delta\, ABC$ is an isosceles triangle

Where, $AB = AC$

To prove : $\angle B = \angle C$

Construction : Draw bisector AD of $\angle A$, which meets BC at D.

Proof : In $\Delta\, ABD$ and $\Delta\, ACD$,

$AB = AC$ (Given)

$\angle BAD = \angle CAD$ (By construction)

$AD = AD$ (Common side)

By Side Angle Side rule, $\Delta\, ABD \cong \Delta\, ACD$

Since, corresponding angles of congruent triangles are equal.

$\angle B = \angle C$ Hence proved.

Theorem 7.4 : If two angles in a triangle are equal, then their opposite sides will be also equal.

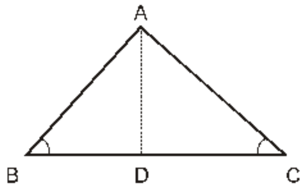


Fig. 7.14

Given : $\Delta\, ABC$ in which $\angle B = \angle C$

To prove : $AB = AC$

Construction : Draw AD, the bisector of $\angle BAC$

Proof : In $\Delta\, ABD$ and $\Delta\, ACD$

$\angle B = \angle C$ (Given)

$AD = AD$ (common side)

$\angle BAD = \angle CAD$ (by construction)

By Angle Side Angle rule,

$\Delta\, ABD \cong \Delta\, ACD$

Thus, corresponding sides $AB = AC$ Hence proved.

Illustrative Examples

Example 6. In ΔABC, the bisector AD of $\angle A$, is perpendicular to side BC. Show that ΔABC is an isosceles triangle.

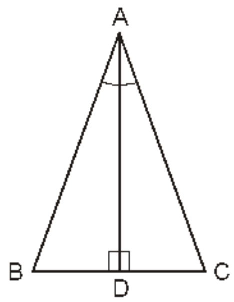


Fig. 7.15

Sol : In ΔABD and ΔACD

$\angle BAD = \angle CAD$ (given that AD is bisector of $\angle A$)

AD = AD (Common side)

$\angle ADB = \angle ADC = 90°$ (Given)

ΔABD $\cong$ ΔACD (By ASA rule)

Therefore, AB = AC

Thus, ΔABC is an isosceles triangle.

Example 7. According to Fig. 7.16, $ABCD$ is a square and ΔCDE is an equilateral triangle, then prove that $AE = BE$.

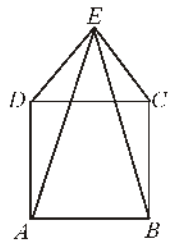


Fig. 7.16

Sol : Given, $ABCD$ is a square and $\Delta\, CDE$ is an equilateral triangle.

To prove : $AE = BE$

Proof : $\Delta\, CDE$ is an equilateral triangle

Thus, $CD = DE = CE$ ... (1)

$\angle DEC = \angle EDC = \angle DCE = 60°$ ... (2)

and $ABCD$ is a square, so

$\angle ADC = \angle BCD = 90°$

Adding $\angle EDC$ to both sides

$\angle ADC + \angle EDC = \angle BCD + \angle EDC$

$\Rightarrow \quad \angle EDA = \angle ECB \quad \ldots(3)$

Now, in $\Delta\, ADE$ and $\Delta\, BCE$

$AD = BC$ (sides of square)

$\angle EDA = \angle ECB$ [From (3)]

$DE = EC$ [From (1)]

By Side-Angle-Side rule $\Delta\, ADE \cong \Delta\, BCE$.

Thus, corresponding sides $AE = BE$ Hence proved

Example 8. Prove that the medians, which bisects equal sides of an isosceles triangle, are equal.

Sol. Given : In an Isosceles $\Delta ABC, D$ and E are mid points of equal sides AB and AC.

To prove : $BE = CD$

Proof : $\Delta\, ABC$ is an isosceles triangle whose sides AB and AC are equal.

$AB = AC \quad \ldots(1)$

and $\angle ABC = \angle ACB \quad \ldots(2)$

and D and E are the mid points of sides AB and AC .

Thus, $DB = DA = EC = AE \quad \ldots(3)$

Now, in $\Delta\, BCD$ and $\Delta\, BCE$

$BC = BC$ (Common side)

$\angle DBC = \angle ECB$ [From (2)]

$BD = CE$ [From (3)]

By Side-Angle-Side rule $\Delta\, BCD \cong \Delta\, BCE$

Corresponding sides will be equal i.e.,

$CD = BE$ or $BE = CD$ Hence proved.

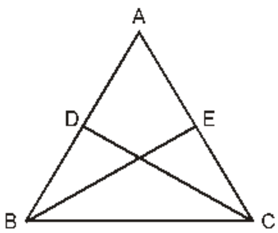


Fig. 7.17

Example 9. In Fig. 7.18, $AB = AC$ and D is a point in ΔABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$.

Sol. Given : In $\Delta\, ABC$, $AB = AC$ and $\angle DBC = \angle DCB$.

To prove : AD is the bisector of $\angle BAC$.

i.e. $\angle BAD = \angle CAD$

Proof : In $\Delta\, BDC$, $\angle DBC = \angle DCB$ so their opposite sides will be same.

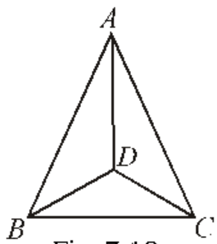


Fig. 7.18

$CD = BD \quad \ldots(1)$

Now, in $\Delta\, ABD$ and $\Delta\, ACD$

$BD = CD$ [From (1)]

$AD = AD$ [Common side]

$AB = AC$ (Given)

By Side-Side-Side rule, $\Delta\, ABD \cong \Delta\, ACD$.

So, corresponding angles will be same. i.e., $\angle BAD = \angle CAD$.

Thus, AD is the bisector of $\angle BAC$. Hence proved.

Example 10. If perpendiculars drawn from mid-point of a side of a triangle to the other two sides are equal, then prove that triangle will be isosceles.

Sol. Given : D is the mid point of side BC of ΔABC . DE and DF are the perpendiculars on AC and AB respectively, and $DE = DF$.

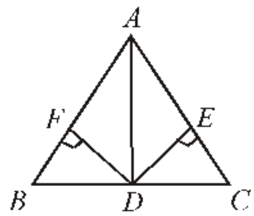


Fig. 7.19

To prove : ΔABC is an isosceles triangle, i.e., $AB = AC$

Construction : Join AD

Proof : In $\Delta\, BDF$ and $\Delta\, CDE$

Hypotenuse $BD =$ Hypotenuse CD (Given)

$\angle DFB = \angle DEC = 90°$

and $DF = DE$ (Given)

By Right Angle Hypotenuse Side rule

$\Delta\, BDF \cong \Delta CDE$.

Thus, corresponding angles $\angle B = \angle C$ and oppostie sides to two equal angles will be equal i.e., $AB = AC$. Hence proved

Example 11. In an isosceles triangle ABC , $AB = AC$ and D,E,F are mid-points of sides BC, AC and AB , then prove that $DE = DF$.

Sol. : According to Fig. 7.20 in $\Delta\, ABC$

$AB = AC$ ...(1)

And D, E, F are the mid-points of sides BC, AC and AB respectively

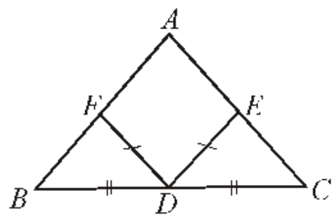


Fig. 7.20

So, in $\Delta\, BDF$ and $\Delta\, CDE$

$BD = CD$ [D is the mid-point of side BC]

$CE = BF$ [Given $AB = AC$]

and $\angle B = \angle C$ [Angles opposite to equal sides are equal]

By Side-Angle-Side-rule

$\Delta\, BDF \cong \Delta\, CDE$

Thus, or $DE = DF$ Hence Proved

Example 12. In Fig. 7.21, ABC is a right-angled triangle, in which $\angle$B = 90°, such that $\angle BCA = 2\angle BAC$. Show that hypotenuse AC = 2 BC.

Sol. Given : A right angled ABC such that $\angle B = 90°$ and $\angle BCA = 2\angle BAC$

To prove : $AC = 2BC$

Construction : Produce **CB** upto **D** such that **BC = BD** and join **AD**

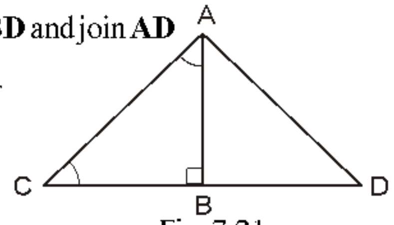


Fig. 7.21

Proof : Let $\angle BAC = x$, then $\angle BCA = 2\angle BAC = 2x$.

In right triangle ABC

$$\angle BAC + \angle BCA + \angle ABC = 180°$$

$\Rightarrow \quad x + 2x + 90° = 180°$

$\Rightarrow \quad 3x = 180° - 90°$

$\Rightarrow \quad x = 30°$

$\therefore \quad \angle BCA = 2 \times 30° = 60°$ and $\angle BAC = 30°$ ...(i)

In ΔABC and ΔABD

BC = BD (By construction)

$\angle ABC = \angle ABD$ (Each is $90°$)

AB = AB [Common side]

$\therefore \quad \Delta ABC \cong \Delta ABD$ (By SAS rule)

$\Rightarrow \quad \angle CAB = \angle DAB$

and AC = AD

Now $\angle BAC = \angle BAD = 30°$ [Using (1)]

$\therefore \quad \angle A = 2 \times 30° = 60°$

and AC = AD

$\Rightarrow \quad \angle C = \angle D$ [Angles opp. to equal sides are equal]

$\Rightarrow \quad \angle C = \angle D = 60°$

In ΔACD,

$$\angle A = \angle C = \angle D = 60°$$

$\Rightarrow \quad \Delta ACD$ is an equilateral triangle.

$\Rightarrow \quad$ AC = CD = AD ...(2)

But BC = BD

$\therefore \quad$ CD = BC + BD.

$\Rightarrow \quad$ CD = BC + BC

$\Rightarrow \quad$ CD = 2 BC

$\Rightarrow \quad$ AC = 2 BC. [Using (2)] Hence Proved.

Exercise 7.2

1. In Fig. 7.22 $AB = AC$ and $\angle B = 58°$ then find the value of $\angle A$

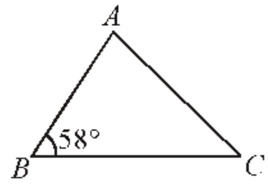


Fig. 7.22

2. In Fig. 7.23 $AD = BD$ and $\angle C = \angle E$, then prove that $BC = AE$.

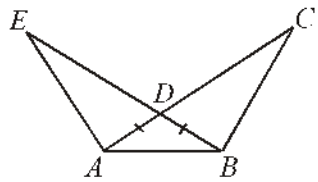


Fig. 7.23

3. If AD is the median of an isosceles triangle ABC and $\angle A = 120°$ and $AB = AC$, then find the value of $\angle ADB$

4. If the bisector of any angle of a triangle also bisects the opposite side then prove that the triangle is an isosceles triangle.

5. In Fig. 7.24, $AB = AC$ and $BE = CD$, then prove that : $AD = AE$.

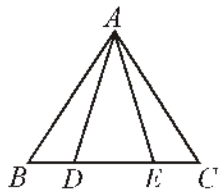


Fig. 7.24

6. E and F are two points on sides AD and BC respectively of square ABCD, such that $AF = BE$. Show that :

   (i) $\angle BAF = \angle ABE$ (ii) $BF = AE$

7. AD and BC are two equal perpendiculars on a line segment AB (see fig. 7.25). Show that CD bisects line segment AB.

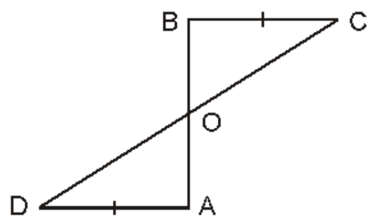


Fig. 7.25

8. The bisectors of angles B and C of an isosceles triangle with $AB = AC$, intersect each other at point O. Produce BO upto point M. Prove that : $\angle MOC = \angle ABC$

9. Line ℓ bisects angle A and B is any point on line ℓ. BP and BQ are the perpendiculars drawn on the sides of angle A from point B (See Fig. 7.26)

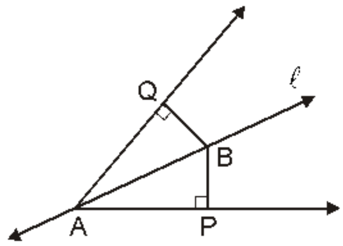


Fig. 7.26

(i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ it means point B is equidistant from the sides of $\angle A$.

11. In Fig. 7.27 $AC = AE, AB = AD$ and $\angle BAD = \angle EAC$. Show that : $BC = DE$

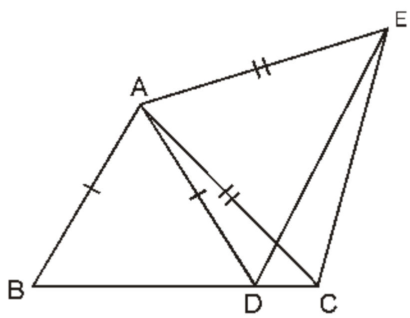


Fig. 7.27

12. In right triangle ABC, angle C is right angle M is the mid-point of hypotenuse AB Join C to M and produce it upto D such that DM = CM. Join point D and B (see fig. 7.28). Show that :

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

7.5 Some other concepts for the Congruence of Triangles

If three angles of a triangle are equal to three angles of another triangle, then it is not necessary that these two triangles are congruent.

If three sides of triangle are equal to three sides of another triangle then according to you whether these triangles be congruent? Definitely, they will be congruent. Now, we prove this theorem by using obtained result.

Theorem 7.5 : Side, Side, Side-Rule (S S S Rule)

If three sides of a triangle are equal to the corresponding three sides of other triangle, then two triangles are congruent.

Given : Corresponding sides of Δ ABC and $\Delta\, DEF$ are same, *i.e.*,

$$AB - DE; BC - EF \quad \text{and} \quad AC = DF$$

To prove : $\Delta ABC \cong \Delta DEF$

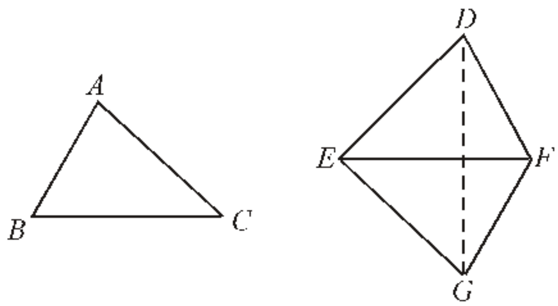


Fig. 7.29

Construction : Draw a line segment EG in the opposite side of $\Delta\, DEF$ such that $EG - AB$ and $\angle ABC = \angle FEG$. Join GE and DG.

Proof: In $\Delta\, ABC$ and ΔGEF

$AB - GE$ (By construction)

$\angle\, ABC = \angle\, GEF$ (By construction)

$BC - EF$ (Given)

By Side-Angle-Side rule, $\Delta ABC \cong \Delta DEF$

So, corresponding angles and corresponding sides of congruent triangles are equal.

$$\angle\, A = \angle\, G;\ AB = GF \qquad \ldots(1)$$

Now, $AB - EG$ (By construction, and $AB - DE$ (Given)

So $\quad EG - DE \qquad \ldots(2)$

Similarly, $AC = GF$ from equation (1) and $AC - DF$ (Given)

$\therefore \qquad GF = DF \qquad ...(3)$

$\Rightarrow$ In ΔEDG, angles opposite to equal sides EG and DE are same.

$$\angle EDG = \angle EGD \qquad ...(4)$$

Similarly, in ΔFDG, angles opposite to equal sides GF and DF are same.

$$\angle GDF = \angle DGF \qquad ...(5)$$

Adding equations (4) and (5)

$$\angle EDG + \angle GDF = \angle EGD + \angle DGF$$

$$\Rightarrow \qquad \angle D = \angle G \qquad ...(6)$$

But from equation (1)

$$\angle A = \angle G \qquad ...(7)$$

From equation (6) and (7)

$$\angle A = \angle D \qquad ...(8)$$

Thus, in ΔABC and ΔDEF

$AB = DE$ (Given)

$\angle A = \angle D$ [From (8)]

$AC = DF$ (Given)

By SAS rule,

$\Delta \ \Delta ABC \cong \Delta DEF$ Hence Proved

Now, we try to verify this theorem by the following activity :

Construct two triangles each of which have sides 4 cm, 3.5 cm and 3 cm.

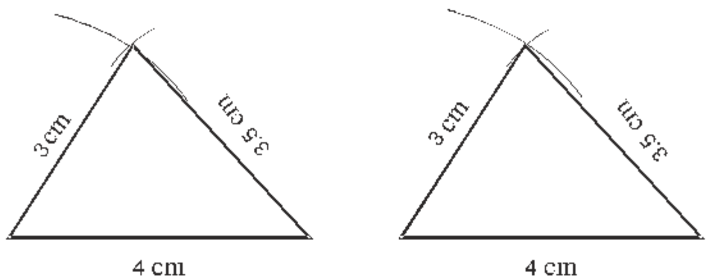


Fig. 7.30

Now cut them and place one of them on the other. What do you see? By keeping equal sides in mind when we placed one upon the other triangle they cover each other completely. This is possible only when two triangles are congruent.

It means two triangles are congruent.

You have already observed SAS rule of congruence. By SAS rule of congruence pair of equal angles may be between pair of corresponding sides. If not so, then two triangles may not be congruent.

Let verify it by an activity.

Draw two right angled triangles in which each one of hypotenuse is 5 cm and a side is 4 cm (See Fig. 7.31).

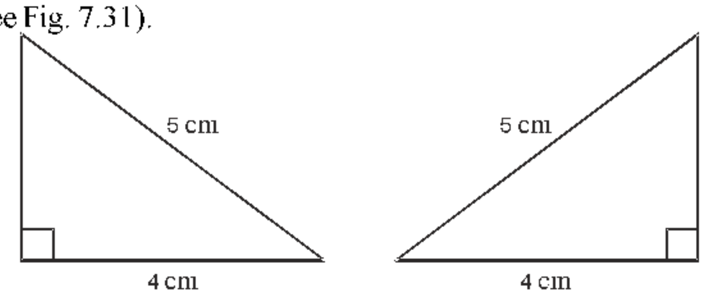


Fig. 7.31

Cut them and place one on the other such that equal sides coincide. If necessary, rotate them. What do you see? You see, on placing one on the other, they exactly cover each other so they are congruent. Repeat this activity by taking differe of right angled triangles. What do you see? You will see that if their hypotenuse and one pair of side are equal, then two right angled triangles will be congruent.

Note that, in this condition right angle is not the angle included in the hypotenuse and side.

In this way we conclude an important fact for right angled triangles which can be proved by theorem.

Theorem 7.6. Right Hypotenuse Side Rule (RHS congruence rule) :

Two right triangles are congruent if and only if the hypotenuse and a side of one triangle are equal to the hypotenuse and the corresponding side of the other triangle.

Given : In two right triangles, ABC and DEF

$$\angle B = \angle E = 90^\circ$$

Hypotenuse $\quad AC$ = hypotenuse DF

and $\quad$ side AB = side DE

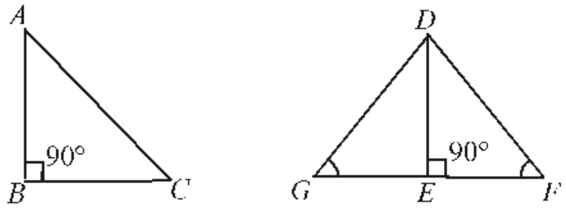


Fig. 7.32

To prove : $\Delta ABC \cong \Delta DEF$

Construction : In ΔDEF, produce E upto G such that $GE = BC$ and join G to D.

Proof : Here, $\angle DEF = 90°$

$\therefore$ and $\angle$ DEG = 90° ...(1)

Now, in ΔABC and Δ DEG

$AB = DE$ (Given)

$BC = GE$ (By construction)

$\angle ABC = \angle DEC = 90°$ [From (i)]

By Side-Angle-Side rule, ΔABC and ΔDEG are congruent. So, their corresponding sides and corresponding angles will be equal.

$\therefore$ $AC = DG$ and $\angle$ C = $\angle$ G ...(2)

But it is given that $AC = DF$ ...(3)

From equations (2) and (3),

$DG = DF$ ...(4)

$\because$ In ΔDGF, angles opposite to equal sides ($DG = DF$) will be equal

$\angle G = \angle F$ ...(5)

From equations (2) and (5), $\angle$ C = $\angle F$ ... (6)

Now, in ΔABC and ΔDEF

$AB = DE$ (Given)

$\angle$ C = $\angle$ F [from (6)]

and $\angle ABC = \angle DEF = 90°$ (Given)

By Angle-Angle-Side rule,

$\Delta ABC \cong \Delta DEF$ **Hence Proved**

Illustrative Examples

Example 13. ***AB* is a line segment and points *P* and *Q* are situated on the opposite side of AB such that each of then is equidistant from *A* and *B*. (see fig. 7.33). Show that line segment *PQ* is the perpendicular bisector of the line segment AB.**

Solution : Here, $PA = PB$ and $QA = QB$ *is* given. We have to show that PQ $\perp$ AB and PQ bisects AB. Let PQ interests line segment AB at C.

You can see two congruent triangles in this figure? Let us take ΔPAQ and ΔPBQ.

AP = BP (Given)

$AQ = BQ$ (Given)

PQ = PQ (Common side)

$\therefore$ $\Delta PQA \cong \Delta PBQ$ (BySSS rule)

So, $\angle$ APQ = $\angle$ BPQ

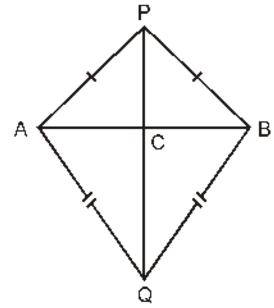


Fig. 7.33

In Δ PAC and Δ PBC, we get

AP = BP (Given)

$\angle$ APC = $\angle$ BPC ($\angle$ APQ = $\angle$ BPQ has been proved)

PC = PC (Common side)

$\therefore$ $\Delta PAC \cong \Delta PBC$

and $\angle$ ACP = $\angle$ BCP

and AC = CB

Now, $\angle$ACP + $\angle$BCP = 180° (Linear pair) ...(1)

So, 2 $\angle$ ACP = 180°

or, $\angle$ ACP = 90° ...(2)

From equations (1) and (2), we conclude that line PQ is perpendicular bisector of AB.

Note that without proving congruence of $APAQ$ and $APBQ$, we cannot show Δ APQ – Δ BPQ, whereas AP – BP (given) PC – PC (common) and $\angle$ PAC = $\angle$ PBC.

(Opposite angles of equal sides in Δ PAB). We obtained this by SSA rule which is not always acceptable for the congruence of triangles and angle is not (included) between the equal sides. Let us take some other examples.

Example 14: *P* is *a* point equidistant from two lines *l* and *m* intersecting at point *A* (see fig 7.34). Show that line *AP* bisects the angle between them.

Solution : It is given that lines l and m intersect at A. Let $PB \perp l$ and $PC \perp m$.

It is given that PB – PC ($\because$ P is equidistant from l and m.)

To Prove : $\angle$ ***PAB*** = $\angle$ ***PAC***

Proof: Now, in Δ PAB and Δ PAC

PB – PC (Given)

$\angle PBA - \angle PCA - 90°$ ($PB \perp l$ and $PC \perp m$)

$PA - PA$ (Common hypotenuse)

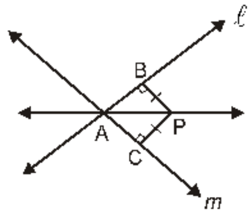


Fig. 7.34

$\therefore$ $\Delta PAB \cong \Delta PAC$ (ByRHSrule)

Therefore $\angle$ PAB = $\angle$ PAC **Hence proved**

Exercise 7.3

1. $\Delta\, ABC$ and $\Delta\, DBC$ are two isosceles triangles on the same base BC such that vertices A and D are situated on the same side of BC (see Fig. 7.35). If AD is extended to intersect BC at P, then show that

   (i) Δ ABD $\cong$ Δ ACD

   (ii) $\Delta\, ABP \cong \Delta$ ACP

   (iii) AP bisects both $\angle$A and $\angle D$

   (iv) AP is the perpendicular bisector of line segment BC.

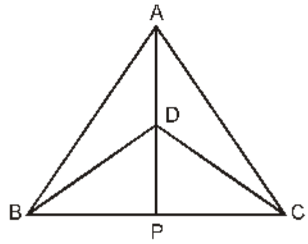


Fig. 7.35

2. AD is an altitude of an isosceles triangle ABC in which $AB = AC$.

   Show that :

   (i) AD bisects line segment BC.

   (ii) AD bisects $\angle$A

3. Two sides AB and BC and median AM of a $\Delta\, ABC$ are respectively equal to the corresponding sides PQ and QR and median PN of other triangle (see Fig. 7.36). Show that:

   (*i*) $\Delta\, ABM \cong \Delta\, PQN$

   (*ii*) $\Delta\, ABC \cong \Delta\, PQR$

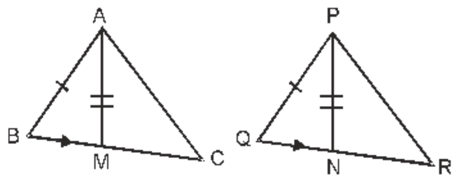


Fig. 7.36

4. BE and CF are two equal altitudes of a triangle ABC. By using RHS rule of congruence, prove that $\Delta\, ABC$ *is* an isosceles triangle.

5. ABC is an isosceles triangle in which $AB = AC$. By drawing $AP \perp BC$, show that $\angle$B = $\angle$C

7.8. Inequalities of a triangle :

In the previous chapter, you have studied about scalene triangles, isosceles triangle and equilateral triangle on the basis of sides of triangle and acute angled triangle, right-angled triangle and obtuse-angled triangle on the basis of angles.

Did you ever think that if measure of sides of triangle are change, then angles also change and if angles of a triangle are change then measure of sides are also change

Why?

Let us try to understand this by the following activity and theorems.

Theorem 7.7. ***If two sides of a triangle are unequal, then angle opposite to longest side is greater than the angle opposite to smaller side.***

Given : In ΔABC, $AB > AC$

To Prove $\angle C > \angle B$

Construction : Draw line CD from vertex C such that $AC - AD$.

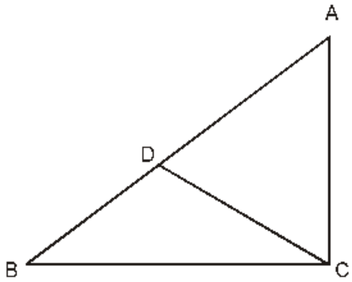


Fig. 7.37

Proof : $AC - AD$ (By construction)

$\therefore$ Their opposite angles will be same.

$\therefore$ $\angle ACD - \angle ADC$ ...(1)

$\because$ $\angle ADC$ is exterior angle of ΔBDC

$\therefore$ $\angle ADC > \angle B$ ...(2)

From equations (1) and (2) $\angle ACD > \angle B$ ...(3)

From figure $\angle ACB > \angle ACD$ ...(4)

From equations (3) and (4), $\angle ACB > \angle ACD > \angle B$

$\Rightarrow$ $\angle ACB > \angle B$

Thus, $\angle C > \angle B$ **Hence Proved**

Theorem 7.8. (Converse of Theorem 7.7)

In a triangle, side opposite to greatest angle is greater than the side opposite to smaller angle.

Given : In ΔABC, $\angle B > \angle C$

To Prove: AC > AB

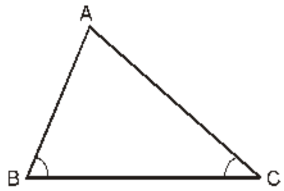


Fig. 7.38

Proof : Following are the three possiblities for sides AC and AB of Δ ABC. Out of them only one is possible.

(i) $AC - AB$

(ii) AC $\angle$ AS, and

(iii) $AC > AB$

Condition : (i) When $AC = AB$

If $AC - AB$, then in Δ ABC, angles opposite to euqal sides will be equal i.e., $\angle B - \angle C$ which is impossible because it is given that $\angle B > \angle C$

Thus, $AC \neq AB$

Condition : (ii) When $AC < AB$

We know that angle opposite to longest side is greater.

$\therefore$ $AC < AB \Rightarrow AB > AC$

$\Rightarrow$ $\angle C > \angle B$

Which is contradiction of given statement

$\therefore$ $AC < AB$

Condition : (iii) Since, AC is neither less than nor equal to AB

AC must $> AB$

Hence, $AC > AB$ is true. **Hence Proved**

Theorem 7.9 : *Sum of any two sides of a triangle is greater than its third side.*

Given : A triangle ABC

To Prove :

(i) $AB \quad BC > AC$

(ii) $BC + AC > AB$

(iii) $AC + AB > BC$

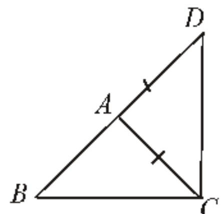


Fig. 7.39

Construction : Produce BA upto D such that AD = AC

Proof: In ADC

$AD - AC$ (By construction)

$\therefore$ Angle opposite to equal sides will be equal.

$\angle ACD = \angle ADC$ ...(1)

and $\angle BCD > \angle ACD$ ...(2)

From equations (1) and (2)

$\angle BCD > \angle ADC - \angle BDC$

$\therefore$ $BD > BC$ [$\because$ Side opposite to greater angle is longer]

Thus $BD > BC$

$\Rightarrow$ $BA \mid AD > BC$ [$\because BD - BA + AD$]

$\Rightarrow$ $BA + AC > BC$ [$\because AD - AC$, by construction]

Similarly, we can prove that

$AB + BC > AC$

$BC \quad AC > AB$ Hence Proved

7.7. Lines and Perpendicular Distance from an External Point:

Distance between a line and its external only point is equal to the length of perpendicular drawn from that point on the line.

Theorem 7.10. ***Out of the all line segments drawn from an external point to a straight line (line segment), then the perpendicular line segment is the shortest.***

AB is a line and C is an external point not lying on it. $CE \perp AB$ and D is any point on AB other than E.

To prove :$CE < CD$

Proof : *In* $\Delta\, CED$

$\angle CED - 90°$ $[\because\ CE \perp AB]$

In $\Delta\, CED,\ \angle CED - 90°$

$\angle CDE \mid \angle CDE\ 90°$

$\Rightarrow$ $\angle CDE < \angle CED$

$\Rightarrow$ $\angle CED > \angle CDE$

$\Rightarrow$ $CD > CE$

[Side opposite to greater angle is longer]

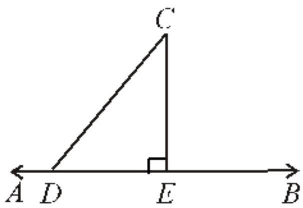


Fig. 7.40

And $\angle DCE < \angle CED$

$\Rightarrow$ $\angle CED > \angle DCE$

$\Rightarrow$ $CD > DE$

It means out of all line segments drawn from an external point to a straight line, the perpendicular line segment is the shortest. **Hence proved**

Illustrative Examples

Example 15 : In Fig. 7.41, AD is the median of Δ ABC, then prove that $AB + AC > 2AD$

Or

Prove that the sum of two sides of a triangle is more than twice the median draw on third side.

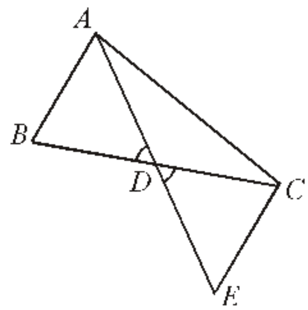


Fig. 7.41

Sol : Given : AD is the median of $\Delta\, ABC$

To Prove : $AB \quad AC > 2\,AD$

Construction : According to figure, produce AD upto E

such that $DE - AD$. Join C to E.

Proof : In $\Delta\, ADB$ and $\Delta\, EDC$

$AD - DE$ (By construction)

$BD - DC$ (Given)

$\angle ADB - \angle EDC$ (Vertically opposite angles)

$\Rightarrow$ By Side Angle Side rule, $\Delta ADB \cong \Delta EDC$

$$AB - CE$$

Now in, Δ ACE

$$AC - CE > AE$$

$\Rightarrow$ $AC + AB > AE$ $[\because CE - AB]$

$\Rightarrow$ $AC - AB > 2\,AD$ $[\because AE - 2AD]$ **Hence Proved**

Example 16. If *ABCD* is a quadrilateral, then prove that

(i) *AB* + *BC* + *CD* + *DA* >2 *AC*

(ii) *AB* + *BC* + *CD* + *DA* > *AC* + *BD*

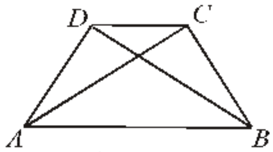


Fig. 7.42

Solution : Given : According to figure 7.42, *ABCD is a* quadrilateral.

To Prove : (i) $AB + BC + CD + DA > 2\,AC$

(ii) $AB - BC + CD - DA > AC - BD$

Construction : Join diagonals *AC* and *BD*.

Proof : We know that the sum of two sides of a triangle is more than the third side. So

In Δ ABC, AB + BC > AC ...(1)

In Δ *ADC,* $AD - DC > AC$ ...(2)

In Δ ABD, $AB + AD > BD$ ...(3)

In Δ BCD, $BC + CD > BD$ ...(4)

Adding (1) and (2), we get

AB + BC + AD + CD > 2AC ...(i)

Again, adding (1), (2), (3) and (4), we get

$$2(AB + BC - AD - DC) > 2\,(AC + BD)$$

$\Rightarrow$ AB + BC + AD + DC > AC + BD ...(ii) **Hence Proved**

Example 17 : In Fig. 7.43, *O* is any point in the interior of ΔABC, then prove that AB + AC > *OB* + *OC*.

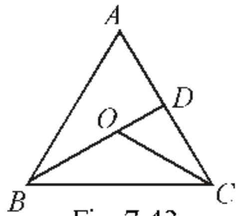


Fig. 7.43

Solution : Given : *O* is an interior point in ΔABC.

To Prove : AB + AC > OB + *OC*

Construction : Produce *BO,* which meets AC at *D*.

Proof : We know that in a triangle, sum of two sides is more than the third side.

$\therefore$ *In* Δ *ABD*, $AB + AD > BD$

$\Rightarrow$ $AB + AD > OB + OD$ ...(1)

Similarly, in $\Delta OCD, OD + DC > OC$ ...(2)

Adding (1) and (2), we get

$$AB + AD + OD - DC > OB - OD + OC$$

$\Rightarrow$ $$AB + (AD + DC) > OB + OC$$

$\Rightarrow$ $$AB + AC > OB + OC$$ **Hence Proved**

Important points

In this chapter, you have studied the following points.

1. Two figures are congruent, if they have same size and same measure.
2. Two circles of same radius are congruent.
3. Two squares of same sides are congruent.
4. If ΔABC and ΔPQR are congruent under the correspondence $A \leftrightarrow P, B \leftrightarrow Q$ and $C \leftrightarrow R$, then in notation form, they are written *as* $\Delta ABC \cong \Delta PQR$.
5. If two sides and one included angle of one triangle are equal to the corresponding two sides and included angle of other triangle, then two triangles are congruent. (SAS rule of congruence)
6. If two angles and included side of a triangle are equal to the corresponding two angles and included side of other triangle, then the two triangles are congruent. (ASA rule of congruence)
7. If two angles and one side of a triangle are equal to the corresponding two angles and one side of other triangle, then two triangles are congruent. (AAS rule of congruence)
8. Angles opposite to equal sides of a triangle are equal.
9. Side opposite to equal angles of a triangle are equal.
10. Each angle of an equilateral triangle is 60°.
11. If the three sides of a triangle are equal to corresponding three sides of other triangle, then two triangles are congruent. (SSS rule of congruence)
12. 'If in two right triangles, hypotenuse and one side of a triangle are equal to hypotenuse and one side of other triangle, then two triangles are congruent. (RHS rule of congruence)
13. In a triangle, angle opposite to longer side is greater.
14. In a triangle, side oposite to greater angle is longer.
15. In a triangle, sum of two sides is greater than the third side.

Miscellaneous Exercise - 7

Multiple Choice Questions (1 to 16)

1. Which one of the following is not the condition of congruence of triangles.

   (a) SAS (b) ASA (c) SSA (d) SSS

2. If $AB - QR, BC - PR$ and $CA = PQ$, then :

   (a) $\Delta ABC \cong \Delta PQR$ (b) $\Delta CBA \cong \Delta PRQ$

   (c) $\Delta BAC \cong \Delta RPQ$ (d) $\Delta PQR \cong \Delta BCA$

3. In $\Delta\, ABC,\ AB - AC$ and $\angle B - 50°$, then $\angle C$ is equal to :

   (a) 40° (b) 50° (c) 80° (d) 130°

4. In $\Delta\, ABC,\ BC - AB$ and $\angle B - 80°$, then $\angle A$ is equal to :

   (a) 80° (b) 40° (c) 50° (d) 100°

5. In $\Delta\, PQR,\ \angle R - \angle$P and QR = 4cm and $PR - 5$ cm, then lenght of PQ is :

   (a) 4 cm (b) 5 cm (c) 2 cm (d) 2.5 cm

6. D is a point situated on side BC of Δ ABC, such that AD bisects $\angle BAC$, then.

   (a) $BD - CD$ (b) $BA > BD$ (c) $BD > BA$ (d) $CD > CA$

7. It is given that $\Delta\, ABC - \Delta\, FDE$ and $AB - 5$ cm, $\angle B - 40°$ and $\angle A - 80°$. Which one of the following is true?

   (a) DF = 5 cm, $\angle$ F = 60° (b) $DF - 5cm, \angle E - 60°$

   (c) DE = 5cm, $\angle$ E = 60° (d) $DE - 5cm,\ \angle D - 40°$

8. Lenghts of two sides of a triangle are 5 cm and 1.5 cm. Following cannot be the length of its third side.

   (a) 3.6 cm (b) 4.1 cm (c) 3.8 cm (d) 3.4 cm

9. In Δ PQP if $\angle$ R $\angle$Q then

   (a) $QR > PR$ (b) $PQ > PR$ (c) $PQ < PR$ (d) $QR < PR$

10. In triangles ABC and $PQR,\ AB - AC,\ \angle C - \angle P$ and $\angle B - \angle Q$. These triangles are :

    (a) Isosceles but not congruent (b) Isosceles and congruent

    (c) Congruent but not isosceles (d) Neither congruent nor isosceles

11. In triangles ABC and DEF, $AB - FD$ and $\angle A - \angle D$. Two triangles are congruent by SAS rule if :

    (a) $BC - EF$ (b) AC = DE (c) $AC - EF$ (d) $BC - FE$

12. In right triangle ABC, if $\angle C$ is a right angle then longest side will be.

    (a) AB (b) 5C

(c) CA (d) None of theite

13. Difference of two sides of a triangle to its third side is :

(a) More (b) same (c) less (d) half

14. If two sides of a triangle are unequal, then angle opposite to longest side is :

(a) Greater (b) smaller (c) equal (d) half

15. The perimeter of triangle is then sum of its medians.

(a) More (b) Less (c) Equal (d) Half

16. The sum of three altitudes of triangles is its perimeter :

(a) more (b) equal (c) half (d) less

17. If in ΔABC, $AB - AC$ and $\angle A < 60°$ then write the relation between BC and AC.

18. In Fig 7.44, write the relation between AB and AC.

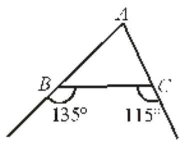


Fig. 7.44

19. In any triangle ABC, $\angle A > \angle B$ and $\angle B > \angle C$, then what will be the smallest side?

20. Find all the angles of an equilateral triangle.

21. P is any point lie on the bisector of $\angle$ABC. If through P, a line is drawn parallel to BA meets line BC at Q then prove that ΔBPQ is an isosceles triangle.

22. ABC is a right angled triangle in which $AB - AC$. Bisector of $\angle$ A meets BC at D. Prove that $BC - 2\,AD$.

23. ΔABC and ΔDEC are lie on the same base BC such that points A and D are on the opposite side of BC, and $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC.

24. ABC'is an isosceles triangle, in which $AC - BC$. AD and BE are altitudes on BC and AC respectively. Prove that $AE - BD$.

25. Prove that sum of any two sides of a triangle is more than twice the corrosponding median of third side.

26. In a ΔABC, D is the mid-point of side AC such that $BD - \frac{1}{2}AC$. Show that $\angle ABC$ is a right angle.

27. In a right triangle prove that line segment joining the mid point of hypotenuse to its opposite vertex is half of the hypotenuse.

28. In Fig. 7.45, if $AB - AC$, then write the relation between ABand AD.

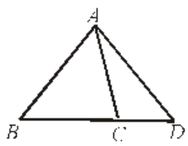


Fig. 7.45

29. AD is a median of ΔABC. Is it true that AB + BC + CA >$2AD$. Give reason for you answer.

30. M is any point on side BC of ΔABC such that AM is the bisector of $\angle BAC$. Is it true that perimeter of triangle is more than 2 AM? Give reason for your answer.

31. Q is any point situated on side SR of ΔPSR such that $PQ - PR$. Prove that : PS > PQ.

32. S is any point situated on side QR of ΔPQR. Show that $PQ - QR + RP > 2PS$

33. D is any point situated on side AC of Δ ABC, with AB = AC. Show that $CD < BD$.

34. In Fig. 7.46. $\angle B > \angle A$ and $\angle D > \angle E$, then prove that $AE > BD$.

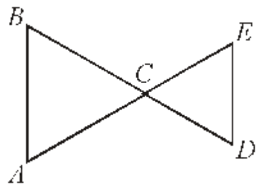


Fig. 7.46

35. In ΔABC, $AB > AC$ and D is any point on side BC prove that $AB > AD$.

36. Prove that sum of three sides of a triangle is more than the sum of its three medians. [Hint: use example 1]

37. In Fig. 7.47. O is the interior point in a triangle, then prove that :
$(BC + AB - AC) < 2\ (OA - OB -$ OC)

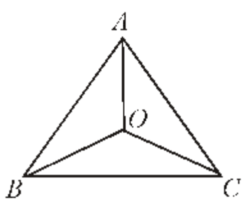


Fig. 7.47

38. Prove that sum of three altitudes of triangle is less then perimeter of triangle.

39. Prove that difference of any two sides of any triangle is less than its third side.

40. Bisectors of $\angle B$ and $\angle C$ of an isosecles triangle with AB = AC, intersect each other at point O. show that adjacent $\angle ABC$ is equal to exterior $\angle BOC$.

41. In Fig. 7.48 AD is the bisector of $\angle BAC$. Prove that AB > BD.

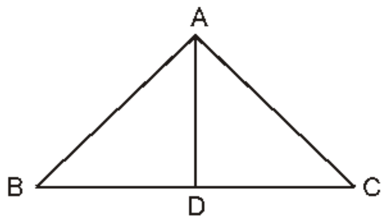


Fig. 7.48

Answer

Exercise 7.1

1. QR: These will be congruent by ASA

2. RP: These will be congruent by AAS

3. No, angle should be between the two sides.

4. No, sides should be corresponding.

5. No, BC should be equal to PQ

6. Yes, these are corresponding sides.

Exercise 7.2

1. 64° **3.** 90°

Miscellaneous Exercise -7

1. C	**2.** B	**3.** B	**4.** C	**5.** A	**6.** B
7. B	**8.** D	**9.** B	**10.** A	**11.** B	**12.** A
13. C	**14.** A	**15.** A	**16.** D		

17. BC < AC

18. AB > AC

19. AB

20. 60, 60, 60

28. AD > AB

29. Yes AB + BD > AD and AC + CD > AD

30. Yes AB + BM >AM and AC + CM > AM

8

Construction of Triangles

8.1 Introduction

Every triangle has six components—three sides and three angles. We have already studied about the essential condition for congruency of triangles. To construct a triangle three components are required i.e. :

(i) Three sides, or

(ii) Two sides and angle between them, or

(iii) Two angles and one side, or

(iv) In right angled triangle hypotenuse and one side

Remark :

(i) If all three angles are given, then construction of triangle is not possible,

(ii) If two sides and their one opposite acute angle be given, then construction of triangle is in ambiguous situation.

In this chapter, we will learn to construct triangles in various conditions.

Construction 8.1 : Construction of triangle whose three sides are given

Construct a triangle ABC whose sides a = 5 cm, b = 4 cm and c – 3.5 cm

Here, 'a' stands for side opposite to $\angle A$; 'b' stands for side opposite to $\angle B$; 'c' stands for side opposite to $\angle C$.

Before the construction of the triangle, we will draw a rough sketch of the triangle of given measurements. On the basis of that we will draw the required triangle.

Construction :

(i) Draw a line segment $BC - a = 5$ cm

(ii) Taking B as centre and 3.5 cm as radius draw an arc.

(iii) Taking C as centre and 4 cm as radius draw another arc which cuts the previous arc at A.

(iv) Join A to B and C

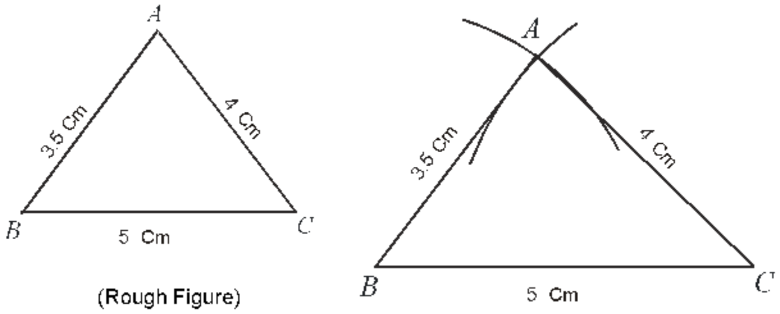


Fig. 8.01

Thus, ABC is the required triangle.

Exercise 8.1

1. Construct a triangle ABC in which $AB = 4$ cm, $BC - 5$ cm and $CA - 6$ cm.
2. Two points A and B are at a distance 6.5 cm to each other. Find the position of point C which is at a distance of 7 cm and 6 cm from points A and B respectively.
3. Construct a triangle ABC in which sides $a = 6.5$ cm, $b = 7.2$ cm and $c = 8$ cm. Draw the bisector of $\angle$B, which meets AC at point M.
4. Construct a Δ ABC such that $a - 7$ cm, $b = 5$ cm and $c - 4$ cm. Draw a perpendicular from A on BC.
5. Construct an equilateral triangle ABC whose each side is 5.5 cm.
6. Construct an isosceles triangle whose base is 3 cm and equal sides are 5 cm.

Construction 8.2

Construction of a triangle whose two sides and angle between them is given

Construct a triangle ABC whose sides a = 4.5 cm, $c - 3.5$ cm and $\angle$B $-$ 45°.

Construction :

(i) Draw a line segment $BC - a - 4.5$ cm

(ii) At point B with the help of ruler and compass draw $\angle$DBC = 45° with BC

(iii) Taking B *as a* centre and 3.5 cm as radius, draw an arc cutting BA from BD.

(iv) Join A to C.

Thus, Δ ABC is the required triangle.

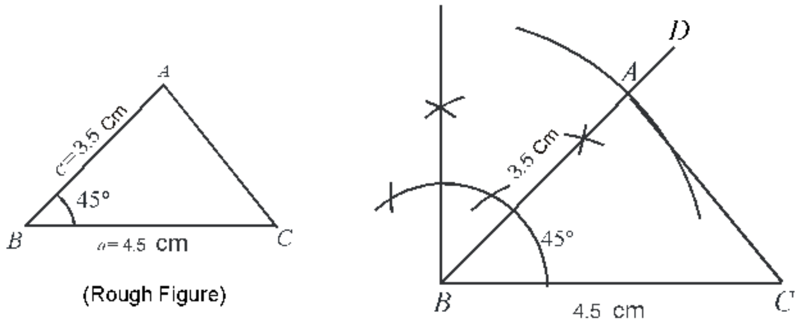


Fig. 8.02

Exercise 8.2

1. Construct a triangle ABC in which $a = 4$ cm, $b = 5$ cm and $\angle C = 60°$.
2. Construct a triangle LMN in which $\angle L = 120°$, $LM - 4$ cm, $LN - 5$ cm.
3. Construct a triangle ABC in which side AB = AC = 8 cm and $\angle A - 75°$. Also draw the bisector of $\angle B$ which meets the opposite side.
4. Construct an isosceles triangle whose vertex angle is 120° and each equal sides is of length 5.5 cm.

Construction 8.3

Construction of a triangle whose one side and two angles are given :

Construct a ΔABC in which $\angle B - 60°$, $\angle C = 75°$ and side $BC - 4.6$ cm.

Construction :

(i) Draw a line segment $BC = 4.6$ cm

(ii) With the help of ruler and compass, at point B, draw $\angle EBC - 60°$ and at point C, draw $\angle FCB = 75°$ with BC.

(iii) BE and CF intersect at point A. Thus, Δ ABC is the required triangle.

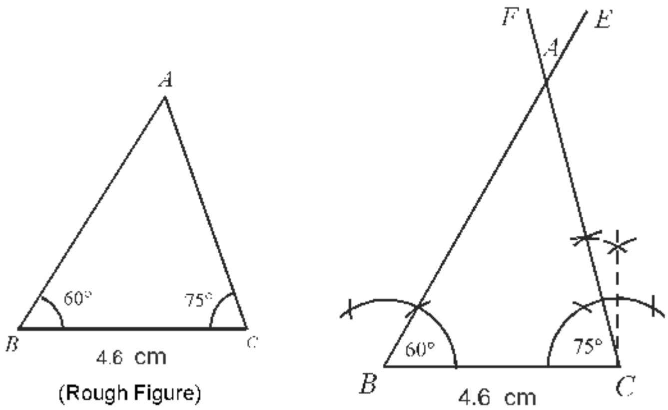


Fig. 8.03

Remark :

If one angle is given at the ends B and C on side BC and a vertex angle is given, then to find the base angle, we can subtract the sum of base and vertex angle from 180° and can get the other base angle, then construct the triangle by above geometric construction.

Exercise 8.3

1. Construct a triangle PQR in which QR – 8 cm, $\angle Q = 120°$ and $\angle R = 30°$.
2. Construct a triangle ABC in which b – 7 cm, $\angle$ A = 90° and $\angle$ C = 60°.
3. Construct an isosceles triangle whose base is 4 cm and vertex angle is 30°. Draw perpendicular from the vertex on base.

Construction 8.4

Construction of a right angled triangle in which hypotenuse and one other side be given.

Construct a right-angled triangle whose hypotenuse $AC = 6.5$ cm and side AB – 3.5 cm.

Construction :

(i) Draw a line segment BD of suitable length and at B draw $\angle DBE$ – 90°.

(ii) Taking B as centre and 3.5 cm as radius, draw an arc which cuts BE at A.

(iii) Taking A as centre and 6.5 cm as radius, draw an arc which cuts BD at C.

(iv) Join A and C.

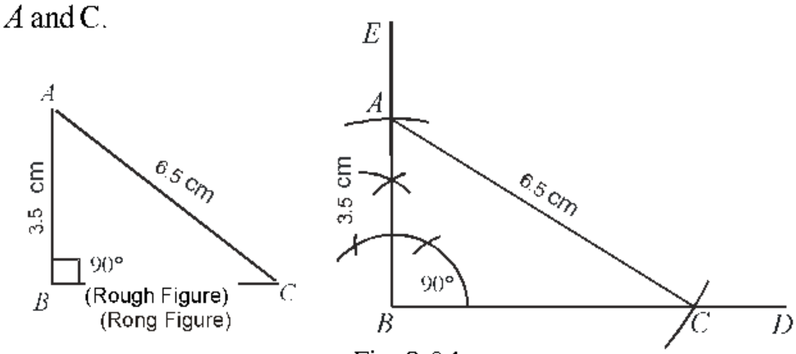


Fig. 8.04

Thus, ABC is the required triangle.

Exercise 8.4

1. Construct a right angled triangle whose hypotenuse is 5 cm and other side is 3 cm.
2. Construct a triangle ABC in which $\angle A$ – 90°, side $AC = 5.4$ cm and hypotenuse BC – 10cm.
3. Construct a right angle triangle ABC in which $\angle A$ – 90° and side a = 10 cm and side b = 6 cm. Draw a perpendicular on hypotenuse from vertex A.

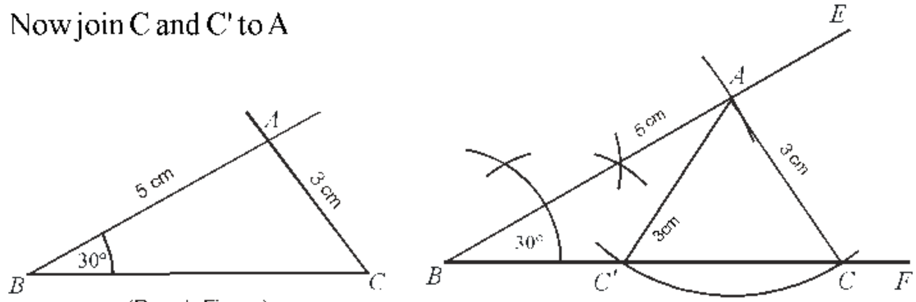
Construction 8.5

Construct of a triangle whose two sides and an angle opposite to one of the given side, is given.

Construct a triangle ΔABC in which $AB = 5$ cm, $AC = 3$ cm, and $\angle B = 30°$.

Construction :

1. Draw a line segment BF. Draw an angle $\angle$EBF = 30° at point B
2. With B as centre and 5 cm as radius, draw an arc which will intersect BE at point A.
3. With A as centre and 3 cm as radius, draw an arc which intersect BF at C and C'. Now join C and C' to A



(Rough Figure)

Fig. 8.05

4. Hence, ΔABC and $\Delta ABC'$ are the required triangles.

Various conditions related to above construction

Condition 1 : When length of side b is less than the length of perpendicular p drawn from A to BF. Then taking A as centre and radius as b, drawn arc does not cut BF. In this condition construction of triangle is not possible (Fig 8.06)

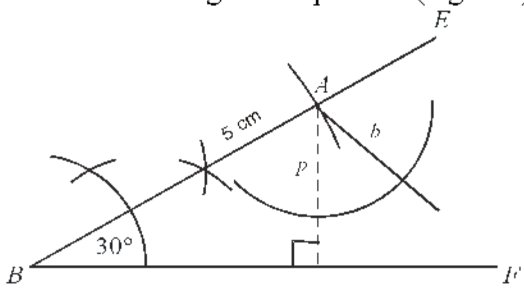


Fig. 8.06

Condition 2 : When length of side b is equal to the length of perpendicular p, then taking A as centre and b as radius, draw an arc will touch BF. In this condition only one triangle can be constructed which will be right-angled triangle (Fig. 8.07).

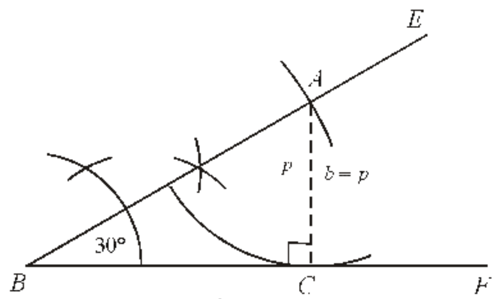


Fig. 8.07

Condition 3 : If side b is greater than perpendicular p and equal to c, then an isosceles triangle can be made in which side AB and AC are equal (Fig. 8.08).

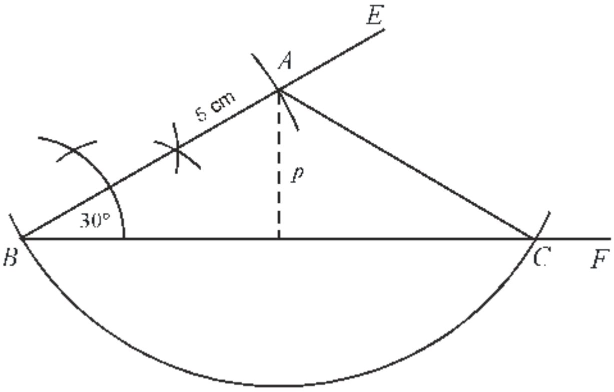


Fig. 8.08

Condition 4 : If side b, is greater than both the perpendicular p and side c, then arc will intersects line BF towards the right side of point B, at point C. In this condition, only one triangle ABC can be constructed. In this condition when arc CB *is* extended then it will intersect at C'. But Δ ABC' is not a triangle because in which $\angle ABC' \neq 30°$ (Fig. 8.09).

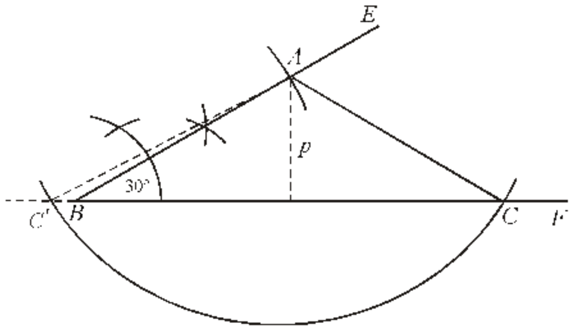


Fig. 8.09

Condition 5 : If side b is greater than perpendicular p and smaller than side c, then arc will intersect BF at different points, and so ΔABC and Δ ABC' are required triangles shown as in (Fig. 8.05)

Remark:

From above conditions, it is clear that, if $\angle$ B is not acute angle, then there is only one triangle is possible because in this condition side b is greater than side c. If $\angle$ B is an acute angle, then above any condition may be possible.

Exercise 8.5

1. Construct a $\angle$ XYZ in which $\angle$ XYZ = 60°, XY = 5 cm and XZ = 4.5 cm. How many triangles of such type can be drawn?
2. Construct a triangle PQR in which $\angle$ PQR = 45°, side PQ = 6 cm and PR = 5 cm.

3. Construct a triangle ABC in which a = 5.4 cm, b = 6.8 and $\angle A = 45^\circ$. Also find the measurement of side AB.

8.02 Difficult Construction of Triangles

Illustrative Example

Example 1. Construct a triangle ABC, when base BC = 6 cm, $\angle ABC = 60^\circ$ and AB + AC = 7 cm.

Solution : Draw a rough sketch of given measurements. Extend BA to point D such that AC = AD so BD = BA + AC. Join D to C Draw perpendicular bisector of DC. Then find the Δ ABC.

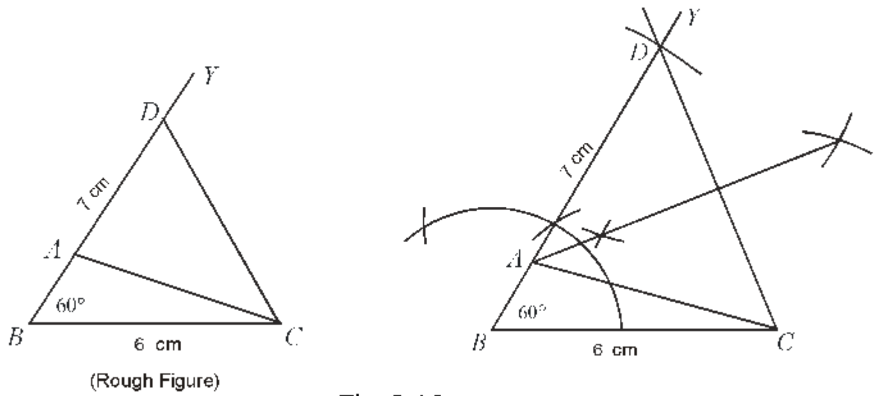


Fig. 8.10

Construction :

(i) Draw the BC = 6 cm and at point B, construct $\angle YBC = 60^\circ$.

(ii) Cut a line segment BD = 7 cm equal to (BA + AC) from BY.

(iii) Join D to C

(iv) Draw perpendicular bisector of DC which intersects BD at point A.

(v) Join A and C.

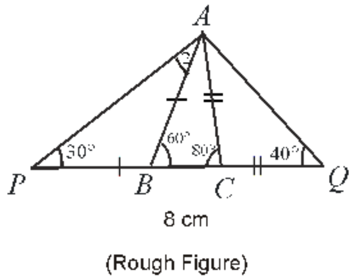
Thus, ABC is required triangle in which BA + AC = 7 cm

Note : A lies on the perpendicular bisector of CD, so AD = DC.

Example 2. Construct a triangle in which AB + BC + CA = 8 cm, $\angle B = 60^\circ$ and $\angle C = 80^\circ$

Solution : Draw the rough sketch with the given measurements

Extend BC in both side such that BP = AB and CQ = AC. Join AP and AQ, obtain Δ ABP and Δ ACQ.



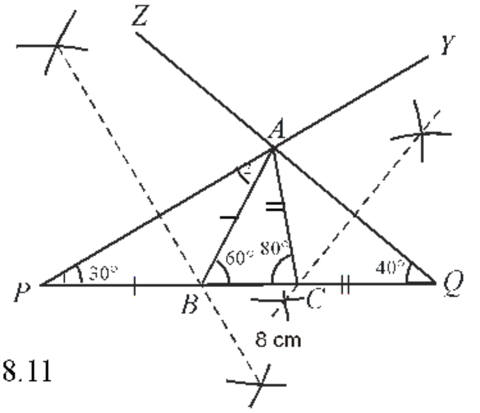


Fig. 8.11

Construction :

(i) Draw a line segment PQ = 8 cm.

(ii) At point P and Q, construct $\angle$YPQ = 30° and $\angle$ZQP = 40° respectively.

(iii) PY and QZ intersects at point A

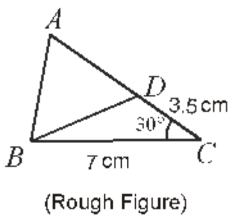
(iv) Draw perpendicular bisectors of AP and AQ which intersects PQ at point B and C respectively.

(v) Join AB and AC (Fig. 8.11)

Thus, ABC is the required triangle.

Example 3. Construct a triangle AB in which BC = 7 cm, b – c = 3.5 cm and $\angle$ C = 30°

Solution : Draw a rough sketch figure with the given dimensions.



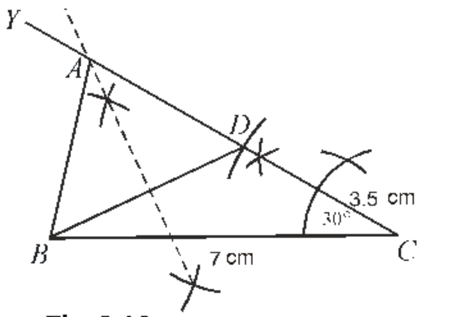


Fig. 8.12

In the figure, AC is bigger then AB. To get required triangle cut line segment AB from AC.

Steps of construction :

(i) Draw a line segment BC = 7 cm and at point C, Construct $\angle$YCB = 30°.

(ii) Cut the line segment CD equal to b – c = 3.5 cm from CY.

(iii) Join BD and draw the perpendicular bisector of BD.

(iv) Let it intersect CY at point A. Join AB.

Thus, ABC is the required triangle.

Example 4. Construct a triangle ABC in which BC = 6 cm, and medians AD and CF are 9 cm and 7.5 cm respectively.

Solution : Draw the rough sketch with the given measurements.

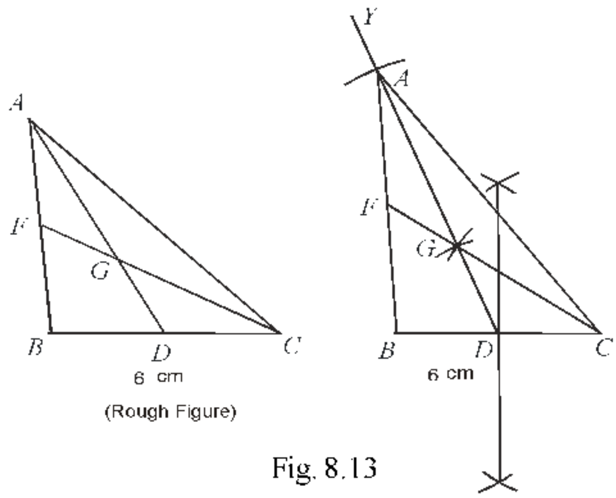


Fig. 8.13

$DC = \frac{1}{2}BC = \frac{1}{2}\times 6 = 3$ cm

$GC = \frac{2}{3}CF = \frac{2}{3}\times 7\cdot 5 = 5$ cm

$GD = \frac{1}{3}\times 9 = 3$ cm

Construction :

Draw $BC = 6$ cm. Bisect BC at D. Taking D as a centre and radius $\left[GD = \frac{1}{3}AD\right]$ $= 3$ cm, draw an arc. Similarly, taking C as centre and radius $\left[GC = \frac{2}{3}CF\right]$ $= 5$ cm, draw another arc, which will intersect the first arc at point G.

Extend DG such that $DA = 9$ cm. Join AB and AC.

Thus, ABC is the required triangle.

Example 5. Construct a triangle ABC whose medians are 3.6 cm, 2.7 cm and 4.2 cm respectively.

Solution : Draw a rough sketch with given measurement

From figure, $OB = \frac{2}{3}\times BE = \frac{2}{3}\times 2\cdot 7 = 1\cdot 8$ cm

$OC = \frac{2}{3}\times CF = \frac{2}{3}\times 4\cdot 2 = 2\cdot 8$ cm

$OA = \frac{2}{3}\times AD = \frac{2}{3}\times 3\cdot 6 = 2\cdot 4$ cm

Complete the figure by extending AD in such a way that $KC = OB$. Find the mid-point of OK say D and complete the triangle ABC.

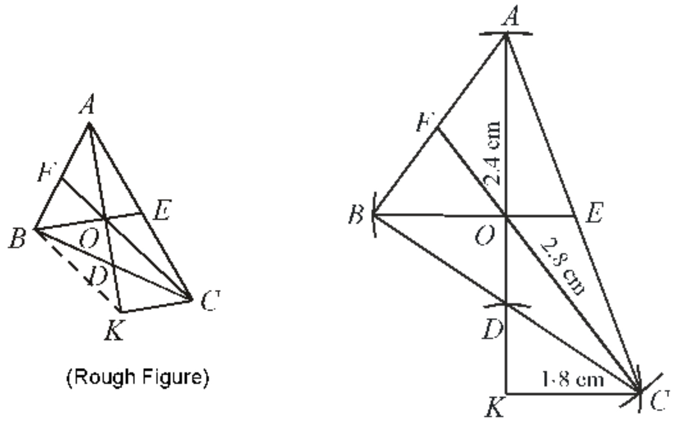


Fig. 8.14

Construction :

Construct triangle OKC in such a way that $OK - AO = 2.4$ cm, $OC = 2.8$ cm and $KC - OB - 1.8$ cm. Find the mid-point of OK say D. Extend KD forward such that $AD - 3.6$ cm. Extend CD backward such that $CD - BD$. Join AB and AC.

Thus, ΔABC is required triangle whose median $AD = 3.6$ cm, $BE - 2.7$ cm and $CF -$ 4.2 cm.

Exercise 8.6

1. Construct a triangle ABC in which $BC - 7$ cm, $\angle C = 50°$ and $AC \quad AB - 8$ cm.
2. Construct a triangle PQR in which $PQ - 6$ cm, $\angle Q = 60°$ and $PQ - PR = 8$ cm.
3. Construct a triangle PQR in which $QR = 5$ cm, $\angle R - 40°$ and $PR - PQ - 1$ cm.
4. Construct a triangle ABC whose perimeter is 12 cm and base angles are 50° and 70°.
5. Construct a triangle ABC in which $BC = 6$cm, medians AD and CF are 6 cm and 7.5 cm respectively.
6. Construct a triangle ABCwhose, three medians are 3 cm, 2.7 cm and 6 cm respectively.

Important Points

To construct a triangle, following components are necessarily known:

(i) All three sides (side-side-side).

or

(ii) Two angles and one side (angle-side-angle).

or

(iii) Two sides and angle between them (side-angle-side).

or

(iv) In right angled triangle, hypotenuse and one side. (hypotenuse- side)

2. In the following conditions, construction of triangle is not possible,

(i) If all three angles are given.

(ii) If two sides and one of the angles opposite to them be acute angle given (In this condition, two triangles can be made but the final triangle cannot be decided) therefore, condition is not clear.

Miscellaneous Exercise-8

1. Construct a triangle whose perimeter is 12 cm and ratio of sides is 1 : 2 : 3.
2. Construct a triangle ABC in which $\angle B - 90°$, $\angle C - 60°$ and $c - 5$ cm.
3. Construct a right angled triangle ABC in which hypotenuse BC is 8.2 cm and one side is 4.2 cm
4. Construct a triangle ABC *in* which $\angle B - 45°$, $\angle C - 60°$ and perpendicular from A on BC is AD and its length is 4 cm.
5. Construct a triangle ABC in which $a - 5.6$ cm, $b \quad c - 10.2$ cm and $\angle B \quad \angle C - 30°$

   (Hint: Vertex angle B is $90° + \frac{1}{2}(\angle B - \angle C) = 105°$)
6. Construct a triangle whose all three medians are 4.2 cm, 4.8 cm and 5.4 cm.
7. Construct an isosceles triangle whose height is 6 cm and equal sides are 7 cm each. Find the measurement of base.

Answers

Exercise 8.5

3. 2.8 cm, 6.8 cm

Miscellaneous Exercise-8

7. 7 cm

9

Quadrilateral

9.01 Introduction

In chapters 5 and 6 you have studied, about many properties of triangles. You know that a triangle is formed by joining three non-collinear points.

Now, we mark the groups of 4-4 points on paper and join them one by one in some order and see that how many possible figures can be found?

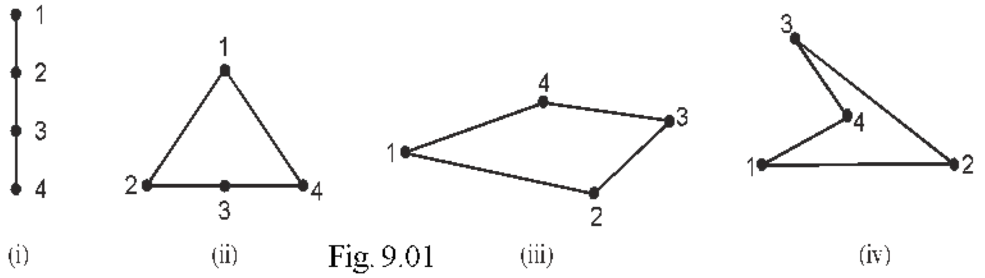


Fig. 9.01

Possible figures like 9.01 (i), (ii), (iii) and (iv) can be formed. In this chapter, we will study figures like 9.01 (iii), which we call quadrilateral.

9.02 Quadrilateral

A figure enclosed by four line segments is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices. Like in Fig. 9.02, $PQRS$ is a quadrilateral where PQ, QR, RS and SP are four sides, P, Q, R and S are four vertices and $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are four angles.

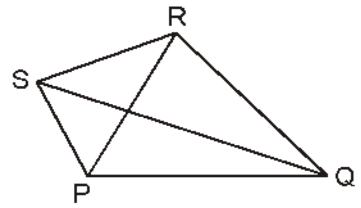


Fig. 9.02

Opposite Sides and Opposite Angles : In Fig. 9.02 RS is the opposite side of PQ and QR is the opposite side of PS. $\angle P$ is opposite of $\angle R$ and $\angle Q$

is opposite of $\angle$ S.

In Fig. 9. 02, pair of **adjacent sides** are PQ, QR and PS, SR. Similarly, SP, PQ and SR, RQ are also pair of adjacent sides.

Diagonal : Line joining opposite vertices is called diagonal. In Fig 9.02, PR and QS are the diagonals of quadrilateral $PQRS$.

9.03 Sum of the Angles of Quadrilateral

Sum of all four angles of a quadrilateral is 4 right angles (360°). We have learnt this property of quadrilateral in chapter 5 through Corollary 4.

9.04 Types of Quadrilateral

- **Kite** : In Fig 9.03 $WXYZ$ is a quadrilateral whose two pairs of adjecent sides *i. e.*, WX, XY and WZ, YZ are equal. It is called kite. Such quadrilateral whose two pair of adjacent sides are equal, is called kite.

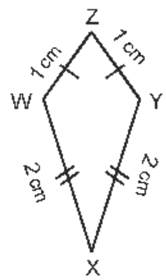


Fig. 9.03
Kite

- **Trapezium** : In Fig 9.04, $ABCD$ is a quadrilateral whose one pair of opposite sides AB and DC are parallel. This quadrilateral is known as trapezium.

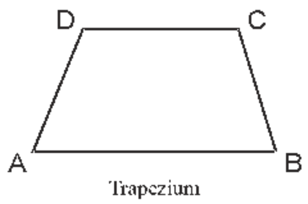


Fig. 9.04

- **Parallelogram :** In Fig. 9.05, $PQRS$ is a parallelogram whose two pair of opposite sides PQ, RS and PS, QR are parallel

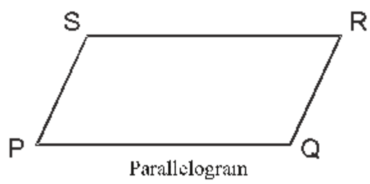
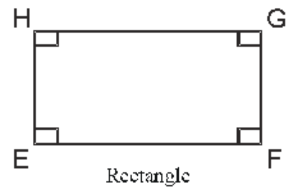


Fig. 9.05

A parallelogram is a trapezium but a trapezium is not a parallelogram.

- **Rectangle :** In Fig 9.06, $EFGH$ is a special parallelogram called rectangle whose each angle is 90°.



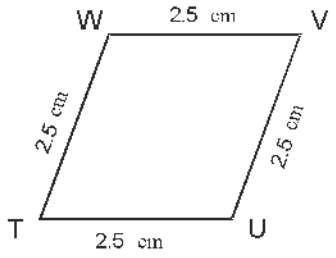
Rectangle

Fig. 9.06

A rectangle is a parallelogram but a parallelogram is not necessarily a rectangle.

A rectangle is a trapezium but a trapezium is not rectangle.

- **Rhombus :** In fig. 9.07, *TUVW* is a special parallelogram called rhombus, whose each side is equal in measure.



Rhombus

Fig. 9.07

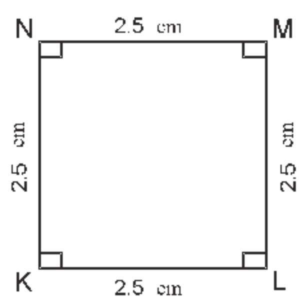
Such parallelogram whose each side is equal, is known as rhombus.

A rhombus is a parallelogram but a parallelogram is not necessarily a rhombus.

A rhombus is a parallelogram, but a parallelogram is not a rhombus.

- **Square** In Fig, 9.08, *KLMN* is a special rectangle called square whose all sides are equal or a special parallelogram whose each side is equal and each angle is 90°.

A square is a trapezium but a trapezium is not a square. A square is a parallelogram but a parallelogram is not necessarily a square. A square is a rectangle but a rectangle is not necessarily a square. A square is a rhombus but a rhombus is not necessarily a square.



Square

Fig. 9.08

9.05 Properties of Parallelogram

Theorem 9.1. ***The diagonal of a parallelogram divides it into two congruent triangles.***

Given : $ABCD$ is a parallelogram and BD its diagonal.

To Prove : $\Delta ABD \cong \Delta BCD$

Proof: In Fig. 9.09, $ABCD$ *is* a parallelogram.

$AB \parallel CD$ and BD is a transversal

$\therefore$ $\angle ABD = \angle CDB$ (Alternate interior angles) ...(i)

$AD \parallel BC$ and BD is a transversal

$\therefore$ $\angle ADB = \angle CBD$ (Alternate interior angles) ...(ii)

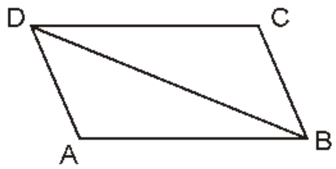


Fig. 9.09

Now, in ΔABD and ΔCDB

$\angle ABD - \angle CDB$ [From equ. (i)]

$BD = BD$ [Common]

$\angle ADB - \angle CBD$ [From equ (ii)]

$\therefore$ $\Delta ABD \cong \Delta CDB$ (By ASA congrunce rule)

Hence proved

Theorem 9.2. ***Opposite sides of a parallelogram are equal.***

Given : In Fig. 9.09, $ABCD$ is a parallelogram.

To Prove : $AB - CD$ and $AD - BC$

Construction : Draw a diagonal BD.

Proof : From theorem 9.1, $\Delta ABD \cong \Delta CDB$

Since, corresponding parts of a congruent triangle are equal.

$AB - CD$ and $AD - BC$ **Hence Proved**

Theorem 9.3. (Converse of Theorem 9.2)

If each pair of opposite sides of a quadrilateral be equal, then it is a parallelogram.

Given : $ABCD$ is a quadrilateral whose opposite sides $AB - CD$ and $BC - AD$.

To prove : $ABCD$ is a parallelogram.

Construction : Join A to C.

Proof: In ΔABC and ΔCDA,

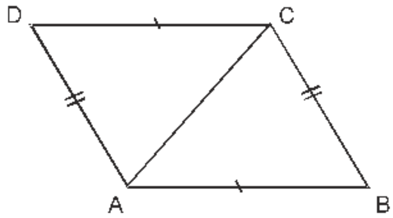


Fig. 9.10

$AB - CD$ (Given)

$BC - AD$ (Given)

$AC = AC$ (Common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By SSS congruence rule]

or $\angle CAB = \angle ACD$ (CPCT)

Transversal line AC intersects two lines AB and CD such that alternate interior angles $\angle CAB$ and $\angle ACD$ are equal

$\therefore$ $AB \parallel DC$ ...(i)

and $\angle ACB - \angle CAD$ (CPCT)

Transversal line AC intersects two lines BC and AD such that interior angles $\angle ACB$ and $\angle CAD$ are equal.

$\therefore$ $BC \parallel AD$ ...(ii)

Thus, from (i) and (ii), quadrilateral $ABCD$ is a parallelogram. **Hence Proved**

Theorem 9.4. *Opposite angles of a parallelogram are equal.*

Given : ABCD is a parallelogram.

To Prove : $\angle A = \angle C$ and $\angle B = \angle D$

Proof: $ABCD$ is a parallelogram.

$AB \parallel DC$ and $AD \parallel CB$

Since, sum of the interior angles on the same side of a transversal is 180°.

$\therefore$ $\angle A + \angle D = 180°$ ...(1)

and $\angle C + \angle D = 180^0$ ...(2)

From (1) and (2)

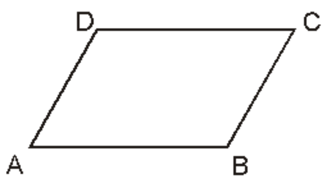


Fig. 9.11

$$\angle A + \angle D = \angle C + \angle D$$

$\therefore$ $\qquad \angle A = \angle C$

Similarly, we can prove $\quad \angle B = \angle D$ **Hence Proved.**

Now, converse of this theorem is also true? Let us prove it.

Theorem 9.5. ***If opposite angles of a quadrilateral are equal, then it is a parallelogram.***

Given : A quadrilateral ABCD in which

$$\angle A - \angle C \quad \text{and} \quad \angle B = \angle D$$

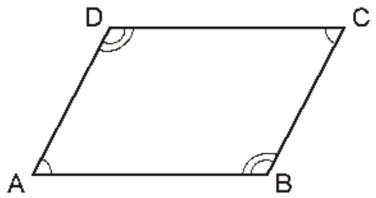


Fig. 9.12

To Prove : ABCD is a parallelogram.

Proof : In a quadrilateral $ABCD$

$\angle A - \angle C$ ...(i)

and $\quad \angle B = \angle D$ ...(ii)

on adding (1) and (2), we get

$\angle A + \angle B = \angle C + \angle D$ ...(1)

But $\quad \angle A + \angle B + \angle C + \angle D - 360^\circ$ ...(2)

From (1) and (2), we get

$$\angle A + \angle B - \angle C \quad \angle D - 180^\circ$$

$$\angle A + \angle B - 180^\circ$$

A transversal line AB intersects two lines AD and BC such that sum of consecutive interior angles is 180°

$\therefore$ $\qquad AD\,|\,BC$

$\angle C + \angle D - 180^\circ \quad \Rightarrow \quad \angle A + \angle D = 180^\circ \qquad [\because \angle C = \angle A]$

A transversal line AD intersects two lines AB and CD such that sum of consecutive interior angles is 180°.

$AB \,||\, DC$ ...(6)

From (5) and (6), we get

Thus, ABCD is a parallelogram. **Hence Proved**

Theorem 9.6. ***Diagonals of a parallelogram bisect each other.***

Given : A parallelogram $ABCD$ whose diagonals AC and BD intersect each other at O.

To Prove : $OA - OC$ and $OB = OD$

Proof : $\because ABCD is$ a parallelogram.

$\therefore$ $AD \parallel$ BC and tansversal BD intersects them.

$\angle ADB - \angle DBC$ (Alternate interior angles)

$\Rightarrow$ $\angle ADO = \angle OBC$ ...(1)

Again $AD \parallel BC$ and transversal AC intersects them

$\angle$ DAC $= \angle BCA$ (Alternate interior angle)

$\Rightarrow$ $\angle DAO = \angle$ BCO ...(2)

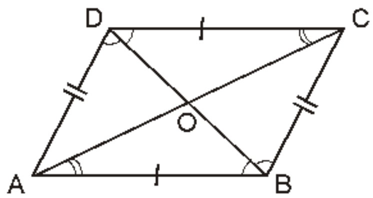


Fig. 9.13

In ΔAOD and ΔCOB

$\angle ADO - \angle CBO$ [From equ (1)]

$AD - BC$ [Opposite side of a parallelogram]

$\angle ADO = \angle BCO$ [From equ (2)]

$\Delta AOD \cong \Delta COB$ [By ASA congruency rule]

Since, corresponding parts of congruent triangles are equal,

so, $OD - OB$

and $OA = OC$ **Hence Proved**

Theorem 9.7. (Converse of Theorem 9.6)

If diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given : A quadrilateral $ABCD$ whose diagonals AC and BD bisect each other at O, *i.e.*, $OA - OC$ and $OB - OD$

To Prove : $ABCD$ is a parallelogram.

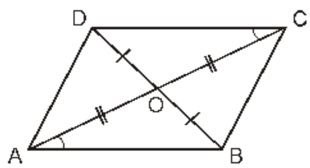


Fig. 9.14

Proof : In ΔAOB and ΔCOD,

$OA - OC$ (Given)

$\angle AOB - \angle COD$ (Vertically opposite angles)

and $OB = OD$ (Given)

$\Delta AOB \cong \Delta COD$ (By SAS congruency rule)

Since, corresponding parts of congruent triangles are equal so

$\angle OAB = \angle$ OCD $\Rightarrow \angle CAB - \angle ACD$

Transversal AC intersects two lines AB and DC such that alternate interior angles CAB and ACD are equal.

$$AB \parallel CD$$

Similarly, we can prove $\quad AD \parallel BC$

$ABCD$ is a parallelogram. **Hence Proved**

Theorem 9.8. ***A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.***

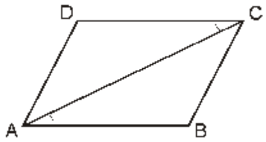


Fig. 9.15

Given : A quadrilateral $ABCD$, in which $AB \parallel DC$ and $AB = DC$

To Prove : $ABCD$ is a parallelogram.

Construction : Join A to C.

Proove : $AB \parallel DC$ and AC is a transversal

$\angle BAC = \angle DCA$ (Alternate interior angles) ...(1)

Now, in ΔABC and ΔCDA

$AB = DC$ (Given)

$\angle BAC = \angle DCA$ [From (1)]

$AC = AC$ (Common)

$\Delta ABC \cong \Delta CDA$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

so $\quad \angle ACB = \angle CAD$

Now, AD and BC both intersects by transversal AC such that alternate angles

$\angle ACB$ *and are equal* $\angle CAD$

$\therefore \quad AD \parallel BC$ ...(2)

and $\quad AB \parallel CD$ (Given)

Thus, $ABCD$ *is* a parallelogram. **Hence Proved**

Illustrative Examples

Example 1. **Two line segments AC and BD bisect each other at point P. Prove that $ABCD$ is a parallelogram.**

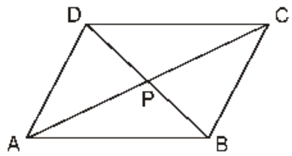


Fig. 9.16

Solution : **Given** AC and BD intersect each other at point P.

To Prove : $ABCD$ is a parallelogram.

Construction : Join AB, BC, CD and DA respectively.

Proof : In ΔAPB and ΔCPD

From figure $ABCD$ is a quadrilateral in which AC and BD are diagonals.

Since, $AP = PC$ and $BP = PD$ (Given)

So, AC and BD bisects each other.

$\therefore \quad$ From theorem 9.7, $ABCD$ is a parallelogram. **Hence Proved**

Example 2. **In a parallelogram ABCD, it is not defined that bisectors of $\angle A$ and $\angle B$ intersect at point P. Prove that $\angle APB = 90°$.**

Solution : Given : In Fig. 9.17, bisectors of adjacent angles $\angle A$ and $\angle B$, intersect at P.

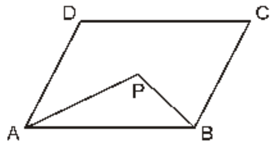


Fig. 9.17

To Prove : $\angle APB - 90°$

Proof : We know that the sum of the adjecent angles of a parallelogram is 180°

$$\therefore \qquad \angle A + \angle B - 180° \qquad ...(1)$$

$$\angle PAB = \frac{1}{2}\angle A \qquad [AP \text{ is the bisector of } \angle A]$$

and $$\angle PBA = \frac{1}{2}\angle B \qquad [BP \text{ is the bisector of } \angle B]$$

$$\therefore \qquad \angle PAB + \angle PBA = \frac{1}{2}(\angle A + \angle B) \qquad ...(2)$$

From (1) and (2), we have

$$\angle PAB + \angle PBA = 90° \qquad ...(3)$$

In Δ PAB, sum of the angles of a triangle is 180°.

$$\therefore \qquad \angle PAB + \angle PBA + \angle APB = 180°$$

From (3) and (4)

$$\Rightarrow \qquad \angle APB = 90°$$ **Hence Proved**

Example 3. Two points P and Q are situated on diagonal BD of a parallelogram $ABCD$ such that $DQ = BP$. Prove $APCQ$ is a parallelogram.

Solution :

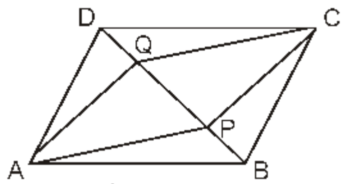


Fig. 9.18

Given : Two points P and Q are situated on diagonal BD of a parallelogram $ABCD$ such that $BP = QD$.

To Prove : $APCQ$ is a parallelogram.

Proof : In ΔAQD and Δ CPB.

$AD = BC$ [Opposite sides a of parallelogram]

$\angle ADQ = \angle CBP$ (Alternate interior angles)

$QD - BP$ (Given)

$\Delta AQD \cong \Delta CPB$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal

$$AQ - CP \qquad ...(1)$$

Similarly, $\Delta CQD \cong \Delta APB$

and $CQ - AP$ ...(2)

From (1) and (2), $APCQ$ is a parallelogram. **Hence Proved**

Example 4. Diagonals of a quadrilateral $ABCD$ intersect each other at point O such that $OA : OC = 3 : 2$. Is $ABCD$ *a* parallelogram? Clearify with reason.

Solution : Given, $OA : OC - 3 : 2$

$$\Rightarrow \qquad OA \neq OC$$

Thus, $ABCD$ is not a parallelogram because diagonals of a quadrilateral $ABCD$ do not bisect each other. **Hence Proved**

Example 5. The angle of a quadrilateral are in ratio 3:4:4:7. Find all the anlges of the quadrilateral.

Solution : Let the angles of a quadrilateral be $3x$, $4x$, $4x$ and $7x$.

Since, the sum of the angles of a quadrilateral is 360°,

$$\therefore \qquad 3x + 4x + 4x + 7x = 360^\circ$$

$$\Rightarrow \qquad 18x = 360^\circ,$$

$$\Rightarrow \qquad x - 20^\circ$$

$\therefore$ Required angles are

$$3x = 3 \times 20^\circ = 60^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

$$7x = 7 \times 20^\circ = 140^\circ$$

Thus, the angles are 60°, 80°, 80° and 140°.

Example 6. A diagonal of a parallelogram bisects its one angle. Prove that this diagonal will also bisect its apposite angle.

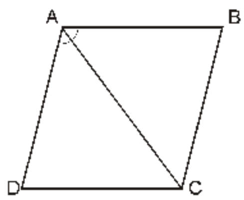


Fig. 9.19

Solution : Let us draw a Fig 9.19 as per given conditions. In which diagonal AC bisects $\angle BAD$ of a parallelogram $ABCD$.

Given: $\angle BAC = \angle DAC$

To prove : $\angle BCA - \angle DCA$

Proof : Given, $AB \quad CD$ and AC is a transversal.

$\therefore \quad \angle BAC - \angle DCA$ (Alternate interior angles) ...(1)

Similarly, $\angle DAC = \angle BCA$ (Alternate interior angles, as $AD \ | BC$)...(2)

But $\angle BCA = \angle DAC$ (Given) ...(3)

From (1), (2) and (3), we get

$$\angle BCA = \angle DCA$$ **Hence Proved**

Example 7. PQ and RS are two equal and parallel line segment. Point M which is not on side PQ or RS, is joined with Q and S respectively. A line parallel to QM and passing through P is drawn, and another line parallel to SM and passing through R is drawn. These lines meet at N. Prove tha line segments MN and PQ are equal and parallel to each other.

Solution : Draw a diagram according to the (Fig. 9.20) given conditions

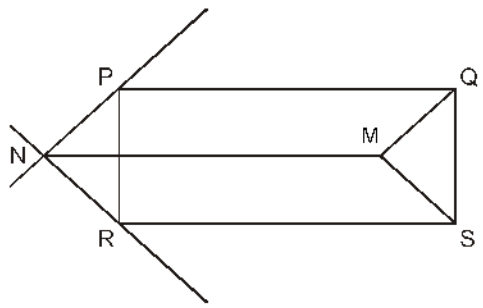


Fig. 9.20

Given that PQ $= RS$ and $PQ \,||\, RS$

$\therefore$ $PQSR$ is a parallelogram.

$\therefore$ $PR = QS$ and $PR \quad QS$ ...(1)

Now $PR \,|\, QS$

$\therefore$ $\angle \text{RPQ} + \angle \text{PQS} = 180^\circ$ (Interior angles of the same side of a transversal)

$\Rightarrow$ $\angle \text{RPQ} + \angle \text{PQM} + \angle \text{MQS} = 180^\circ$ ...(2)

Also $PN \,||\, QM$ (By construction)

$\therefore$ $\angle \text{NPR} + \angle \text{RPQ} + \angle \text{PQM} = 180^\circ$ [From (2) and (3)] ...(3)

$\Rightarrow$ $\angle NPR - \angle RPQ + \angle \text{PQM} = \angle \text{RPQ} + \angle \text{PQM} + \angle \text{MQS}$

$\therefore$ $\angle NPR - \angle MQS$ ...(4)

Similarly, $\angle NRP = \angle \text{MSQ}$ ...(5)

In Δ PNR and Δ QMS

$PR - RS$ [From (1)]

$\angle$ NPR $= \angle$ MQS [From (4)]

$\angle NRP - \angle MSQ$ [From (5)]

$\therefore$ $\Delta \text{PNR} \cong \Delta \text{QMS}$ [By ASA, (1), (4) and (5)]

So, $PN - QM$ and $NR =$ MS

Since, PN || QM, so $PQMN$ is a parallelogram.

Thus, $MN - PQ$ and MN || PQ. **Hence Proved**

Exercise 9.1

1. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.
2. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O, where $OA = 3$ cm and $OD = 2$ cm. Find the lengths of AC and BD.

3. The diagonals of a parallelogram are perpendicular to each other. Is this satement true? Give reason in support of your answer.
4. State whether the angles 110°, 80°, 70° and 95°, are angles of a quadrilateral? Why and why not?
5. Are all the angles of a quadrilateral, be obtuse angles? Give reason for your answer.
6. One angle of a quadrilateral is 108° and other three angles are equal. Find the measure of each of the three equal angles.
7. $ABCD$ is a trapezium in which $AB \quad DC$ and $\angle A - \angle B - 45°$. Find $\angle C$ and $\angle D$ of this trapezium.
8. The angle between two altitudes, drawn from the vertex of an obtuse angle to the opposite sides of a parallelogram is 60°. Find all the angles of this parallelogram.
9. Points E and F lie on diagonal AC of a parallelogram $ABCD$ such that $AE = CF$. Show that $BFDE$ is a parallelogram.
10. In Fig. 9.21, $ABCD$ is a parallelogram. AQ and CP are the bisectors of $\angle$A and $\angle$C respectively. Prove that $APCQ$ is a parallelogram.

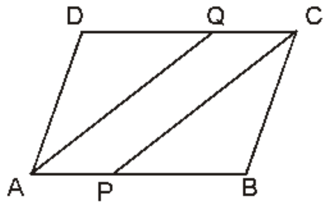


Fig. 9.21

11. In Fig. 9.22, $ABCD$ and $AFEB$ are parallelograms. Prove that $CDFE$ is a parallelogram.

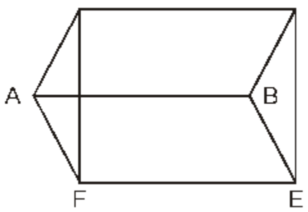


Fig. 9.22

12. The perpendiculars AP and CQ are drawn from points A and C respectively on diagonal BD of a parallelogram $ABCD$. Prove that $AP - CQ$.
13. In Fig. 9.23. $ABCD$ is a quadrilateral in which $AB \| DC$ and $AD - BC$. Prove that $\angle A - \angle B$.

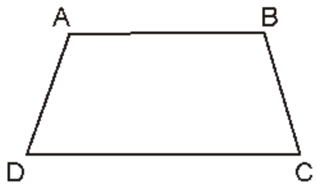


Fig. 9.23

14. In Fig. 9.24, $ABCD$ *is* a parallelogram. P and Q are the mid points of opposite sides AB and CD. Prove that $PRQS$ is a parallelogram.

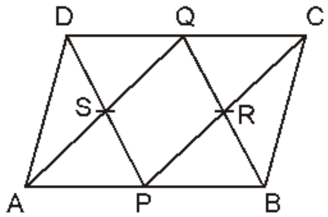


Fig. 9.24

9.06 Special Parallelogram and their Properties

You have studied different types of parallelogram in this chapter and you have also verified some properties.

Let us try to understand some existing properties of those specific parallelograms with the help of some theorems.

Theorem 9.9. ***If diagonals of a parallelogram are equal, then it is a rectangle.***

Given : $ABCD$ is a parallelogram in which $AC - BD$.

To Prove : $ABCD$ is a rectangle.

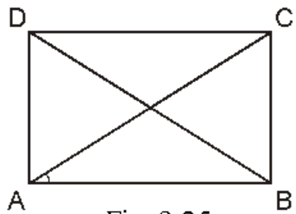


Fig. 9.25

Proof : In ΔABC and ΔBAD

$BC - AD$ (Opp. sides of parallelogram)

$AB - AB$ (Common side)

$AC - BD$ (Given)

By SSS property of congruency

$$\Delta ABC \cong \Delta BAD$$

Since, corresponding angles of congruent triangles are equal,

$\therefore \quad \angle DAB + \angle CBA = 180°$

$\Rightarrow \quad \angle DAB + \angle DAB - 180°$

$\therefore \quad \angle DAB - \angle CBA - 90°$

Thus, $ABCD$ *is* a rectangle. **Hence Proved**

Converse : ***Diagonals of a rectangular are equal to each other.***

Theorem 9.10. ***If the diagonals of a parallelogram are perpendicular to each other, then it is a rhombus.***

Given : Diagonals AC and BD of parallelogram $ABCD$ are perpendicular to each other.

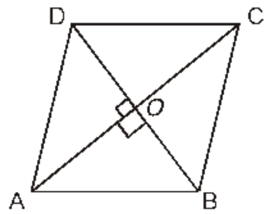


Fig. 9.26

To prove : $ABCD$ is a rhombus

Proof : In ΔAOB and ΔCOB

$OB - OB$ (Common)

$\angle AOB = \angle BOC = 90°$ (Given)

$AO = CO$ (Diagonals of parallelogram bisect each other)

$\Delta AOB \cong \Delta COB$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

$AB = BC$

Thus, $ABCD$ *is* a rhombus. **Hence Proved**

Converse : ***Diagonals of a rhombus are perpendicular to each other.***

Theorem 9.11. ***If the diagonals of a parallelogram are equal and perpendicular to each other, then it is a square.***

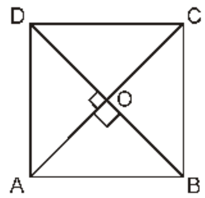


Fig. 9.27

Given : $ABCD$ is a parallelogram where $AC - BD$ and

$AC \perp BD$

To Prove : $ABCD$ *is* a square.

Proof: $\because$ Diagonals of a parallelogram bisect each other

$BO - OD$ ...(1)

In ΔABO and ΔADO

$BO = OD$ [From (1)]

$\therefore$ $\angle AOB - \angle AOD - 90°$ (Given)

$AO - AO$ (Common)

$\Delta ABO - \Delta ADO$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

$\therefore$ $AB - AD$ ...(2)

Now, in ΔABD and ΔBAC

$BD - AC$ (Given)

$AB = AB$ (Common)

$AD = BC$ (Opposite sides of parallelogram)

$\Delta ABD \cong \Delta BAC$ (By SSS congruency)

Since, corresponding parts of congruent triangles are equal,

$\therefore$ $\angle DAB - \angle CBA$ ...(3)

We know that the sum of two adjacent angles of a parallelogram is 180°,

$\therefore$ $\angle DAB + \angle CBA = 180°$ ...(4)

From (3) and (4), we get

$\angle DAB = \angle CBA = 90°$ ...(5)

And from (2) and (5), we get

ABCD is a square. **Hence Proved**

Converse : ***Diagonals of a square are equal and perpendicular to each other.***

9.07 Mid-Poit Theorem

You have studied about some properties of triangles and quadrilaterals. Now let us study another result which is related to the mid-point theorem.

Theorem 9.12. ***A line segment joining the mid points of any two sides of a triangle, is half and parallel to its third side.***

Given : In ΔABC, points D and E are mid-points of sides AB and AC respectively.

To Prove : (i) $DE \,|\, BC$, and (ii) $DE = \frac{1}{2}BC$

Construction : Extend DE to F where $EF = DE$. Join C to F.

Proof : In Δ ADE and Δ CFE

$AE = CE$ (Given)

$\angle AED = \angle CEF$ (Vertically opposite angles)

$DE = EF$ (By construction)

$\Delta ADE \cong \Delta CFE$ (By SAS congruency rule]

Since, corresponding parts of congruent triangles are equal.

$\therefore$ $AD = CF$ ...(1)

and $\angle EAD = \angle ECF$

Transversal AC intersects lines AB and CF such that alternate angles EAD and ECF are equal.

$\therefore$ $AD \,||\, CF$ and $BD \,||\, CF$

But $AD = BD$ (Given) ...(3)

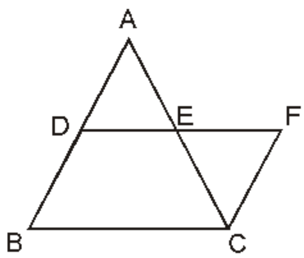


Fig. 9.28

From (1) and (3), we get

$BD = CF$ and $BD \quad CF$

$\therefore$ $BCFD$ is a paralellogram.

$\therefore$ DF = BC and $DF \quad BC$

$\Rightarrow$ $\frac{1}{2}DF = \frac{1}{2}BC$

$\Rightarrow$ $DE = EF = \frac{1}{2}DF$. (By construction)

$\Rightarrow \qquad\qquad\qquad\qquad \mathrm{DE} = \frac{1}{2}\mathrm{BC}$

Thus, $\qquad\qquad\qquad\qquad \mathrm{DE} = \frac{1}{2}\mathrm{BC}$ and $\mathrm{DE} \parallel \mathrm{BC}$ **Hence Proved**

Theorem 9.14. (Converse of Theorem 9.12)

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

Given : In ΔABC, D is the mid-point of side AB in Fig. 9.29 $DE \quad BC$ and it intersects AC at point E.

To Prove : $AE = EC$

Construction : Draw CF parallel to BD which intersects DE (extended) at point F

Proof : $\because$ $\qquad BC \parallel DF \qquad$ (Given)

and $\qquad BD \quad CF \qquad$ (By construction)

$\therefore \qquad BCFD$ is a parallelogram.

$\therefore \qquad BD = CF$

But $\qquad BD = AD \qquad$ (Given)

$\therefore \qquad AD - CF \qquad$ ...(1)

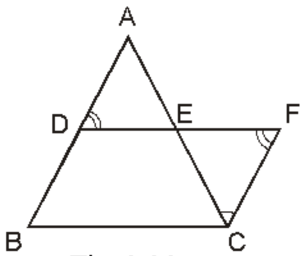


Fig. 9.29

Now, in $\Delta\, ADE$ and $\Delta\, CFE$

$\angle ADE - \angle CFE$ (Alternate interior angles)

$AD = CF \qquad$ [From (1)]

$\angle DAE - \angle ECF \qquad$ [Alternate interior angles]

$\Delta\,\mathrm{ADE} \cong \Delta\,\mathrm{CFE} \qquad$ (By ASA congruency rule)

$AE = CE \qquad$ **Hence Proved**

9.08. Intercept

If there are two lines l_1 and l_2 in a plane and line l_3 intersects these lines on different points A and B, then the line segment AB is called intercept made by lines l_1 and l_2 on line l_3

Here, will prove the following theorem for three parallel lines. This theorem can be extended for more than three lines.

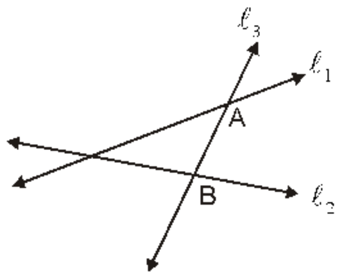


Fig. 9.30

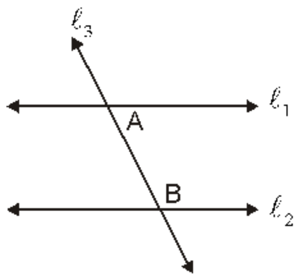


Fig. 9.31

Theorem 9.15. ***If there are three or more than three parallel lines and intercepts made by them on a transversal line are equal, then the corresponding intercepts on the other transversal will be also equal.***

Given : In Fig 9.32, l, m, n are three parallel lines and two transversals lines l_1 and l_2 intersect them on points A, B, C and D, E, F respectively and $AB = BC$.

To Prove : $DE = EF$

Construction : Draw $GH \parallel l_1$ which passes through E.

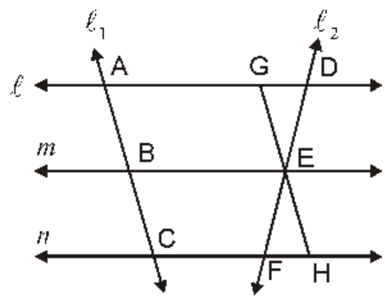


Fig. 9.32

Proof : $\because$ l and m are parallel lines

$\therefore$ $AG \parallel BE$

and $AB \parallel GE$ (By construction)

$\therefore$ $ABEG$ is a parallelogram.

$\therefore$ $AB = GE$ ...(1)

Similarly, $BCHE$ is a parallelogram.

$\therefore$ $BC = EH$ ...(2)

Given $AB = BC$

$\therefore$ $GE = EH$ ...(3)

Again $l \parallel n$ and transversal l_2 intersects them.

$\angle GDE = \angle EFH$ (Alternate interior angles)...(4)

Now, in ΔGDE and ΔHFE

$\angle GDE = \angle EFH$ [From (4)]

$GE = EH$ [From (3)]

$\angle GED = \angle HEF$ [Vertically opposite angles]

$\Delta GDE \cong \Delta HFE$ (By ASA congruency rule)

Since, corresponsding parts of congruent triangles are equal,

Thus, $DE = EF$ **Hence Proved**

Illustrative Examples

Example 8. ***D*, *E* and *F* are respectively mid-points of sides BC, CA and *AB* of are equilateral triangle *ABC*. Prove that ΔDEF is an equilateral triangle.**

Solution :

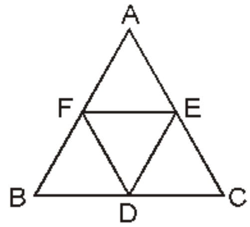


Fig. 9.33

Given : ΔABC in which D, E, F are the mid-points of sides EC, CA and AB respectively.

To Prove : ΔDEF is an equilateral triangle.

Proof : In ΔABC, D, E and F are the mid - points of sides BC, CA and AB respectively.

$$\therefore \qquad DE = \frac{1}{2}AB \qquad ...(1)$$

$$EF = \frac{1}{2}BC \qquad ...(2)$$

$$FD = \frac{1}{2}AC \qquad ...(3)$$

But ΔABC is an equilateral triangle .

$$AB = BC = CA \qquad ...(4)$$

From (1), (2), (3) and (4)

$$DE - EF - FD$$

Thus, ΔDEF is an equilateral triangle. **Hence Proved**

Example 9. **Prove that a line, joining the mid-points of the diagonals of a trapezium, will be parallel to its parallel sides and half of their difference.**

Solution : Given : A trapezium $ABCD$ in which $AB \mid DC$, F and G are mid-points of diagonals AC and BD respectively.

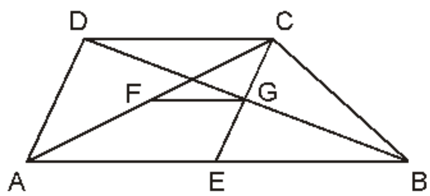


Fig. 9.34

To Prove : (i) $FG \quad AB$

(ii) $FG = \frac{1}{2}(AB - CD)$

Construction : Join CG and extend it such that it will meet AB at E.

Proof : In ΔCDG and ΔEBG

$\angle CDG = \angle EBG$	(Alternate interior angles)
$DG - GB$	(Given)
$\angle DCG - \angle BEG$	(Alternate interior angles)
$\Delta CDG \cong \Delta EBG$	(By ASA cong:uency rule)

Since, corresponding parts of two congruent triangles are equal.

$\therefore \qquad CG = EG \qquad ...(1)$

and $\qquad CD = EB \qquad ...(2)$

Now in ΔACE, F and G are the mid points of sides AC and CE respect ively.

$$FG \parallel AE \text{ and } FG = \frac{1}{2}AE \qquad ...(3)$$

But $\qquad AE = AB - EB$

$$AE = AB - CD \qquad \text{[From (2)]} \qquad ...(4)$$

From (3) and (4), we get

$$FG = \frac{1}{2}AE = \frac{1}{2}(AB - CD)$$

and $\qquad FG \parallel AE$

Thus, $\qquad FG \parallel AB$ and $FG = \frac{1}{2}(AB - CD)$ **Honce Proved**

Example 10. Prove that the quadrilateral obtained by joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

Solution :

Given : In Fig 9. 35 $ABCD$ is a quadrilateral where P, Q, R and S are the mid-points of its consecutive sides respectively.

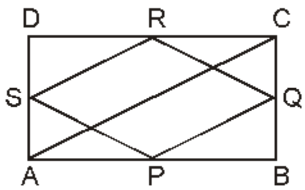


Fig. 9.35

To Prove : $PQRS$ is a parallelogram.

Construction : Join AC.

Proof : In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \qquad ...(1)$$

In ΔADC, S and R are mid points of sides AD and DC respectively.

$$\therefore \qquad SR \parallel AC \text{ and } SR = \frac{1}{2}AC \qquad ...(2)$$

From (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

Thus, in quadrilateral $PQRS$ one pair of opposite sides are equal and parallel.

Thus, $PQRS$ is a parallelogram

Example 11. In Fig 9.36, X and Y are respectively mid points of opposite sides AD and BC *of a* parallelogram $ABCD$. Also, BX and DY intersect line AC at points P and Q respectively. Show that AP = PQ = QC.

Solution : In fig. 9.36, X and Y are the mid-points of sides AD and BC respectively of a parallelogram $ABCD$.

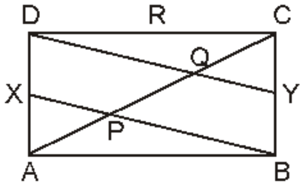


Fig. 9.36

$\therefore \quad DX = \frac{1}{2}AD$ and $BY = \frac{1}{2}CB$

But $ABCD$ is a parallelogram.

$\therefore$ AD = BC and AD || BC [Opposite sides of a parallelogram]

$\Rightarrow \quad \frac{1}{2}AD = \frac{1}{2}BC$ and AD || BC

$\Rightarrow$ DX = BY and DX || YB

$\because$ One pair of opposite sides of quadrilateral XBYD are equal and parallel

$\therefore$ XBYD is a parallelogram

$\Rightarrow$ PX || QD

We know that the segment drawn from the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In Δ CBP, Y is the mid-point of BC and YQ || BP

$\therefore$ Q is the mid-point of CP

$\Rightarrow$ P is the mid-point of AQ

$\Rightarrow$ AP = PQ ...(i)

Simlilary CQ = PQ ...(ii)

AP = PQ = QC [From (i) and (ii)]

Example 12. In Fig. 9.37 AY and CX are respectively the bisectors of opposite angles A and C of parallelogram ABCD. Show that

AY || CX

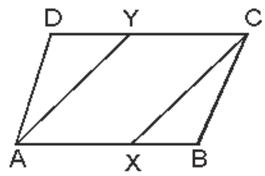


Fig. 9.37

Solution : $\angle A = \angle C$ (opposite angles of parallelogram)

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle C$

[$\because$ AY and CX are the bisectors of $\angle$ A and $\angle$ C respectively]

$\therefore$ AB || CD (opposite sides of parallelogram)

Also AX || YC and transversal CX intersects them

$\angle YAX = \angle YCX$ ...(i)

A transversal line CX intersects two parallel lines AB and CD.

So, $\angle CXB = \angle YCX$ ($\because$ alternative angles) ... (ii)

From (i) and (ii) $\angle YAX = \angle CXB$

because the corresponding angles are equal.

$\therefore$ AX || YC (because sum of interior angles on one side of a transversal line is $180°$)

$\therefore$ AY || CX **Hence Proved**

Example 13. Show that a quadrilateral, formed by joining the mid-points of the sides of a rhombus, in the same order, is a rectangle.

Solution : Let ABCD be a rhombus and P,Q,R,S are mid-points of sides AB, BC, CD and DA respectively. (Fig. 9.38). Join AC and BD.

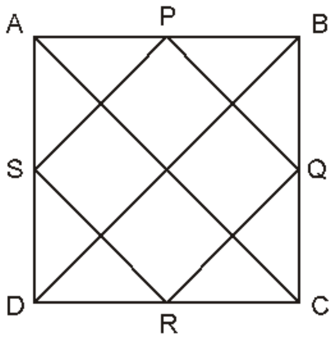


Fig. 9.38

$\therefore$ From ΔABD, we get

$$SP = \frac{1}{2}BD \text{ and } SP \parallel DB$$

(S and P are the mid points of sides AB and AD respectively)

Similarly, $RQ = \frac{1}{2}BD$ and $RQ \parallel BD$

$\therefore$ $SP = RQ$ and $SP \parallel RQ$

$\therefore$ PQRS is a parallelogram.

Also $AC \perp BD$ (Diagonals of a rhombus are perpendicular to each other)

PQ || AC (In Δ BAC, P and Q are mid points of sides AB and BC respectively)

SP || BD ...(1)

In Δ ABC, P and Q are the mid point of AB and BC respectively

$\therefore$ PQ || AC ...(2)

From (1) and (2) $SP \perp PQ$ [$\because AC \perp BD$]

$\therefore$ $\angle SPQ = 90°$

$\therefore$ PQRS is a rectangle. **Hence Proved**

Exercise 9.2

1. In Fig. 9.39 ABCD and AEFG are two parallelograms. If $\angle C = 55°$, then determine $\angle F$.

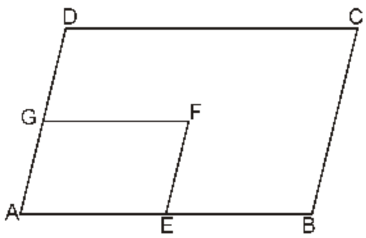


Fig. 9.39

2. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.
3. Can all the angles of a quadrilateral be right angles? Give reason of your answer.
4. The diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35^\circ$, then determine $\angle B$.
5. Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, then determine CD.
6. ABCD is a rhombus in which altitude from D on AB, bisects AB. Find the angles of the rhombus.
7. In a triangle ABC, lines RQ, PR and QP are respectively drawn parallel to lines BC, CA and AB passing through points A, B and C, as shown in fig. 9.40 Show that $BC = \frac{1}{2} = QR$.

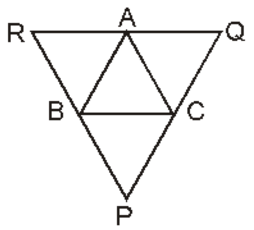


Fig. 9.40

8. D, E and F are respectively the mid points of sides BC, CA and AB respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.
9. Points P and Q are respectively taken on opposite sides AB and CD of a parallelogram $ABCD$ such that $AP = CQ$ (Fig 9.41). Show that AC and PQ bisect each other.

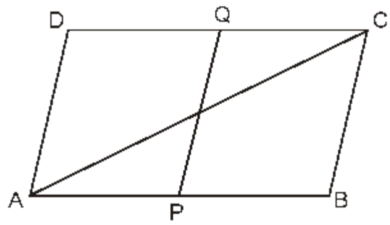


Fig. 9.41

10. E is the mid-point of side AD of a trapezium $ABCD$ in which $AB \parallel DC$. Through point E a line is drawn parallel to AB, intersects BC at F. Show that F is the mid-point of BC. [Hint: Join AC].
11. In ΔABC, $AB - 5$cm, BC = 8cm and CA = 7cm. If D and E are mid-points of AB and BC respectively, then find the length of DE.
12. In Fig 9.42, it is given that $BDEF$ and $FDCE$ are parallelograms. Can you say that $BD - CD$? Why and why not?

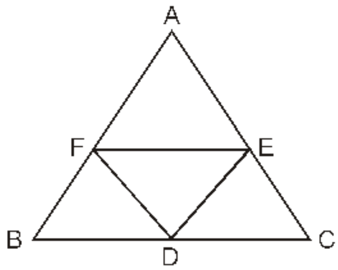


Fig. 9.42

13. In Fig. 9.43, D, E and F are mid points of sides BC, CA and AB respectively. If AB – 4.3 cm, BC – 5.6 cm and AC – 3.9 cm, then find the perimeter of the following :
(i) ΔDEF (ii) quadrilateral $BDEF$

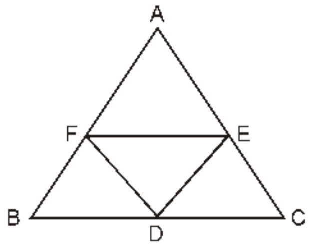


Fig. 9.43

14. Prove that a quadrilateral obtained by joining the mid points of consecutive sides of a square, is also a square.

15. The diagonals of a quadrilateral are perpendicular to each other. Prove that a quadrilateral obtained by joining the mid-points of its sides is a rectangle.

16. Prove that in a right-angled triangle, the bisecting median of hypotenuse is half of the hypotenuse.

17. Prove that a rhombus is obtained by joining the mid-points of the pairs of opposite sides of a rectangle.

Constructions of Quadrilaterals

9.09. Quadrilateral

A plane figure enclosed by four line segment, is called a quadrilateral. The line joining its opposite vertices is called its diagonal.

In Fig. 9.44 AB, BC, CD and DA are its four sides. A, B, C and D are its vertices and AC and BD are the diagonals of quadrilateral $ABCD$.

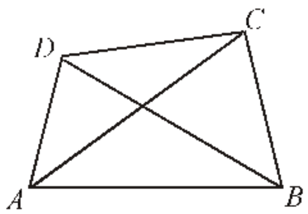


Fig. 9.44

9.10. Construction of Quadrilateral

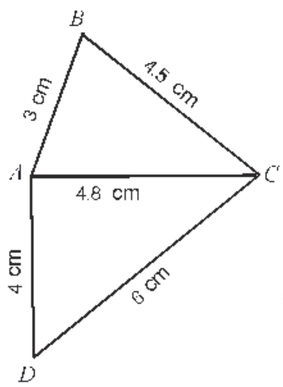
When we have to construct a quadrilateral, a rough sketch should be drawn and mark the given facts. Generally, there is specific importance of diagonal in construction of a quadrilateral. So it should be considered by drawing a diagonal in rough sketch definitly. Is any triangle can be construct by it? It should be seen by drawing quadrilateral after formation of triangle. Construction of a quadrilateral can be completed by drawing triangle. It is not necessary to draw a diagonal in each condition. Some times without drawing any diagonal, quadrilateral can be constructed.

It can be understand clearly by the constructions given in this chapter.

Construction 9.16 : Construction of a quadrilateral when four sides and a diagonal are given.

Construct a quadrilateral $ABCD$ in which AB – 3 cm, BC – 4.5 cm, CD = 6 cm, DA = 4 cm and AC = 4.8 *cm*.

Construction : First draw a rough sketch on the basis of given measures and mark them on it.



(Rough Figure)

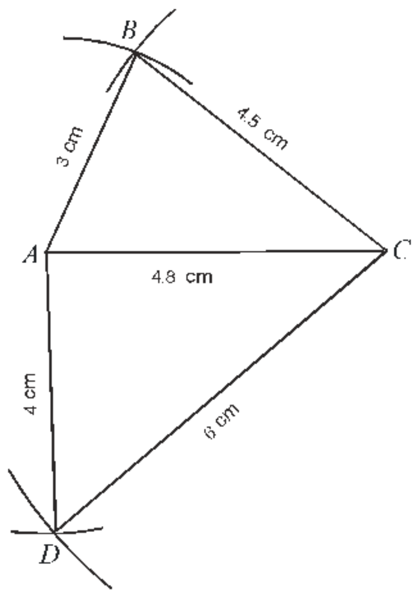


Fig. 9.45

According to the rough sketch, draw AC – 4.8 cm. Construct a triangle ABC by drawing an arc of radius 3 cm from A and arc of radius 4.5 cm from C. Similarly, complete Δ ACD by drawing arc of length equal to AD and CD.

Thus, $ABCD$ *is* the required quadrilateral.

Construction 9.17. ***Construction of a Quadrilateral in which four sides and one angle are given.***

Construct a quadrilateral $ABCD$ in which $AB = 4.8$ cm, $BC = 3.5$ cm, $CD = 4.5$ cm, DA = 4 cm and $\angle A = 60°$

Construction : Draw a rough sketch according to the given measures and mark them.

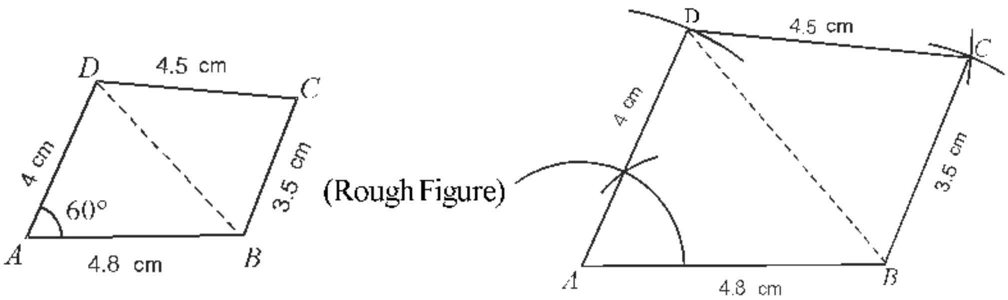


Fig. 9.46

Draw line segment $AB = 4.8$ cm. Draw $\angle DAB = 60°$ at point A and cut $AD = 4$ cm from A. Draw two arcs from point D and B of raddi 4.5 cm and 3.5 cm respectively they intersect at point C. Join DC and BC. Thus, $ABCD$ is a required quadrilateral.

Construction 9.18. ***Construction of a quadrilateral when three sides an two diagonals are given.***

Construct a quadrilateral $ABCD$ in which $AB = 5.5$ cm, $BC = 3.3$ cm, $AD = 4.6$ cm and diagonals $AC = 5.7$ cm and $BD = 6$ cm.

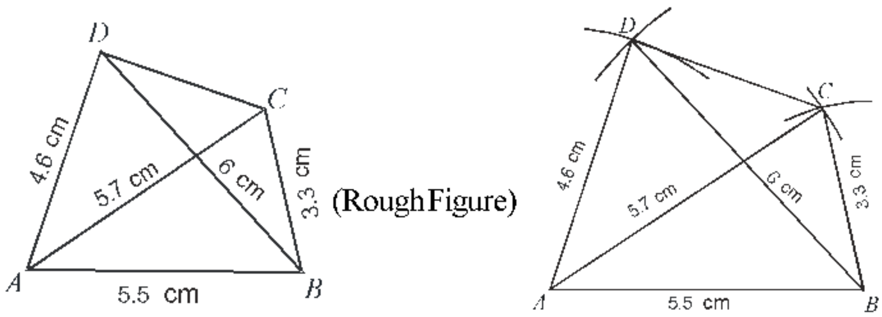


Fig. 9.47

Construction : Draw a rough sketch according to the given measures and mark them. According to rough sketch draw line segment $AB = 5.5$ cm. Construct a ΔABD by drawing the arcs of radii 4.6 cm and 6 cm from points A and B respectively. Similarly again, construct ΔABC by drawing arcs of radii 5.7 cm and 3.3 cm from points A and B. Join C and D.

Thus, $ABCD$ is the required quadrilateral.

Construction 9.19. ***Construction of a quadrilateral when three sides and two angles between them are given.***

Construct a quadrilateral $ABCD$ in which AB – 3.5 cm, BC – 5 cm, CD = 5.5 cm, $\angle$ B = 120° and $\angle$ C = 60°.

Construction : Draw a rough sketch according to given measures and mark them.

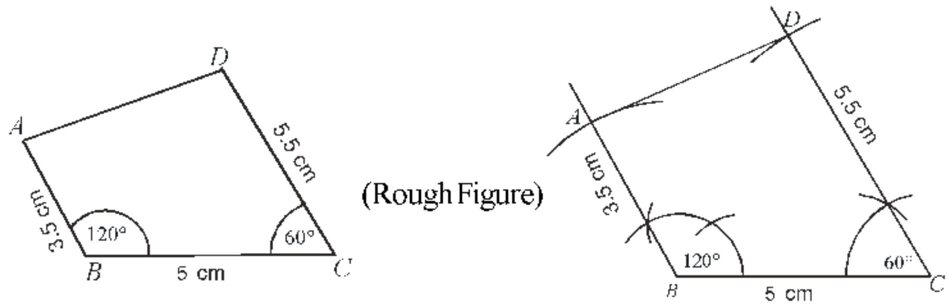


Fig. 9.48

Draw line segment BC = 5 cm. Draw $\angle$ B = $\angle$120° and $\angle C = \angle$ 60° at points B and C respectively with BC and cut the given lengths AB – 3.5 cm and CD – 5.5 cm from B and C respectively. Join A and D to obtained the quadrilateral.

Thus $ABCD$ is required quadrilateral.

Construction 9.20. ***Construction of a quadrilateral when two consecutive sides and angle between them and other two angles are given.***

Construct a quadrilateral $ABCD$ in which AB – 5 cm, AD – 5.3 cm, $\angle$ A= 60°, $\angle$ C = 105° and $\angle$ D = 90°.

Construction : $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \qquad 60° + 105° + \angle B + 90° = 360° \qquad \Rightarrow \qquad 255° + \angle B = 360°$

Draw a rough sketch according to given measures and mark them.

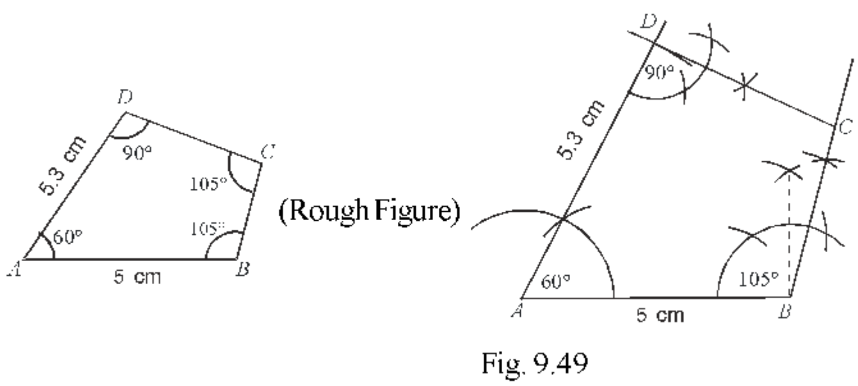


Fig. 9.49

Draw line segment $AB = 5$ cm. At points A and B, draw $\angle A = 60°$ and $\angle B = 105°$ with AB. From A cut, $AD = 5.3$ cm and at D, draw $\angle D = 90°$ with AD.
Thus, $ABCD$ is the required quadrilateral.

Exercise 9.3

Construct quadrilaterals for the following given facts with description of steps of construction.

1. In a quadrilateral $ABCD$, $AB – 3.5 cm$, $BC = 4.8$ cm, $CD – 5.1$ cm, $AD = 4.4$ cm and a diagonal $AC − 5.9$ cm.
2. In a quadrilateral $PQRS$, $PQ − 4$ cm, $QR = 3$ cm, $QS − 4.8$ cm, PS = 3.5 cm and $PR – 5$ cm.
3. In, quadrilateral $ABCD$, $AB – 4$ cm, $BC = 4.5$ cm, $CD – 3.5$ cm, $AD – 3$ cm and $\angle A = 60^{\circ}$
4. In, quadrilateral $ABCD$, $AB − 3.5$ cm, $BC − 3$ cm, $AD − 2.5$ cm, $AC = 4.5$ cm and $BD = 4$ cm.
5. In, quadrilateral PQRS, $PQ – 3$ cm, QR = 4 cm, PS = 4.5 cm, $PR – 6$ cm and QS = 5.5 cm.
6. In, quadrilateral $ABCD$, $AB − BC − 3.0$ cm, $AD − 5$ cm, $\angle A = 90°$ and $\angle B − 120°$.
7. In, quadrilateral $ABCD$, $AB − 3.8$ cm, $BC − 2.5$ cm, $CD − 4.5$ cm and $\angle B − 30°$ and $\angle C = 150°$.
8. In, quadrilateral $PQRS$, $PQ – 3$ cm, QR = 3.5 cm, $\angle Q = 90°$ and $\angle P = 105°$ and $\angle R = 120^{\circ}$.
9. In, quadrilateral $PQRS$, $PQ = 2.5$ cm, $QR − 3.7$ cm, $\angle Q = 120°$, $\angle S = 60°$ and $\angle R = 90^{\circ}$.

9.11 Constructions of Parallelograms and Rectangles

Before the construction of parallelogram, rectangle, square and rhombus, it is necessary to know the following facts.

1. In a parallelogram : (i) Opposite sides are equal,
   (ii) opposite angles are equal,
   (iii) Diagonals bisect each other,
   (iv) Each diagonal bisects a parallelogram into two congruent triangles.
2. In a rectangle : (i) Each angle is right angle,
   (ii) Opposite sides are equal.
   (iii) Diagonals are equal.
   (iv) Diagonals bisect each other.
3. In a square : (i) All four sides are equal.
   (ii) Each angle is right angle.
   (iii) Diagonals are equal.

(iv) Diagonals bisect each other at right angle,

(v) Each diagonal makes an angle of $45°$ with sides.

4. In a rhombus : (i) All four sides are equal.

(ii) Opposite angles are equal.

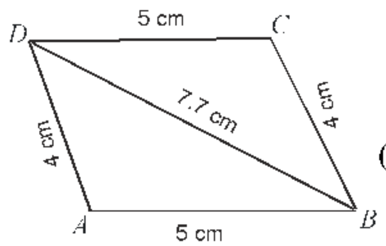
(iii) Diagonals bisect each other at right angle.

(iv) Diagonals are bisectors of the vertex angles.

Construction 9.21 . ***Construction of a parallelogram when two sides and one diagonal are given.***

Construct a parallelogram $ABCD$ *if* $AB = 5$ cm, $BC = 4$ cm and $BD - 7.7$ cm.

Construction : Draw rough sketch according to given measures and mark them.



(Rough Figure)

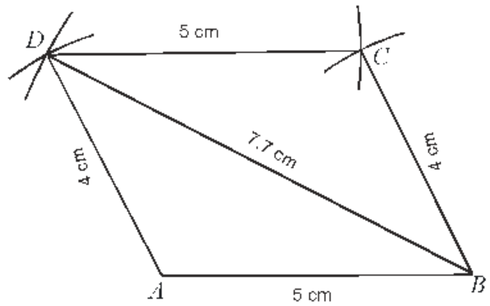


Fig. 9.50

Draw a line segment $AB - 5$ cm. Construct ΔABD by drawing arcs of radii 4 cm and 7.7 cm from points A and B respectively. Similarly, construct Δ BCD by drawing arcs of radii 4 cm and 5 cm from points B and D respectively.

Thus, $ABCD$ *is* the required parallelogram.

Construction 9.22. ***Construction of a parallelogram when a side and two diagonals are given.***

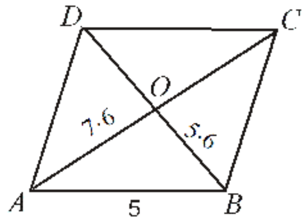
Construct a parallelogram $ABCD$ in which $AB = 5$ cm, diagonal $AC - 7.6$ cm and diagonal $BD = 5.6$ cm.

Hint: In a parallelogram, diagonal bisect each other

$$\therefore \qquad AO = OC = \frac{1}{2}AC \text{ and } BO = OD = \frac{1}{2}BD.$$

$$AO = \frac{1}{2} \times 7.6 = 3.8\,cm \text{ and } BO = \frac{1}{2} \times 5.6 = 2.8\,cm.$$

Construction: Draw a rough sketch according to given measures and mark them.



(Rough Figure)

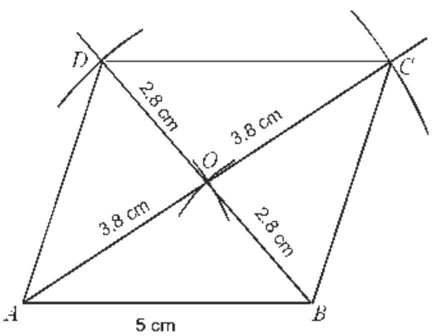


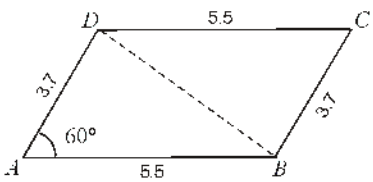
Fig. 9.51

Draw a line segment $AB - 5$ cm. Construct a ΔAOB by drawing arcs of radii 3.8 cm or half of diagonal AC and 2.8 cm or, half of diagonal BD. Extend AO and BO so that $AC - 7.6$ cm and $BD = 5.6$ cm. Join BC, CD and AD. Thus, $ABCD$ *is* the required parallelogram.

Construction 9.23. ***Construction of a parallelogram in which two adjecent sides and angle between them are given.***

Construct a parallelogram ABCD where $AB - 5.5$ cm, $BC - 3.7$ cm and $\angle A - 60°$.

Construction : Draw a rough sketch according to given measures and mark them.



(Rough Figure)

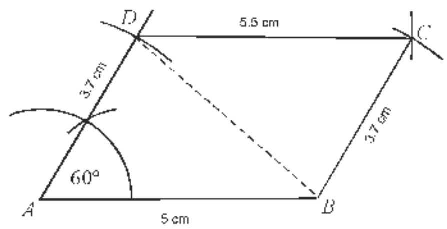


Fig. 9.52

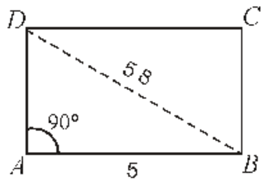
Draw a line segment $AB - 5.5$ cm. Construct $\angle BAM - 60°$ at point A and make a ΔABD by taking AD = 3.7 cm. Similarly, by taking $BC = 3.7$ cm and DC = 5.5 cm construct Δ BDC.

Thus, $ABCD$ *is* a required parallelogram.

Construction 9.24. ***Construction of a rectangle whose diagonal and one side are given.***

Construct a rectangle in which diagonal $BD - 5.8$ cm and one side $AB - 5$ cm.

Construction : Draw a rough sketch according to given measures and mark them.



(Rough Figure)

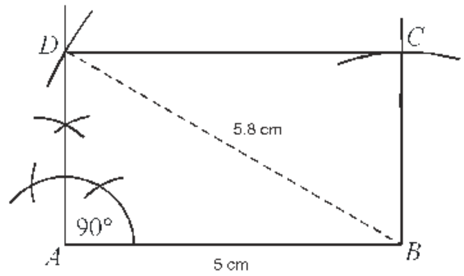


Fig. 9.53

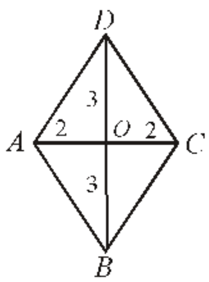
Draw a line segment AB – 5 cm. At point A, draw $\angle A - 90°$. Taking B as centre. draw an arc of radius $BD - 5.8$ cm which intersect at D. Now taking D and B as centres draw two arcs of radii AB and AD respectively which intersect, at C. Join CD and BC.

Thus, ABCD is a required rectangle.

Construction 9.25: ***Construction of rhombus when two diagonals are given.***

Construct a rhombus whose diagonals are 4 cm and 6 cm respectively.

Construction : Draw a rough sketch of given measures and mark them.



(Rough Figure)

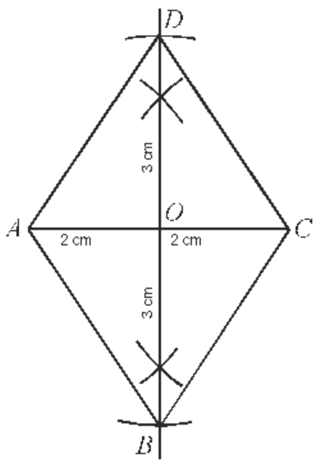


Fig. 9.54

Draw diagonal $AC - 4$ cm. Draw perpendicular bisector of AC which meets AC at O. Take O as centre and draw two arcs .of radius equal to half of the length of other diagonal $BD = \left(\frac{1}{2} \times 6 = 3cm\right)$ both sides of AC. These arcs intersect perpendicular bisector at B and D. Join AB, BC, CD and AD.

Thus ABCDis a required rhombus.

Construction 9.26 : ***Construction of a square whose diagonals are given.***

Construct a square whose diagonal is 5 cm.

Construction : Draw a rough sketch of given measures and mark them.

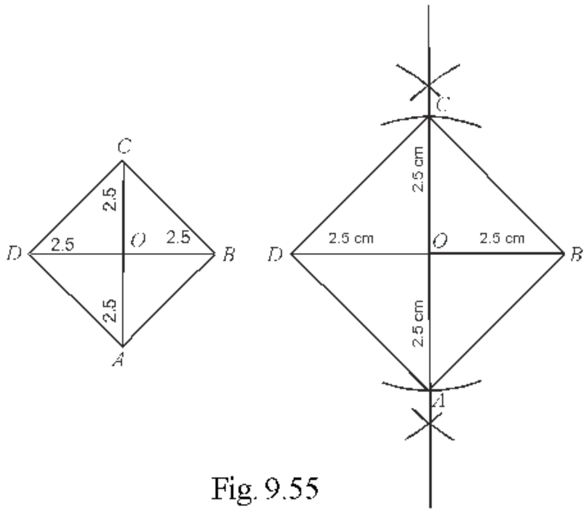


Fig. 9.55

Draw a diagonal $BD = 5$ cm. Draw its perpendicular bisector which meets BD at O. Take O as centre and cut $OC = OA = 2.5$ cm. Join AB, BC, CD and DA.

Thus, $ABCD$ is the required square.

Exercise 9.4

1. Construct a parallelogram $ABCD$ in which AB = 4.7 cm, $BC = 3.5$ cm and $AC = 7$cm.
2. Construct a parallelogram $PQRS$ in which $PQ = 5$ cm, diagonal $PR = 7.6$ cm and diagonal $QS = 5.6$ cm.
3. Construct a parallelogram ABCD whose two sides are 4.6 cm and 3 cm respectively and angle between them is 60°.
4. Construct a rectangle $ABCD$ in which $AB = 6$ cm and diagonal $AC = 10$ cm.
5. Construct a rhombus $ABCD$ whose diagonals $AC = 7$ cm and $BD = 5$ cm.
6. Construct a square $ABCD$ whose diagonal is 6 cm.

Construction 9.27 : ***Construction of a Trapezium***

(a) ***When four sides of a trapezium are given and in which two sides are parallel.***

Construct a trapezium $ABCD$ in which $AB = 7$ cm, $BC = 6$ cm, $CD = 4$ cm, $DA = 5$ cm and AB || CD.

Construction : Draw a rough sketch based on given measures and mark all the lengths.

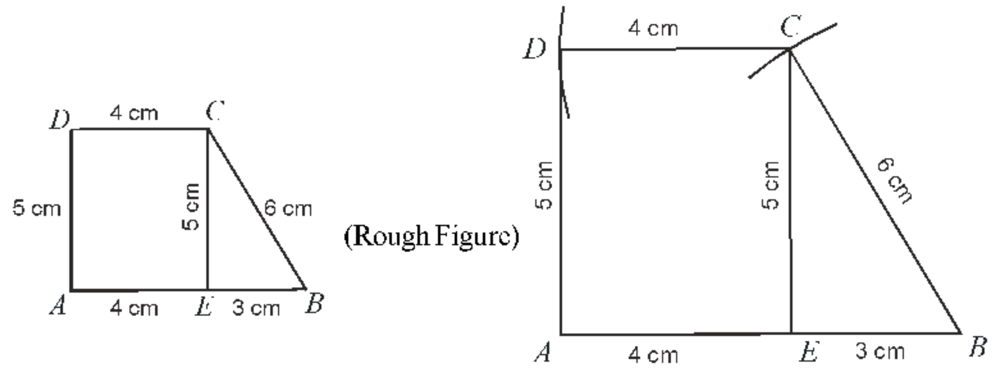


Fig. 9.56

Mark a point E on line AB such that $AE - DC - 4$ cm. Draw a line segment AB = 7 cm and mark point E such that $AE - 4$ cm. Take E and B as centres draw two arcs radii 5 cm (=AD) and 6 cm (= BC) respectively which intersect each other at point C. Again taking A and C as centres, draw two arcs of radii 5 cm and 4 cm such that they intersect each other at D. Join B to C, C to D and A to D to construct a complete quadrilateral.

Thus, $ABCD$ is the required trapezium.

(b) ***Construction of Trapezium if three sides and one angle are given and it is also given that which sides are parallel.***

Construct a trapezium $ABCD$ in which $AB \parallel CD$, $\angle B - 90°$, AB = 4 cm, BC = 2.8 cm, $AD - 3.5$ cm.

Construction : Draw a rough sketch of trapeÐium and mark all the given measures.

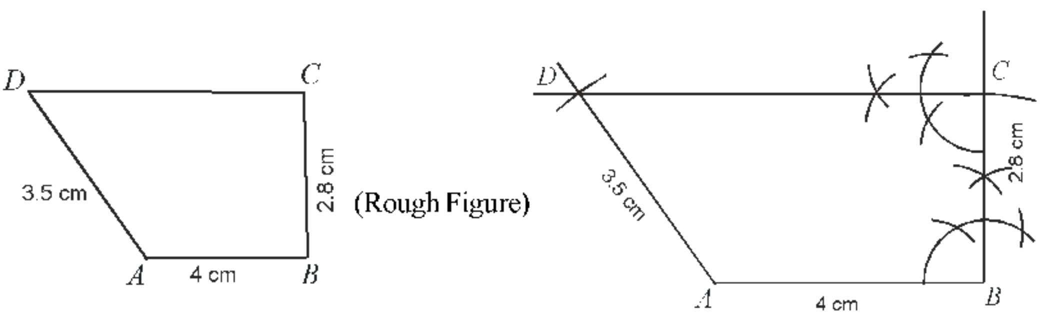


Fig. 9.57

For fair construction, take $AB - 4$ cm. Construct $\angle B = 90°$ at B. Cut a point at a distance of 2.8 cm from line making right angle and mark it as C. Again construct $\angle C - 90°$ at point C. ($\because$ AB CD and $\angle B - 90°$, $\angle C - 90°$)

Taking A as a, centre, cut a point on a line drawn perpendicular at point C and mark it as D. Join A and D to construct a quadrilateral $ABCD$.

Thus, $ABCD$ is a required trapezium.

Exercise 9.5

1. Construct a trapezium $ABCD$ in which $AB \mid CD, AB = 4$ cm, $BC - 2.3$ cm, $CD - 2.8$ cm and $DA - 1.9$ cm.
2. Construct a trapezium $PQRS$ in which $PQ \mid SR, PQ - 6$ cm, $RS = 3$ cm, PS = 3 cm and $QR - 5$ cm.
3. Construct a trapezium $ABCD$ in which $AB \mid CD$, $AB - 8$ cm, $BC - 6$ cm, $CD = 4$ cm and $\angle B = 75°$.
4. Construct a trapezium $ABCD$ in which $AB \quad CD$, $AB - 4$ cm, $BC = 4$ cm, $AD = 5$ cm and $\angle B = 90°$.

IMPORTANT POINTS

1. Sum of all the angles of a quadrilateral is 360°.
2. A diagonal of a parallelogram divides it into two congruent triangles.
3. In a parallelogram :

   (i) opposite angles are equal.

   (ii) opposite sides are equal.

   (iii) diagonals bisect each other.
4. Any quadrilateral is a parallelogram, if :

   (i) its opposite angles are equal,

   (ii) its opposite sides are equal.

   (iii) its diagonals bisect each other.

   (iv) a pair of opposite sides is equal and parallel.
5. Diagonals of a rectangle bisect each other and are equal and vice-versa.
6. Diagonals of a rhombus, bisect each other at right angles and vice-versa.
7. Diagonals of a square bisect each other at right angles and are equal and vice-versa.
8. A line segment joining mid-points of two sides of a triangle, is parallel to third side and half of it.
9. A line passing through the mid-point of one side of a triangle side and parallel to another side, bisects the third side.
10. A quadrilateral, obtained by joining the mid-points of the sides of a quadrilateral, in a order, is a parallelogram.
11. It is necessary for the construction of a quadrilateral:

    (i) four sides and a diagonal are given.

    (ii) four sides and an angle are given.

    (iii) three sides and two diagonals are given,

(iv) three sides and angle between them are given.

(v) two adjecent sides and angle between them and other two angles are given.

12. It is necessary for the construction of a parallelogram :

(i) two adjecent sides and a diagonal are given.

(ii) one side and two diagonals are given.

(iii) two adjecent sides and two angle between them are given.

13. It is necessary for the construction of a rectangle :

(i) two adjecent sides are given.

(ii) a side and a diagonal are given.

14. For construction of square, it is necessary that:

(i) a side is given.

(ii) diagonal is given.

15. For the construction of rhombus, it is necessary that :

(i) measure of a side and an angle between two adjecent sides,

(ii) diagonals are given.

16. For the construction of a trapezium, it is necessary that :

(i) four sides are given and parallel sides are known.

(ii) three sides and an angle are given and parallel sides are known.

17. Opposite sides of a parallelogram are equal and parallel and opposite angles a equal.

18. Four sides of a square are equal and each angle is right angle. Diagonals of a square are equal and bisect each other at right angles.

19. Opposite sides of a rectangle are equal and each angle is right angle.

20. In a rhombus, four sides are equal and opposite angles are equal. Diagonals bisect each other at right angles.

21. In a trapezium, only one pair of opposite sides be parallel.

Miscellaneous Exercise-9

Write the correct answers of each of the following questions (From question 1 to 15)

1. Three angles of a quadrilateral are 75°, 90° and 75°. Its fourth angle is :

(a) 90° (b)95° (c) 105° (d) 120°

2. A diagonal of rectangle is inclined at an angle of 25° with a side. The acute angle between its diagonals is :

(a) 55° (b) 50° (c) 40° (d) 25°

3. $ABCD$ is a rhombus in which $\angle ACB = 40°$. Then $\angle ADB$ is :

(a) 40° (b) 45° (c) 50° (d) 60°

4. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rectangle if :

(a) $PQRS$ is a rectangle

(b) $PQRS$ is a parallelogram

(c) Diagonals of $PQRS$ are perpendicular to each other.

(d) Diagonals of $PQRS$ are equal.

5. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rhombus if:

(a) $PQRS$ is a rhombus

(b) $PQRS$ is a parallelogram

(c) Diagonals of $PQRS$ are perpendicular to each other

(d) Diagonals of $PQRS$ are equal.

6. If the ratio of angles A, B, C and D of a quadrilateral ABCD, taking in this order is 3 : 7:6:4. Then $ABCD$ is a :

(a) rhombus (b) parallelogram (c) trapezium (d) kite

7. In a quadrilateral $ABCD$, bisectors of $\angle A$ and $\angle B$, intersect each other at P. bisectors of $\angle B$ and $\angle C$ at Q, bisectors of $\angle$C and $\angle$ D at R, and bisectors of $\angle$ D and $\angle A$ intersect each other at S. Then $PQRS$ is a :

(a) rectangle (b) rhombus (c) parallelogram

(d) quadrilateral whose opposite angles are supplementry.

8. If AP and CQD are two parallel lines, then bisectors of $\angle APQ$, $\angle BPQ$, $\angle CQP$ and $\angle$ PQD make:

(a) a square (b) a rhombus (c) a rectangle (d) any other parallelogram

9. Joining the mid-points of the sides of rhombus, in a order, obtained a figure is :

(a) a rhombus (b) a rectangle (c) a square (d) any parallelogram

10. D and E are mid points of side AB and AC respectively of a ΔABC and O is any point on side BC. O is joined with A. If P and Q are mid points of OB and OC respectively, then $DEQP$ is a :

(a) square (b) rectangle (c) rhombus (d) parallelogram

11. The figure obtained by joining the mid-points of the sides of a quadrilateral ABCD in order, is only a square if:

(a) $ABCD$ is a rhombus

(b) Diagonals of $ABCD$ are equal

(c) Diagonals of $ABCD$ are equal and perpendicular to each other

(d) Diagonals of $ABCD$ are perpendicular to each other

12. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O. If $\angle DAC - 32°$ and $\angle AOB - 70°$, then $\angle$DBC is :

(a) 24° (b) 86° (c) 38° (d) 32°

13. Which of the following statements is not true for a parallelogram :

(a) Opposite sides are equal

(b) Opposite angles are equal

(c) Opposite angles are bisected by diagoanls

(d) Diagonals bisect each other.

14. D and E are mid points of sides AB and AC respectively of a ΔABC. DE is extended upto F. To prove that CF is equal to the line segment DA and parallel, we are required an other information, which is :

(a) $\angle DAE - \angle EFC$ (b) $AE - EF$

(c) $DE - EF$ (d) $\angle ADE - \angle ECF$

15. Diagonals of a parallelogram $ABCD$ intersect at point O. If $\angle$ BOC = 90° and $\angle BDC - 50°$, then $\angle OAB$ is :

(a) 90° (b)50° (c)40° (d) 10°

16. $ABCD$ *is* a parallelogram. If its diagonals are equal, then find the value of $\angle ABC$.

17. Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Given reason for your answer.

18. Three angles of a quadrilateral $ABCD$ are equal. Is this a parallelogram?

19. In quadrilateral $ABCD$, $\angle A + \angle D - 180°$. What specific name can be given to this quadrilateral?

20. All angles of a quadrilateral are equal. What specific name is given to this quadrilateral?

21. Diagonals of rectangle are equal and perpendicular to each other. Is this statement true? Give reason for your answer.

22. Any square, inside an isosceles right-angled triangle is such that one angle is common in both square and triangle. Show that the vertex of the square, which opposite to the vertex of the common angle, bisects the hypotenuse.

23. In a parallelogram $ABCD$, AB = 10 cm and $AD - 6$ cm. Bisector of $\angle A$ meets DC at E and producing AE and BC meet at F. Find the length of CF.

24. P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively in which $AC - BD$ and $AC \perp BD$. Prove that $PQRS$ is a square.

25. A diagonal of a parallelogram, bisects one of its angle. Prove that this parallelogram is a rhombus.

26. $ABCD$ is a quadrilateral in which $AB \mid DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

27. E is the mid point of median AD of ΔABC. BE *is* extended to meet AC at F. Show that

$$AF = \frac{1}{3} AC.$$

28. Show that a quadrilateral formed by joining the mid points of the consecutive side of a square is also a square.

29. Prove that a quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

30. P and Q are two points on opposite sides AD and BC of parallelogram $ABCD$ such that diagonal PQ passes through O, the point of intersection of diagonals AC and BD. Prove that PQ is bisected at point O.

31. $ABCD$ *is a* rectangle whose diagonal BD bisects $\angle B$. Show that $ABCD$ *is* a square.

32. D, E and F are respectively the mid-points of sides AB, BC and CA of a ΔABC. Prove that by joining the points D, E and F triangle ABC divided into four congruent triangles.

33. Prove that the line joining the mid-points of diagonals of a trapezium, is parallel to the parallel sides of that trapezium.

34. P is the mid point of side CD of a parallelogram $ABCD$. *A* line drawn passing through C and parallel to PA meets AB *at* Q and extended DA at R. Prove that $DA - AR$ and $CQ - QR$.

35. Construct a quadrilateral $ABCD$ in which AB – 3.7 cm, BC – 3 cm, CD – 5 cm, AD = 4 cm and $\angle$ A = 90°.

36. Construct a quadrilateral $ABCD$ in which $AB - AD$ – 3.2 cm, BC – 2.5 cm, AC – 4 cm and BD = 5 cm.

37. Construct a quadrilateral $PQRS$ in which PQ = 3.5 cm, QR = 3.5 cm, $\angle$ P = 60°, $\angle$ Q = 105° and $\angle$ S = 75°.

38. Construct a rhombus whose one side is 3.6 cm and one angle is 60°.

39. Construct a square in which $AB + BC + CD - DA$ – 12.8 cm.

40. Construct a trapezium in which $AB \mid CD$, AB – 5 cm, BC – 3 cm, AD = 3.3 cm and distance between parallel sides is 2.5 cm.

41. Construct a rhombus $ABCD$ in which AB = 6 cm and $\angle A = 120°$.

42. Construct a trapezium where AB – 2.3 cm, BC – 3.4 cm, CD = 5.4 cm, DA – 3.7 cm and AB || CD.

43. Construct a rhombus $ABCD$ whose diagonals are 5.6 cm and 7.2 cm.

44. Construct a rectangle $ABCD$ in which AB – 4.5 cm and BD = 6 cm.

Answers

EXERCISE 9.1

1. 36°, 60°, 108°, 156°
2. AC = 6 cm, BD = 4 cm
3. No, diagonals of a parallelogram bisects each other.
4. No, sum of the angles of a quadrilateral should be 360°.
5. No, sum of angles will be greater than 360° which is not possible for a quadrilateral.
6. 84°
7. Each angle is 135°
8. 120°, 60°, 120°, 60°

EXERCISE 9.2

1. 55°
2. No, sum of all angles will be less than 360°.
3. Yes, that will be rectangle or square.
4. 145°
5. 4 cm
6. 60, 120, 60, 120
13. (i) 6.9 cm (ii) 9.9 cm

Miscellaneous Exercise 9

1. D	2. B	3. C	4. D
5. C	6. B	7. D	8. C
9. B	10. D	11. C	12. C
13. C	14. C	15. C	

16. 90°, this quadrilateral is a rectangle.
17. This statement is not true because diagoanls of a rhombus are perpendicular to each other but they are not equal.
18. It is not necessary to be parallelogram.
19. Parallelogram.
20. Rectangle or square
21. No, diagonals of rectangle are equal but not perpendicular to each other.
23. 4 cm

10

Area of Triangles and Quadrilaterls

10.01 Introduction

We know that the study of Geometry, orginated with the measurement of land in the process of recasting boundaries and distribution of the fields. For example, Kartik distributes his trapezium shaped field by joining the mid points of non-parallel sides between his two daughters. (See Fig 10.01). Is this distribution equal in area? To get answer to this type of problems, there is a need to have a look at area of plane figures.

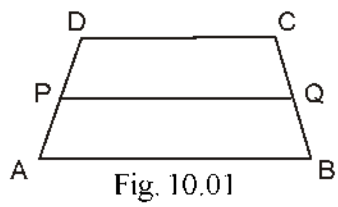


Fig. 10.01

10.02 Area

The part of plane enclosed by a simple closed figure, is called the plane region of that figure and magnitude or measure of this plane region is called the area of that field. This magnitude or measure is always expressed with the help of some units, for example 6 square cm (cm^2), 9 square metre (m^2), 12 hectare etc.

In chapter 7, we have studied about congruent figures. If two plane figures are same in shape and measure, they are called congruent. If they are cut and put on any plane, then both the figures have an equal plane region on that plane that means, two congruent figures are equal in areas. Is its converse also true? Let us try to understand the following activities.

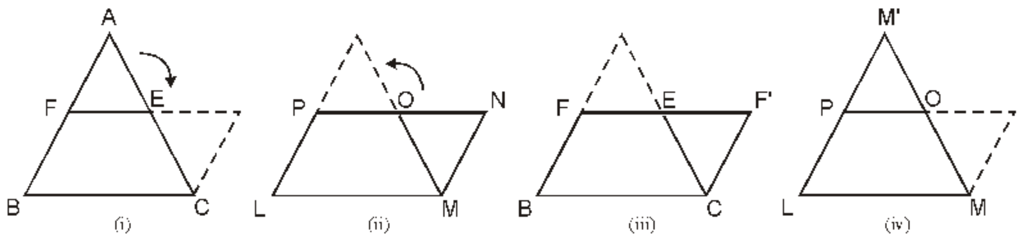


Fig. 10.02

Step-1

Prepare a carbon copy of Fig 10.02 (i), (iii) by leaving the dotted part with the help of pencil and scale on a white paper, by using carbon paper putting under this page of book. From prepared carbon copies, cut ΔAEF of fig 10.02 (i) and according to 10.02 (ii), keeping E on the same point, move ΔAEF so that A reach to point C and paste. Now we will get a quadrilateral $BCF'F$. Similarly, from Fig. 10.02 (iii), cut a ΔMNO and at O still, move such that N comes to P and paste as in Fig. 10.02 (iv). In this way a $\Delta LMM'$ will be obtained.

Step-2

Now put quadrilateral $BCF'F$ on Fig. 10.02 (iii) and $\Delta LMM'$ on Fig. 10.02 (i). Are they cover each other completely? Yes, they are covering. It means that quadrilateral $BCF'F \cong$ quadrilateral $LMNP$ and $\Delta LMM' \cong \Delta BCA$. Are these congruent figures equal in area? Definitely they are equal. But converse of it that ΔABC and quadrilateral $BCF'F$ and quadrilateral $LMNP$ and $\Delta LMM'$ which are equal in area. Area Δ ABC, quadrilateral $BCF'F$ and quadrilateral $LMNP$, $\Delta LMM'$ are congruent? Undoubtebly no.

So, we can say that ***congruent figures are equal in area but the figures equal in area need not necessarily congruent.***

If quadrilateral $BCF'F \cong$ quadrilateral $LMNP$ and $\Delta LMM' \cong \Delta BCA$, then we write it as :

$$ar(BCF'F) = ar(LMNP) \text{ and } ar(LMM') = ar(BCA)$$

Let us see the Fig 10.03. You may observe that plane region figure T is made by adding two figures X and Y in plane region. Now, you can easily see that

Area of figure T = Area of figure X + Area of figure Y

OR $\quad ar(T) = ar(X) + ar(Y)$

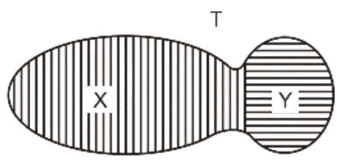


Fig. 10.03

In the previous classes, you have studied the formulae for finding the areas of different figures such as reactangle, square, triangle, etc. In this chapter, you will learn about the relation between area of geometrical figures under the condition when they lie on the same base and between the same pair of parallel lines. By studying this we will try to understand the deep knowledge of these formulae.

10.03 Figures Made on Same Base and Between Pair of Same Lines

Look at the following figures 10.04.

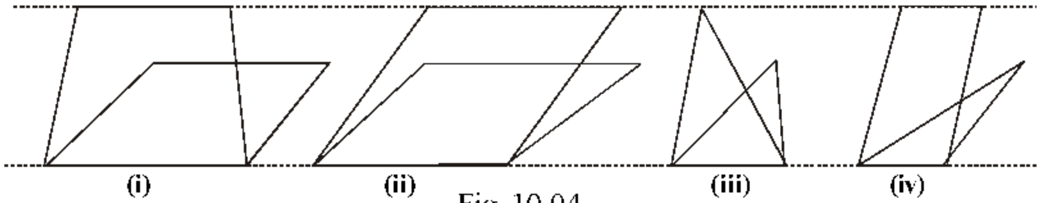


Fig. 10.04

In Fig. 10.04 (i), (ii), (iii) and (iv) in each case two figures are made on the same base but they are not made between same dotted wall of pair of parallel lines.

Now see the figures 10.05 given below.

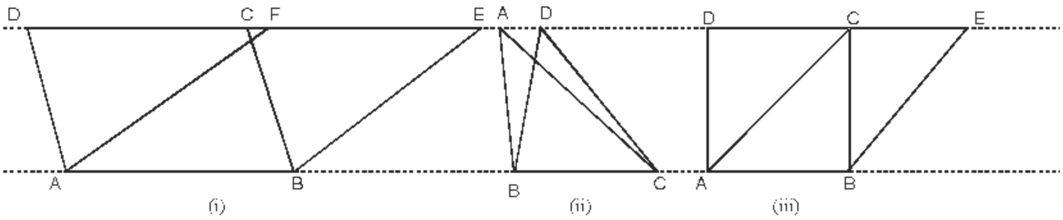


Fig. 10.05

In Fig. 10.05 in each case two figures are between two parallel lines. Here, in Fig. 10.05 (i) parallelogram $ABCD$ and $ABEF$ are on same base AB and between same parallel lines AB and DE. In Fig. 10.05 (ii), $\Delta\, ABC$ and Δ BCD are on same base BC and between same parallel lines BC and AD. Similarly, in Fig. 10.05 (iii) a square $ABCD$ and a parallelogram $ABEC$ are on same base AB and between same parallel lines AB and DE.

In Fig. 10.05 (i), (ii), (iii) are such figures that are said to be made on same base and between same parallel lines. In all the figures the bases are common in the two figures and the opposite-vertex of common base, is on the line drawn parallel to the base in each figure.

Now in article 10.06, keeping in mind the knowledge gained till now, which group of figures in fig. (i), (ii), (iii) and (iv) is made on the same base and between same parallel lines?

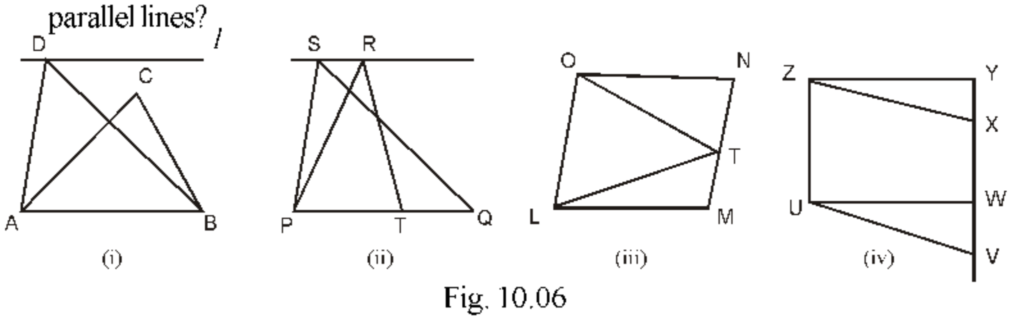


Fig. 10.06

Let us discuss

In Fig. 10.06 (i), $\Delta\, ABC$ and $\Delta\, ABD$ have common base AB, but the vertex C of $\Delta\, ABC$ does not lie on the line l which is parallel to base AB.

In Fig. 10.06 (ii) PT is the base of ΔPTR and PQ is base of ΔPQS means there is no common base of both the triangles but both the triangles are made in between two parallel lines PQ and SR.

In Fig. 10.06 (iii), parallelogram $LMNO$ and ΔLTO are on the same base LO and between the parallel lines LO MN. Similarly, in Fig. 10.06 (iv), parallelogram $UVXZ$ and rectangle $UWYZ$ are made on same base UZ and between a pair of parallel lines UZ and VY. In this way Fig. 10.06 (i), (ii) are not in category of the figures, made on same base and between same parallel lines while Fig. 10.06 (iii) and (iv) are said to be in this category.

EXERCISE 10.1

1. Which of the following figures are lying on the same base and between the same parallel lines? Write common base and pair of parallel lines in such a case.

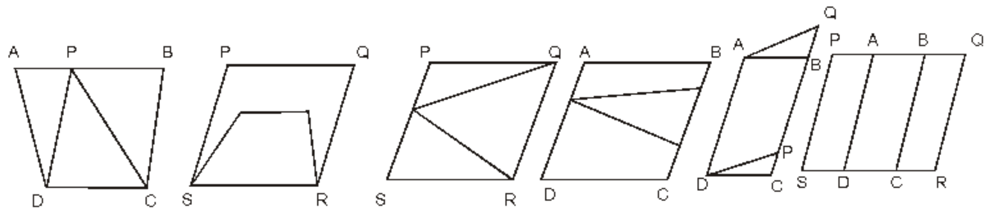


Fig. 10.07

2. Draw the following figures, on the same base and between the same parallel lines-

(i) An obtuse angled triangle and a trapezium.

(ii) A parallelogram and an isosceles triangle.

(iii) A square and a parallelogram.

(iv) A rectangle and a rhombus.

(v) A rhombus and a parallelogram.

Activity 10.2

Step-1

Make two carbon copies of a parallelogram by keeping two carbon papers between three white papers and labell their vertices by A, B, C, D. Mark a point P on side CD by pressing such that it also appears on carbon copies.

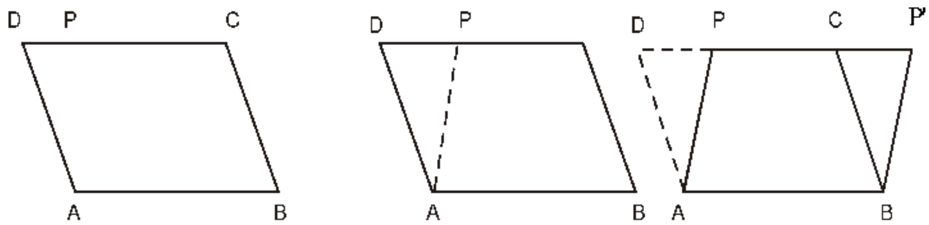


Fig. 10.08

Step-2

(i) Cut original copy and paste it on a page of your exercise note book.

(ii) Cut ΔAPD made by joining P to A on carbon copy. Paste ΔAPD on otherside of carbon copy, in such a way that after cutting side AD should coincide with the side BC of trapezium $ABCP$. Keep in mind that A should be on B and D on C.

Thus we are getting two new paralellograms $ABCD$ and $ABP'P$. Paste one of these two quadrilaterals on your exercise note book, on same page as fig. 10.08 (iii).

Step-3

Paste another new parallelogram $ABP'P$ on original copy such that side AB of both parallelograms should coincide. (See Fig 10.09).

In a new figure two parallelograms $ABCD$ and $ABP'P$ are made on same base and between a pair of same parallel lines.

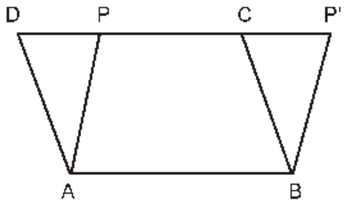


Fig. 10.09

Can you say that parallelogram $ABCD$ and $ABP'P$ are equal in area? Let us see.

$\because$ $\Delta APD \cong \Delta BP'C$

(ΔAPD is pasted after cutting)

$\therefore$ $ar(APD) = ar(BP'C)$

Adding ar ($ABCP$) on both sides, we get

ar (APD) − ar ($ABCP$) = ar ($ABCP$) − ar ($BP'C$)

or ar ($ABCD$) = ar ($ABP'P$)

$\Rightarrow$ Both parallelograms which are made on same base AB and between the parallel lines (AB | DP'), are equal in area.

Let us try to prove this result by any other method.

Theorem 10.1. ***Two parallelograms, made on same base and between the same parallelal lines, are equal in areas.***

Given : Two parallelograms $ABCD$ and $ABFE$ whose base is AB and are between two parallel lines AB and DF.

To Prove : Area of parallelogram $ABCD$ − Area of parallelogram $ABFE$

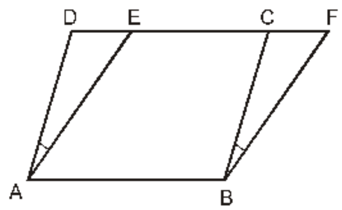


Fig. 10.10

Proof : In ΔADE and ΔBCF

$AE = BF$ (Opposite sides of parallelogram $ABFE$)

$\angle DAE = \angle CBF$ (Corresponding angles)

$AD = BC$ (Opposite sides of parallelogram $ABCD$)

$\Delta ADE \cong \Delta BCF$ (By SAS congruency rule)

$\therefore$ ar (ΔADE) = ar (ΔBCF)

Adding ar (ABC) on both sides, we get

ar (ΔADE) + ar ($ABCE$) = ar (BCF) + ar ($ABCE$)

$\therefore$ ar ($ABCD$) = ar ($ABFE$) **Hence Proved**

Corollary 1.

A parallelogram and a rectangle are lying on same base and between two parallel lines, then their areas are equal and area of parallelogram is equal to the product of its base and distance between two parallel lines.

Given : In Fig. 10.11, $ABCD$ is a parallelogram and $EFCD$ is a rectangle. Also $AL \perp DC$

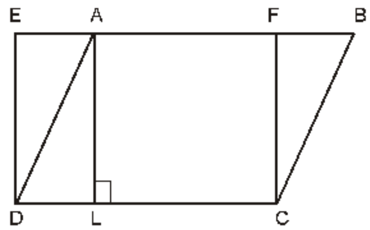


Fig. 10.11

To Prove : (i) ar ($ABCD$) = ar ($EFCD$)

(ii) $ar(ABCD) = DC \times AL$

Proof : (i) As rectangle is also a parallelogram,

$\therefore$ ar ($ABCD$) = ar ($EFCD$) . . . (i)

(ii) $\because$ Area of rectangle = length × breadth

$\therefore$ $ar\ (EFCD) = DC \times FC$

$\therefore$ $ar\ (ABCD) = DC \times FC$ [From (i)] ...(ii)

$\because$ $AL \perp DC$ (given)

So, $AFCL$ is also a rectangle.

$\therefore$ $AL = FC$ ... (iii)

Thus, ar ($ABCD$) = $DC \times AL$ [From (ii) and (iii)] **Hence Proved**

Corollary 2.

If a triangle and a parallelogram are made on the same base and between pair of same parallel lines, then area of the triangle is half of the area of parallelogram.

Given : ΔABP and parallelogram $ABCD$ are made on same base AB and between same parallel lines AB and PC.

To Prove : $ar(PAB) = \frac{1}{2} ar(ABCD)$

Construction : Draw $BQ \parallel AP$.

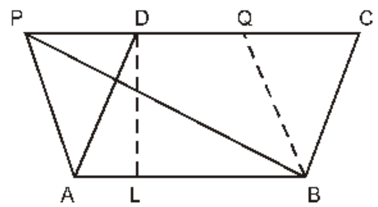


Fig. 10.12

Proof :

$\therefore$ $AB \mid CD$ (given)

$\therefore$ $AB \mid PQ$

and $AP \mid BQ$ (By construction)

$\therefore$ $ABQP$ is a parallelogram.

$\therefore$ ar (ABCD) = ar (ABQP) (By theorem 10.1)

And $\Delta ABP \cong \Delta QPB$ (A diagonal B divides a parallelogram in two congruent triangles)

$\therefore$ $ar(ABP) - \text{ar}(QPB)$

$$= \frac{1}{2} ar(ABQP)$$

$\Rightarrow$ $ar(ABP) = \frac{1}{2} ar(ABCD)$ **Hence Proved**

Corollary 3.

$$\textbf{\textit{Area of triangle}} = \frac{1}{2} \times \textbf{\textit{base}} \times \textbf{\textit{height}}$$

From Fig. 10.12, if DL $\perp$ AB,

$\therefore$ ar(ABCD) $- AB \times DL$ then by corollary

But $ar(PAB) = \frac{1}{2} ar(ABCD)$ (From corollary 2)

$\therefore$ $ar(PAB) = \frac{1}{2} AB \times DL$

or Area of triangle $= \frac{1}{2} \times$ base $\times$ height **Proved**

EXERCISE 10.2

1. In Fig 10.13, $ABCD$ is a parallelogram, in which AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, $AE - 8$ cm and $CF - 10$ cm, then find the value of AD.

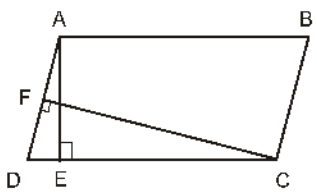


Fig. 10.13

2. If E, F, G and H are respectively the mid-points of sides of a parallelogram. Show that $\text{ar}(\text{EFGH}) = \frac{1}{2}\text{ar}(\text{ABCD})$

3. P and Q are respectively points lying on the side DC and AD respectively of a parallelogram $ABCD$. Show that ar (APB) = ar (BQC).

4. In Fig. 10. 14, P is any point in interior of a parallelogram $ABCD$. Show that :

(i) $\text{ar}(\text{APB}) + \text{ar}(\text{PCD}) = \frac{1}{2}\text{ar}(\text{ABCD})$

(ii) $\text{ar}(\text{APD}) + \text{ar}(\text{PBC}) = \text{ar}(\text{APB}) + \text{ar}(\text{PCD})$

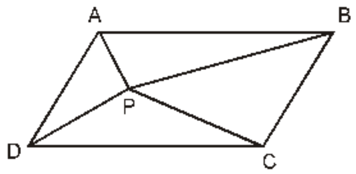


Fig. 10.14

5. In Fig. 10. 15, $PQRS$ and $ABRS$ are parallelograms and X is any point on side TR. Show that :

(i) $\text{ar}(\text{PQRS}) = \text{ar}(\text{ABRS})$

(ii) $\text{ar}(\text{AXS}) = \frac{1}{2}\text{ar}(\text{PQRS})$

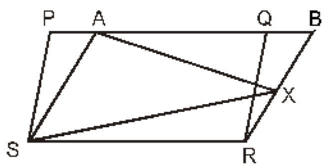


Fig. 10.15

6. A farmer had a field in the form of a parallelogram $PQRS$. He took any point A situated on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portion of the field separately. How should he do it?

10.4. Triangles on the same base and between same parallel lines :

Theorem 10.2. Area *of triangles, on same base and between same parallel lines, are equal.*

Given : Δ ABC and Δ DBC are on base BC and between parallel lines BC and AF.

To Prove : ar(Δ ABC) − ar(Δ DBC)

Construction : From point C draw two lines CE and CF parallel to AB and BD respectively.

Proof : $ABCE$ and $DBCF$ are between same parallel lines BC and AF.

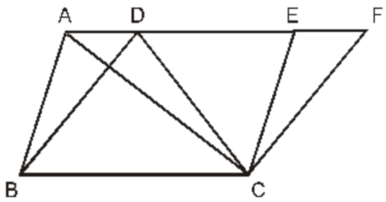


Fig. 10.16

$\therefore$ ar ($ABCE$) = ar ($DBCF$) ...(1)

[From theorem 10.1]

$\because$ AC *is* a diagonal of parallelogram $ABCE$,

$\therefore$ $$\text{ar}(\Delta\ \text{ABC}) = \frac{1}{2}\ \text{ar}\ (ABCE) \qquad ...(2)$$

Similarly, DC is a diagonal of parallelogram $DBCF$,

$$\text{ar}\ (DBC) - \frac{1}{2}\ \text{ar}\ (DBCE) \qquad ...\ (3)$$

From (1), (2) and (3), we get

ar (Δ ABC) − ar (Δ DBC) **Proved**

Theorem 10.3. ***If area of two triangles are equal and one side of a triangle is equal to one side of other triangle, then their corresponding altitudes are equal.***

Given : In Δ ABC and Δ DEF

(i) ar (Δ ABC) − ar (Δ DEF)

(ii) $BC - EF$

To Prove : Altitude AN = Altitude DM

Proof : In Δ ABC, AN is the altitude to the corresponding side BC.

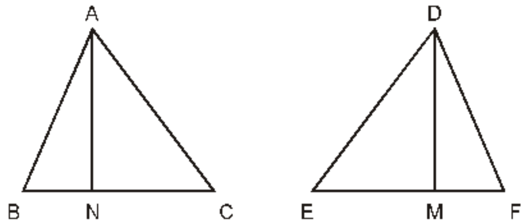


Fig. 10.17

$\therefore$ $ar(\Delta\text{ABC}) = \frac{1}{2} \times \text{BC} \times \text{AN}$ [By corollary 3] ...(1)

Similarly ar $(\Delta\ \text{DEF}) = \frac{1}{2} \times \text{EF} \times \text{DM}$ [By corollary 3]] ... (2)

But, given ar (Δ ABC) − ar (Δ DEF)

$\therefore$ $$\frac{1}{2} \times \text{BC} \times \text{AN} = \frac{1}{2} \times \text{EF} \times \text{DM}$$

But $BC = EF$ (Given)

$\therefore$ $AN = DM$ **Hence Proved**

10.5. Baudhayan Theorem

Baudhayan gave us a very important result on a right angled triangle which is known as Baudhayan Theorem. This theorem is also famous by the name of Pythagorus Theorem. Now we will prove it.

Theorem 10.4. ***In a right angled triangle, square made on hypotenuse, is equal to the sum of the squares made on other two sides.***

Given : In ΔABC, $\angle C = 90°$, and squares on sides AB, BC and CA are $ADEB$, $CBFG$ and $ACHK$ respectively.

To Prove : Square $ADEB$ = Square $ACHK$ + Square $CBFG$

Construction : From point C draw $CM \parallel BE$ which intersects AB at L. Join BK and CD.

Proof : $\angle BAD = \angle CAK = 90°$

Adding $\angle$ CAB both sides

$$\angle BAD + \angle CAB = \angle CAK + \angle CAB$$

$$\Rightarrow \quad \angle CAD = \angle BAK \qquad ...(1)$$

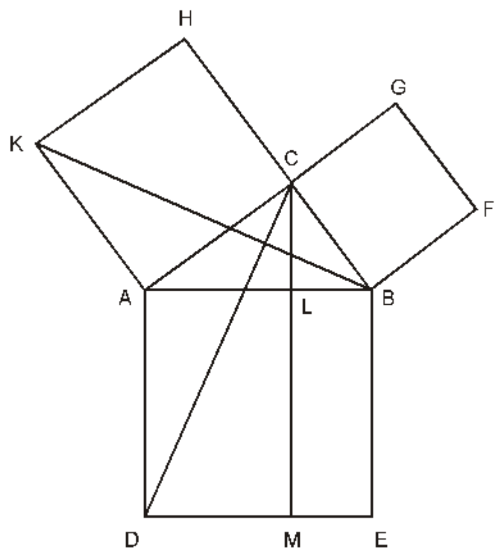


Fig. 10.18

In ΔBAK and ΔDAC, we get

$AB = AD$

(Sides of a square $ABED$)

$\angle BAK = \angle$ DAC [From (1)]

$AK = AC$

[Sides of square $ACHK$]

$$\Delta BAK \cong \Delta CAD$$

(By SAS congruency rule) ...(2)

But $\angle BCA = \angle ACH = 90°$

$\angle BCA + \angle ACH = 180°$

$\Rightarrow BCH$ is a straight line.

$CH \parallel AK$ (Opposite sides of square $ACHK$)

ΔBAK and square $ACHK$ *are* on same base AK and between same parallel lines AK and BH.

$\therefore$ $ar\ (\Delta BAK) = \frac{1}{2}$ ar (square $ACHK$) ...(3)

Similarly, ΔADC and rectangle $ADML$ are on same base AD and between same parallel lines AD and CM.

$\therefore$ ar ($\Delta\ CAD$) = $\frac{1}{2}$ ar (rectangle $ADML$) ... (4)

$\therefore$ From (2), (3) and (4), we get

ar ($\Delta\ CAD$) – ar ($\Delta\ BAK$) – $\frac{1}{2}$ ar of square ($ACHK$) – $\frac{1}{2}$ ar (of rectangle $ADML$)

$\therefore$ ar (square $ACHK$) – ar (rectangle $ADML$) ... (5)

Similarly, ar (square $CBFG$) − ar (rectangle $LMEB$) ...(6)

Adding (5) and (6), we get

ar (square $ACHK$) + ar (square $CBFG$) = ar (rectangle $ADML$) − ar (rectangle $LMEB$)

$\therefore$ ar (square ADEB) = ar (square ACHK) + ar (square CBFG) **Proved**

Theorem 10.5. (Converse of Baudhayan Theorem)

In a triangle, if square of a side is equal to the sum of the squares of other two sides, angle opposite to this side, is a right angle.

Given : In $\Delta\ ABC$, $AB^2 \quad BC^2 = AC^2$

To Prove : $\angle$ B = 90°

Construction : Construct a $\Delta\ PQR$ such that $\angle$ Q = 90°, $PQ - AB$ and $QR - BC$

Proof : In $\Delta\ PQR$ by Baudhayan theorem

$PR^2 - PQ^2 + QR^2$

But $PQ - AB$ and $QR = BC$.

$\therefore$ $PR^2 - AB^2 + BC^2$ ...(1)

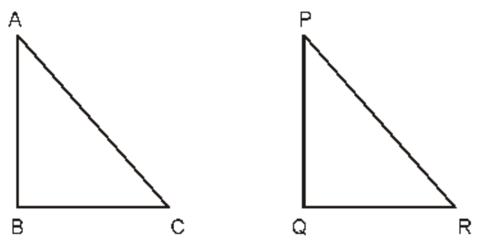


Fig. 10.19

But given that

$$AC^2 = AB^2 + BC^2 \qquad ...(2)$$

From (1) and (2)

$$PR^2 = AC^2 \Rightarrow PR = AC \qquad ...(3)$$

Now in $\Delta\, ABC$ and $\Delta\, PQR$, we get

$PQ = AB$ (By construction)

$QR = BC$ (By construction)

$PR = AC$ [From (3)]

$\Delta\, ABC \cong \Delta\, PQR$ (By SSS congruency rule)

$\therefore \quad \angle B = \angle Q = 90^\circ$

But $\quad \angle Q = 90^\circ$

$\therefore \quad \angle B = 90^\circ$ **Hence Proved**

Illustrative Examples

Example 1. *PQRS* is a square. *T* and *U* are the mid-points of PS and *QR* respectively (Fig. 10.20). Find the area of ΔOTS. If PQ = 8 cm and *O* is the point of intersection of *TU* and QS.

Solution : $PS = PQ = 8$ cm and $TU \quad PQ$

$$\therefore \quad ST = \frac{1}{2}PS = \frac{1}{2}\times 8 = 4 \text{ cm}$$

also $\quad PQ = TU = 8$ cm and $PQ \parallel TU$

In ΔPQS, T is the mid point of PS and $TO \parallel PQ$ them

$$OT = \frac{1}{2}PQ.$$

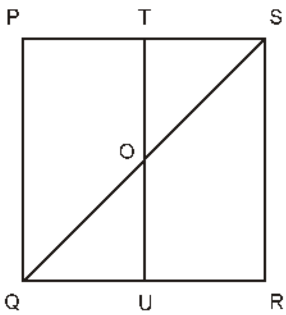


Fig. 10.20

$$\therefore \quad OT = \frac{1}{2}TU = \frac{1}{2}\times 8 = 4 \text{ cm}$$

$$\therefore \quad ar(\Delta\, OTS) = \frac{1}{2}\times OT \times TS$$

[$\Delta\, OTS$ is a right angle triangle]

$$= \frac{1}{2}\times 4 \times 4 \text{ cm}^2 = 8 \text{ cm}^2$$

Exercise 2. *ABCD* is a parallelogram and BC is produced upto *Q* such that AD = *CQ* (Fig 10.21). If *AQ* intersects side *DC* at P. Then show that : ar(ΔBPC) = ar(ΔDPQ)

Solution : ar(ACP) = ar(BCP) ...(1)

[Triangles made on same base PC and between same parallel lines PC and AB]

also ar (ADC) – ar (ADQ) ...(2)

[Triangles made on same base AD and between same parallel lines AD and BQ]

ar (ADC) - ar (ADP) − ar (ADQ) - ar (ADP)

ar (APC) = ar (DPQ) ...(3)

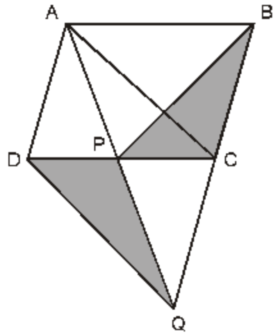


Fig. 10.21

From (1) and (3)

$\therefore$ ar (BCP) – ar (DPQ) **Hence Proved**

Example 3. In Fig. 10.22 *ABCD* is a parallelogram. Point P and *Q* divide side BC in three equal parts. Prove that ar (APQ) = ar (*DPQ*) = $\frac{1}{6}$ ar (ABCD).

Solution : Draw PR and QS parallel to AB from points P and Q respectively (Fig. 10.22).

Now, $PQRS$ is a parallelogram

whose base $PQ - \frac{1}{3}$ BC.

Now ar (APD) = $\frac{1}{2}$ ar $(ABCD)$

[triangle and parallelogram $ABCD$ lie on same base AD and between parallel lines AD and BC] ...(1)

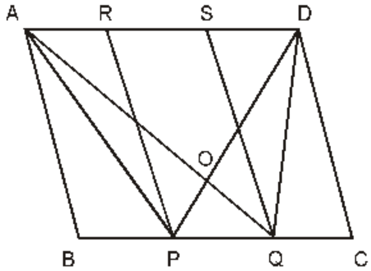


Fig. 10.22

also $\quad \text{ar}(\text{AQD}) = \frac{1}{2}\text{ar}(\text{ABCD})$ ...(2)

From (1) and (2), we get

ar (APD) = ar (AQD) ...(3)

Subtracting ar (AOD) from both sides, we get

ar (APD) - ar (AOD) = ar (AQD) - ar (AOD)

$\therefore$ ar (APO) = ar (OQD) ...(4)

Adding ar (OPQ) to both sides, we get

ar (APO) + ar (OPQ) − ar (OQD) + ar (OPQ)

ar (APQ) = ar DPQ

$\because$ ar (APQ) = $\frac{1}{2}$ ar (PQRS) and $\qquad ar\,(DPQ) = \frac{1}{2}\,ar(PQRS)$

now $\qquad \text{ar (PQRS)} = \frac{1}{3}\,ar\,(ABCD)$

$$\therefore \qquad \text{ar}(\text{APQ}) = \text{ar}(\text{DPQ}) = \frac{1}{2}\text{ar}(\text{PQRS}) = \frac{1}{2}\times\frac{1}{3}\text{ar}(\text{ABCD})$$

$$= \frac{1}{6}\text{ar}(\text{ABCD})$$

Hence Proved

Example 4. **In Fig. 10.23 *l*, *m* and n are lines such that *l* || *m* and line *n* intersects line *l* at P and line *m* at *Q*. *ABCD* is a quadrilateral such that vertex *A* is situated on line *l*, vertices C and D are situated on line *m* and *AD* || *n*. Show that :**

ar (ABCQ) = ar (*ABCDP*)

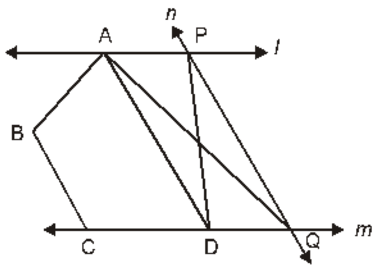


Fig. 10.23

Solution : $ar\,(ADP) = ar\,(ADQ)$ ...(1)

[On same base AD and between same parallel lines AD and PQ]

Adding ar $(ABCD)$ to both sides of (1), we get

ar (ADP) + ar $(ABCD)$ = ar (ADQ) + ar $(ABCD)$

$ar\,(ABCDP) = ar\,(ABCQ)$ **Hence Proved**

Example 5. **In Fig 10.24, *BD* || CA, *E* is the mid-point of CA and $BD = \frac{1}{2}$ CA.**

Prove that ar (ABC) = 2 ar (*DBC*).

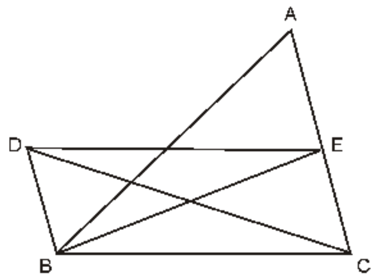


Fig. 10.24

Solution : Join DE. Here $BCED$ is a parallelogram.

$\because \qquad BD = CE \text{ and } BD \parallel CE$

$\therefore \qquad ar\,(DBC) = \text{ar (EBC)}$ ...(1)

(On same base BC and between same parallels BC and DE)

BE is median in Δ ABC.

$$\therefore \qquad \text{ar (EBC)} = \frac{1}{2}\,\text{ar (ABC)}$$

Now ar (ABC) = ar (EBC) + ar (ABE)

$\therefore \qquad$ ar (ABC) = 2 ar (EBC)

$\therefore \qquad$ ar (ABC) = 2 ar (DBC) $\qquad$ [using (1)] $\qquad$ **Hence Proved**

Example 6. In an acute angled Δ ABC, $\angle B$ is an acute angle. Therefore, all angles will be less than 90°. AD is perpendicular on BC. Prove that :
$AB^2 = AC^2 + BC^2 - 2\ BC \times DC$

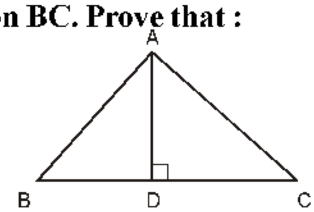


Fig. 10.25

Solution : Given : ΔABC, AD $\perp$ BC

To Prove :

$$AB^2 - AC^2 + BC^2 - 2\ BC \times DC$$

Proof : In Δ ABD, $\angle D - 90°$

$\therefore$ $AB^2 = AD^2 \quad BD^2$ (By Budhayan Theorem)

$\Rightarrow$ $AB^2 - AD^2 \quad (BC \quad DC)^2$

$= AD^2 + BC^2 + DC^2 - 2\ BC \times DC$

$= (AD^2 - DC^2) - BC^2 - 2\ BC \times DC$ ...(1)

Also in Δ ADC, $\angle$ D = 90^0

$\therefore$ $AC^2 = AD^2 + DC^2$ ...(2)

From (1) and (2), we get

$$AB^2 - AC^2 \quad BC^2 \quad 2\ BC \times DC$$ **Hence Proved**

Example 7. Prove that the sum of the squares of the sides of a rhombus, is equal to the sum of the squares of its diagonals.

Solution :

Given : Diagonals AC and BD of a rhombus $ABCD$ interest at point O.

To Prove : $AB^2 + BC^2 \quad CD^2 + DA^2 - AC^2 \quad BD^2$

Proof : We know that diagonals of a rhombus intersect each other at right angles.

Therefore, in $\Delta\ AOB$, $OA^2 - OB^2 - AB^2$ ...(1)

Similarly, in Δ BOC, $OB^2 - OC^2 - BC^2$ ...(2)

In Δ COD, $OC^2 + OD^2 - CD^2$ ...(3)

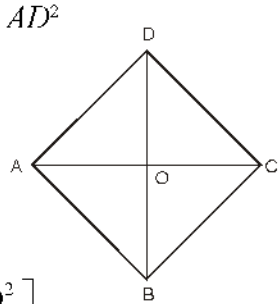
And in Δ AOD, $OA^2 \quad OD^2 - AD^2$ ...(4)

Adding (1), (2), (3) and (4), we get

$$2(OA^2 + OB^2 - OC^2 - OD^2) - AB^2 + BC^2 + CD^2 + AD^2$$

$$\because \quad OA - OC - \frac{1}{2} AC$$

$$\text{and} \quad OB - OD - \frac{1}{2} BD$$



$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{AC^2}{4} + \frac{AC^2}{4} + \frac{BD^2}{4} + \frac{BD^2}{4}\right]$$

Fig. 10.26

$$\therefore \quad AB^2 \quad BC^2 + CD^2 \quad AD^2 - 2\left[\frac{AC^2}{2} + \frac{BD^2}{2}\right]$$

$$- AC^2 - BD^2$$ **Hence Proved**

EXERCISE 10.3

Write true or false and give reason to your answer :

1. $ABCD$ is a parallelogram and X is the mid-point of AB. If $ar(AXCD) = 24\text{cm}^2$, then ar (ABC) = 24 cm^2.
2. $PQRS$ is a rectangle which is inside a quadrant of a circle of radius 13 cm. A is any point on side PQ. If $PS - 5$ cm, then $ar(RAS) - 30\text{cm}^2$.
3. $PQRS$ is a parallelogram whose area is 180 cm^2 and A is any point on diagonal QS. Then area of ΔASR is 90 cm^2.
4. ABC and BDE are two equilateral triangles such that D *is* the mid-point of side BC. Then ar $(BDE) - \frac{1}{4}$ ar (ABC).
5. In Fig 10.27, $ABCD$ and $EFGD$ are two parallelograms and G is the mid-point of side CD. Show that :

   ar (DPC) = $\frac{1}{2}$ ar (EFGD)

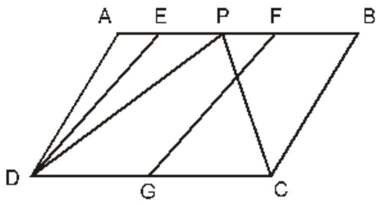


Fig. 10.27

6. In a trapezium $ABCD$, $AB \mid CD$ and L is the mid-point of side BC. A line PQ || AD is drawn through L which meets AB on P and extended DC at Q (Fig. 10.28). Prove that : ar $(ABCD)$ *ar* $(APQD)$.

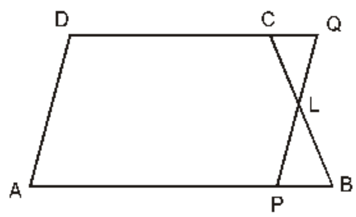


Fig. 10.28

7. If the mid-points of any quadrilateral are joined in a order, then prove that area of

such obtained parallelogram is half of the area of the given quadrilateral (Fig. 10.29).
[Hint: Join BD and draw perpendicular from A on BD.]

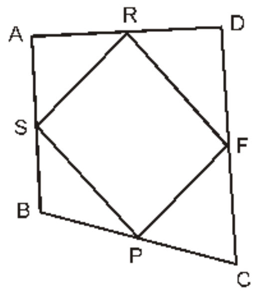


Fig. 10.29

8. A man walks 10 m in east and then 30 m in the north side. Find his distance from initial point.
9. A ladder is placed with a wall such that its lower end at a distance from wall is 7 m. If its other end at reached to the window height of 24 m. Find the length of the ladder.
10. Two poles of height 7 m and 12 m are standing on a plane ground. If distance between their feet is 12 m. Find the distance between upper ends of poles.
11. Find the length of altitude and area of an equilateral triangle whose length of side is a.
12. Find the length of diagonal of a square whose each side is 4 m.
13. If an equilateral triangle ABC, AD is perpendicular on BC then prove that $3AB^2 = 4\,AD^2$
14. O is any point inside a rectangle $ABCD$. Prove that $OB^2 + OD^2 = OA^2 + OC^2$
15. In an obtuse triangle ABC, $\angle$ C is an obtuse angle. $AD \perp BC$ meats BC at D on extending farward. Prove that :

$$AB^2 = AC^2 + BC^2 + 2BC \times CD$$

Important Points

1. If $\Delta ABC \cong \Delta PQR$, then ar ($\Delta\, ABC$) = ar ($\Delta$ PQR). Total area R of plane figure $ABCD$ is equal to the sum of area of triangular fields R_1 and R_2 or ar (R) = ar (R_1) + ar(R_2) [Figure 10.30]

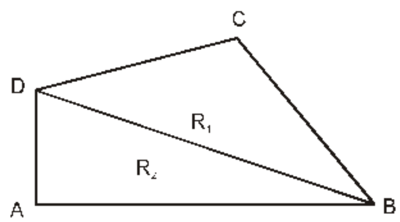


Fig. 10.30

2. Area of two congruent figures are equal but converse of it is not true always.

3. A diagonal of a parallelogram, divides it into two triangles of equal area.

(i) Areas of parallelograms made on same base and between same parallel lines, are equal.

(ii) A parallelogram and a rectangle made on same base and between same parallel lines are equal in areas.

4. Parallelograms made on same base and between same parallel lines, are equal in areas.

5. Triangles made on same base and between same parallel lines are equal in areas.

6. Corresponding altitudes of triangles having equal bases and equal areas, are equal.

7. If a triangle and a parallelogram are made on same base and between same parallel lines, then area of triangle is half of the area of parallelogram.

Miscellaneous Exercise 10

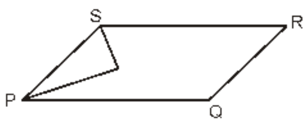
Write correct answer in each of the following :

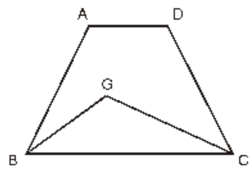
1. Median of a triangle divides it into two :

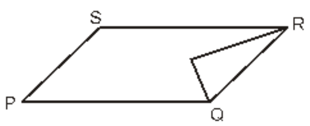
(a) triangles of equal areas (b) congruent triangles

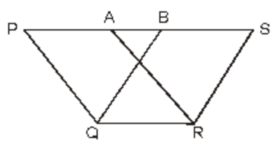
(c) right angled triangles (d) isosceles triangles

2. In which of following figures, you find two polygons made on same base and between same parallel lines :

(a) 

(b) 

(c) 

(d) 

3. The figure, made by joining mid-points of adjecent sides 8 cm and 6 cm of a rectangle is :

(a) a rectangle of area 24 cm^2 (b) a square of area 25 cm^2

(c) a trapezium of area 24 cm^2 (d) a rhombus of area 24 cm^2

4. In Fig. 10.31, area of parallelogram ABCD is :

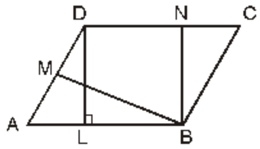


Fig. 10.31

(a) AB × BM (b) BC × BN

(c) DC × DL (d) AD × DL

5. In fig. 10.32, if parallelogram ABCD and rectangle ABEM are of equal areas, then :

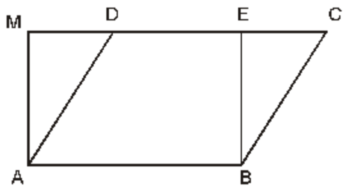


Fig. 10.32

(a) perimeter of ABCD = Perimeter of ABEM

(b) perimeter of ABCD < perimeter of ABEM

(c) perimeter of ABCD > perimeter of ABEM

(d) perimeter of ABCD = $\frac{1}{2}$ (Perimeter of ABEM)

6. Mid points of the sides of a triangle make a simple quadrilateral by taking with any vertex as fourth point, whose area is equal to :

(a) $\frac{1}{2}$ ar (ABC) (b) $\frac{1}{3}$ ar (ABC)

(c) $\frac{1}{4}$ ar (ABC) (d) ar (ABC)

7. Two parallelogram are on same base and between same parallel lines. Ratio of their areas is :

(a) 1 : 2 (b) 1 : 1

(c) 2 : 1 (d) 3 : 1

8. ABCD is a quadrilateral whose diagonal AC divides it into two parts of equal areas, then ABCD :

(a) is a rectangle (b) is always a rhombus

(c) is a parallelogram (d) is all of these

9. A triangle and a parallelogram are on same base and between same parallel lines, then ratio of areas of triangle with area of parallelogram is :

(a) 1 : 3 (b) 1 : 2

(c) 3 : 1 (d) 1 : 4

10. ABCD is a trapezium whose sides AB = a cm and DC = b cm (Fig. 10.33) E and F are mid-points of non-parallel sides. Ratio of ar (ABFE) and ar (EFCD) is :

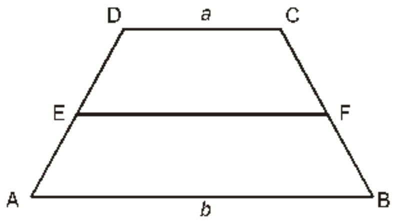


Fig. 10.33

(a) a : b (b) (3a + b) : (a + 3b)

(c) (a + 3b) : (3a + b) (d) (2a + b) : (3a + b)

11. If P is any point on median AD of ΔABC, then ar (ABP) $\neq$ ar(ACP).

12. If in fig. 10.34, PQRS and EFRS are two parallelogram, then ar (MFR) = $\frac{1}{2}$ ar (PQRS).

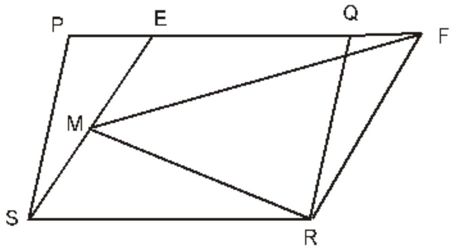


Fig. 10.34

13. In Fig. 10.35 PSDA is a parallelogram. Points Q and R on PS are taken such that PQ = QR = RS and PA || QB || RC.

Prove that ar (PQE) = ar (CFD)

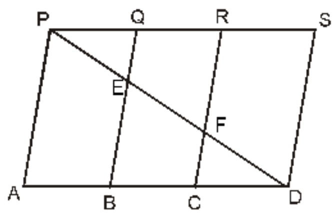


Fig. 10.35

14. X and Y are two points on side LN of ΔLMN such that LX = XY = YN. Through X a line is drawn parallel to LM, which meets MN at Z. (see Fig. 10.36). Prove that

: ar (LZY) = ar (MZYX)

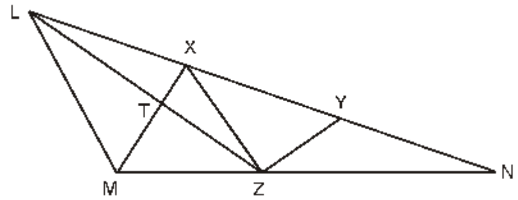


Fig. 10.36

15. Area of parallelogram ABCD is $90\,\text{cm}^2$ [Fig. 10.37]. Find the area of

(i) ar (ABEF) (ii) ar (ABD) (iii) ar (BEF)

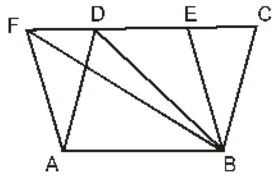


Fig. 10.37

16. In $\Delta ABC, D$ is the mid-point of side AB and P is any point on side BC. If line segment CQ || PD meets side AB at Q (Fig. 10.38), then prove that :

ar (BPQ) = $\frac{1}{2}$ ar (ABC)

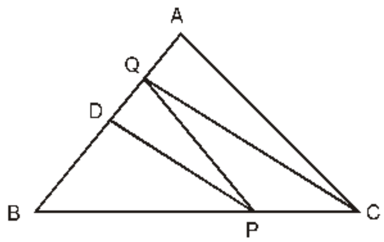


Fig. 10.38

17. ABCD is a square. E and F are respectively mid-points of sides BC and CD. If R is the mid-point of line segment EF (Fig. 10.39), then prove that : $ar(\Delta AER) = ar(\Delta AFR)$

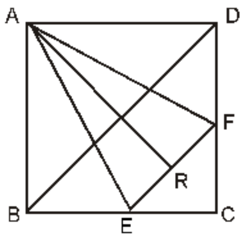


Fig. 10.39

18. O is any point on diagonal PR of a parallelogram PQRS (Fig. 10.40). Prove ar (PSO) = ar (PQO)

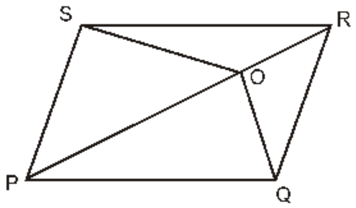


Fig. 10.40

19. ABCD is a parallelogram in which side BC is extended upto point E such that CE = BC (Fig. 10.41). AE intersects side CD at F. If ar (DFB) is 3 cm^2, then find the area of parallelogram ABCD.

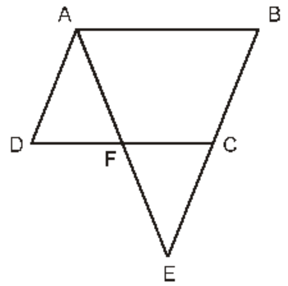


Fig. 10.41

20. Point E is taken on side BC of a parallelogram ABCD. AE and DC are extended so that they meet at F. Prove that ar (ADF) = ar (ABFC).

21. Diagonals of a parallelogram ABCD intersect at O. A line is drawn from O which meets AD at P and BC at Q. Show that PQ divides this parallelogram into two parts of equal areas.

22. Medians BE and CF of a ΔABC intersect each other at point G. Prove that area of ΔGBC is equal to the area of quadrilateral AFGE.

23. In Fig. 10.42 CD || AE and CY || BA. Prove that : ar (CBX) = ar (AXY)

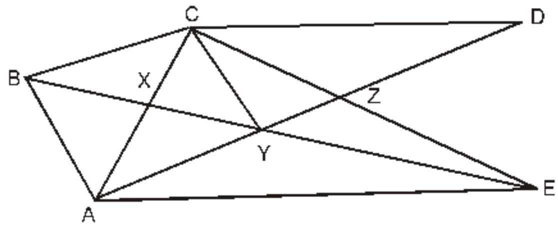


Fig. 10.42

24. ABCD is a trapezium in which AB || CD, CD = 30 cm and AB = 50 cm. If, X and Y are mid-points of AD and BC respectively, then prove that :

ar (DCYX) = $\frac{7}{9}$ ar (XYBA).

25. In ΔABC, L and M are point on sides AB and AC respectively such that LM || BC. Prove that : ar (LOB) = ar (MOC) is LC and BM intersect at O.

26. In Fig. 10.43 ABCDE is a pentagon. BP drawn parallel to AC, meets extended DC at P and EQ drawn parallel to AD meets extended CD at Q.
Prove that :
ar (ABCDE) = ar (APQ).

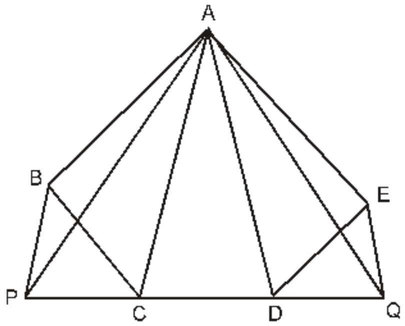


Fig. 10.43

27. If medians of a triangle ABC meet at point G, then prove that :

ar (AGB) = ar (AGC) = ar (BGC) = $\frac{1}{3}$ ar (ABC).

28. In figure 10.44 X and Y are respectively the mid points of sides AC and AB .QP || BC and CYQ and BXP are striaght lines. Show that : ar (ABP) = ar (ACQ)

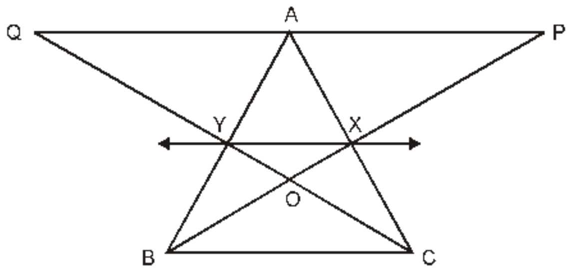


Fig. 10.44

29. In Fig. 10.45, ABCD and AEFD are two parallelogram. Prove that : ar (PEA) = ar (QFD).
[Hint : Join PD]

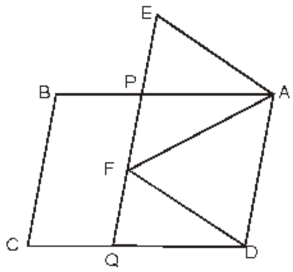


Fig. 10.44

Answers

EXERCISE 10.1

1. (i) DC and DC || AB ; (iii) QR, QR || PS ; (v) AD, AD || QC

EXERCISE 10.2

1. 12.8 cm

EXERCISE 10.3

1. Flase $ar(A \times CD) = ar(ABCD) - ar(BC \times) = 48 - 12 = 36\,\text{cm}^2$

2. True $SR = \sqrt{13^2 - 5^2} = 12ar(PAS) = \frac{1}{2}ar(PQRS) = 30\,\text{cm}^2$

3. Flase area of $\Delta QRS = 90\,\text{cm}^2$ and $ar(ASR) < ar(QRS)$

4. True $\frac{ar(BDE)}{ar(ABC)} = \frac{\sqrt{3}(BD)^2}{\sqrt{3}(BC)^2} = \frac{1}{4}$

4

5. Flase $ar(DPC) = \frac{1}{2}(ABCD) = ar(EFGD)$

6. $10\sqrt{10}$ m

9. 25 m

10. 13 m

11. $\frac{\sqrt{3}}{a}a, \frac{\sqrt{3}}{4}a^2$

12. $4\sqrt{2}$

Miscellaneous Exercise 10

1. A	**2.** D	**3.** D	**4.** C
5. C	**6.** A	**7.** D	**8.** D
9. B	**10.** B		

11. Flase: $ar(ABD) = (ACD)$ and $ar(PBD) = ar(PCD)$ $\therefore$ $ar(ABP) = ar(ACP)$

12. True: $ar(PQRS) = ar(EFRS) = 2ar(MFR)$

15. (i) 90 cm^2 ; (ii) 45 cm^2 ; (iii) 45 cm^2

19. 13 cm^3

11

Area of Plane Figures

11.01 Introduction

We know that a closed figure bounded by three line segments is called a triangle and a closed figure bounded by four line segments is called quadrilateral. The part of the plane enclosed by a simple closed figure is called plane region.

In the previous classes we have studied about the area of plane figures such as triangles and quadrilaterals.

In this chapter, we will find the area of triangle, quadrilateral, rectangular paths and area of four walls, with the use of Heron's formula.

11.02 Area of Triangle

We know that, area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$. Using this formula, we can find the base and height of a triangle as :

$$\text{Base of a triangle} = \frac{2 \times \text{Area}}{\text{Height}}$$

$$\text{Height of a triangle} = \frac{2 \times \text{Area}}{\text{Base}}$$

From the above formulae, height of triangle is compulsory to find the area of a triangle. If sides of a triangle are given and height is not given then the area of triangle can be found by using Heron's formula.

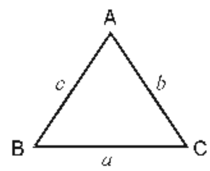


Fig. 11.01

Heron's Formula

(1) If a, b, c are respective three sides of any triangle then, area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{a+b+c}{2} = \frac{\text{Perimeter of triangle}}{2}$ =Semi-perimeter of the triangle

(2) Area of an Isosceles Triangle

If the length of two equal sides of an isosceles triangle is 'a' and length of third side is b, then semi-perimeter of isosceles triangle will be :

$$s = \frac{a+a+b}{2} = \frac{2a+b}{2}$$

By Heron's formula

Area of isosceles triangle

$$= \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{2a+b}{2}-a\right)\left[\frac{2a+b}{2}-b\right]\left[\frac{2a+b}{2}-a\right]}$$

$$= \sqrt{\left(\frac{2a+b}{2}\right)\times\left(\frac{b}{2}\right)\times\left(\frac{2a-b}{2}\right)\times\left(\frac{b}{2}\right)}$$

$$= \frac{b}{4}\sqrt{(2a+b)(2a-b)}$$

$$= \frac{b}{4}\sqrt{4a^2-b^2}$$

$\therefore$ Area of isosceles triangle $= \frac{b}{4}\sqrt{4a^2-b^2}$ sq. units.

(3) Area of an Equilateral Triangle

If a is the side of an equilateral triangle, then its semi-perimeter will be :

$$s = \frac{a+a+a}{2} = \frac{3a}{2}$$

$\therefore$ By Heron's formula

Area of equilater triangle

$$= \sqrt{\frac{3a}{2}\left[\frac{3a}{2}-a\right]\left[\frac{3a}{2}-a\right]\left[\frac{3a}{2}-a\right]}$$

$$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$$

$$= \frac{a^2\sqrt{3}}{4} \text{ sq. units}$$

(4) Area of a Right-angled Triangle :

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (\text{Product of sides containing a right angle})$$

$\therefore$ Area of right angled triangle $= \frac{1}{2} \times a \times b$ sq. units.

Illustrative Examples

Example 1. Find the area of a triangle whose sides are 8 cm, 15 cm and 17 cm.

Sol. Let a = 8 cm, b = 15 cm, c = 17 cm

$$\therefore \text{ Semi perimeter } = s = \frac{a+b+c}{2} = \frac{8+15+17}{2} = \frac{40}{2} = 20\,\text{cm}$$

$$\therefore \text{ Area of a triangle } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{20(20-8)(20-15)(20-17)} \text{ sq. cm}$$

$$= \sqrt{20 \times 12 \times 5 \times 3} \text{ sq. cm}$$

$$= \sqrt{100 \times 36} \text{ sq. cm}$$

= 60 sq. cm

$\therefore$ Area of a triangle = 60 sq. cm.

Example 2. Each of two equal sides of an isoscles triangle is 7 cm and the third side is 6 cm. Find the area of the triangle.

Sol. Let a = 7 cm, b = 6 cm

$$\text{Area of triangle } = \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{6}{4}\sqrt{4 \times 49 - 36} \text{ sq. cm}$$

$$= \frac{6}{4}\sqrt{196 - 36} \text{ sq. cm}$$

$$= \frac{6}{4}\sqrt{160} \text{ sq. cm}$$

$$= \frac{6}{4} \times 4 \times \sqrt{10} \text{ sq. cm}$$

$$= 6\sqrt{10} \text{ sq. cm}$$

Example 3. The base of a triangle is 24 cm and its height is 12 cm. Find the area of the triangle.

Sol. Base of the triangle = 24 cm and height = 12 cm

$$\text{Area of triangle} = \frac{1}{2} \times \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 24 \times 12$$

$$= 144 \text{ cm}^2.$$

Example 4. Find the area of an equilateral triangle whose one side is 8 cm.

Sol. Let a = 8 cm

$$\therefore \quad \text{Area of equilateral triangle} = \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4} \times (8)^2$$

$$= \frac{\sqrt{3}}{4} \times 64 = 16\sqrt{3} \text{ cm}^2$$

Example 5. Two sides of a triangle are 7 cm and 8 cm respectively. If the perimeter of the triangle is 24 cm, then find the area of the triangle.

Sol. Let a = 7 cm, b = 8 cm

$$\text{Perimeter} = a + b + c = 24 \text{ cm}$$

$$\therefore \quad \text{Third side, } c = 24 - 7 - 8 = 9 \text{ cm}$$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{24}{2} = 12 \text{ cm}$$

$\therefore$ Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-7)(12-8)(12-9)} \text{ cm}^2$$

$$= \sqrt{12 \times 5 \times 4 \times 3}$$

$$= \sqrt{12 \times 5 \times 4 \times 3}$$

$$= 12\sqrt{5} \text{ cm}^2$$

Exercise 11.1

1. If the base of a triangle is 20 cm and its height is 6 cm then find the area of the triangle.
2. A triangle whose sides are 15 cm, 25cm and 30cm respectively, then find the area of the triangle.
3. In an isoscels triangle each equal side is of 8 cm and third side is 4 cm, then find its area.
4. An equilateral triangle whose side is 20 cm, then find its area.
5. A triangle whose two sides are 8 cm and 15 cm and its perimeter is 40 cm then find the area of the triangle.
6. A triangular table whose sides are in the ratio 3 : 4 : 5 and perimeter is 36 m then find the area of the table.
7. A field whose shape is triangular. Its sides are 20m, 51m and 37m, then how many small beds of measure of 2×3 m^2 can be plotted in the field?

11.03 Area of a Quadrilateral

A plane figure enclosed by four line segments is called quadrilateral. Any quadrilateral can be divided into two triangles by drawing its diagonals. In Fig 11.02 diagonal AC of a quadrilateral ABCD divides it into two triangles ABC and ACD. Therefore, area of quadrilateral ABCD will be the sum of the areas of both the triangles. From vertices B and D draw perpendicular BE and DF on diagonal AC respectively of quadrilateral ABCD.

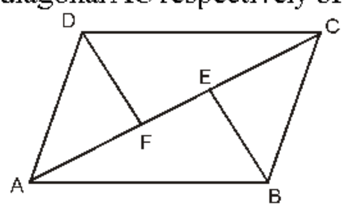


Fig. 11.02

$\therefore$ Area of Δ ABC $= \frac{1}{2} \times AC \times BE$

and Area of Δ ACD $= \frac{1}{2} \times AC \times DF$

$\therefore$ Area of Quadrilateral ABCD = Area of Δ ABC + Area of Δ ACD

$$= \frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times DF$$

$$= \frac{1}{2} \times AC \times (BE + DF)$$

$\therefore$ Area of a quadrilateral $= \frac{1}{2} \times$ diagonal $\times$ (Sum of perpendiculars drawn on the diagonal)

Area of a Parallelogram

A quadrilateral whose opposite sides are equal and parallel to each other is called parallelogram.

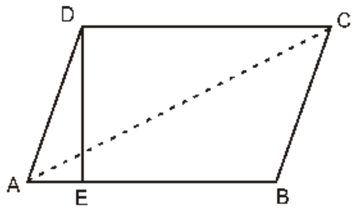


Fig. 11.03

(a) Area of parallelogram

= base $\times$ height = AB $\times$ DE

(b) Diagonal AC divides parallelogram ABCD into two triangles ABC and ADC of equal areas.

$\therefore$ Area of parallelogram = 2 × (Area of ΔABC)

Illustrative Examples

Example 6. A diagonal of a quadrilateral is 10 cm and length of perpendiculars drawn of quadrilateral on diagonal from opposite vertices are 6 cm and 4 cm respectively, then find the area of the quadrilateral.

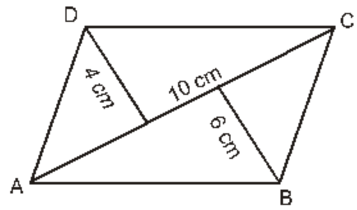


Fig. 11.04

Sol. Area of quadrilateral

$= \frac{1}{2} \times \text{diagonal} \times (\text{Sum of perpendicular drawn on the diagonal})$

$= \frac{1}{2} \times 10 \times (6+4)$ sq. cm.

$= \frac{1}{2} \times 10 \times 10$ sq. cm.

$= 50$ sq. cm.

Example 7. Find the area of a parallelogram whose height is 15 m and base is 10 m.

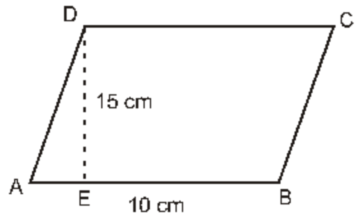


Fig. 11.05

Sol. Base = 10 m

Height = 15 m

Area of parallelogram = base × height

= 10 × 15

= 150 m^2

Example 8. Find the area of a quadrilateral ABCD whose diagonal AC = 15 cm and sides AB = 7 cm, BC = 12cm, CD = 12 cm and DA = 9 cm.

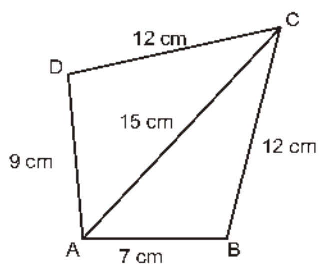


Fig. 11.06

Sol. According to figure

Area of quadritateral

ABCD = ar (ΔABC) + ar (ΔACD)

In ΔABC, AB = 7 cm, BC = 12 cm

and AC = 15 cm

$\therefore$ Semi-perimeter, $s = \frac{7+12+15}{2} = 17$ cm

$\therefore$ Area of $\Delta ABC = \sqrt{17 \times (17-7) \times (17-12) \times (17-15)}$

$= \sqrt{17 \times 10 \times 5 \times 2}$

$= \sqrt{1700}$

$= 10\sqrt{17}$

$= 10 \times 4.12 = 41.2 \text{ cm}^2$

In ΔACD, AC = 15 cm, CD = 12 cm and DA = 9 cm

$\therefore \quad s = \frac{15+12+9}{2} = \frac{36}{2} = 18 \text{ cm}$

$\therefore \quad$ Area of $\Delta ACD = \sqrt{18(18-15)\times(18-12)(18-9)}$

$= \sqrt{18 \times 3 \times 6 \times 9}$

$= 54 \text{ cm}^2$

$\therefore \quad$ Area of quadrilateral = (41.2 + 54.0)

$= 95.2 \text{ cm}^2$

Example 9. Two adjacent sides of a parallelogram are 5 cm and 4 cm repectively and diagonal is 7 cm. Find the area of the parallelogram.

Sol. According to figure 11.07, area of quadrilateral ABCD = 2 × (Area of ΔABC)

In ΔABC

AB = 5 cm, BC = 4 cm, AC = 7 cm

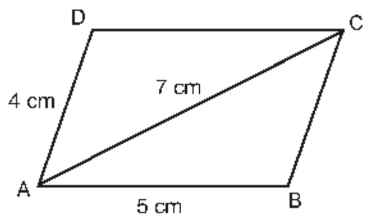


Fig. 11.07

$\therefore \quad s = \frac{5+4+7}{2} = \frac{16}{2} = 8 \text{ cm}$

From Heron's formula, area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-5)(8-4)(8-7)}$

$= \sqrt{8 \times 3 \times 4 \times 1}$

$= 4\sqrt{6} \text{ cm}^2$

$\therefore \quad$ Area of parallelogram = 2 × area of ΔABC

$= 2 \times 4\sqrt{6} \text{ cm}^2$

$= 8\sqrt{6} \text{ cm}^2$

Exercise 11.2

1. Find the area of the quadrilateral whose diagonal is 12 cm and lengths of perpendiculars drawn from opposite vertices to the diagonal are 7 cm and 8 cm respectively.
2. Area of a parallelogram shaped play ground is 2000 m^2. If its base is 50m, then find the breadth of the play ground.
3. Find the area of a quadrilateral whose sides are AB = 3 cm, BC = 4 cm, CD = 6 cm and DA = 5 cm and diagonal is AC = 5 cm.
4. Find the area of quadrilateral whose sides are 9 cm, 40 cm, 28 cm and 15 cm and angle between first two side is a right angle.

5. Find the area of a parallelogram whose two adjacent sides are 50 cm and 40 cm respectively and diagonal is 30 cm.
6. Find the area of a parallelogram whose a diagonal is 5.2 cm and the length of each perpendicular from the opposite vertices to the diagonal is 3.5 cm.
7. A plot of land has a shape of a parallelogram. It is to be covered by mud. Find the cost of spreading mud at the rate of ₹ 100 per square metre while the adjacent sides of the plot are 39 m and 25 m and the diagonal is 56 m.

11.04 Area of different Quadrilaterals

(1) Area of a cyclic Quadrilateral

A quadrilateral whose all four vertics are on the circumference of a circle is called cyclic quadrilateral. Opposite angles of a cyclic quadrilateral are supplementary from figure 11.08. ABCD is a cyclic quadrilateral whose sides ae a, b, c and d respectively.

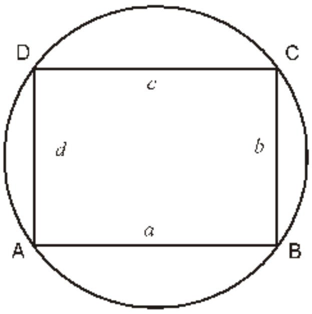


Fig. 11.08

$\therefore$ Semi perimeter $s = \dfrac{a+b+c+d}{2}$

$\therefore$ Area of cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

(2) Area of a Rhombus

A quadrilateral whose all four sides are equal and diagonals bisect each other at right angle, is called a rhombus.

$\therefore$ Area of Rhombus $= \dfrac{1}{2} \times$ (Product of diagonals)

(3) Area of a Trapezium

A quadrilateral whose only two sides are parallel, is called trapezium. In fig. 11.09, ABCD is a trapezium whose sides AB and CD are parallel and distance between the parallel sides is DE. Here, DE is perpendicular on AB and DB is the diagonal.

Therefore, area of trapezium ABCD

= Area of $\Delta ABD +$ Area of ΔBCD

$= \dfrac{1}{2} AB \times DE + \dfrac{1}{2} \times DC \times DE$

$= \dfrac{1}{2} \times DE \times (AB + DC)$

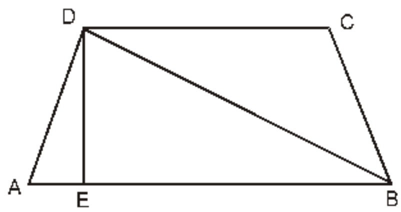


Fig. 11.09

Area of trapezium $= \dfrac{1}{2} \times$ sum of parallel sides $\times$ distance between parallel sides

Illustrative Examples

Example 10. Find the area of a cyclic quadrilateral whose sides are 36 cm, 77 cm, 75 cm and 40 cm.

Sol. Let a = 36 cm, b = 77 cm, c = 75 cm and d = 40 cm

$\therefore$ Semi perimeter is a cyclic quadrilateral

$$s = \frac{a+b+c+d}{2}$$

$$= \frac{36+77+75+40}{2}$$

$$= \frac{228}{2}$$

= 114 cm

$\therefore$ Area of a cyclic quadrilateral

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$= \sqrt{(114-36)(114-77)(114-75)(114-40)}$$

$$= \sqrt{78 \times 37 \times 39 \times 74}$$

$$= \sqrt{2 \times 39 \times 39 \times 37 \times 37 \times 2}$$

= 2 × 39 × 37

= 2886 cm^2

Example 11. The length of diagonals of a rhombus are 20 cm and 30 cm respectively, then find its area.

Sol. Area of rhombus = $\frac{1}{2}$ × (product of length of diagonals)

$$= \frac{1}{2} \times 20 \times 30$$

= 300 cm^2

Example 12. Find the area of a trapezium whose parallel sides are 65 cm and 50 cm respectively and non-parallel sides are 20 cm and 25 cm respectively.

Sol. In fig. 11.10 ABCD is a trapezium whose parallel sides are AB = 65 cm and DC = 50 cm and non-prallel sides are AD = 20 cm and BC = 25 cm

Here EB = AB – AE = AB – DC

= (65 – 50) cm

= 15 cm

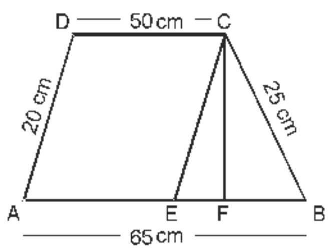


Fig. 11.10

$\therefore$ Semi perimeter of ΔBEC

$$s = \frac{15+20+25}{2} = 30cm$$

$\therefore$ Area of ΔBEC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{30(30-15)(30-20)(30-25)}$

$= \sqrt{30 \times 15 \times 10 \times 5}$

$= \sqrt{22500}$

$= 150 \text{ cm}^2$

Height of $\Delta BEC = CF = \dfrac{2 \times Area\,of\ \Delta BCE}{base\ BE}$

$= \dfrac{2 \times 150}{15} = 20\,cm$

$\therefore$ Area of parallelogram AECD = AE × CF

= 50 × 20

= 1000 cm^2

$\therefore$ Area of trapezium ABCD = area of parallelogram AECD + area of ΔEBC

= 1000 cm^2 + 150 cm^2

= 1150 cm^2

Example 13. Find the area of trapezium whose parallel sides are 32 cm and 37 cm respectively and the distance between parallel sides is 20 cm.

Sol. Area of trapezium = $\frac{1}{2}$ × (Sum of parallel lines) × (distance between parallel lines)

$= \frac{1}{2} \times (32 + 37) \times 20$

$= \frac{1}{2} \times 69 \times 20$

= 690 cm^2

Exerice 11.3

1. Sides of a cyclic quadrilateral shaped ground are 72 m, 154 m 80m and 150m respectively. Find its area. Find the total expenditure of the cost of tiling is ₹ 5 per square metre.
2. Diagonals of a rhombus are 25 cm and 42 cm. Find its area and perimeter.
3. Perimeter of a rhombus is 40 m and length of its diagonal is 12m, then find its area.
4. A trapezium shaped field whose parallel sides are 42 m and 30 m and the other sides are 18 m and 18m. Find its area.
5. If the area of a trapezium is 350 cm^2 and lengths of its parallel sides are 26 cm and 44 cm, then find the distance between the parallel sides.

6. A table has a shape of a trapezium. The parallel side of the table are 8 m and 16 m. If area of the table is 108 m^2 then find the width (distance between parallel sides) of the table.

11.5 Area of a Rectangle, Square and four walls

1. In fig. 11.11 length of the rectangular is a and breadth is b, therefore perimeter of rectangle = total length of all four sides

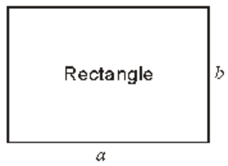


Fig. 11.11

= 2 × (length + breadth)

= 2 (a + b)

Area of rectangle = length × breadth

= a × b

2. If side of a square is 'a', then perimeter of square = 4 × side = 4a

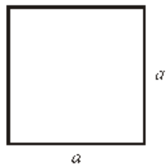


Fig. 11.12

area of square = $(\text{side})^2 = a^2$

3. Area of four walls of a room or rectangular tank = 2 × (length + breadth) × height

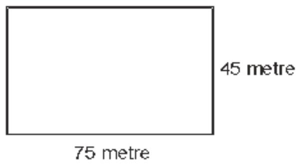


Fig. 11.13

∴ Area of four walls of a room = Peremeter × height

Perimeter of floor of a room = 2 × (length + breadth)

Illustrative Examples

Example 14. The length of a rectangular ground is 75 m and breadth is 45 m, then find the area and perimeter of the ground.

Sol. Length = 75 cm and breadth = 45 m

∴ Area of a rectangular ground

= length × breadth

= 75 × 45

= 3375 m^2

Perimeter of rectangular ground = 2 (length + breadth)

= 2 (75 + 45)

= 2 × 120

= 240 m

Example 15. Distance covered to take 5 rounds of a rectangular field is 600 m. If the breadth of the field is 25 m, then find its length.

Sol. Perimeter of field = $\frac{600}{5} m$

= 120 m

or $\quad 2(a + b) = 120$

or $\quad 2(a + 25) = 120$

or $\quad a + 25 = 60$

or $\quad a = 60 - 25$

or $\quad a = 35$ m

$\therefore$ Length of field = 35 m.

Example 16. Length of a square field is 120 m. Find its perimeter and area.

Sol. Side of a square field = 120 m

$\therefore$ Perimeter of field $= 4 \times$ side

$= 4 \times 120$

$= 480$ m

Area of field = $(\text{side})^2$

$= (120)^2$

$= 14400 \text{m}^2$

Example 17. Length of a rectangular field is 35 m and breadth is 20m. It is to be tiled. If the measures of a tile is 7 cm × 5cm then how many tiles will be required?

Sol. Length of rectangular field = 35 m

Breadth of rectangular field = 20 m

$\therefore$ Area of field $= 35 \times 20$

$= 700 \text{ m}^2$

Length of tile = 7 cm = 0.07 m

Breadth of tile = 5 cm = 0.05 m

$\therefore$ Area of a tile $= 0.07 \times 0.05$

$= 0.0035 \text{ m}^2$

$\therefore$ Required number of tiles

$$= \frac{area\ of\ field}{are\ of\ tile}$$

$$= \frac{700}{0.0035}$$

$$= \frac{7000000}{35}$$

$= 200000.$

Example 18. Area of a square ground is 625 m^2. There is 2.5 m wide path outside around it. Find the area of the path. If 50 cm long square pieces are to be paved in the path, then find how many square pieces are required.

Sol. As per question

Area of square ground = 625 m^2

$\therefore$ Length of the ground $= \sqrt{Area} = \sqrt{625} = 25$ m

In Fig. 11.14, there is 2.5 m path surrounding the ground outside

$\therefore$ Outer length with path = 25 + 2.5 + 2.5
= 30 m

$\therefore$ Area of ground with path = $(30)^2$
900 m^2

$\therefore$ Area of path = Area of ground with path – Area of ground
= 900 – 625
= 275 m^2

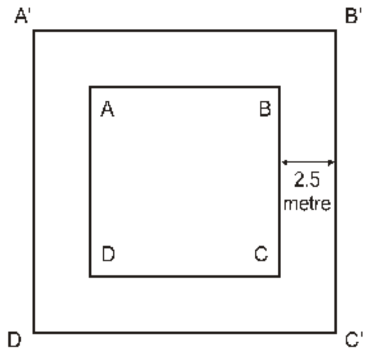


Fig. 11.14

Length of the square piece to be paved in path = 50 cm
= 0.50 m

$\therefore$ Area of a square piece = 0.50 × 0.50
= 0.2500 m^2

$\therefore$ Required number of square pieces

$$= \frac{area\ of\ path}{area\ of\ a\ square\ piece}$$

$$= \frac{275}{0.25} = \frac{27500}{25}$$

= 1100

Example 19. A rectangular ground is 40 m long and 30 m wide. A 3m wide path, parallel to its length and width, is made is the middle of the ground. find the expenditure of paving the concrete on this path at the rate of Rs 200 per square meter.

Sol. As per question

Length of ground = 40 m

Width of ground = 30 m

$\therefore$ Area of ground = 40 × 30
= 1200 m^2

In fig. 11.15

Area of path IJKL parallel to length
40 × 3
= 120 m^2

Area of path EFGH parallel to breadth = 30 × 3 = 90 m^2

Area of shaded part PQRS = 3 × 3 = 9 m^2

Area of total path = 120 + 90 – 9 = 201 m^2

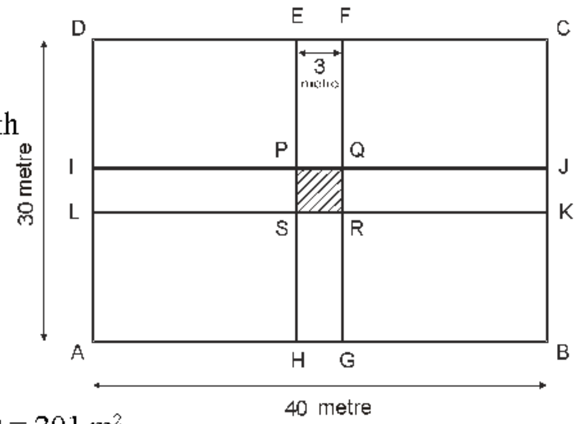


Fig. 11.15

$\therefore$ Expenditure at the rate of ₹ 200 per square metre of paving concrete

= ₹ 200 × 201

= ₹ 40200

$\therefore$ Expenditure = ₹ 40200

Example 20. Find the area of a field based on given measure in figure.

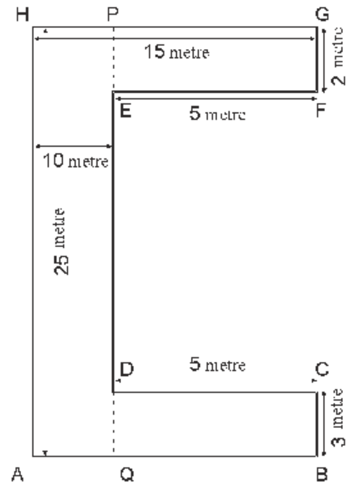


Fig. 11.16

Sol. We can divide the given figure in the form of rectangular, therefore

(i) Area of rectangle GFEP = EF × GF

$= 5 \times 2$

$= 10 \text{ m}^2$

(ii) Area of rectangle BCDQ = QB × BC

$= 5 \times 3$

$= 15 \text{ m}^2$

(iii) Area of rectangle HPQA = HA × HP

$= 25 \times 10$

$= 250 \text{ m}^2$

$\therefore$ Area of given field = area GFEP + area BCDQ + area HPQA

$= (10 + 15 + 250) \text{ m}^2$

$= 275 \text{ m}^2$.

Example 21. A rectangular room is 7m long, 6 m wide and 3.5m high. If there are 3 doors of size 2m × 1.5m and 4 windows of size 1.25m × 1m then find the expenditure of colouring of the walls at the rate of ₹ 4 and 45 paise per square metre.

Sol. Length of room = 7 m

Breadth of room = 6 m

and height of the room = 3.5m

$\therefore$ Area of four walls of room $= 2\,(l + b) \times h$

$= 2(7 + 6) \times 3.5 \text{ m}^2$

$= 91 \text{ m}^2$

Area of one door $= 2 \times 1.5 = 3 \text{ m}^2$

Area of 3 door $= 3 \times 3 \text{ m}^2$

$= 9 \text{ m}^2$

Area of one window $= 1.25 \times 1 \text{ m}^2$

Area of 4 windows $= 1.25 \text{ x } 4 \text{ m}^2$

$= 5.00 \text{ m}^2$

$\therefore$ Remaining area of four walls excluding doors and windows

$= [91 - (9+5)]$

$= (91 - 9 - 5)$

= (91 – 14)

= 77 m^2

$\therefore$ Expenditure of colouring of the walls

= ₹ 77 × 4.50

= ₹ 346.50

Example 22. A rectangular tank of water is 12 m long, 6 m wide and 2 m deep. Find the cost of repairing its four walls and floor at the rate of ₹ 15 per square metre.

Sol. Length of rectangular tank = 12 m

Breadth of rectangular tank = 6 m

Deapth of rectangular tank = 2 m

$\therefore$ Area of four walls of tank

= 2 × (length + breadth) × deapth

= 2 × (12 + 6) × 2

= 72 m^2

Area of floor of tank = length × breadth

= 12 × 6 m^2

= 72 m^2

$\therefore$ Total area of four walls and floor

= (72 + 72) m^2

= 144 m^2

$\therefore$ Cost of repairing the four walls and floor = 144 × 15

= ₹ 2160

Exercise - 11.4

1. Find the area and perimeter of rectangles of following measure:
   (i) length = 9.5 m, breadth = 7.5 m
   (ii) length = 125 m, breadth = 75 m
   (iii) length = 12.5 cm, breadth = 7.5 cm
2. Find the area and perimeter of the square of following measure of a side :
   (i) 5.3 m (ii) 8.5 m (iii) 9.6 m
3. After running 4 rounds to a square field, a distance covered is 16 km. Find the length and area of the ground.
4. A rectangular field is 75 m long and 45 m wide. How many beds 5 m long and 3 m wide can be made in it?
5. The length of a room is 10 m and breadth is 5 m. How many square pieces of area 50 cm^2 are required on its floor? Find the expenditure of paving the square pieces on the floor at the rate of ₹ 20 per piece.

6. Area of a square ground is 2025 m^2. There is 3.5 m wide path outside around it then find the area of the path.
7. A rectangular room is 40 m long and 25 m wide. There is a road 2.5 m wide around it. Tiles of size 50 cm × 70 cm are to be paved on it. Find the number of pieces.
8. A rectangular ground is 60 m long and 40 m wide. There is a 4 m wide path parallel to its length and width in side the ground. find the expenditure of spreading mud on the path at the rate of ₹100 per square metre.
9. Find the area and perimeter of the given diagram

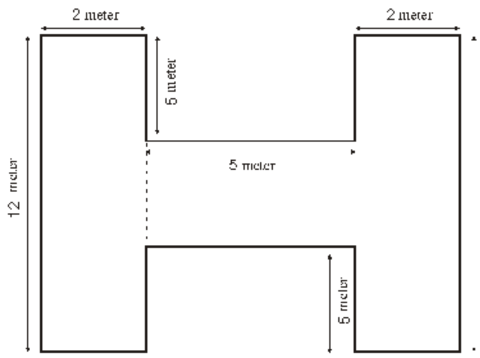


Fig. 11.17

10. The length, breadth and height of a room are respectively 15.35 m, 4.65 m and 6.50 m. If there are 4 doors of size 1.5m × 1.3 m and 3 windows of size 1.5m × 1.2m, then find the expenditure on whitewashing the room at the rate of ₹ 4.30 per square metre.
11. A water tank is 10m long, 8m wide and 2m deep. Find the expenditure of repairing its four walls and floor at the rate of ₹ 15 per square metre.

Important Points

1. Area of triangle = $\frac{1}{2}$ × base × height
2. Area of triangle by Heron's formula

   $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units

   where $s = \frac{a+b+c}{2}$

   $= \frac{perimeter\ of\ triangle}{2}$

   = semi perimeter, where a, b, c are side of a triangle.
3. Area of an isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$ sq. units

   Where a is one of the equal sides of a triangle and b is other side.
4. Area of an equilateral triangle $= \frac{a^2\sqrt{3}}{4}$ sq. units, where a is the side of triangle.
5. Area of right angled triangle = $\frac{1}{2}$ × (product of sides containing a right angle)

   $= \frac{1}{2}$ × base × height
6. Area of quadrilateral = $\frac{1}{2}$ × diagonal × (Sum of perpendiculars drawn on diagonal)
7. Area of parallelogram = base × height
8. Area of a cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

   where a, b, c, d are sides and $s = \frac{a+b+c+d}{2}$
9. Area of rhombus = $\frac{1}{2}$ × Products of lengths of diagonals
10. Area of trapezium = $\frac{1}{2}$ × (Sum of parallel sides) × distance between two parallel sides.
11. Area of rectangle = length × breadth
12. Area of a square = $(\text{side})^2$
13. Area of four walls = 2 × (length + breadth) × height

Miscelleneous Exercise - II

1. If the side of an equilateral triangle is 8 cm, then area of the triangle will be :

(A) $16\sqrt{3}$ cm^2 (B) $8\sqrt{3}$ cm^2

(C) $64\sqrt{3}$ cm^2 (D) $4\sqrt{3}$ cm^2 ( )

2. The sides of a triangle are 40 cm, 70 cm and 90 cm, area of triangle will be :

(A) $600\sqrt{5}$ cm^2 (B) $500\sqrt{6}$ cm^2

(C) $482\sqrt{5}$ cm^2 (D) $60\sqrt{5}$ cm^2 ( )

3. Each equal side of an issoceles triangle is 6 cm and the other sides is 8 cm, then area of triangle will be :

(A) $8\sqrt{5}$ cm^2 (B) $5\sqrt{8}$ cm^2

(C) $3\sqrt{55}$ cm^2 (D) $3\sqrt{8}$ cm^2 ( )

4. The perimeter of an equilateral triangle is 60 cm, then its area will be :

(A) $400\sqrt{3}$ cm^2 (B) $100\sqrt{3}$ cm^2

(C) $50\sqrt{3}$ cm^2 (D) $200\sqrt{3}$ cm^2 ( )

5. Area of a right- triangle is 6 cm^2 and its base is 9 cm, then the length of perpendicular will be :

(A) 8cm (B) 4 cm (C) 16 cm (D) 32 cm ( )

6. If the side of a square is 10 cm, then its perimeter will be :

(A) 20 cm (B) 10 cm (C) 40 cm (D) 30 cm ( )

7. If the diagonals of a rhombus are 8 cm and 6 cm, then its area will be :

(A) 48 cm^2 (B) 24 cm^2 (C) 12 cm^2 (D) 96 cm^2 ( )

8. If the perimeter of room is 40 m and height is 4m, then area of its four walls will be:

(A) 40 m^2 (B) 80 m^2 (C) 120 m^2 (D) 160 m^2 ( )

9. What will be the length of side of an equilateral triangle whose area is $9\sqrt{3}$ cm^2

____

10. Write the formula to find the area of cyclic quadrilateral.

____

11. Area of a square is 144 m^2 are, then write its perimeter.

____

12. If the base of a parallelogram is 18m and area is 174.60 m^2, then write its height.

____

13. Write the area of a quadrilateral whose diagonal is 6 cm and the sum of internal perpendiculars is 12 cm.

14. The ratio of the sides of a triangle is 25 : 17 : 12 and perimeter is 540 m, then find the area of triangle.

---

15. Area of an issoceles triangle is 12 cm^2 Find its base, if the length of its equal sides is 5 cm.

---

16. The perimeter of any triangle is 40 cm. If its two sides are 8 cm and 15 cm respectively, then find its area and also find the length of perpendicular, drawn from a vertex to the longest side.

---

17. The perimeter of a rhombus is 146 cm and length of a diagonal is 55 cm, then find the area of the rhombus.
18. There is sufficient grass for eating in a rhombus shaped grass field for 18 cows. If each side of this rhombus is 30 m and longest diagonal is 48 m, how much area of this field will be available for each cow to eat grass?
19. An umbrella is made by joining 10 triangular pieces of two different coloured cloths. The measures of each piece is 20 cm, 50 cm and 50 cm. How much cloth is used in umbrella.
20. Ratio of two parallel sides of a trapezium is 16 : 5. Which is cut out of a rectangle whose sides are 63 m and 5m. If the area of a trapezium is $\frac{4}{15}$ th part of the area of a rectangle trapezium.
21. Area of a rectangular field is 4356 m^2 and length of the ground is 99m. There is a 4.5 m wide road in the middle of the ground parallel to length and breadth. How many square pieces of side 1.50 m will be required to cover the road.
22. A room is 8m50 cm long and 6m 50cm wide. How long a carpet 25 cm wide required to cover its floor? Find the cost of carpet at the rate of Rs. 20 per meter.

Answer

Exercise 11.1

1.	60 square cm	**2.**	$50\sqrt{14}$ cm^2
3.	$4\sqrt{15}$ square cm	**4.**	$100\sqrt{3}$ cm^2
5.	60 cm^2	**6.**	54 m^2
7.	51 beds		

Exercise 11.2

1.	90 square cm	**2.**	40 m
3.	18 square cm	**4.**	306 square cm
5.	1200 square cm	**6.**	18.2 cm^2
7.	₹ 84000		

Exercise 11.3

1.	11544 m^2, ₹ 57720	**2.**	525 cm^2, 97.72 cm
3.	96 m^2	**4.**	610.9 m^2
5.	10 cm	**6.**	9 m

Exercise 11.4

1.	(i) 71.25 m^2, 34 m		(ii) 9375 m^2, 400 m
	(iii) 93.75 cm^2, 40 cm		
2.	(i) 28.09 m^2, 21.2 m		(ii) 72.25 m^2, 34 m
	(iii) 92.16 m^2, 38.4 m		
3.	250 m, 62500 m^2	**4.**	225 beds
5.	1000 square piece, ₹ 20000	**6.**	679 m^2
7.	1000 square piece	**8.**	₹ 38400
9.	58 m^2	**10.**	₹ 839.12
11.	₹ 2280		

Miscellaneous Exercise - 11

1. (A)

2. (A)

3. (A)

4. (B)

5. (A)

6. (C)

7. (B)

8. (D)

9. 6 cm

11. 480 m

12. 9.7 m

13. 36 cm^2

14. 9000 m^2

15. 8 cm

16. 60 cm^2, $7\frac{1}{17}$ cm

17. 1320 cm^2

18. 48 m^2

19. $2000\sqrt{6}$ cm^2

20. 25.6 m, 8 m

21. 277 square pieces

22. 221 m, ₹ 4420

12

Surface Area and Volume of Cube and Cuboid

12.01 Introduction

In the previous chapters, we have learnt to find the areas of some plane figures like triangle, quadrilateral and rectangle. In this chapter, we will learn to find the surface area and volume of the solid figures like brick, match box, chalk box and iron box etc.

The shape and volume of all these objects are definite. These figures are three dimensional solid figures, it means these figures have length, breadth and height. The surface area of any solid figures means sum of areas of all surfaces. The space occupied by any solid is called volume. The area is measured in square unit and volume is measured in cubic unit.

12.02 Cuboid

A bundle of rectangular sheet of paper placed one by one to a height, takes a shape that's called a cuboid. Each face of it is rectangular therefore a cuboid is also called a rectangular solid. There are 6 faces, 8 vertics and 12 edges in cuboid. For example room, box and brick etc. Opposite faces of a cuboid are equal to each other. To find the surface area of a cuboid, we have to find the area of its 6 faces.

Area of the face ABCD = Area of the face A' B' C' D' = length $\times$ breadth

Area of the face ADD'A' = Area of the face BCC'B'= height $\times$ breadth

Area of the face ABB'A' = Area of the face DCC'B' = length $\times$ height

$\therefore$ Total surface area of cuboid

= 2 [length $\times$ breadth + height $\times$ breadth + length $\times$ height] square units

$= 2\,[l \times b + b \times h + h \times l]$

$= 2\,[l b + bh + hl]$

12.03 Cube

If each face of a cuboid is of the shape of a square means its length, breadth and height all are equal, then it is called a cube.

Every face of a cube is square.

$\because$ Surface area of a face of the cube = $(\text{side})^2$

$\therefore$ Surface area of the 6 faces = $6\,(\text{side})^2$

$\Rightarrow$ Total surface area of a cube = $6\,(\text{side})^2$ square unit

12.04 Diagonal of Cube and Cuboid

A line joining the opposite vertics of two parallel faces of either cube or cuboid is called diagonal. There are four diagonal in a cube or cuboid.

Length of diagonal of a cuboid =

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$$

$$= \sqrt{\ell^2 + b^2 + h^2} \text{ unit}$$

Length of diagonal of a cube = $\sqrt{3}$ (length of side) unit

Illustrative Examples

Example 1. The length, breadth and height of a room are 5 m, 3.5m and 4 m respectively. Find the total surface area of the room.

Solution : Length of the room = 5 m

Breadth of the room = 3.5 m

Height of the room = 4m

$\therefore$ Total surface area of the room

$= 2\,[l \times b + b \times h + h \times l]$

$= 2\,[5 \times 3.5 + 3.5 \times 4 + 4 \times 5]$

$= 2\,[51.5]\ cm^2$

$= 103\ cm^2$

Example 2. Find the maximum length of a rod that can be placed inside a 12 m long, 9 m wide and 8 m high room.

Solution : Length of room = 12 m

Breadth of room = 9 m

Height of room = 8 m

$\therefore$ Maximum length of rod, that can be placed inside the room will be equal to its diagonal.

so, length of rod = diagonal of room = $\sqrt{(\ell^2) + (b^2) + (h^2)}$

$$= \sqrt{(12)^2 + (9)^2 + (8)^2} \text{ m}$$

$$= \sqrt{144 + 81 + 64} \text{ m}$$

$$= \sqrt{289} \text{ m}$$

= 17 m

Example 3. The side of a cube is 12 cm. Find the total surface area of cube.

Solution : Total surface area of a cube = 6 (side)^2

= 6 × (12)^2

= 6 × 144 cm^2

= 864 cm^2

Example 4. A box is 1 m long, 60 cm wide and 40 cm heigh. Find the expenditure of colouring its all outer side without its bottom at the rate of Rs. 20 per square meter.

Solution : Lenght of box = 1m = 100 cm

Breadth of box = 60 cm

Height of box = 40 cm

Total surface area of box except bottom

= $(l \times b) + 2\,[b \times h + h \times l]$

= (100× 60) + 2 [60 × 40 + 40 ×100]

= 6000 + 2 [2400 + 4000]

= 6000 + 2 [6400]

= 6000 + 12800 cm^2

= 18800 cm^2

= 1.88 m^2

Rate of colouring = Rs. 20 per square meter

Thus, total expenditure on colouring

= Rs. 1.88 × 20

= Rs. 37.60

= Rs. 37.60

Example 5. The length, breadth and height of a box of tin is 15 cm, 10 cm and 8 cm respectively. Such 20 boxes are to be made. Find the area of tin used in it. If the cost of tin is Rs. 50 per square meter, then find the total cost of tin used in boxes.

Solution : Length of a box = 15 cm

Breadth of a box = 10 cm

Height of box = 8 cm

Total surface area of a box = 2 [$l \times b + b \times h + h \times l$]

$= 2[15\times10 + 10 \times 8 + 8 \times 15]cm^2$

$= 2\ [150 + 80 + 120]cm^2$

$= 2\ [350]\ cm^2$

$= 700\ cm^2$

Area of tin used in 20 such boxes

$= 20 \times 700\ cm^2$

$= 14000\ cm^2$

$= 1.4\ m^2$

$\therefore$ Cost of tin used in 20 boxes = Rs. 1.4×50 = Rs. 70

Exercise 12.1

1. The length, breadth and height of a wooden box is 1m, 60m and 40 cm respectively. find the outer surface area of the wooden box.
2. The measure of a box are 40 cm, 30 cm and 20 cm respectively. How much square cm cloth is required to make the cover of box?
3. The length of a room is 5m, breadth is 3.5 m and height is 4 cm. Find the total expenditure of white washing on the four walls and roof at the rate of ₹ 15 per square meter.
4. The side of a cubical chalk box is 4 cm. Find the total surface area of chalk box and length of its diagonal.
5. Total surface area of a cube is 1014 m^2. find the length of the side of cube.
6. A wooden box with lid is made of 2.5 cm thick wood. Inner length, breadth and height of box are 1 m, 65 cm and 55 cm. Find the total expenditure of colouring its outer surface area at the rate of ₹ 15 per square meter.
7. Each face of a cube is 100 cm^2. If cube is cut by a plane, parallel to its base, in two equal parts, then find the total surface area of each equal part.
8. A box without lid, is made of 3 cm thick wood whose external length, breadth and height are 146 cm, 116 cm and 83 cm respectively. find total surface area of inner side.

12.05 Volume of Cube and Cuboid

We know that each solid object occupies some space. Measure of this occupied space is called volume of that object. If object is hollow, then it can be filled with air or liquid. This liquid takes shape of that object (pot). In this condition, the amount of liquid filled inside the pot, is called the capacity of the pot.

Capacity of a pot, is the volume of the liquid that can be filled inside the pot. Its unit is cubic unit.

Volume of cube and cuboid can be find by following formulae.

Volume of cuboid = (length × breadth× height) cubic unit

Volume of cube = $(\text{side})^3$ cubic unit

Illustrative Examples

Example 6. Volume of a cube is 216 cubic meter. Find its side

Solution : Let side of cube is x meter

$\therefore$ Volume of cube = $(\text{side})^3$

$\Rightarrow$ $216 = x^3$

$\Rightarrow$ $x^3 = 6 \times 6 \times 6 = (6)^3$

$\Rightarrow$ $x = 6$

$\therefore$ side of cube = 6 cm

Example 7. A water tank is 6m long, 5 m wide and 4.5 m deep. How many litres of water can be filled in it. (1 litre = 1000 cubic cm)

Solution : Length of water tank = 6 m = 600 cm

Breadth of water tank = 5 m = 500 cm

Height of water tank = 4.5 m = 450 cm

$\therefore$ volume of tank = $600 \times 500 \times 450$ cm^3

$$= \frac{600 \times 500 \times 450}{1000} \text{ litres}$$

= 135000 litres

$\therefore$ 13500 litres water can be filled in the tank.

Example 8. The length of a box is 30 cm, breadth 20 cm and height is 6 cm. How many cassets of size 10 cm × 5 cm × 2 cm can be placed inside it.

Solution : Length of box = 30 cm

Breadth = 20 cm

Height = 6 cm

$\therefore$ Volume of box = $30 \times 20 \times 6$ cm^3

Volume of a casset = $10 \times 5 \times 2 = 100$ cm^3

$\therefore$ Number of cassets = $\dfrac{Volume\ of\ box}{Volume\ of\ a\ casset}$

$$= \frac{30 \times 20 \times 6}{10 \times 5 \times 2}$$

= 36

$\therefore$ Number of cassets = 36

Example 9. A wooden box is made of 1 cm thick wood. If its outer length, breadth and height are 75 cm, 60 cm and 40 cm, then find the volume of the wood used in the box.

Solution : Length of the box = 75 cm

Breadth = 60 cm

Height = 40 cm

Outer volume of the box = $75 \times 60 \times 40$

$= 180000 \text{ cm}^3$

Thickness of wood = 1 cm

$\therefore$ Inner length of the box = $(75 - 2 \times 1)$ cm

= 73 cm

Inner breadth of the box = $(60 - 2\times1)$ cm

= 58 cm

Inner height of the box = $(40 - 2 \times 1)$ cm

= 38 cm

$\therefore$ Inner volume of the box = $73 \times 58\times38$

$= 160892 \text{ cm}^3$

$\therefore$ Volume of wood used in the box

= Outer volume – Inner volume

= 180000 – 160892

= 19108 cm^3

Example 10. How many bricks of measure 25 cm × 16cm × 10cm are required to make a wall of size 20 m long, 5m wide and 50 cm thick while there is a door of size 2m × 1.5m and two windows of size 1.5m × 1m? Find the cost of bricks at the rate of ₹ 280 per thousand.

Solution : Volume of wall = $20 \times 5 \times 0.5 = 50 \text{ m}^3$

Volume of blank place left for a door and two windows

= (length × breadth × thickness) of door +

2 (length × breadth × thickness) of windows

$= [2\times1.5\times0.5+2\,(1.5\times1\times0.5)]\text{m}^3$

$= [1.5+1.5]\text{m}^3$

$= 3\text{m}^3$

$\therefore$ Volume of wall where bricks will be used = $50 - 3 = 47 \text{ m}^3$

Volume of a brick = $(25\times16\times10) \text{ cm}^3$

$$= \left(\frac{25}{100}\times\frac{16}{100}\times\frac{10}{100}\right) \text{m}^3$$

$$= \frac{4000}{1000000} = \frac{1}{2500} \text{ m}^3$$

$\therefore$ Number of bricks $= \dfrac{47}{\frac{1}{2500}} = 47 \times 2500 = 117500$

$\therefore$ Cost of bricks $= 117500 \times \dfrac{280}{1000} =$ ₹ 3290

Exercise 12.2

1. The measure of a match box is 3cm × 2cm × 1cm. Find the volume of a packet of such 12 match boxes.
2. Perimeter of a face of a cube is 24 cm. Find the volume of the cube.
3. Core of three cubes of metal are 3 cm, 5cm and 4cm respectively, By melting all there, a new cube is made. Find the volume of new cube and the length of the core of this cube.
4. The length of a water tank is 2.5m and breadth in 2m. It contains 1500 litres water in it. Find the deapth of the tank.
5. The length of a wall is 4m, breadth 15 cm and height is 3m. How many bricks of size 20cm × 10cm ×8cm are required to make a wall. If the cost of bricks is ₹ 120 per thousand, then find the total cost of bricks.
6. There is a water tank of size 20m ×15m×6m in a village. How many litres of water can be filled in it? If 1000 litre of water is used daily from it. Then for how many days it will be sufficient?
7. The length of a wall in 8m and height is 4m. Wall is 35 cm thick. There is one door of size 2m×1m and two windows each of size 1.20m × 1m. Find the expenditure of making wall at the rate of ₹ 1500 per cubic meter.
8. How many bricks of size 25cm×15cm×6cm are required to make a wall 5 m long, 2m high and 50cm thick, if 10% place is occupied by cement?
9. The mud taken out from a pond, is spread equally in a field. If the pit dug out in pond is 200 m long, 50m wide and 0.75m deep, then how much high, will be the lend of field ? The area of the field is 1500 square meter.
10. The outer length, breadth and height of the wooden box with lid is 1.25m, 0.80 m and 0.55m. Thickness of wood is 2.5 cm. If the weight of 1 cubic metre wood is 250 kg. Find the toal weight of box.

Important Points

1. If the side of a cube is a, then total surface area of cube = $6a^2$

   Volume of cube = a^3

   Diagonal of cube $= a\sqrt{3}$

2. If length is a, breadth is b and height is c of a cuboid, then total surface area of a cuboid

   $= 2[ab + bc + ac]$

   Volume of cuboid = $a \times b \times c$

   Diagonal of cuboid $= \sqrt{a^2 + b^2 + c^2}$

3. Units related to volume are :

   1 litre = 1000 cubic centimetre

   1000 litre = 1 cubic metre = 1 kilo litre

   1 cubic centimetre = 1000 cubic millimetre

   1 cubic metre = 1000000 centimetre

Miscellaneous Exercise - 12

1. If the volume of a cube is 125m^3, then side of a cube will be :

   (A) 7m (B) 6m (C) 5m (D) 2m

2. If the volume of a cube is 1331 cubic centimeter, then surface area of cube is -

   (A) 762cm^2 (B) 726cm^2 (C) 426cm^2 (D) 468cm^2

3. Length, breadth and height of a cuboid is 4m, 3m and 2m, surface area of cuboid will be

   (A) 25m^2 (B) 26m^2 (C) 52m^2 (D) 62m^2

4. Diagonal of a cuboid of measure 8m×7m×6m is

   (A) 12.2m (B) 12.02m (C) 14.2m (D) 14.02m

5. Side of a cube is 5 cm. Diagonal of cube is

   (A) $4\sqrt{3}$ cm (B) $2\sqrt{3}$ cm (C) $5\sqrt{3}$ cm (D) 5cm

6. If the volume of a cuboid is 400 cm^3 and area of its base is 80 cm^2, then height of cuboid is

   (A) 7 cm (B) 6 cm (C) 4 cm (D) 5 cm

7. Measures of a cuboid are 15 cm × 12cm × 6 cm. How many cubes of side 3 cm can be made by melting this cuboid?

____

8. Edge of two cubical dice is 2 cm. A solid is made by pasting a face of each dice. Find the total surface area of solid.

____

9. If the edges of three cubes are 3 cm, 4 cm and 5cm respectively, then find side of the cube made by these cubes.

____

10. An empty cistern is 4m long and 3m wide. How many cubic meter of water is filled in it so that the height of water become 2 m?

____

11. There is 8 litre water in a cubical pot. Find the total surface area of pot.

12. The measures of any godown are 60m × 25m × 10m. How many maximum wooden crates can be put inside the store of size 1.5m × 1.25m × 0.5 m?

13. A river of 3m deep and 40m wide, is flowing at the rate of 2km/hour and fells into the sea. How much water will fell in a minute, in sea?

14. If length, breadth and height of a right angular parallel hexagon are in the ratio 6 : 5 : 4 and its total surface area is 33300 square centimeter. Find the volume of right angular paralllel hexagon.

15. A plot is 20m long and 15m wide. Digging the land 10m, breadth 6m and depth 5m, from outside of the plot, is spread over the plot. Find the height of the mud spread over in the plot.

Answers

Exercise 12.1

1. 2.28 m^2

2. 5200 cm^2

3. ₹1282.5

4. 96 cm^2, $4\sqrt{3}$ cm

5. 13 cm

6. ₹ 53.55

7. 400 cm^2

8. 55400 cm^2

Exercise 12.2

1. 72 cm^3

2. 216 cm^3

3. 216 cm^3, 6 cm

4. 30 cm

5. 1125 bricks, Rs. 135

6. 1800000 litres, 1800 days

7. ₹ 14490

8. 2000 bricks

9. 50 cm

10. 25 kg.

Miscellaneous Exercise - 12

1. (C)

2. (B)

3. (C)

4. (A)

5. (C)

6. (D)

7. 40 cubes

8. 80 cm^2

9. 6 cm

10. 24 m^3

11. 2400 cm^2

12. 16000 crate

13. 4000 m^3

14. 405000 cm^3

15. 1m

13

Angles and their Measurement

13.01 Trigonometry

The word "trigonometry" is derived from a combination of three words 'tri', 'gon', and metron. 'Tri' means three, 'gon' means sides and 'metron' means ' a measure'. Thus, trigonometry deals with the measurment of sides (and angles) of a triangle. The relations between sides and angles of a triangle are used to find distance, height and areas etc. which can not be easily measured. We used the trigonometry in finding the distance of the earth from the sun and moon. Field like physics. Navy or Engineering the knowledge of trigonometry is very useful.

13.02 Positive and Negative distances

XOX' and YOY' are two mutually perpendicular lines, intersecting at O. Thus, the plane is divided into four parts. In such case

(i) The distance measured form O along OX are considered positive and the distance measured along OX' are considered negative.

(ii) The distances measured form O along OY are considered positive and those measured along OY' are considered negative. XOY, YOX', $X'OY'$ and $Y'OX$ are called the first, second, third and fourth quadrants respectively. It should be noted that this order is in anticlockwise direction.

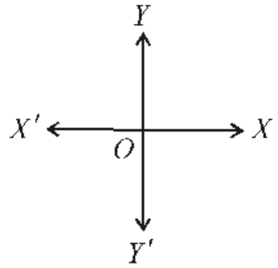


Fig. 13.01

13.03 Angle

The amount of rotation produced by the revolving the ray in moving form its initial position OX to the present position OA is called angle. Thus, in fig. 13.02, XOA is an angle. We use the symbol $\angle$ to denote an angle. Generally the capital letters A, B, C, ... of the English alphabet are used to denote the vertices of the angles and the measures of the angles are denoted by α (alpha), β(beta), γ(gamma), θ(theta), ϕ(phi), ψ(psi) etc.

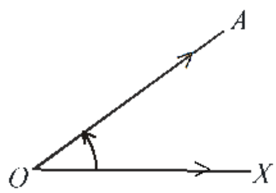


Fig. 13.02

Positive and Negative Angles :

If the ray OA, starting form its initial position OX revolves in the anticlockwise direction, the angle so formed is considered positive. In fig. 13.03, $\angle XOA$ is a positive angle.

If the ray OA, starting from its initial position OX revolves in clockwise direction, the angle so formed is considered negative. In fig. 13.03, $\angle XOA'$ is a negative angle.

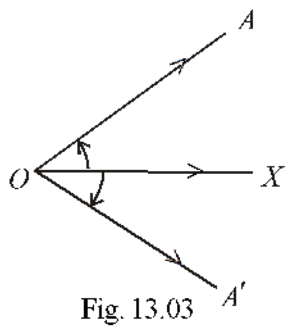


Fig. 13.03

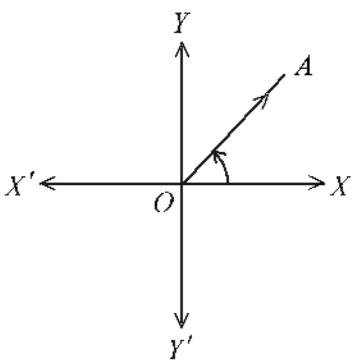
13.04 Angles of any magnitude :

(i) If the revolving ray OA, starting from its initial position OX revolving in anticlockwise direction and makes certain angle in the first quadrant, then this angle lies between $0°$ and $90°$. For example $\angle XOA$ in fig. 13.04 (i).

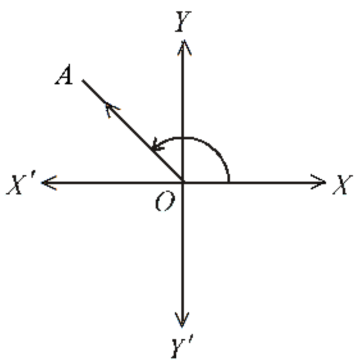
(ii) The angle in which the revolving ray OA, starting from its initial position OX and revolving in anticlockwise direction, makes in the second quadrant lies between $90°$ and $180°$. For example $\angle XOA$ in fig. 13.04 (ii).

(iii) The angle in which the revolving ray OA, starting form its initial positive OX and revolving in anticlockwise direction, makes in the third quadrant lies between $180°$ and $270°$. For example $\angle XOA$ in fig. 13.04 (iii).

(iv) The angle in which the revolving ray OA, starting form its initial positive OX and revolving in anticlockwise direction makes, in the fourth quadrant lies between $270°$ and $360°$. For example $\angle XOA$ in fig, 13.04 (iv).



(i)



(ii)

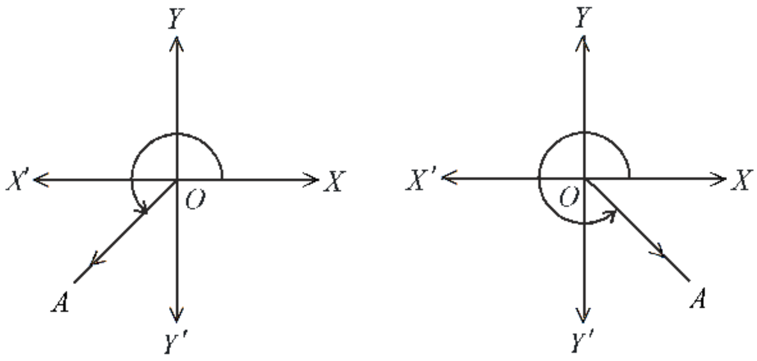


Fig. 13.04

If the ray OA, moving in anticlockwise direction, makes a complete revolution and comes back to its original position OX, then it describes an angle of $360°$.

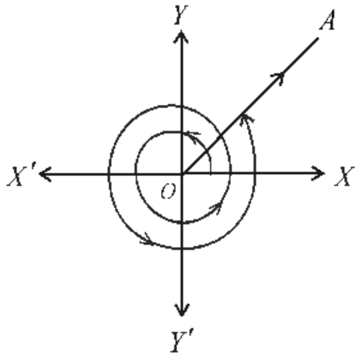


Fig. 13.05

So far we have seen that the maximum measure of an angle can be $360°$ or 4 right angles but angles greater than $360°$ are also possible. The revolving ray, revolving about its original position, described an angle of $360°$ in each complete rotation.

If the revolving ray, starting from its original position OX and revolving about the point O, make two complete rotations, then it describes an angle of $2 \times 360° = 720°$. If, after two complete rotations, come back to the position OA again then the angle so described $= 2 \times 360° + \angle XOA$ (Fig. 13.05).

Example 1. Display an angle of $390°$ with the help of a figure.

Solution : $390° = 1 \times 360° + 30°$

The revolving ray OA, starting form its initial position OX and revolving in anticlockwise direction point O makes one

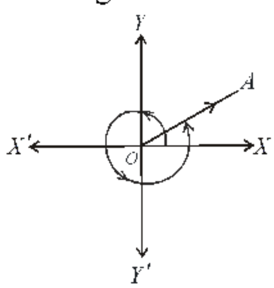


Fig. 13.06

complete rotation and moves through $30°$ in the same direction and comes to the position OA in the first quadrant, as shown in figure 13.06.

Example 2. Display an angle of $-750°$ with the help of a figure.

Solution : $-750° = -(2 \times 360°) - 30°$

The revolving ray OA, starting form its initial position OX and revolving in clockwise direction about the point O, makes two complete rotation and moves in the same direction through an angle of $30°$. Thus its final position in the fourth quadrant will be as shown in the figure 13.07.

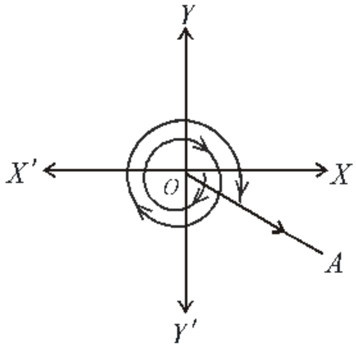


Fig. 13.07

13.05 Measurement of Angles

(i) Sexagesimal system

(ii) Centesimal system

(iii) Circular system

(i) Sexagesimal system : In this system, angles are measured in degrees, minutes and seconds. They are related as follows :

$$1 \text{ right angle } = \text{ ninety degrees } = 90°$$

$$1 \text{degree } (1°) = \text{ sixty minutes } = 60'$$

$$1 \text{ minute } (1') = \text{sixtey seconds } = 60''$$

(ii) Centesimal system : This system is also known as French system. In this system, the angles are measured in grades, minutes and seconds. They are related as follows :

$$1 \text{ right angle } = 100 \text{ grade } = 100^g$$

$$1 \text{ grade } (1^g) = 100 \text{ minutes} = 100'$$

$$\text{one minute } (1') = 100 \text{ seconds} = 100''$$

Note : This system is not in practice.

(iii) Circular system : In this system the unit of angle is radian. "The angle subtended by an arc of a circle, whose length is equal to radius, at the centre of the circle, is called one radian". Let O be the centre and r be the radius of the circle. Draw an arc AB whose length is r. Thus the $\angle AOB$ is called one radian. The angle of 1 radian is denoted as 1^c. Hence in Fig. 13.08, $\angle AOB = 1^c$.

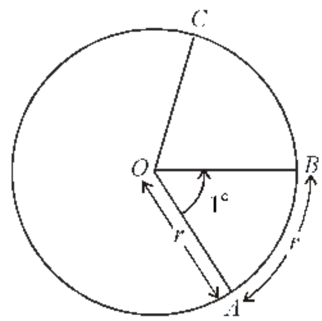


Fig. 13.08

Let C be any other point on the circumference of the circle.

Then

$$\frac{\angle AOC}{\angle AOB} = \frac{\text{arc } AC}{\text{arc } AB}$$

or $$\frac{\angle AOC}{\text{arc } AC} = \frac{\angle AOB}{\text{arc } AB}$$

$$\angle AOC = \frac{\text{arc } AC}{\text{arc } AB} \times \angle AOB$$

$$= \frac{\text{arc } AC}{\text{arc } AB} \times 1^c$$

$$= \frac{\text{arc } AC}{\text{arc } AB} \text{ radian} \qquad \cdots(1)$$

If $\angle AOC = \theta^c$ (θ radian) and arc $AC = x$ then from equation (1)

$$\theta^c = \frac{x}{r} = \frac{\text{arc}}{\text{radius}} \text{ radian}$$

13.06 The Value of π

The ratio of the circumference and the diameter of a circle is always constant. This constant quantity is denoted by the Greek letter π upto 8 places of decimal is 3.14159265.

In fractional form, its value is considered as $\frac{22}{7}$.

13.07 Value of 1 radian

We know that

$$\pi = \frac{\text{circumference of the circle}}{\text{diameter of the circle}}$$

If the radius of the circle is r, its diameter is $2r$ and its circumference is $2\pi r$. We also known that there is a definite relationship between the length of an arc and the angle subtended by that arc at the centre form Fig. 13.08.

$$\frac{\angle AOB}{360^\circ} = \frac{\text{arc } AB}{\text{circumference of the circle}}$$

or $$\frac{\angle AOB}{\text{arc } AB} = \frac{360^\circ}{\text{circumference}}$$

or $$\frac{1^c}{r} = \frac{360^\circ}{2\pi r}$$

or $$1^c = \frac{180^\circ}{\pi} \qquad \ldots(1)$$

or $\quad 1^c = 57°17'45''$ apporximately

(Taking $\pi = 3.1416$)

Thus we find that the value of radian does not depend on the radius of the circle. Hence the value of one radian is constant for all circles.

From equation (1)

$$\pi^c = 180°$$

In practice, we leave '*c*' and instead of writing π^c, we simply write π.

Hence $180° = \pi$.

Example 3. Convert 60° into radians.

Solution : We know that $180° = \pi$ radian

$$\therefore \quad 1° = \frac{\pi}{180} \text{ radian}$$

$$\therefore \quad 60° = \frac{\pi}{3} \text{ radian.}$$

Example 4. Convert $\frac{\pi}{4}$ radian into degrees.

Solution : We know that $1^c = \frac{180°}{\pi}$

$$\therefore \quad \frac{\pi}{4} = \frac{180°}{4} = 45°$$

Example 5. Find the length of the arc subtending an angle of $\frac{\pi}{3}$ radians at the centre of the circle. $\therefore$

Solution : If r be the radius of the circle and x be the length of the arc subtending an angle of $\frac{\pi}{3}$ radians at the centre, then

$$\frac{\pi}{3} = \frac{x}{r}$$

or $\quad x = \frac{\pi}{3} r.$

Example 6. Find the angle subtended by the whole of the circumference at the centre of the circle.

Solution : We known that the circumference of a circle of radius r is $2\pi r$ and the angle subtended by an arc of length r at the centre is 1 radian.

$\because$ Angle subtended at the centre by an arc of length r = 1 radian

$\therefore$ Angle subtended at the centre by an arc of length 1 $= \frac{1}{r}$ radian

$\therefore$ Angle subtended at the centre by an arc of length $2\pi r = \frac{1}{r} \times 2\pi r$ radian $= 2\pi$ radian.

Hence the whole circumference of a circle subtends an angle of 2π radians at the centre.

Example 7. How much time does the minute hand of a watch take to describe an angle of $\frac{3\pi}{2}$ radians.

Solution : We know that

4 right angles = 2π radians

The time taken by the minute hand of the watch to describe an angle of 2π radians = 1 hour

$\therefore$ The time for describing 1 radian $= \frac{1}{2\pi}$ hours

$\therefore$ Time for describing an angle of $\frac{3\pi}{2}$ radian $= \frac{1}{2\pi} \times \frac{3\pi}{2}$ hours

$$= \frac{3}{4} \text{ hours } = \frac{3}{4} \times 60 \text{ minutes } = 45 \text{ minutes}$$

Important Points

1. When the revolving ray revolves in anticlockwise direction, the angle thus formed is positive and if it revolves in clockwise direction, the angle so formed is negative.
2. The following system are used to measure angles :
   (i) Sexagesimal system
   (ii) Centesimal system
   (iii) Circular system.
3. In sexagesimal system, the unit of angle is 'degree'
   1 right angle =90°, $1° = 60'$ and $1' = 60''$.
4. The angle subtended at the centre of the circle by an arc whose length is equal to its radius is called an angle of one radian.
5. The angle subtended at the centre by the circumference of the circle is 2π radians.
6. The relation between the sexagesimal and circular system is $D = \left(\frac{180°}{\pi}\right)R$, where D and R are the measures of the angle in degrees and radians respectively.

Miscellaneous Exercise 13

Objective questions [1-5]

1. The line describing an angle of $750°$, lies in :

   (A) First quadrant (B) Second quadrant

   (C) Third quadrant (D) Fourth quadrant

2. The number of radians in angle $30°$ is :

   (A) $\frac{\pi}{3}$ radian (B) $\frac{\pi}{4}$ radian

   (C) $\frac{\pi}{6}$ radian (D) $\frac{3\pi}{4}$ radian

3. The value of $\frac{3\pi}{4}$ in sexagesimal system is :

   (A) $75°$ (B) $135°$

   (C) $120°$ (D) $220°$

4. How much time the minute hand of a watch will take to describe an angle of $\frac{\pi}{6}$ radians :

   (A) 10 minutes (B) 20 minutes

   (C) 15 minutes (D) 5 minutes

5. The value of the angle, in radian, subtended at the centre of the circle of radius 100 meters by an arc of length 25π meters is :

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

   (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

6. In which quadrant does the revolving ray lie when it makes the following angles :

   (i) $240°$ (ii) $425°$ (iii) $-580°$

   (iv) $1280°$ (v) $-980°$

7. Convert the following angles in radians :

   (i) $45°$ (ii) $120°$

   (iii) $135°$ (iv) $540°$

8. Express the following angles in sexagesimal system :

(i) $\frac{\pi}{2}$ (ii) $\frac{2\pi}{5}$

(iii) $\frac{5}{6}\pi$ (iv) $\frac{\pi}{15}$

9. Find the angle in radians, subtended at the centre of a circle of radius 5 cm by an arc of the circle whose length is 12cm.

10. How much time the minute hand of a watch will take to describe an angle of $\frac{3\pi}{2}$ radians ?

11. How much time the minute hand of a watch will take to describe an angle of $120°$?

12. In a circle, the angle subtended at the centre by an arc of length 10 cm is $60°$. Find the radius of the circle.

13. If the minute hand of a watch has revolved through 30 right angles, just after the mid day, then what is the time by the watch ?

14. The angles of a triangle are in ratio 2 : 3 : 4. Find the all three angles in radians.

15. Express $\frac{3}{5}\pi$ into sexagesimal system.

Answers

Miscellaneous Exercise 13

1. (A) 2. (C) 3. (B)

4. (D) 5. (A)

6. (i) Third (ii) First (iii) Second (iv) Third (v) Second

7. (i) $\frac{\pi}{4}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{3\pi}{4}$ (iv) 3π

8. (i) $90°$ (ii) $72°$ (iii) $150°$ (iv) $12°$

9. $\frac{12}{5}$ radian

10. 45 minute

11. 20 minute

12. $\frac{30}{\pi}$ cm

13. 7.30 P.M.

14. $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$

15. $108°$

❑

14

Trigonometric Ratios of Acute Angles

14.01 Right Angled Triangle :

In previous chapters we have studied about the triangle in which one angle is right angle, such triangles are called right triangles. Fig. 14.01 is a right triangle in which $\angle B$ is a right angle. The side opposite the right angle is called hypotenuse. Hence in right angled triangle ΔABC, AC is hypotenuse. Regarding other two angles of the right angled triangle, the side which makes angle with the hypotenuse is called base or adjacent side and the side opposite to this angle is called perpendeicular. In Fig. 14.01 for $\angle C$, side CB is base AB is perpendicular. Similarly for $\angle A$ side AB is base and side BC is perpendicular. In right angled triangle the angles other than the right angle are acute angles. The relation between the sides of a right triangle is "Square on hypotenuse is equal to the sum of squares on other two sides" known as Baudhayan Theorem. In brief the above theroem may be read as

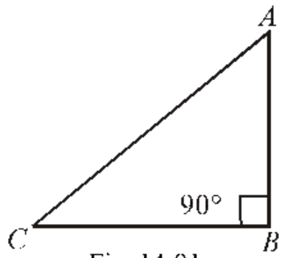


Fig. 14.01

$$AC^2 = AB^2 + BC^2$$

Clearly if out of three sides AB, BC, and AC two are given then third can easily be obtained.

Example 1. **In right angled triangle ABC give the names of sides corresponding to angle θ and ϕ .**

Solution : In ΔABC, $\angle B$ is a right angle, so AC is hypotenuse. Now for angle θ BC is base and AB is perpendicular. Similarly for angle ϕ side AB is base and side BC is perpendicular.

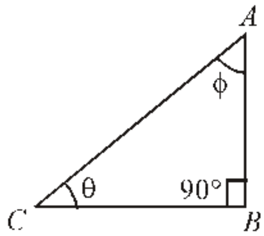


Fig. 14.02

Example 2. **In triangle ABC, $\angle B$ is right angle. If $AB = 4$ cm and $AC = 5$ cm then find BC.**

Solution : Draw rough figure of ΔABC as per the given specification.

$\angle B = 90°$

$AC = 5$ cm

$AB = 4$ cm

From Baudhayan theorem

$$AC^2 = AB^2 + BC^2$$

or $\quad (5)^2 = (4)^2 + BC^2$

or $\quad BC^2 = 25 - 16$

or $\quad BC^2 = 9$

Hence $\quad BC = 3$ cm.

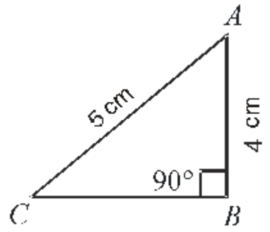


Fig. 14.03

14.02 Trigonometric Ratios of an Acutre Angle :

In right angled triangle the ratio of any two sides is called trigonometrical ratio.

Let triangle ABC be a right angled triangle in which $\angle ABC$ is a right angle and $\angle CAB = \theta$, then

(i) $\dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \text{sine}\,\theta$ or $\sin\theta$

In brief sine θ is written as $\sin\theta$

(ii) $\dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \text{cosine}\,\theta$ or $\cos\theta$

In brief cosine θ or is written as $\cos\theta$.

(iii) $\dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB} = \text{tangent}\,\theta$ or $\tan\theta$

In brief tangent θ is written as $\tan\theta$

(iv) $\dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{AB}{BC} = \text{cotangent}\,\theta$ or $\cot\theta$

In brief cotangent θ is written as $\cot\theta$.

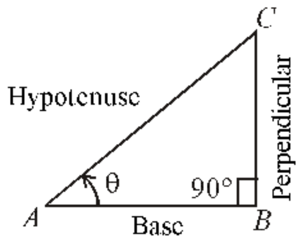


Fig. 14.04

(v) $\dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB} = \text{secant}\,\theta$ or $\sec\theta$

In brief secant θ is written as $\sec\theta$.

(vi) $\dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{BC} = \text{cosecant}\,\theta$ or $\operatorname{cosec}\theta$

In brief cosecant θ is written as $\operatorname{cosec}\theta$.

Note : (i) Suppose the revolving line AX moves in anticlock wise direction keeping the vertex A fixed and makes an acute angle $\angle XAP = \theta$. Draw perpendiculars CB, C_1B_1 and C_2B_2 from the points C, C_1, C_2 respectively on AX.

We find that the triangle CAB, C_1AB_1, C_2AB_2 etc. are similar. Therefore

$$\sin\theta = \frac{BC}{AC} = \frac{B_1C_1}{AC_1} = \frac{B_2C_2}{AC_2} = \cdots$$

and $$\cos\theta = \frac{AB}{AC} = \frac{AB_1}{AC_1} = \frac{AB_2}{AC_2} = \cdots$$

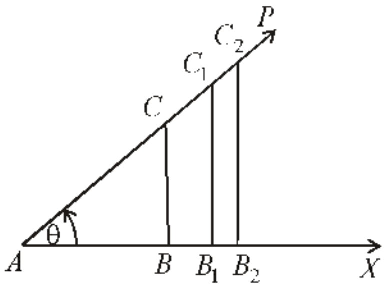


Fig. 14.05

From this we observe that the value of $\sin\theta$ or $\cos\theta$ remains unchanged in every case, i.e. these values do not depend on the position of the point P on the revolving line. Similarly the other trigonometrical ratios also do not depend on the position of point P on the revolving line.

Hence the trigonometrical ratio depend on the acute angle θ not on the size of the right angled triangle. Since for every acute angle θ, the value of the trigonometrical ratio is unique, therefore, trigonometrical ratios are also called as trigonometrical functions.

(ii) $\sin\theta, \cos\theta, \tan\theta, \ldots$ does not mean the multiplication of $\sin$ or $\cos$ or $\tan$ or $\cdots$ by θ

i.e.,
$$\sin\theta \neq \sin\times\theta$$
$$\cos\theta \neq \cos\times\theta$$
$$\tan\theta \neq \tan\times\theta$$

(iii) Trigonometrical ratios of any positive acute angle are always positive.

Illustrative Examples

Example 3. In triangle ABC angle B is a right angle, find all trigonometrical ratios of the angle A.

Solution : From Fig. 14.06, side AC is hypotenuse and side opposite to $\angle A$ is BC. Therefore in ΔABC side AB is base, BC is perpendicular and AC is hypotenuse.

$\therefore$ $$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

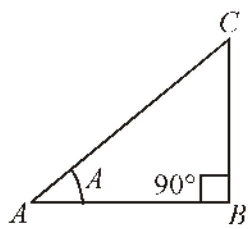


Fig. 14.06

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\text{cosec}\, A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}$$

Example 4. In triangle ABC angle C is a right angle and $AB = 25$ cm and $BC = 24$ cm then find all trigononmetric ratios of the angle A.

Solution : $\because$ ΔABC is a right triangle.

$$\therefore \quad AC = \sqrt{AB^2 - BC^2}$$

$$= \sqrt{(25)^2 - (24)^2}$$

$$= \sqrt{49}$$

$$= 7 \text{ cm}$$

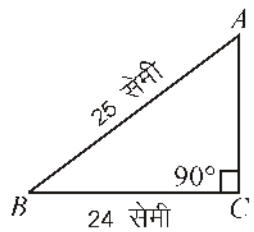


Fig. 14.07

$$\therefore \quad \sin A = \frac{BC}{AB} = \frac{24}{25}$$

$$\cos A = \frac{AC}{AB} = \frac{7}{25}$$

$$\tan A = \frac{BC}{AC} = \frac{24}{7}$$

$$\cot A = \frac{AC}{BC} = \frac{7}{24}$$

$$\sec A = \frac{AB}{AC} = \frac{25}{7}$$

$$\text{cosec}\, A = \frac{AB}{BC} = \frac{25}{24}$$

Example 5. If $\sin\theta = \frac{3}{5}$, then find remaining trigonometrical ratios of θ .

Solution : We draw a right ΔABC where $\sin\theta = \frac{3}{5}$ (Fig. 14.08) i.e., perpendicular AB and hypotenuse AC are in the ratio 3 : 5. Let $AB = 3k$ and $AC = 5k$, where $k > 0$, which is proportionality constant.

Hence form Baudhayan theorem

$$BC^2 = AC^2 - AB^2 = (5k)^2 - (3k)^2 = 16k^2$$

$$\therefore \quad BC = \pm 4k$$

Hence angle θ is an acute angle, therefore BC will be positive.

$$BC = 4k$$

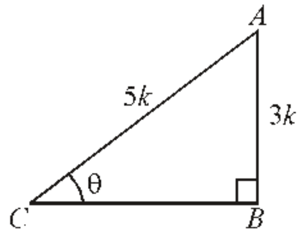


Fig. 14.08

$$\therefore \qquad \cos\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan\theta = \frac{AB}{BC} = \frac{3k}{4k} = \frac{3}{4}$$

$$\cot\theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

$$\sec\theta = \frac{AC}{BC} = \frac{5k}{4k} = \frac{5}{4}$$

$$\text{co sec}\,\theta = \frac{AC}{AB} = \frac{5k}{3k} = \frac{5}{3}.$$

Example 6. If $\sec\theta = \frac{13}{12}$ then find the value of $\frac{1-\tan\theta}{1+\tan\theta}$.

Solution : Draw a right angled triangle ABC where $\sec\theta = \frac{13}{12}$ (Fig. 14.09)

i.e., hypotenuse AC and base BC are in the ratio of 13 : 12.
Let AC = 13k, BC = 12k, where k > 0, which is proportionality constant.
Hence form Baudhayan theorem

$$AB^2 = AC^2 - BC^2 = (13k)^2 - (12k)^2 = 25k^2$$

$$\therefore \qquad AB = \pm 5k$$

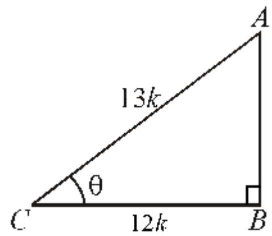


Fig. 14.09

Since angle θ is an acute angle, therefore AB will be positive.

$$\therefore \qquad AB = 5k$$

$$\therefore \qquad \tan\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$$

Now $$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\frac{5}{12}}{1+\frac{5}{12}} = \frac{7}{17}$$

Example 7. If $\operatorname{cosec} A = 2$, then find the value of $\cot A + \frac{\sin A}{1+\cos A}$.

Solution : $\because \ \operatorname{cosec} A = \frac{2}{1}$

Draw a right angled triangle ABC in which hypotenuse AC and perpendicular BC are in the ratio 2 : 1 (Fig. 14.10).

Let AC = 2k, BC = k

where k > 0, is the proportionality constant.

Hence from Baudhayan theroem

$$AB^2 = AC^2 - BC^2$$

or $$AB^2 = (2k)^2 - k^2 = 3k^2$$

or $$AB = \pm\sqrt{3}k$$

Since angle A is an acute angle, therefore AB will be positive.

$$\therefore \quad AB = \sqrt{3}k$$

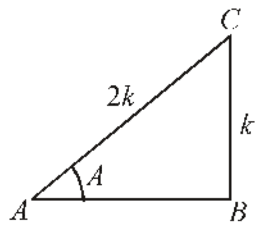


Fig. 14.10

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cot A = \frac{AB}{BC} = \frac{\sqrt{3}k}{k} = \sqrt{3}$$

Now

$$\cot A + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

$$= \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$

$$= \sqrt{3} + \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \sqrt{3} + 2 - \sqrt{3} = 2$$

Exercise 14.1

1. If in ΔABC, $\angle A = 90°$, $a = 25$ cm, $b = 7$ cm then find all the trigonometric ratio of $\angle B$ and $\angle C$.

2. If in ΔABC , $\angle B = 90°$, $a = 12$ cm, $b = 13$ cm then find all the trigonometrical ratios of $\angle A$ and $\angle C$.

3. If $\tan A = \sqrt{2} - 1$ then prove that $\sin A \cos A = \frac{1}{2\sqrt{2}}$.

4. If $\sin A = \frac{1}{3}$ then find the value of $\cos A \operatorname{cosec} A + \tan A \sec A$.

5. If $\cos\theta = \frac{8}{17}$ then find all the remaining trigonometrical ratios.

6. If $\cos A = \frac{5}{13}$ then find the value of $\frac{\operatorname{cosec} A}{\cos A + \operatorname{cosec} A}$.

7. If $5\tan\theta = 4$ then find the value of $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}$.

8. In ΔABC , $\angle C = 90°$ and if $\cot A = \sqrt{3}$ and $\cot B = \frac{1}{\sqrt{3}}$ then prove that $\sin A \cos B + \cos A \sin B = 1$.

9. If $16\cot A = 12$ then find the value of $\frac{\sin A + \cos A}{\sin A - \cos A}$.

10. In Fig. 14.13, $AD = DB$ and $\angle B = 90°$ than find the value of the following :

    (i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$

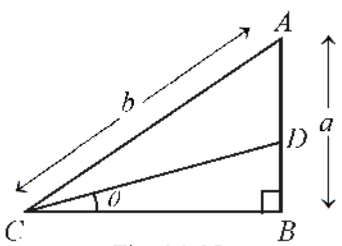


Fig. 14.13

14.03 Relation between Trigonometic Ratios :

In any right angle triangle OMP, PM is perpendicular, OM is base and OP is hypotenuse for the $\angle\theta$.

(i) $\sin\theta \operatorname{cosec}\theta = 1$

$$\sin\theta = \frac{PM}{OP} \qquad \cdots (1)$$

and $$\operatorname{cosec}\theta = \frac{OP}{PM} \qquad \cdots (2)$$

Multiplying (1) and (2)

$$\sin\theta \cdot \operatorname{cosec}\theta = \frac{PM}{OP} \times \frac{OP}{PM} = 1$$

i.e., $\sin\theta \operatorname{cosec}\theta = 1$

$$\Rightarrow \quad \sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

or $$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

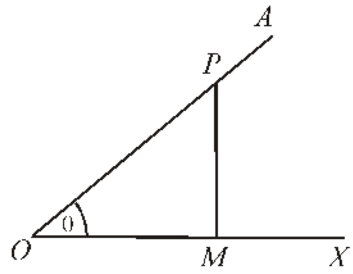


Fig. 14.14

Hence $\sin\theta$ and $\operatorname{cosec}\theta$ are reciprocal of each other.

(ii) $\cos\theta \cdot \sec\theta = 1$

From Fig. (14.14)

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OM}{OP} \qquad \cdots (3)$$

and $$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OP}{OM} \qquad \cdots (4)$$

Multiplying (3) and (4)

$$\cos\theta \cdot \sec\theta = \frac{OM}{OP} \cdot \frac{OP}{OM} = 1$$

i.e., $\cos\theta \cdot \sec\theta = 1$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sec\theta} \text{ या } \sec\theta = \frac{1}{\cos\theta}$$

Hence $\cos\theta$ and $\sec\theta$ are reciprocal of each other.

(iii) $\tan\theta \cdot \cot\theta = 1$

From Fig. 14.14

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PM}{OM} \qquad \cdots (5)$$

and $$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{PM} \qquad \cdots (6)$$

Multiplying (5) and (6)

$$\tan\theta \cdot \cot\theta = \frac{PM}{OM} \times \frac{OM}{PM} = 1$$

i.e., $\tan\theta \cdot \cot\theta = 1$

$$\Rightarrow \qquad \tan\theta = \frac{1}{\cot\theta} \text{ or } \cot\theta = \frac{1}{\tan\theta}$$

Hence $\tan\theta$ and $\cot\theta$ are reciprocal of each other.

(iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

From equation (1) and (3)

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{PM}{OP}}{\frac{OM}{OP}} = \frac{PM}{OP} \times \frac{OP}{OM} = \frac{PM}{OM} = \frac{\text{Perpendicular}}{\text{Base}} = \tan\theta$$

i.e., $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(v) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

From equation (1) and (3)

$$\frac{\cos\theta}{\sin\theta} = \frac{\frac{OM}{OP}}{\frac{PM}{OP}} = \frac{OM}{OP} \times \frac{OP}{PM} = \frac{OM}{PM} = \frac{\text{Base}}{\text{Perpendicular}} = \cot\theta$$

i.e., $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(vi) $\sin^2\theta + \cos^2\theta = 1$

From equation (1) and (3)

$$\sin\theta = \frac{PM}{OP} \text{ एवं } \cos\theta = \frac{OM}{OP}$$

Squaring and adding

$$\sin^2\theta + \cos^2\theta = \left(\frac{PM}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2$$

$$= \frac{PM^2 + OM^2}{OP^2} = \frac{OP^2}{OP^2} = 1 \qquad \text{[from Baudhayan theorem (Fig. 14.14)]}$$

(vii) $1 + \tan^2\theta = \sec^2\theta$

From Fig. 14.14

$$\tan\theta = \frac{PM}{OM}$$

$$\therefore \quad 1+\tan^2\theta = 1+\frac{PM^2}{OM^2} = \frac{OM^2+PM^2}{OM^2}$$

$$= \frac{OP^2}{OM^2} \quad \text{[From Baudhayab theroem]}$$

$$[\because \sec\theta = \frac{OP}{OM}, \text{ Fig. (14.14)}]$$

or $\quad 1+\tan^2\theta = \sec^2\theta$

Aliter :

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing both sides by $\cos^2\theta$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \quad \tan^2\theta + 1 = \sec^2\theta \qquad \left(\because \frac{\sin\theta}{\cos\theta} = \tan\theta, \frac{1}{\cos\theta} = \sec\theta\right)$$

(viii) $1+\cot^2\theta = \operatorname{cosec}^2\theta$

From Fig. 14.14

$$\cot\theta = \frac{OM}{PM}$$

$$\therefore \quad 1+\cot^2\theta = 1+\left(\frac{OM}{PM}\right)^2 = \frac{PM^2+OM^2}{PM^2}$$

$$= \frac{OP^2}{PM^2} \qquad \text{(from Baudhayan theorem)}$$

$$= \operatorname{cosec}^2\theta \quad \left(\because \operatorname{cosec}\theta = \frac{OP}{PM} \text{ from fig. 14.14}\right)$$

Aliter :

We know that

$$\cos^2\theta + \sin^2\theta = 1$$

Dividing both sides by $\sin^2\theta$

$$\left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 = \frac{1}{\sin^2\theta}$$

$$\Rightarrow \quad \cot^2\theta + 1 = \operatorname{cosec}^2\theta \quad \left(\because \frac{\cos\theta}{\sin\theta} = \cot\theta, \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right)$$

Note : $(\sin\theta)^2$ is always written as $\sin^2\theta$ and same is read as 'sign square theta'

i.e., $(\sin\theta)^2 = \sin^2\theta \neq \sin\theta^2$

Other trigonometrical ratios are also to be treated in the same manner

Illustrative Examples

Example 8. If $\cos\theta = \frac{5}{13}$ then find the value of $\sin\theta$, $\tan\theta$ with the help of relations between the trigonometrical ratios when θ is an acute angle.

Solution : We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Putting $\cos\theta = \frac{5}{13}$

$$\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$$

$$\Rightarrow \quad \sin^2\theta = 1 - \frac{25}{169}$$

$$\Rightarrow \quad \sin^2\theta = \frac{144}{169}$$

$$\Rightarrow \quad \sin\theta = \pm\frac{12}{13}$$

$$\Rightarrow \quad \sin\theta = \frac{12}{13} \qquad (\because \theta \text{ is an acute angle})$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{5}$$

Example 9. If $\tan\theta = \sqrt{3}$ then find all the trigonometric ratios using relations between the trigonometric ratios, when θ is an acute angle.

Solution : we know that

$$1 + \tan^2\theta = \sec^2\theta$$

or $$\sec^2\theta = 1 + \left(\sqrt{3}\right)^2$$
$$= 1 + 3 = 4$$

$\Rightarrow \quad \sec\theta = \pm 2$

or $\quad \sec\theta = 2 \quad (\because\ \theta$ is an acute angle)

$\because \quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{2}$

Now $\sin\theta = \tan\theta \cdot \cos\theta$

$\Rightarrow \quad \sin\theta = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2},$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\sqrt{3}/2\right)} = \frac{2}{\sqrt{3}},$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}.$

Example 10. If $\operatorname{cosec} A = \sqrt{10}$ then find $\cot A, \sin A, \cos A$ using the relations between the trigonometrical ratios, when θ is an acute angle.

Solution : We know that

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$\Rightarrow \quad \cot^2 A = \operatorname{cosec}^2 A - 1$

$\Rightarrow \quad \cot^2 A = \left(\sqrt{10}\right)^2 - 1 = 10 - 1 = 9$

$\Rightarrow \quad \cot A = 3 \quad (\because\ \theta$ is an acute angle)

Now $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{10}}$

and $\cos A = \cot A \cdot \sin A$

$= 3 \cdot \frac{1}{\sqrt{10}} = \frac{3}{\sqrt{10}}$

Example 11. If $\sec\theta = \frac{17}{8}$ then find all the trigonometrical ratios with the help of relation between the trigonometrical ratios, when θ is an acute angle.

Solution : We know that

$1 + \tan^2\theta = \sec^2\theta$

or $\quad \tan^2\theta = \sec^2\theta - 1$

or $\quad \tan^2\theta = \left(\frac{17}{8}\right)^2 - 1$

$$= \frac{289-64}{64} = \frac{225}{64}$$

$\Rightarrow \quad \tan\theta = \frac{15}{8} \qquad (\because\ \theta \text{ is an acute angle})$

$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{(15/8)} = \frac{8}{15}$

and $\quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{17/8} = \frac{8}{17}.$

Now $\quad \sin\theta = \tan\theta\cos\theta$

$$= \frac{15}{8}\cdot\frac{8}{17}$$

$$= \frac{15}{17}$$

and $\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{15}{17}\right)} = \frac{17}{15}.$

Example 12. If $\sin\theta = \dfrac{a^2-b^2}{a^2+b^2}$ then find the value of $\cos\theta$ and $\tan\theta$ using the relation between the trigonometrical ratios, when θ is an acute angle.

Solution : $\quad \sin\theta = \frac{a^2-b^2}{a^2+b^2}$

Now $\quad \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$

or $\quad \cos^2\theta = 1 - \frac{\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}$

$$= \frac{\left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}$$

$$= \frac{4a^2b^2}{\left(a^2+b^2\right)^2}$$

$$\Rightarrow \quad \cos\theta = \pm\frac{2ab}{\left(a^2+b^2\right)}$$

$$\text{or} \quad \cos\theta = \frac{2ab}{\left(a^2+b^2\right)} \quad (\because\ \theta \text{ is an acute angle})$$

$$\text{Now} \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\dfrac{a^2-b^2}{a^2+b^2}\right)}{\left(\dfrac{2ab}{a^2+b^2}\right)} = \frac{a^2-b^2}{2ab}$$

Exercise 14.2

Solve with the help of relation between trigonometric ratio [Q. 1 to 10]

1. If $\operatorname{cosec} A = \frac{5}{4}$ then find the value $\cot A, \sin A, \cos A$.

2. If $\tan A = \frac{20}{21}$ then find the value $\cos A$ and $\sin A$.

3. If $\sin A = \frac{3}{5}$ then find the value $\cos A$ and $\tan A$.

4. If $\cos B = \frac{1}{3}$ then find remaining trigonometric ratios.

5. If $\sin A = \frac{5}{13}$ then find the value $\cos A$ and $\tan A$.

6. If $\tan A = \sqrt{2} - 1$ then prove that $\sin A \cos A = \frac{1}{2\sqrt{2}}$.

7. If $\tan A = 2$ then find the value $\sec A \sin A + \tan^2 A - \operatorname{cosec} A$.

8. If $\sin\theta = \frac{4}{5}$ then find the value $\frac{4\tan\theta - 5\cos\theta}{\sec\theta + 4\cot\theta}$.

9. If $\cos\theta = \frac{1}{\sqrt{2}}$ then find the value $\sin\theta$ and $\cot\theta$.

10. If $\sec\theta = 2$ then evaluate $\tan\theta, \cos\theta$ and $\sin\theta$.

14.04 Trigonometric Identities :

Such trigonometric relations which are always true for the angles involved and for those angles for which trigonometrical ratios are defined are called trigonometric identities.

The relations defined in article 14.02 and 14.03 are the true for all values of angle θ . Thus these relations are called basic identities.

In trigonometry all relations are not identities, for example $\sin\theta = \cos\theta$ is an equation because this is not true for all values of θ .

To prove the trigonometrical identities one should take care of the following points :

(i) Always start form the difficult side of the identity and making use of basic identities and find second side of the identity.

(ii) If the identity contains the trigonometrical ratios then it is always better to convert these ratios in term of sines and cosines

(iii) If there exists any radical sign then it should be removed.

(iv) In some problems we may use rationalisation.

(v) If it is not possible to obtain one side form the other then simplify both the sides as far as possible and prove them identically equal.

Illustrative Examples

Example 13. Prove the identity :

$$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta.$$

Solution : L.H.S $= \sec^2\theta - \cos^2\theta$

$$= 1 + \tan^2\theta - \left(1 - \sin^2\theta\right) \quad \left[\begin{array}{l} \because \sec^2\theta = 1 + \tan^2\theta \text{ and} \\ \cos^2\theta = 1 - \sin^2\theta \end{array}\right]$$

$= \tan^2\theta + \sin^2\theta$

$=$ R.H.S

Example 14. Prove the identity :

$$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1.$$

Solution : L.H.S $= (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

(converting all ratios into sine and cosine θ

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} \cdot \frac{1}{\sin\theta\cos\theta} \quad \text{(Using basic identities)}$$

$= 1$

$=$ R.H.S

Example 15. Prove the identity :

$$\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta} = \tan\theta + \cot\theta \cdot$$

Solution : L.H.S $= \sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}$

$= \sqrt{\left(1+\tan^2\theta\right)+\left(1+\cot^2\theta\right)}$ [use of trigonometric identities]

$= \sqrt{\tan^2\theta + 2 + \cot^2\theta}$

$= \sqrt{\tan^2\theta + 2\tan\theta\cot\theta + \cot^2}$ $[\because \tan\theta\cot\theta = 1]$

$= \sqrt{\left(\tan\theta + \cot\theta\right)^2}$

$= \tan\theta + \cot\theta$

$=$ R.H.S

Example 16. Prove the identity :

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \mathrm{cosec}\,\theta + \cot\theta \cdot$$

Solution : To remove radical sign multiplying numerator and denominator by $\sqrt{1+\cos\theta}$ in the L.H.S.

L.H.S. $= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \times \sqrt{\frac{1+\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{\left(1+\cos\theta\right)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{\left(1+\cos\theta\right)^2}{\sin^2\theta}}$ $\left[\because 1-\cos^2\theta = \sin^2\theta\right]$

$= \frac{1+\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \mathrm{cosec}\,\theta + \cot\theta$

$=$ R.H.S.

Example 17. Prove the identity :

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 - 2\sec\theta\tan\theta + 2\tan^2\theta \cdot$$

Solution : L.H.S. $= \dfrac{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}$

$$= \frac{1-\sin\theta}{\cos\theta} \times \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \qquad \cdots(1)$$

$$\text{R.H.S.} = 1 - 2\cdot\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} + 2\cdot\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta - 2\sin\theta + 2\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\left(\cos^2\theta + \sin^2\theta\right) - 2\sin\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{1 - 2\sin\theta + \sin^2\theta}{1 - \sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

From equation (1) and (2)

$\therefore$ L.H.S. = R.H.S.

Example 18. Prove the identity :

$$\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}.$$

Solution : On rearranging both the sides

$$\frac{1}{\sec\theta - \tan\theta} + \frac{1}{\sec\theta + \tan\theta} = \frac{2}{\cos\theta} = 2\sec\theta$$

Now, L.H.S. $= \dfrac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \dfrac{2\sec\theta}{\sec^2\theta - \tan^2\theta}$

$= \dfrac{2\sec\theta}{1 + \tan^2\theta - \tan^2\theta}$

$= 2\sec\theta$

= R.H.S.

Example 19. Prove the identity :

$$\sec^6\theta - \tan^6\theta = 1 + 3\tan^2\theta + 3\tan^4\theta.$$

Solution : We know that

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Now, L.H.S. $= (\sec^2\theta)^3 - (\tan^2\theta)^3$

$= (\sec^2\theta - \tan^2\theta)(\sec^4\theta + \sec^2\theta\tan^2\theta + \tan^4\theta)$

$= (1 + \tan^2\theta - \tan^2\theta)\{\sec^2\theta(\sec^2\theta + \tan^2\theta) + \tan^4\theta\}$

$= 1\cdot\{(1+\tan^2\theta)(1 + \tan^2\theta + \tan^2\theta) + \tan^4\theta\}$

$= (1+\tan^2\theta)(1 + 2\tan^2\theta) + \tan^4\theta$

$= 1 + 3\tan^2\theta + 3\tan^4\theta$

= R.H.S.

Exercise 14.3

Prove the following identities :

1. $\cos\theta \cdot \tan\theta = \sin\theta$
2. $(1 - \sin^2\theta)\tan^2\theta = \sin^2\theta$
3. $\dfrac{\cos^2\theta}{\sin\theta} + \sin\theta = \text{cosec}\,\theta$
4. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$
5. $\text{cosec}^6\theta - \cot^6\theta = 1 + 3\,\text{cosec}^2\theta\cot^2\theta$

6. $\sin^2\theta\cos\theta+\tan\theta\sin\theta+\cos^3\theta=\sec\theta$

7. $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}=\sin\theta+\cos\theta$

8. $\dfrac{\text{cosec}\theta}{\text{cosec}-1}+\dfrac{\text{cosec}\theta}{\text{cosec}\theta+1}=2\sec^2\theta$

9. $\dfrac{\sin\theta}{1-\cos\theta}=\dfrac{1+\cos\theta}{\sin\theta}$

10. $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=2\,\text{cosec}\,\theta$

11. $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=\dfrac{1-\sin\theta}{\cos\theta}$

12. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}}=\cot\theta+\text{cosec}\,\theta$

13. $\dfrac{\sqrt{\left(\text{cosec}^2\theta-1\right)}}{\text{cosec}\,\theta}=\cos\theta$

14. $(1+\cot\theta-\text{cosec}\,\theta)(1+\tan\theta+\sec\theta)=2$

Importants Points

1. In right angled triangle

(i) $\sin\theta=\dfrac{\text{Perpendicular}}{\text{Base}}$ (ii) $\cos\theta=\dfrac{\text{Base}}{\text{Hypotenuse}}$

(iii) $\tan\theta=\dfrac{\text{Perpendicular}}{\text{Base}}$ (iv) $\cot\theta=\dfrac{\text{Base}}{\text{Perpendicular}}$

(v) $\sec\theta=\dfrac{\text{Hypotenuse}}{\text{Base}}$ (vi) $\text{cosec}\,\theta=\dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

2. (i) $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ (ii) $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

(iii) $\text{cosec}\,\theta=\dfrac{1}{\sin\theta}$ (iv) $\sec\theta=\dfrac{1}{\cos\theta}$

3. $\sin^2\theta+\cos^2\theta=1$

4. $1+\tan^2\theta=\sec^2\theta$

5. $1+\cot^2\theta=\text{cosec}^2\theta$

Miscellaneous Exercise 14

1. If $\tan\theta = \sqrt{3}$ then the value of $\sin\theta$ is :

   (A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2}{\sqrt{3}}$ (D) 1

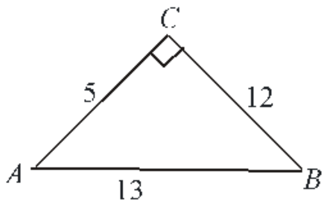
2. If $\sin\theta = \frac{5}{13}$ then the value of $\tan\theta$ is :

   (A) $\frac{5}{12}$ (B) $\frac{12}{13}$ (C) $\frac{13}{12}$ (D) $\frac{12}{5}$

3. If $\sqrt{3}\cos A = \sin A$ then the value of $\cot A$ is :

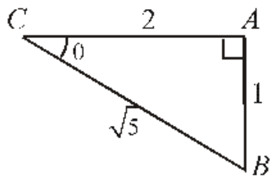
   (A) $\sqrt{3}$ (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) 2

4. In given ΔABC the value of $\cot A$ is :



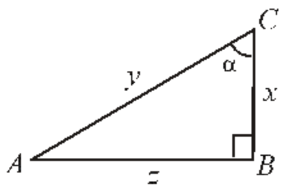
   (A) $\frac{12}{13}$ (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D) $\frac{13}{5}$

5. In given ΔABC the value of $\tan\theta$ is :



   (A) 2 (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{1}{2}$

6. In given ΔABC the value of $\operatorname{cosec}\alpha$ is :



   (A) $\frac{y}{x}$ (B) $\frac{y}{z}$ (C) $\frac{x}{z}$ (D) $\frac{x}{y}$

7. The value of $\sin^2 30° + \cos^2 30°$ is :

   (A) 0 (B) 2 (C) 3 (D) 1

8. The value of $\operatorname{cosec}^2 55° - \cot^2 55°$ is :

(A) 1 (B) 2 (C) 3 (D) 0

9. If $\cot\phi = \frac{20}{21}$ then the value of $\operatorname{cosec}\phi$ is :

(A) $\frac{21}{20}$ (B) $\frac{20}{29}$ (C) $\frac{29}{21}$ (D) $\frac{21}{29}$

10. If in ΔABC , $\angle B = 90°$, $c = 12$ cm $a = 9$ cm then the value of $\cos C$ is :

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{5}{3}$ (D) $\frac{4}{5}$

11. The value of $(\sec 40° + \tan 40°)(\sec 40° - \tan 40°)$ is :

(A) -1 (B) 1 (C) $\cos 40°$ (D) $\sin 40°$

12. The value of $\frac{1}{\sin\theta - \tan\theta}$ is :

(A) $\frac{\cot\theta}{\cos\theta - 1}$ (B) $\frac{\cot\theta}{\cot\theta - \operatorname{cosec}\theta}$

(C) $\operatorname{cosec}\theta - \cot\theta$ (D) $\cot\theta$

13. The value of $\frac{\sec A - 1}{\sec A + 1}$ is:

(A) $\frac{1+\cos A}{1-\cos A}$ (B) $\frac{\cos A - 1}{1+\cos A}$ (C) $\frac{1-\cos A}{1+\cos A}$ (D) $\frac{\cos A - 1}{1-\cos A}$

14. The value of $\cot^2\theta - \frac{1}{\sin^2\theta}$ is :

(A) 2 (B) 1 (C) 0 (D) -1

15. If $\operatorname{cosec}\theta = \frac{41}{40}$ then find the value of $\tan\theta$ and $\cos\theta$.

....

16. If in ΔABC , $\angle B$ is right angle and $AB = 12$ cm and $BC = 5$ cm then find the value of $\sin A, \tan A, \sin C$ and $\cot C$.

....

17. If $\cos\theta = \frac{3}{5}$ then evaluate $\frac{\sin\theta - \cot\theta}{2\tan\theta}$

....

18. If $\cos\theta = \frac{21}{29}$ then evaluate $\frac{\sec\theta}{\tan\theta - \sin\theta}$

....

19. If $\cot A = \sqrt{3}$ then prove that $\sin A\cos B + \cos A\sin B = 1$.

....

Using relation between trigonometric ratios prove the following [Q. 20-24].

20. If $\tan\theta = \frac{4}{3}$ then evaluate $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$.

21. If $\cot\theta = \frac{b}{a}$ then evaluate $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$.

22. If $\operatorname{cosec} A = 2$ then evaluate $\cot A + \frac{\sin A}{1+\cos A}$.

23. If $\cot\theta = \frac{1}{\sqrt{3}}$ then evaluate $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$.

24. If $\sin A = \frac{1}{3}$ then evaluate $\cos A \operatorname{cosec} A + \tan A \sec A$.

Prove the following [Q. 25-27]

25. $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$

26. $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

27. $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \tan A + \sec A$

Prove the following identities [Q. 28-29]

28. $\frac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \tan\alpha\tan\beta$

29. $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

Prove the following [Q. 30 - 35]

30. $\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$

31. $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

32. $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

33. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \tan^2 A + \cot^2 A + 7$

34. $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$

Answers

Exercise 14.1

1. $\sin B = \dfrac{7}{25}$, $\cos B = \dfrac{24}{25}$, $\tan B = \dfrac{7}{24}$

$\operatorname{cosec} B = \dfrac{25}{7}$, $\sec B = \dfrac{25}{24}$, $\cot B = \dfrac{24}{7}$

$\sin C = \dfrac{24}{25}$, $\cos C = \dfrac{7}{25}$, $\tan C = \dfrac{24}{7}$

$\operatorname{cosec} C = \dfrac{25}{24}$, $\sec C = \dfrac{25}{7}$, $\cot C = \dfrac{7}{24}$

2. $\sin A = \dfrac{12}{13}$, $\cos A = \dfrac{5}{13}$, $\tan A = \dfrac{12}{5}$

$\operatorname{cosec} A = \dfrac{13}{12}$, $\sec A = \dfrac{13}{5}$, $\cot A = \dfrac{5}{12}$

$\sin C = \dfrac{5}{13}$, $\cos C = \dfrac{12}{13}$, $\tan C = \dfrac{5}{12}$

$\operatorname{cosec} C = \dfrac{13}{5}$, $\sec C = \dfrac{13}{12}$, $\cot C = \dfrac{12}{5}$

4. $2\sqrt{2} + \dfrac{3}{8}$

5. $\sin\theta = \dfrac{15}{17}$, $\tan\theta = \dfrac{15}{8}$, $\operatorname{cosec}\theta = \dfrac{17}{15}$,

$\sec\theta = \dfrac{17}{8}$, $\cot\theta = \dfrac{8}{15}$

6. $\dfrac{169}{229}$

7. $\dfrac{5}{14}$

9. 7

10. (i) $\dfrac{a}{\sqrt{4b^2 - 3a^2}}$ (ii) $\dfrac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}$ (iii) $\dfrac{a}{2\sqrt{b^2 - a^2}}$

Exercise 14.2

1. $\cot A = \frac{3}{4}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

2. $\cos A = \frac{21}{29}$, $\sin A = \frac{20}{29}$

3. $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$

4. $\sin B = \frac{2\sqrt{2}}{3}$, $\tan B = 2\sqrt{2}$, $\cot B = \frac{1}{2\sqrt{2}}$,

   $\sec B = 3$, $\operatorname{cosec} B = \frac{3}{2\sqrt{2}}$

5. $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$

7. $\frac{12-\sqrt{5}}{2}$

8. $\frac{1}{2}$

9. $\sin\theta = \frac{1}{\sqrt{2}}$, $\cot\theta = 1$

10. $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = \sqrt{3}$

Miscellaneous Exercise 14

1. (B) 2. (A) 3. (C) 4. (B) 5. (D) 6. (B)
7. (D) 8. (A) 9. (C) 10. (A) 11. (B) 12. (A)
13. (C) 14. (D)

15. $\tan\theta = \frac{40}{9}$, $\cos\theta = \frac{9}{41}$ 16. $\frac{5}{13}, \frac{5}{12}, \frac{12}{13}, \frac{5}{12}$ 17. $\frac{3}{160}$

18. $\frac{841}{160}$ 20. 3 21. $\frac{b+a}{b-a}$ 22. 2 24. $\frac{16\sqrt{2}+3}{8}$

❑

15

Statistics

15.01 Introduction :

The branch of science known as Statistics has been used in India form ancient times. Which is evident from the following examples.

In Mahabharata period, king Rituparna while going alone with King Nala for the Swaymbara of Damyanti, estimated accurately the number of leaves and fruits on the basis of sample of a tree. The description of administrative setup based on registration of births and deaths during the region of Chandragupta Maurya (324-300 B.C.) is found in Kautilya's Arthashtra. Similarly many examples of use of statistics in administsative setup and in wars are found. When we observe or notice in newspapers, channels of electronic media, magazines and communication, then we get the factual and comparative informations about the temperature of cities, position of rainfall.

In this chapter we shall try to study, in detail, about collection, classification, tabulation, mean, median and mode of such data. Situation of shares of different companies. We use the data in our whole life. It is very important for us that we know to produce meaningful information from the data.

The branch of mathematics in which we learn about these meaningful information is know as statistics.

15.02 Collection of data :

On the basis of method of collection, data can be divided into two categories :

(1) Primary data (2) Secondary data.

(1) Primary data :

Data which are collected for the first time by the statical investigator or with the help of his workers is called primary data. e.g. The weight and height of the student in class 9th of school. We collect the primary data by following methods.

(i) Direct personal investigation : In this method, the investigator himself contacts the person form whose he/she is to collect information.

(ii) Indirect investigation : When the field of investigation is wide it is not possible for the investigator to go and contact everyone personally. In such a situation he/she may try to collect the information in the following ways :

(a) **Schedule to be filled in by enumerators :** In this method, the inversigator prepares the schedules, gives it to the trained enumerators. Then the enumerator ask the questions to concerned informants and writes the answer in schedules.

(b) **Information through questionnaires to be filled in by informants :** In this method investigator prepares a questionnaire related to investigation and sends to the informants with the object to get information and assurance of information's secrecy.

(c) **Information through local sources or correspondents :** When collection of daily information is required form different places, then investigator appoints local persons or correspondents for reporting the informations.

(d) **Indirect investigation through experts :** In this method the information is not collected form the people directly involved in the investigation. Rather it is collected form people called witnesses who are indirectly involved. If the annual evaluation of the students is to be done without examination, then this can be done by the concerned teacher.

(2) Secondary data :

If this method the data are already collected by a person or a society and these may be in published or unpublished form. These are generally obtained form the following two sources :

(i) **Published sources :** Government, non-government and other investigators collect data on different subjects form time to time and get it published, there main sources are as follows :

(a) **International publication :** Organisations like Unite Nations Organisation (U.N.O.), International Labour Organisation (I.L.O.), International Monetary Fund (I.M.F.) collect the data related with them and get them published for their members.

(b) **Government publication :** A number of Ministries and Departments (The department of Statistics) of central and state government collect data concerning them and published them.

(c) **Semi Government publication :** Municipalities, Corporations, Zila Parishads, Village Panchayats publish data on Education, Electricity, Birth and death and revenue records form time to time.

Similarly Universities, Research Institutes and teachers organisations compile different types of data and get them published.

(ii) **Unpublished sources :** Sometimes data on important subjects is collected by government and non-government institutions but they remain unpublished. Such unpublished matter can be obtained form office files, records, registers or the diaries of research scholars.

Exercise 15.1

1. What are primary and secondary data ? Clarify the difference.
2. Describe the methods of collection of primary data.

15.03 Presentation of Data :

After collection of data, an investigator has to select the suitable method to present the data in a form which is meaningful, easy to understand and gives its main features at a glance.

Data collected in original form is called **'raw data',** e.g. marks obtained by 10 students in a test of science are :

62 75 65 40 35 70 25 20 36 55

Can you obtained maximum and minimum marks form this data? If we arrange the data in ascending order or descending order, then we can easily obtained maximum and minimum marks. Ascending order of data are follows :

20 25 35 36 40 55 62 65 70 75

Here, it is clear that minimum marks is 20 and maximum marks is 75. Difference of maximum and minimum marks of data is called **range.** Thus, here

$75 - 20 = 55$ is the range.

If data are large, then it should be written in tabular form rather than in ascending or descending order.

Presentation of Data in Tabular Form :

In a school, following are the marks obtained by 20 students of class X in half yearly examination, out of 10 marks :

5	4	3	4	5	8	4	3	8	2
4	5	4	3	8	5	3	4	2	8

By seeing this raw data, it is difficult to estimate the level of class, but if we arrange the data in tabular form, then this task will became very easy. *"The number of times an observation occurs in the given data, is called the frequency of observation."* It is denoted as "f".

Following is the method to prepare a table from 06 the above data :

(i) In the first column of the table, we write all the marks from lowest to highest.

(ii) We now look at the first value in the given raw data and put vertical line known as tally mark, in the second column opposite to it.

(iii) When four lines are made opposite any mark, don't make the fifth line in the same way but make a line across the first four ($\cancel{\vert\vert\vert\vert}$)

(iv) If a number occurs six times, then draw again a verticle line and repeat the process. ($\cancel{\vert\vert\vert\vert}\vert$)

(v) After completing the entries of second column, count the number of tally marks against each observation and write in the third column.

Table

Marks	**Tally Marks**	**Frequency (f)**
2	$\vert\vert$	2
3	$\vert\vert\vert\vert$	4
4	$\cancel{\vert\vert\vert\vert}\vert$	6
5	$\vert\vert\vert\vert$	4
8	$\vert\vert\vert\vert$	4

$$\Sigma f = 20$$

This type of frequency distribution is called **ungrouped frequency distribution table.**

If number of observations is very large, then we divide the data in groups. These groups are called **class** and their measure is known as **class-interval** or **class size** or **class width.** Lower number of each class is known as **lower class limit** and maximum number is **upper class limit**. In next example we will understand the presentation in tabular form of data.

During Van Mahotsava 50 plants had planted in each school out of 30 schools.

After one month, following was the number of remaining plants.

22	6	48	0	28	22	17	10	32	6
22	22	28	26	17	36	10	22	28	0
28	22	48	32	10	48	25	36	6	32

Table

No. of Remaining Plants	Tally Marks	No. of students (frequency)
0 — 10	~~IIII~~ III	8
11 — 20	II	2
21 — 30	~~IIII~~ ~~IIII~~ II	12
31 — 40	~~IIII~~	5
41 — 50	III	3
Total		$\sum f = 30$

Presentation of data, by this method is called **'Grouped frequency distribution'**.

We can easily estimate and conclude by inspection of this table.

Exercises 15.2

1. Given below are the weights (in kg.) of 30 students of class X of a school :

34	34	36	37	38	33	34	35	36	37	38	33	34	35	34
33	37	35	34	36	38	36	35	34	35	37	38	34	35	35

Prepare a frequency distribution table for the above data.

2. Given below were the weights (in kg.) of 30 children born in a village :

3.4	3.6	3.0	3.8	3.6	3.8	2.9	3.4	2.9	3.4
3.0	3.4	3.2	3.1	3.2	3.2	3.1	3.2	3.4	3.0
3.1	3.2	3.5	3.7	3.1	3.0	2.9	3.0	3.1	3.2

Prepare a frequency distribution table for the above data.

3. Three coins were tossed 30 times. Each time the number of heads occuring was noted down as follow :

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

4. The blood groups of 30 students of class X are recorded as follows :

AB	O	O	A	B	O	A	O	B	A	O	B	A	O	O
A	AB	O	A	A	O	O	AB	B	A	O	B	A	B	O

Present this data in the form of a frequency distribution table. Find out which is the most common and which is the rarest blood group among these students.

5. Following are the marks obtained by 30 students of class IX in an examination. Use these marks to prepare a frequency table of 5 class size 10.

19	27	40	3	33	41	18	8	20	0	23	49	16	36	14
39	6	12	29	28	22	24	37	10	23	38	35	9	49	23

6. Prepare a frequency distribution table for the following distribution of taking class interval 5.

13	11	8	19	0	44	27	10	8	35	13
27	30	17	43	23	19	43	17	7		

7. The value of π upto 50 decimal places is given below :

3.14159265358979323846264338327950288419716939937510

(i) Prepare a frequency distribution table of the digits from 0 to 9 which comes after the decimal point.

(ii) What are the most and the least frequentely occuring digits ?

8. The distances form the residence of 40 engineers to their working palce (in km) are as follows :

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
3	18	15	12	5	3	8	7	9	7
14	12	9	2	15	6	7	15	12	6

By taking first class interval 0-5 (5 not included) prepare a frequency distribution table of class size 5 for the above data . What main features do you observe from this tabular representation ?

9. It is asked to 30 students that how many hours they have studied in last week ? Result obtained are as follows :

2	3	5	8	6	9	8	7	14	12
6	17	1	15	8	2	12	4	3	10
3	2	6	1	12	5	8	5	8	4

(i) Prepare a frequency distribution table from the above data, taking class size as 5.

(ii) How many students have studied upto 15 hours or more than in one week ?

15.04 Graphical Representation of Data

After the creation of universe, man has developed science and mathematics according to his needs. In this developmenmt, statistics also developed and figures are being used for sending information which is known as graphical representation. Graphs are eye-catching and easier to understand than the actual data. We shall study the following three types of graphical representation of the data :

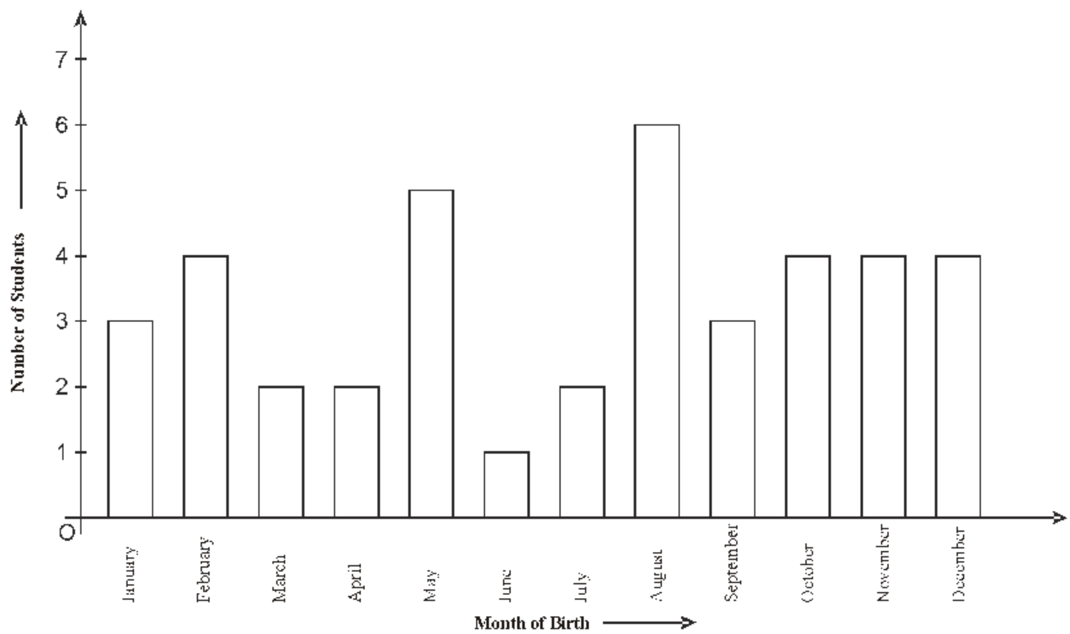
(i) Bar Graph

(ii) Histograms

(iii) Frequency polygon

(i) Bar graphs : A bar graph is a pictorial representation of the numerical data, in which bars or rectangles of uniform width are drawn with equal spaces in between them on the axis. The height of bar is proportional to the numerical data it represents, we can understand this by the following example.

Example 1. In class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained :



(i) How many students were born in the month of May ?

(ii) In which month maximum number of students were born ?

(iii) In which month minimum number of students were born ?

Solution : Here variable is 'month of birth' and the value of variable is "number of students born".

(i) 5 students born in the month of May.

(ii) In August, the maximum number of students were born.

(iii) In June, the minimum number of students were born.

Example 2. The family has planned the following expenditures per month under various heads : monthly income of a family is 20 thousand.

Table - 3

Commodities (variable)	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Children's education	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Preapre a bar graph for the above data.

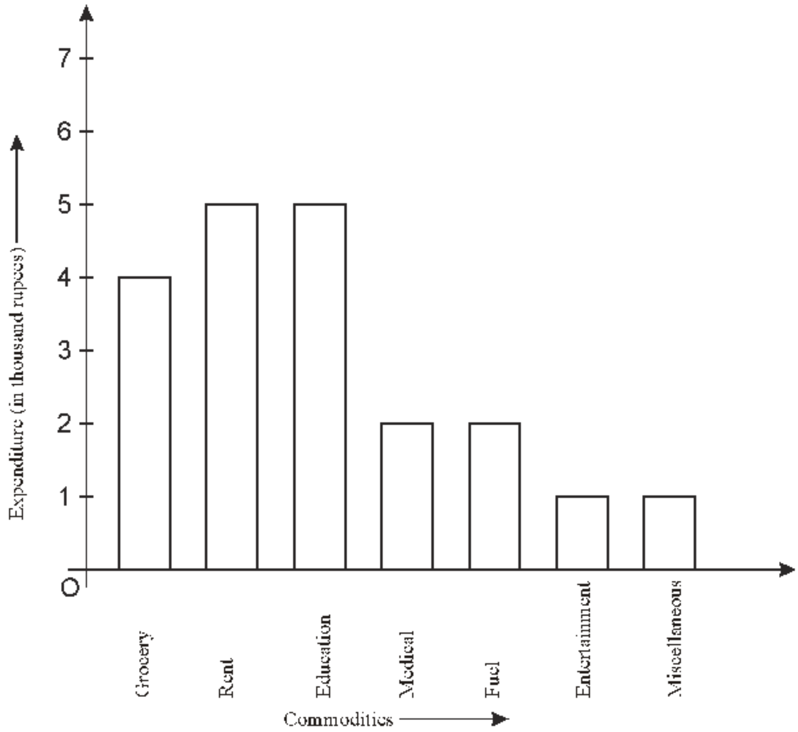
Solution : Following are the steps to prepare a bar graph for the above data :

1. Take any scale and mark the variables on the horizontal axis. Since, the width of the bar is not important, but for clarity, we take equal width for all bars and maintain equal gaps in between. Let one variable be represented by one unit.

2. We repersent the expenditure on the vertical axis. Since the maximum expenditure is 5 thousand rupees, so we can choose the scale as 1 unit = 1000 rupees.

3. To represent first variable (Grocery), we draw a rectangle bar of width 1 unit and height 4 units.

4. Similarly, other variable are represented by giving the unit space in between consecutive bars. (See Fig. 15.02)



Here, we can easily compare the expenditures on different commodities. Thus, this is a best way instead of presenatation of data in tabular form.

(ii) Histogram

A histogram is a graphical representation of a frequency distribution in the form of rectangle with class intervals as bases and heights proportional to corresponsing frequencies. Choosing a suitable scale, mark class-limits on X-axis and frequencies on Y-axis, such that area of so formed rectangles should remain proportional to the corresponding frequencies. Here, we will study the construction of histograms related to four different types of frequency distribution.

(a) When frequency distribution is grouped and continous and with equal class-intervals.

(b) When frequency distribution is grouped and continuous but with unequal class-intervals.

(c) When frequency distribution is grouped but not continuous.

(d) When frequency distribution is ungrouped and mid-point of class-intervals are given.

Now,we will clarify the above statements by the following examples.

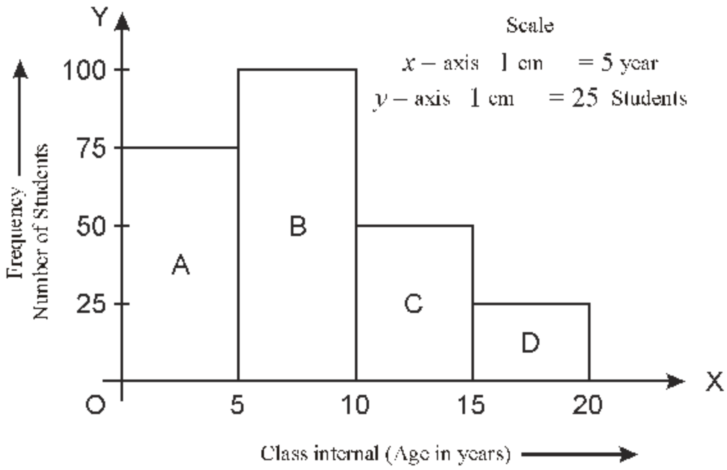
Example 3. **Draw a histogram for the following frequency distribution:**

Table-4

Class-interval (Age in Years)	0 — 5	5 — 10	10 — 15	15 – 20
No. of students	72	103	50	25

Solution : Here, frequency distribution is grouped and continuous and class-size is also same. Therefore, class-interval, *i.e.*, age in years (scale 1cm = 5 years) will be marked on x-axis.

Now, since number of students in class-interval 0-5 is 72. Taking class-interval 0-5 as base and the corresponding frequency as height, we construct rectangle. Similarly, we will construct rectangles B, C, D.



So, it is clear that all the rectangles have same base (1 cm) and height equal to frequency therefore area of rectangles should be proportional to frequency.

Example 4. The weekly wages of the workers of a company is given in the following table. Draw workers histogram for this data.

Table-5

Weekly Wages	1000 — 2000	2000 — 2500	2500 — 3000	3000 — 5000	5000 — 5500
Number of Workers	26	30	20	16	1

Solution : Here, frequency distribution is grouped and continuous but the class-interval is unequal, so to find heights of rectangles following method will be used, in which heights remains proportional to frequencies.

(a) Write minimum class size (h) of class-interval, here $h = 500$

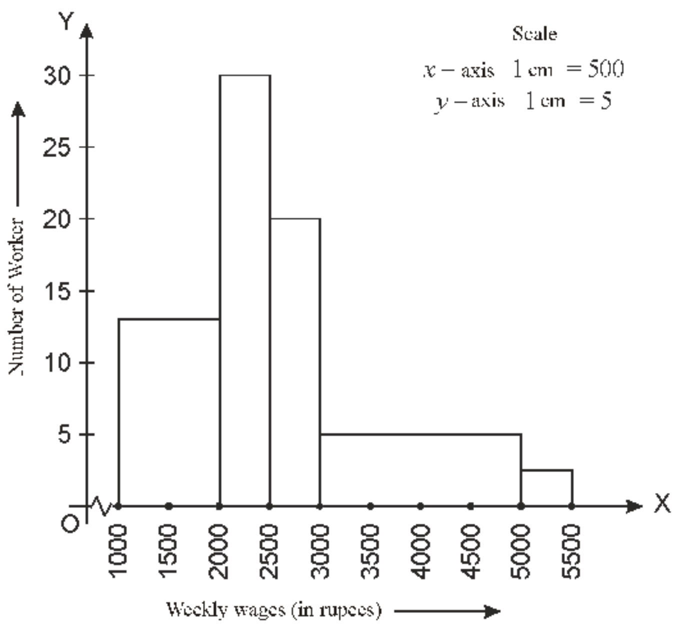
(b) Compute the adjusted frequencies of each class by using the following formula :

$$\text{Adjusted frequency of a class} = \frac{h}{\text{class size}} \times \text{frequency of the class}$$

Thus, we obtain a new table as :

Table-6

Weekly Wages (in Rupees)	No. of Workers	Adjusted Frequencies (Adjusted No. of Workers)
1000 — 2000	26	$\frac{500}{1000} \times 26 = 13$
2000 — 2500	30	$\frac{500}{500} \times 30 = 30$
2500 — 3000	20	$\frac{500}{500} \times 20 = 20$
3000 — 5000	16	$\frac{500}{2000} \times 16 = 4$
5000 — 5500	1	$\frac{500}{500} \times 1 = 1$



Draw class interval on X-axis (scale 1 cm = ₹ 500) and number of workers on Y- axis (scale 1 cm = 5 workers). Since, the first class interval (1000-2000) is starting form 1000 and not zero, so we show it on the graph by making a *kink* or a break on the axis. Now, we construct rectangles A, B, C, D, E with class-limits as bases and respective adjusted frequencies as heights.

Example 5. Draw a histogram for the frequency distribution :

Class-interval	10 — 19	20 — 29	30 — 39	40 — 49	50 — 59
Frequency	2	3	5	7	3

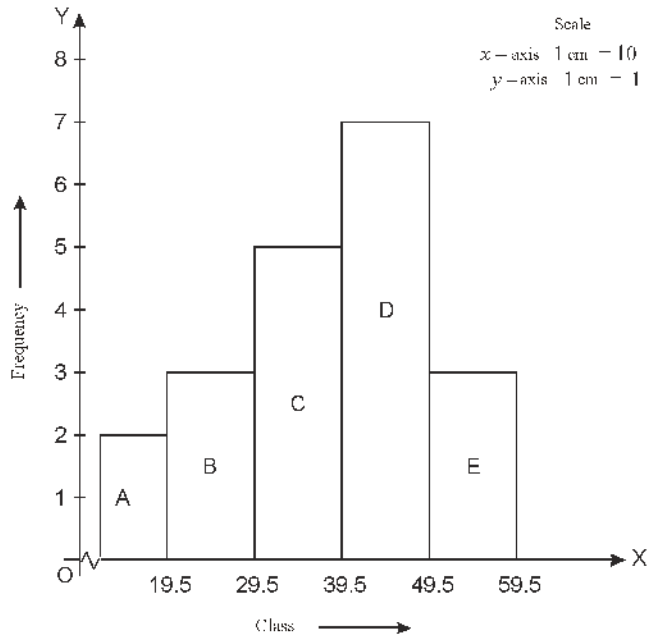
Solution : Here, frequency distribution is grouped but not continuous so we first convert it into continuous frequency distribution. After making certain modifications, we get the following Table-8.

Table - 8

Class-interval	9.5 — 19.5	19.5 — 29.5	29.5 — 39.5	39.5 — 49.5	49.5 — 59.5
Frequency	2	3	5	7	3

Therefore, we will draw class-interval (scale 1cm = 10) on x-axis and frequencies on y-axis (scale 1 cm =1).

Since, the first class interval (9.5-19.5) is starting form 9.5 and not zero, so we show it on the graph by making a kink or a break on the axis. Thus, the required histogram is shown above.



Example 6. Draw a histogram for the following frequency distribution :

Table - 9

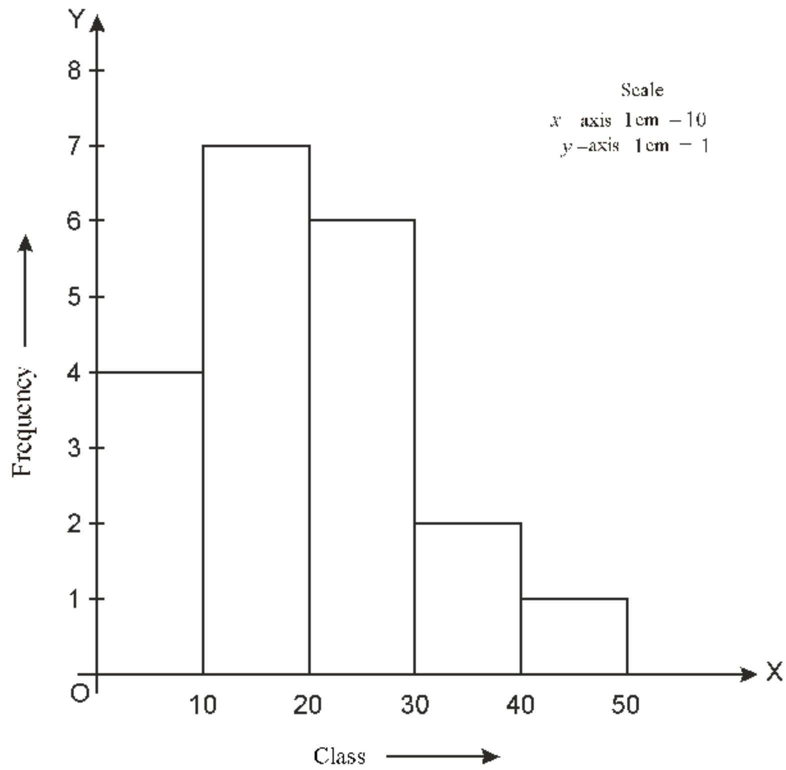
Mid point	5	15	25	35	45
Frequency	4	7	6	2	1

Solution : Here frequency distribution is ungrouped and mid points of the distribution are given. So, after converting in grouped frequency distribution. We get the following table :

Table - 10

Class	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50
Frequency	4	7	6	2	1

$\therefore$ This frequency distribution is grouped and continuous and class-size is also same. So, we will draw class-intervals on X-axis (scale : 1 cm = 10) and frequencies on Y-axis (scale : 1cm = 1) and obtained the follwing histogram.



(3) Frequency Polygon

Frequency polygon is another method of representing frequency distributions graphically. Frequency polygon can be drawn in two ways :

1. By using histogram 2. Without using histogram

1. In order to draw a frequency polygon by using histogram, we may follow the following process :

(i) Draw a histogram for the given frequency distribution.

(ii) Obtain the mid points of the upper horizontal side of each rectangle.

(iii) Join these mid points respectively, by straight lines.

(iv) The mid points at each end are joined to the immediately lower or higher mid-point at zero frequency, *i.e.*, on the X-axis.

(v) Join the mid point of last rectangle to its next possible mid point of class-interval on X-axis.

The figure so obtained is called a frequency polygon.

Remark :

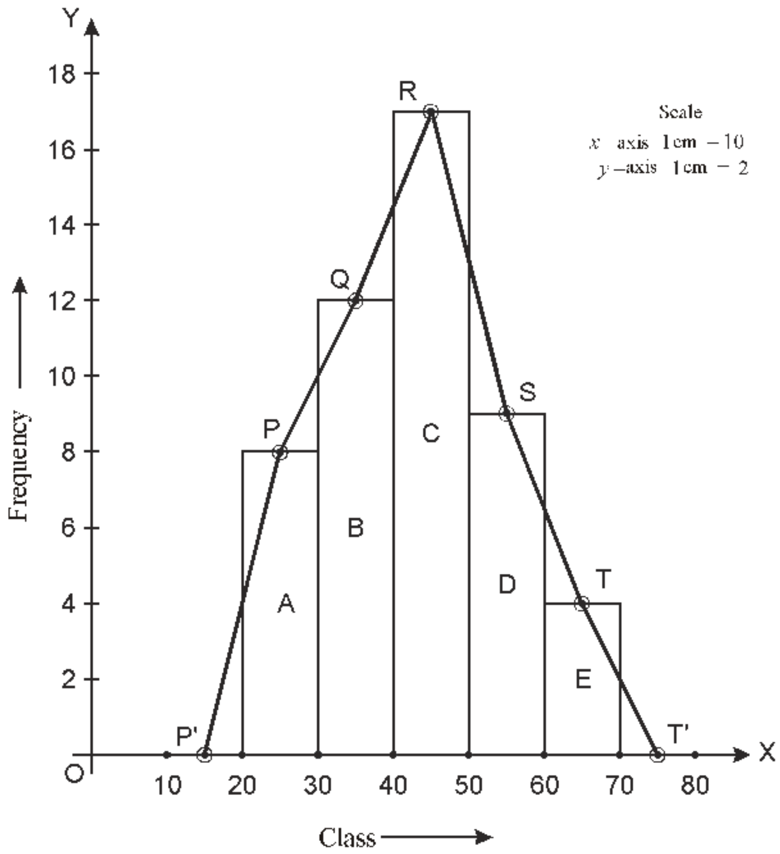
If there is no possiblity of part (iv) & (v) for example, in case of marks obtained by the students in a test, we cannot go below zero and beyond maximum marks on the two sides. In such cases the extreme line-segments are only partly drawn and are brought down vertically so that they meet with the vertical sides of the first and last rectangles.

Example 7. Prepare a frequency polygon by making histogram for the following frequency distribution :

Table-11

Class-interval	20 — 30	30 — 40	40 — 50	50 — 60	60 — 70
Frequency	8	12	17	9	4

Solution : Given frequency distribution is grouped and continous so we draw histogram by the method mentioned earlier. Join the mid points P, Q, R, S, T of the upper horizontal side of rectangle A, B, C, D, E respectively, by straight lines.



Since, here nothing is said about imagined classes so by joining mid points of class intervals from the begining and end points, we get frequency polygon $P' PQRSTT'$

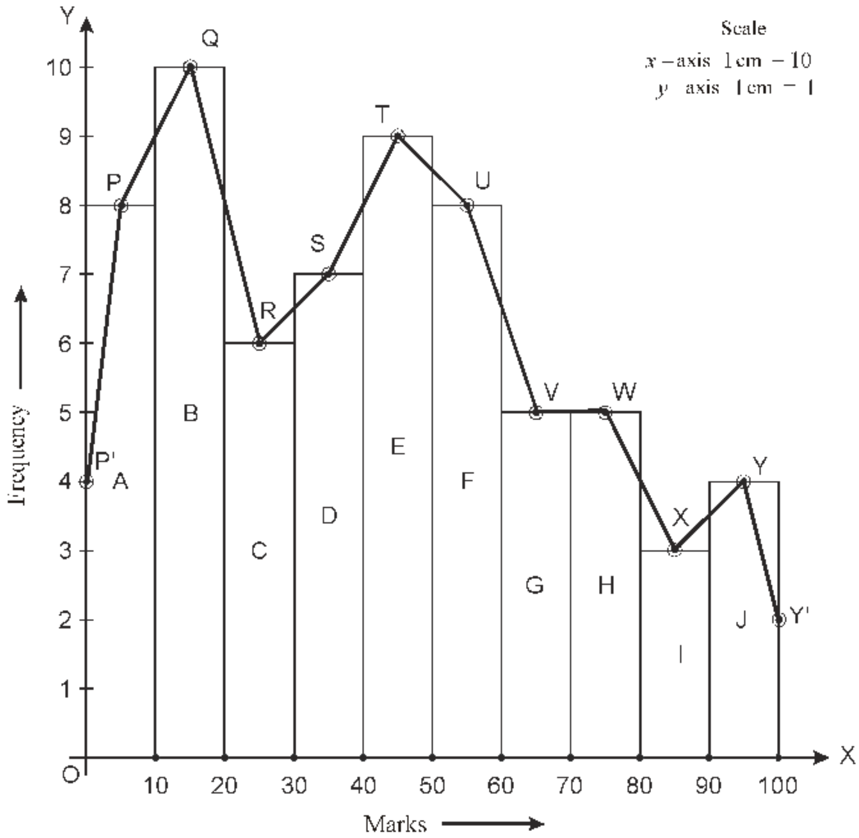
Thus, the required frequency polygon is shown in fig. 15.7.

Example 8. Make a frequency polygon by drawing a histogram for following frequency distribution.

Table - 12

Marks	0 10	10 20	20 30	30 40	40 50	50 60	60 70	70 80	80 90	90 100
No. of Students	8	10	6	7	9	8	8	6	3	4

Solution : Given frequency distribution is grouped and continuous, so we draw histogram according to the method learnt earlier.



Join the mid-points $P, Q, R, S, T, U, V, W, X, Y$ of the upper horizontal side of rectangles $A, B, C, D, E, F, G, H, I, J$ resepectively, by straight lines.

Since, here marks obtained by students cannot below 0 and beyond 100 so imaginary intervals does not exist. The first mid point, i.e., **P** is joined to zero frequency the point where this line segment meets the vertical axis is marked as **P'**. Let **Y** be the mid-point of the class succeeding the last class of the given data. Thus, OP' PQRSTUVWXYY' is the frequency polygon, which is shown in Fig. 15.08.

2. Following is the working method to draw a frequency polygon without making histogram:

(i) If frequency distribution is grouped, then find class mark. Now this frequency distribution will be converted into ungrouped.

(ii) Represent class marks on x-axis on a suitable scale.

(iii) Represent class marks on y-axis on a suitable scale.

(iv) Plot the points (x_1, f_1), (x_2, f_2)...

(v) Join these points by line segments.

(vi) Join these ends by mid points of the imagined classes adjacent to them and thus obtained frequency polygon for given frequency distribution.

(v) Join these points by line segments.

(vi) Join these ends by mid points of the imagined classes adjacent to them and thus obtained frequency polygon for given frequency distribution.

Remark:

1. For any type of frequency distribution, polygon can be made easily.

2. If to make imaginary classes is not possible, then draw vertical lines at the starting point and end point of these class-intervals and join them with the mid point of class-interval. Thus, we will get required frequency polygon.

Example 9: Draw frequency polygon for given frequency distribution.

Table-13

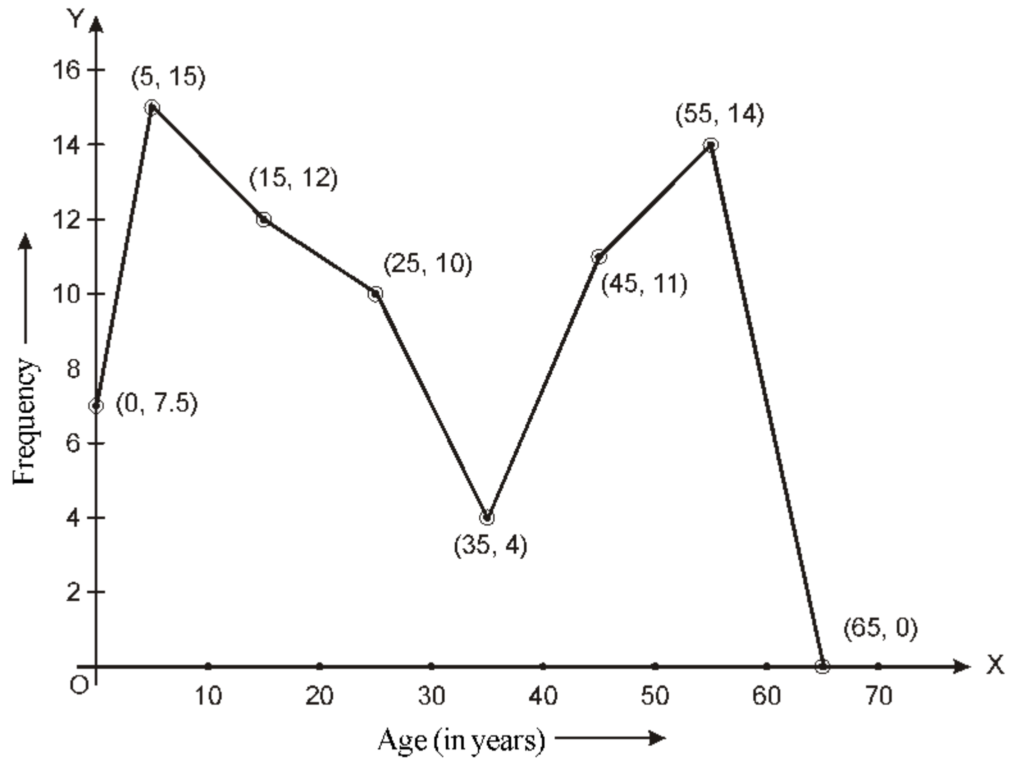
Age (in years)	0 — 10	10 — 20	20— 30	30 — 40	40 — 50	50 — 60
Frequency	15	12	10	4	11	14

Solution : The frequency distribution is grouped and continuous. So, we obtained the following table on the basis of class.

Table-14

Age (in years)	0 — 10	10 — 20	20— 30	30 — 40	40 — 50	50 — 60
Class mark	5	15	25	35	45	55
Frequency	15	12	10	4	11	14

Now, mark the points (5, 15), (15, 12), (25, 10), (35, 4), (45, 11), (55, 14) assuming suitable scale.



Since, age cannot be negative, so instead of imaginary classes, draw vertical lines at the starting point and end point of these class intervals and join them with the mid points of class interval. Thus, we will get required frequency polygon for given frequency distribution as shown in the figure.

Example 10. Draw frequency polygon for the following frequency distribution.

Table-15

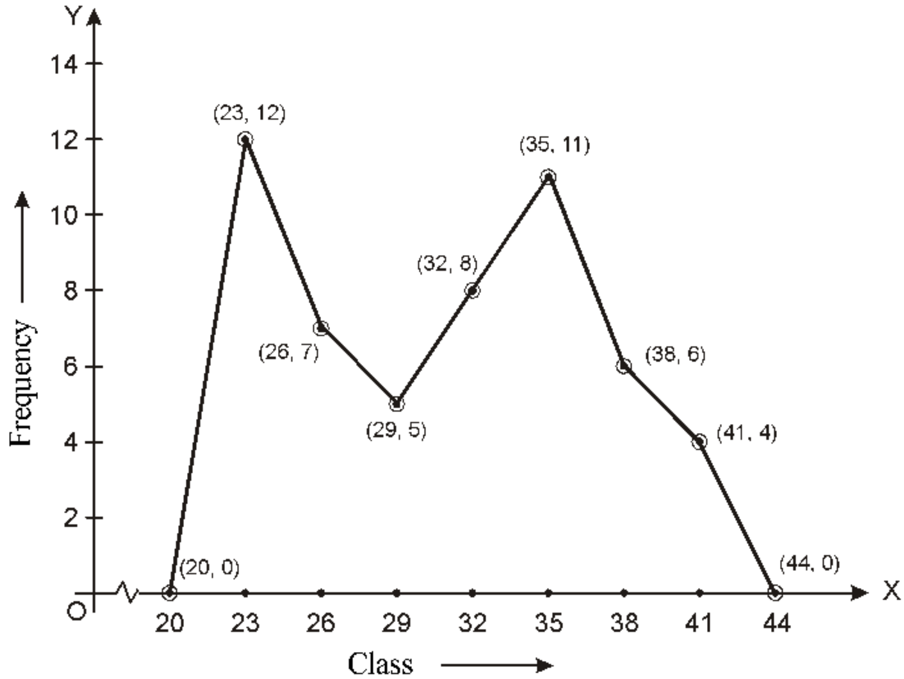
Class	22 — 24	25 — 27	28— 30	31 — 33	34 — 36	37— 39	40 — 42
Frequency	12	7	5	8	11	6	4

Solution : Here, frequency distribution is grouped but not continuous because draw after making continuous class there is not change in class mark. Thus, instead of making continuous, we will find class mark and then draw frequency polygon.

Table - 16

Class	22 — 24	25 — 27	28— 30	31 — 33	34 — 36	37 — 39	40 — 42
Class mark	23	26	29	32	35	38	41
Frequency	12	7	5	8	11	6	4

By assuming suitable scale, we will mark the points (23, 12), (26, 7), (29, 5), (32, 8), (35, 11), (38, 6), (41, 4) on graph paper.



Join points (23, 12) to (20, 0) and (41, 4) to point (44, 0). Thus, we get the required frequency polygon.

Exercise 15.3

1. An organisation have conducted a survey in the whole world to find out the reason behind the diseases and death of females of age group 15-44 (in years) and obtained the following data (in %).

Table -17

S. No.	Reasons	Female death rate (%)
1.	Fertility health stage	31.8
2.	Nerve Psychiatric stage	25.4
3.	Loss	12.4
4.	Heart vessel stage	4.3
5.	Breathign stage	4.1
6.	Other reasons	22.0

(i) Express above informations by bar graph.

(ii) Which stage is the major cause of bad health and death of females in the whole world?

2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indain society is given below:

Table - 18

S.No.	Area	No. of Girls per thousand boys
1.	Scheduled Caste	940
2.	Scheduled Tribe	970
3.	Non SC/Tribe	920
4.	Backward districts	950
5.	Non-backward districts	920
6.	Rural	930
7.	Urban	910

(i) Represent the above informations by bar graph.

(ii) Discuss, what conclusion can be drawn from this graph?

3. In an assembly elections, results of winning seats by various parties are as follows:

Political Party	A	B	C	D	E	F
Winning Seats	75	55	37	29	10	37

(i) Draw a bar graph to represent the results of electron.

(ii) Which party won maximum seats?

From the following tables draw a histogram (4 to 8)

Table - 20

4.

Class	0 — 5	5 — 10	10 — 15	15 — 20	20 — 25
Frequency	18	15	14	8	10

Table - 21

5.

Class	0 — 20	20 — 40	40 — 60	60 — 80	80 — 100
Frequency	5	6	12	4	3

Table - 22

6.

Class	3 — 6	6 — 12	12 — 13	13 — 14	14 — 15
Frequency	150	420	100	110	50

Table - 23

7.

Class	5 — 9	10 — 14	15 — 19	20 — 24
Frequency	3	5	8	2

Table - 24

8.

Class	8	14	20	26	32
Frequency	10	15	25	9	6

9. Draw a frequency polygon with the help of a histogram for the following distribution. Maximum number is 10.

Table - 25

Marks	0 — 5	5 — 10	10 — 15	15 — 20	20 — 25	25 — 30	30 — 35
Frequency	1	2	4	6	5	3	2

10. Draw a frequency polygon with the help of a histogram for the following distribution. Maximum number is 10.

Table - 26

Marks	0 — 2	2 — 4	4 — 6	6 — 8	8 — 10
Number of students	7	8	4	9	2

11. Draw a frequency polygon for the following frequency distribution.

Table - 27

Variable x	5	10	15	20	25	30
Frequency f	2	6	4	1	5	2

12. Draw a frequency polygon for the following frequency distribution.

Table - 28

Production (in tonnes)	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50	50 — 60	60 — 70
Frequency	8	18	23	37	47	26	16

Measures of Central Tendency

In this chapter, we have studied the presentation of data in various forms - by frequency distribution tables, bar graphs, histograms and frequency polygons.

In order to make these data meaningful, there is always need to study all the data. This can be done with the help of measures of central tendency or averages.

Let us consider a situation, in which two students Parveen and Akash have received their test copies.

In test, there were five questions of 10 marks each. Following are the marks obtained by them.

Table - 29

Question Numbers	1	2	3	4	5
Makrs obtained by Praveen	10	8	9	8	7
Makrs obtained by Akash	4	7	10	10	10

Average marks obtained by Praveen $= \frac{42}{5} = 8.4$

Average marks obtained by Akash $= \frac{41}{5} = 8.2$

Since average marks of Praveen was greater than average marks of Akash. So Praveen says his performance is good but Akash was not agree with this. He kept marks obtained by both in ascending order and then obtained mid marks.

Table - 30

Marks obtained by Praveen	7	8	8	9	10
Marks obtained by Akash	4	10	10	10	10

Akash says his mid marks obtained was 10 whereas of Praveen's was 8. So his performance was better than Praveen, but Praveen was not agree with Akash to convince Praveen, Akash make at trick. He says, he had got 10 marks 3 times whereas Praveen have got only once. Thus his performance is better.

In first stage, Praveen has got average marks, that is '**Mean'.** Mid makrs which was used by Akash in his discussion is **Median**. In second discussion Akash has maximum marks got maximum times, is **mode**.

Now, we will discuss deeply on Mean.

The arithmetic mean of a set of observations is equal to their sum divided by the total number of observations and is denoted by $\bar{x}$ and read as (x bar).

Example 11. Daily wages of 5 workers are ₹ 250, ₹ 200, ₹ 225, ₹ 300, ₹ 275. Find their mean.

Solution : Mean of observations

$$(\bar{x}) = \frac{\text{Sum of all observations}}{\text{Total no. of observations}} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{250+200+225+300+275}{5} = \frac{1250}{5} = ₹250$$

Thus, average wages of 5 persons = ₹ 250

If we want to find the mean of 30 persons, we will write it as : $x_1 + x_2 + x_3 ... + x_{30}$ which is a lengthy process. We use Σ for summations. So, $x_1 + x_2 + x_3 ... + x_{30}$ can be written as $\sum_{j=1}^{30} x_j$

$$\overline{x} = \frac{\sum_{j=1}^{30} x_j}{30}$$

Similarly, if number of observation $= n$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_j}{n}$$

Median : Median of the given number of observations is the value of the variable which divides it exactly into two equal parts. So, when data is written in ascending or descending order, then median of ungrouped data is calculated as:

(i) If number of observation (n) is odd, then

$$\text{Median} = \text{value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{observation}$$

For example, if $n = 13$ then

Median will be the value of $\left(\frac{13+1}{2}\right)^{\text{th}}$ observation, i.e., value of 7th observation.

(ii) If number of observation (n) is even, then

$$\text{Median} = \frac{\text{value of } \left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \text{value of } \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

For example, if n = 16, then

$$\text{Median} = \frac{\text{value of } 8^{\text{th}} \text{ observation} + \text{value of } 9^{\text{th}} \text{ observation}}{2}$$

Example 12 : Following are the heights (in cm) of 9 students of a class. Find the median of these data:

155 160 145 149 150 147 152 144 148

Solution : Arranging the given data in ascending order, we have

144 145 147 148 149 150 152 155 160

Here, number of students is 9 i.e., which is an odd number

$$\therefore \quad \text{Median} = \text{value of} \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of} \left(\frac{9+1}{2}\right)^{\text{th}} \text{ observation}$$

$= \text{value of } 5^{\text{th}}$ observation

= 149 cm

Example 13 : Following are the points obtained by a Kabaddi team in various matches:

17 2 7 27 15 5 14 8 10 24

48 10 8 7 18 28

Find the median of the points obtained by the team.

Solution : Arranging the given data in ascending order, we have

2 5 7 7 8 8 10 10 14 15

17 18 24 27 28 48

Here, number of observations is 16 which is an even number.

$$\therefore \qquad \text{Median} = \frac{\left(\frac{16}{2}\right)^{th} \text{observation} + \left(\frac{16}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{8^{th} + 9^{th} \text{observation}}{2}$$

$$= \frac{10+14}{2} = \frac{24}{2} = 12$$

Mode:

Mode is the value of the observation which occurs most frequently. It means a variable with maximum frequency is called mode.

This measure of central tendency is frequently used in readymade garment and shoe industry. With the help of mode, these industries take decision that propduction of which product whould be increased.

Example 14 : Find the mode of the marks obtained by 20 students (out of 10), from the followign data:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

Solution : Arranging the given data in ascending order, we have

2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9, 9, 9, 10, 10

Here, 9 occurs maximum number of time, i.e., 4 times.

Thus, mode is 9.

Exercise 15.4

1. The following number of goals were scored by a team in a series of 10 matches:

2	3	4	5	0	1	3	3	4	3

Find the mean, median and mode of these scores.

2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

41	39	48	52	46	62	54	40	96	52
98	40	42	52	60					

Find the mean, median and mode for above data.

3. The following observations are arranged in ascending order. If the median of the data is 63, then find the vlaue of x :

29 32 48 50 x $x + 2$ 72 78 84 95

4. Find the mode of 14 25 14 28 18 17 18 14 23 22 14 18

5. Find the mean salary of 60 workers, working in a factory, from the following table:

Table - 31

Salary (in ₹)	3000	4000	5000	6000	7000	8000	9000	10000
No. of workers	16	12	10	8	6	4	3	1

Miscellaneous Exericse - 15

1. In a distribution 5, 5, 6, 4, 9, 5, 3, 2, 7, 6, 3, 8, 4 frequency of class interval 3 - 5 is

(A) 3 (B) 4 (C) 6 (D) 7

2. Range of the following distribution will be

3.2 2.8 3.1 2.1 3.2 2.4 2.1 2.8 2.7 2.7

(A) 2.7 (B) 3.1 (C) 2.4 (D) 1.1

3. In the following distribution, number of students of age less than 25 years is

Age (in years)	5-10	10 - 15	15 - 20	20 - 25	25 - 30
No. of students	3	6	8	8	2

(A) 8 (B) 16 (C) 9 (D) 25

4. In a bar graph, height of rectangle is

(A) inversely proportiional to frequency of class

(B) proportional to frequency of class

(C) proportional to class-interval

(D) inversely proportional to class-interval

5. The examaination result of any class of a school can be comparatively studied:

(A) by circular diagram (B) by bar graph

(C) by linear graph (D) all of these

6. The range of distribution 6, 1, 2, 3, 9, 8, 3, 4, 8, 2, 3 will be

(A) 4 (B) 8 (C) 7 (D) 6

7. In the distribution 5, 1, 5, 2, 3, 6, 5, 4 frequency of 5 will be

(A) 1 (B) 2 (C) 3 (D) 4

8. The median of 11, 2, 7, 8, 9, 3, 5 will be

(A) 7 (B) 9 (C) 5 (D) 11

9. The mean of 15, 0, 10, 5 will be

(A) 15 (B) 10 (C) 5 (D) 7.5

10. In 4, 3, 4, 5, 4, 2, 4, 1 mode will be

(A) 1 (B) 2 (C) 5 (D) 4

Very Short Answer Type Question

11. Write the frequency of class-interval 0 - 5 from the following distribution:
3, 2, 0, 10, 8, 5, 13, 5, 6, 6, 0, 14

12. If 7 is the mean of the numbers 5, 8, 4, x, 6, 9 then find the vlaue of x

13. What is range?

14. What is histogram?

15. Make a frequency table of 9, 7, 9, 8, 3, 9, 8, 3, 5, 7, 5, 3.

16. If arithematic mean of a frequency distribution is 15 and $\sum f = 20$ then find the value of $\sum fx$

17. Write median of distribution 5, 2, 3, 7, 5, 4, 3, 2, 1

18. Find median of distribution 12, 1, 6, 4, 10, 8, 1, 4

19. Find mode of distribution 4, 3, 4, 1, 2, 4, 7, 5, 3

Importants Points

1. The data are of two types : (i) primary data (ii) secondary data.
2. The methods of collecting the primary data
   (i) By direct personal (individual) investigation, (ii) By indirect investigation.
3. Indirectly data is obtained by the following ways:
   (i) Filling the list by statisticians, (ii) Filling the list by informer,
   (iii) By local sources of correspondent,
   (iv) Direct oral investigation by the specialists.
4. Data which have already collected whether published or not, is called secondary data.
5. Secondary data may be published or not.
6. Following are the main sources of published data:
   (i) International organisation, (ii) Government publication,
   (iii) Semi-government publication, (iv) Publication of business organisation,
   (v) Publication of research centres, (vi) Paper-magazines,
   (vii) Published papers of research scholars.
7. Generally, x is used for marks is frequency distribution.
8. Range is the difference between maximum and minimum value of variable x.
9. Maximum and minimum value of a class is known as its upper and lower limit.
10. Average of upper and lower limit of a class is called mid vlaue of class or class mark and is represented as x
11. The following graphs are mainly used in statistical analysis:
    (a) Bar graph, (b) Histogram, (c) Frequency polygon
12. Three measures of centeral tendency for ungrouped data are:
    (i) **Mean :** The mean of a set of observation is equal to their sum divided by the total number of observations, i.e.,

    $$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$

    (ii) **Median :** Middle most value (observation) is called median.

    (iii) **Mode :** Observation which occurs maximum times is called mode.

Answer

Exercise 15.1

1. **Primary data :** When an investigator collects data himself with a definite plan or design in his mind, it is called primary data.

Secondary data : Data which are not originally collected rather obtained from published or unpublished sources is known as secondary data.

Exercise 15.2

1.

Variable (x)	33	34	35	36	37	38
Tally makrs	$\vert\vert\vert$	$\cancel{\vert\vert\vert\vert}\vert\vert\vert$	$\cancel{\vert\vert\vert\vert}\vert\vert$	$\vert\vert\vert\vert$	$\vert\vert\vert\vert$	$\vert\vert\vert\vert$
Frequency (f)	3	8	7	4	4	4

2.

Variable (x)	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8
Tally makrs	$\vert\vert\vert$	$\cancel{\vert\vert\vert\vert}$	$\cancel{\vert\vert\vert\vert}$	$\cancel{\vert\vert\vert\vert}\vert$	–	$\cancel{\vert\vert\vert\vert}\vert$	$\vert$	$\vert\vert$	$\vert$	$\vert\vert$
Frequency (f)	3	5	5	6	–	5	1	2	1	2

3.

No. of ends	**Tally makrs**	**Frequency (f)**
0	$\cancel{\vert\vert\vert\vert}\vert$	6
1	$\cancel{\vert\vert\vert\vert}\cancel{\vert\vert\vert\vert}$	10
2	$\cancel{\vert\vert\vert\vert}\vert\vert\vert\vert$	9
3	$\cancel{\vert\vert\vert\vert}$	5
Total		30

4.

Blood group	**Tally makrs**	**No. of students**
A	$\cancel{\vert\vert\vert\vert}\vert\vert\vert\vert$	9
B	$\cancel{\vert\vert\vert\vert}\vert$	6
O	$\cancel{\vert\vert\vert\vert}\cancel{\vert\vert\vert\vert}\vert\vert$	12
AB	$\vert\vert\vert$	3
Total		30

5.

Class interval	**Tally makrs**	**Frequency**
0 — 10	$\cancel{\vert\vert\vert\vert}$	5
10 — 20	$\cancel{\vert\vert\vert\vert}\vert$	6

20 — 30	𝍸||||	9
30 — 40	𝍸|	6
40 — 50	||||	4

6.

Class	**Tally Makrs**	**Frequency (f)**
0 — 5	|	1
5 — 10	|||	3
10 — 15	||||	4
15 — 20	||||	4
20 — 25	|	1
25 — 30	||	2
30 — 35	|	1
35 — 40	|	1
40 — 45	|||	3

7. (i)

Digits	0	1	2	3	4	5	6	7	8	9	Total
Frequency	2	5	5	8	4	5	4	4	5	8	50

(ii) 3 and 9 are the numbers whcih occurs maximum and number 0 ocurs minimum

8.

Distance (in km)	**Tally Marks**	**Frequency**
0 — 5	𝍸	5
5 — 10	𝍸𝍸|	11
10 — 15	𝍸𝍸|	11
15 — 20	𝍸||||	9
20 — 25	|	1
25 — 30	|	1
30 — 35	||	2

40

9.

Number of hours	**Frequency**
0 — 5	10
5 — 10	13
10 — 15	5
15 — 20	2
Total	30

(ii) 2 Children

Exericse 15.3

1. (ii) Fertility health stage

3. (ii) Party A

Exercise 15.4

1. Mean = 2.8, Median = 3, Mode = 3

2. Mean = 54.8, Median = 52, Mode = 52

3. $x = 62$

4. 14

5. Mean salary of 60 workers is ₹ 5083.33.

Miscellaneous Exericse - 15

1. (b) **2.** (d) **3.** (d) **4.** (d) **5.** (b) **6.** (b)

7. (c) **8.** (a) **9.** (d) **10.** (d) **11.** –4 **12.** –10

13. Range is the difference between the maximum and the minimum valeu of variable x.

14. Histogram is a rectangular form of grouped and continuous frequency distribution.

15.

x	3	5	7	8	9
f	3	2	2	2	3

16. 300 **17.** 3 **18.** 5 **19.** 4

16

Percentages

Objective :

Concept of percentage is brought out through the number of fatal accidents.

Content :

In 2009, a total of 7516 accidents were recorded in which 2325 persons lost their lives and 6936 were injured. Calculate percentage of total road accident victims who killed. Of fatal accidents, 1170 pedestrians and 2677 were injured. Calculate percentage of pedestrians-

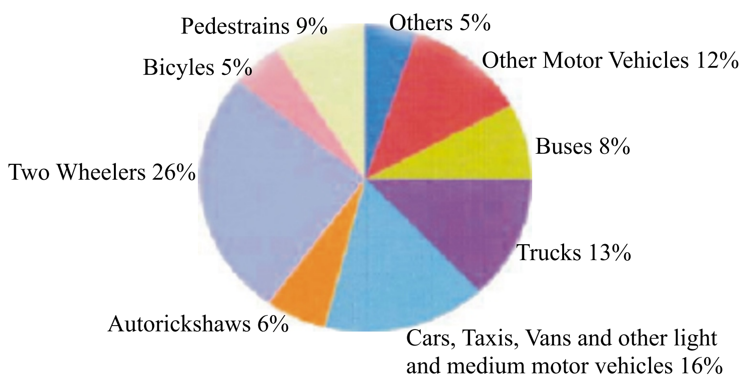
(a) Killed (b) Injured

Pedestrians are the most vulnerable road users. Can you suggest some measures of safety for them? 2-wheeler drivers accounted for 691 of all those killed and 2358 of all the injured victims in 2009. Calculate percentage of two-wheeler killed and injured. Can you suggest some measures of road safety for these drivers?

In 2009, 1993 males were killed and 158 females were killed. By what percentage are more males killed. If 174 children were killed, what is the percentage of children who have lost their lives on Delhi Roads?

Activity :

Profile of people killed on roads in detail is given in the form of a pie chart.



1. Express each percentage in decimal form.
2. If 1,33,938 persons were killed, calculate how many pedestrians were killed?

Circles

Objective :

Circular road signs give order. Identification of common circular road signs.

Content :

You have already studied that -

- Triangles warn
- Rectangles inform

Now we study about a circle. A circular traffic sign gives us order.

Blue circle gives a positive instruction. Draw one such traffic sign.

Red circle gives a negative instruction. Draw one such traffic sign.

Find out two traffic sign which do not follow these rules.

Activity :

Locate all the traffic signs in your locality. Find out what percentage of the traffic signs give order.

Statistics

Objective :

Circular road signs give order. Identification of common circular road sings.

Content :

Bar graphs are pictorial representation of numerical data.

Exercise :

The data of various vehicles for the year 1980 and 1990 in percentage is given below:

Vehicle	1980	1990
Private Car	22.48	21.74
Motor Cycle/Scooter	64.13	67.50
Taxi	1.20	0.58
MCR/TSR	3.82	3.51
Goods Vehicles	6.85	5.61
Buses	1.52	1.06

Draw a bar graph of the above data.

1. Compare the percentage of taxis in the year 1980 and 1920.
2. What is the difference in the percentage of private cars and taxis in the year 1990?

Quadrilaterals

Objective :

Using road signs to determine the area of quadrilaterals.

Content :

Students are introduced to the tropic quadrilaterals in which they can differentiate between a square, a rectangle, a parallelogram, a rhombus etc. traffic signs are in various shapes.

Size of rectangle is 20 inches x 18 inches.

Q 1. Find the cost of making two rectangular boards @ ₹10 per square inch.

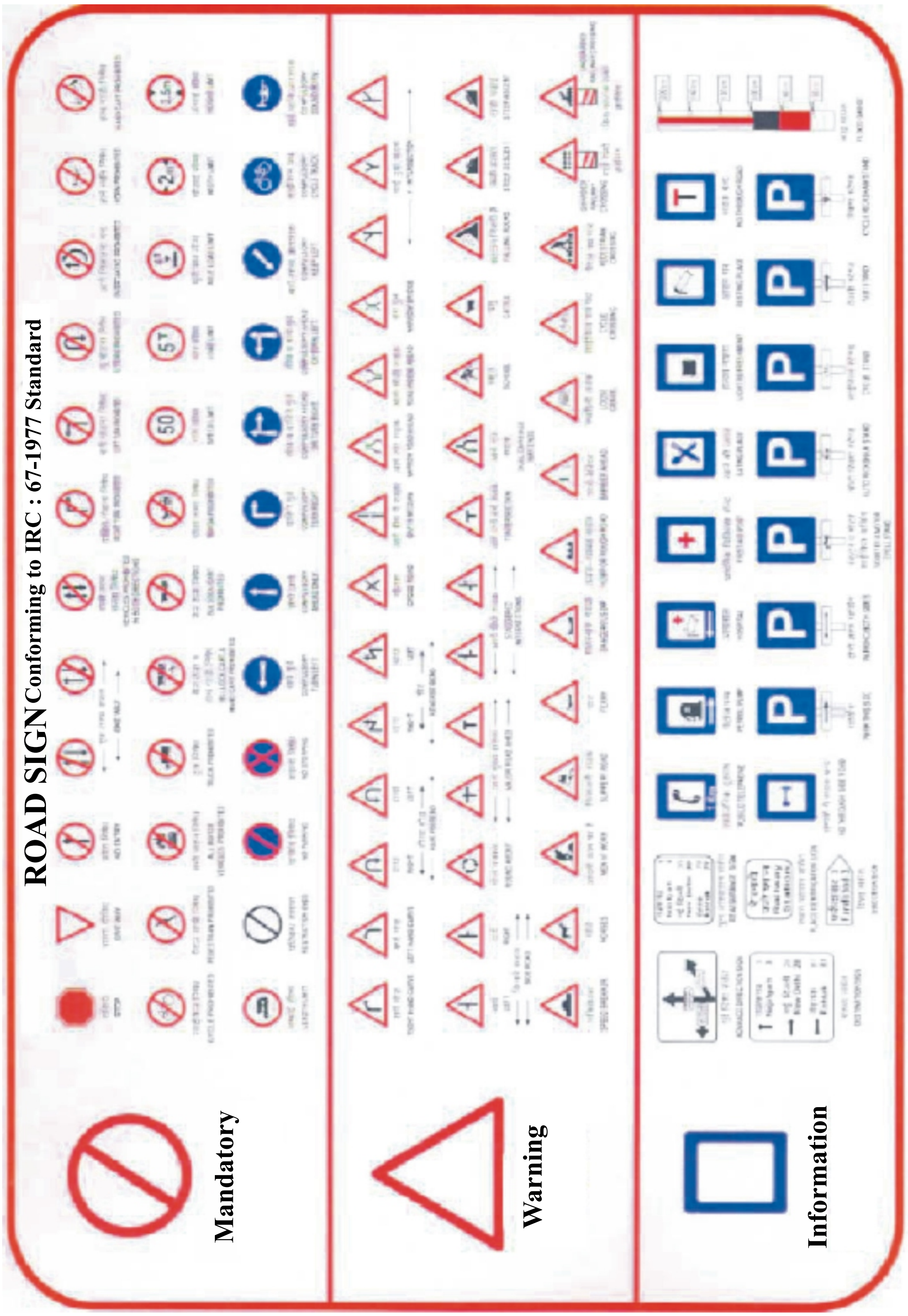
An informatory sign board, rectangular in shape, blue in colour is given below.

Q 2. It has a while square with Red Cross in it. Rectangle is 30"x25"and square is 20"x20". Find the area of blue region.

Activity :

Visit any traffic training institute and look at the board which show traffic signs of all shapes.

Draw these signs on a chart paper and state which of them are squares and which are rectangles. Also state which of them are parallelograms?


ROAD SIGN Conforming to IRC : 67-1977 Standard
Mandatory
Warning
Information

Probability

Objective :

Children will be given the knowledge of probability.

P(e) = number of favourable outcomes/total outcomes

Exercise :

Traffic lights remains green, orange or red for 90 seconds on a busy road crossing. Suresh, while travelling to his office noted that sometimes the traffic signal was red, sometimes green and sometimes orange. He noted this for 10 days are found the following:

SINGLE	RED	ORANGE	GREEN
10	3	3	4

Q 1. Find the probability of Suresh getting:

1. Green light
2. Red light at red light crossing

Q 2. What is the sequence of traffic lights?

Data

Laws and regulations help us to be aware of the road safety system. A specific eligibility of 18 years is given for obtaining a licence and children should not break this law. Over 500 minors were prosecuted in 2009. Find out how many students drive in your class. What is the fraction of the students driving to the overall strength of the class? Compare it with students of other sections and make a bar graph.

The fine for underage driving is Rs. 300 and if a minor is caught driving a vehicle, the owner of the vehicle can be fined an amount of Rs. 1,000. If an underage driver causes any accident, he can be looked under IPC 304 A or IPC 337, and can be sent to juvenile home.

Seat belts help a driver to lead a safe and secure life. 2,55,686 persons were prosecuted for not wearing a helmet in Delhi. **Find out penalty for this violation. What is total challan collected for this offence this year?**

11,084 persons were prosecuted foe not wearing a seat belt in Delhi. **Find out the penalty for this violation. What is the total challan collected for this offence this year?**

Section 177 of Motor Vehicle Act can punish a driver who is not wearing seatbelt with a fine upto Rs. 100.

Drunken driving is a major cause of fatal accidents. 8296 people were prosecuted in 2008 but 12,784 in 2009 in Delhi. **Find out increase in the number of such cases.**

Section 185 of Motor Vehicle Act can punish a driver under the influence of alcohol with a fine upto Rs. 2000 or imprisonment with a term which may extend upto 6 months. With a subsequent offence within 3 years, imprisonment increases to 2 years and a fine of Rs. 3,000.