Class-12

PHYSICS

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Board of Secondary Education Rajasthan, Ajmer

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Board of Secondary Education Rajasthan, Ajmer

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PHYSICS

Class-12

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PHYSICS

Class-12

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PREFACE

The book is based on latest syllabus prescribed by Board of Secondary Education Rajasthan, Ajmer. The presentation of the study material incorporates latest concepts supported by relevent diagrams and graphs. To create interest in the subject many examples from daily life, relating to the physical concepts are cited.

The book is basically a translation work of the hindi version published by Board of Secondary Education Rajasthan, however certain modifications in the diagrams are made where ever possible, as required.

Inspite of best efforts certain errors and omissions might have crept in. Feedback is solicited from teachers and students for further improvement of the book.

(Authers)

Syllabus

PHYSICS

Class-12

Time : **Max. Marks :**

S.No.	Learning Areas	Marks
1.	Theory question Paper -1	56
2.	Sessional	14
3.	Practical Examination	30
4.	**Total**	**100**

It is compulsory to pass theory and practical examination separately.

S.No.	Lessons	Periods	Marks
1.	Electrostatics	28	7
2.	Current Electricity	18	5
3.	Magnetic effect of electric current	17	11
4.	Magnet and magnetic properties of Materials	11	3
5.	Electromagnetic induction and Alternating current	23	7
6.	Optics	33	9
7.	Photoelectric effect and matter waves	12	4
8.	Atomic and Nuclear Physics	18	6
9.	Electronics	20	6
10.	Electromagnetic waves and Communication and contemporary physics	10	4
	Total	**190**	**56**

Unit : I Electrostatics

1. **Electric field :** Electric charge, type of charge and properties, coulomb's law, force between many charges and principal of superposition, electric field, electric field due to a point charge, electric field due to system of charges, electric field lines and their properties, electric dipole, electric dipole moment, electric field due to electric dipole, toque on electric dipole in uniform electric field.

2. **Gauss's Law and its application:-** Electric flux, continuous charge distribution, Gauss's law and its derivation, to find intensity of electric field using Gauss's law (i) Infinite liner charge distribution (linear). (ii) Infinite uniformaly charged non conducting sheet (iii) Uniformaly charged Infiniteconducting plate. (iv) Uniformaly charged spherical shell. (v) Charged metallic sphere, (vi) Uniformaly charged non conducting and force on charged surface. Energy per unit volume in Electric field, equilibrium charged soap bubble.

3. **Electric Potential:-** Electrostatic potential and potential difference, Electric potential due to a point charge,

Electric potential due to system of charges, Electric potential due to electric dipole, equi potential surface. Relation between electric potential and electric field, calculation of potential due to (i) Due to uniformaly spherical shell (ii) Charged conductor (iii) Charged spherical conductor. Potential energy due to system of charges, work done in rotating an electric dipole in an external electric field and potential energy.

4. **Electric Capacitance**:- Conductor and insulator, Bound and free charges in a conductor. Dielectric material and polarization. Capacity of a conductor, capacity of an isolated spherical charged conductor, Capacitor, capacity of a parallel plate capacitor (i) For air or vacuum between the plates (ii) Partially fillled with dielectric (iii) Capacity with different plates of dielectric with different thickness. Combination of capacitors series and parallel. Energy stored in capacitor, charge redistribution and energy loss, when two charged conductors are joined by a conducting wire.

Unit-2 Current Electricity

1. **Current Electricity:-** Electric current, flow of charges within a metallic conductor, drift velocity and mobility and relation with electric current. Ohm's law and its derivation; electrical resistance ohmic and non-ohmic resistance. Resistivity and effect of temperature on it. Carbon resistors and colour code. Series and parallel combination of resistors. Internal resistance of cell. EMF and terminal voltage for a cell. Combination of cells- series and parallel. Electric energy and electric power.

2. **Electric Circuit :-** Kirchhoff's law and their application. Wheatstone bridge, metre bridge. Potentiometer– principal, standardisation and sensitively, application of potentiometer to find (i) Internal resistance of a primary cell. (ii) Comparing EMF of two cells. (iii) To fine value of small resistance. Calibration of an Ammeter and voltmeter.

Unit-3 Magnetic effects of electric current :-

Orested's experience and its conclusions. Biot- Savart's law the direction of magnetic field. Magnetic field due to a straight and infinity long current carrying wire magnetic field due to current carrying current. Comparison of small current loop with magnetic dipole. Helmholtz coil. Force on charge moving in magnetic field Motion of a charge in magnetic field. Principal, construction and limitations of a cyclotron. Force on a current carrying wire in magnetic field. Magnetic force between two parallel currents. Definition of unit of current in SI system ie. 1 ampere. Force and torque on a current loop in uniform magnetic field. (i) Moving coil galvanometer (ii) Pivoted coil galvanometer and conversion into an ammeter and voltmeter. Ampere's law. Application of ampere's law to find magnetic field due to a infinite loop straight current carrying conductor, long cylindrical conductor magnetic field inside infinite long and straight solenoid. Comparison of a solenoid and a bar magnet. Magnetic field at the axis of toroid.

Unit-4 Magnetism and magnetic properties of materials:-

Natural and artificial magnet, properties of a bar magnet, magnetic field lines, Neutail points, magnetic moment, magnetic intensity, Torque on a bar magnet in uniform magnetic field. Geomagnetism, elements of Geomagnetism, magnetism and Gauss's law. Behaviour of materials in magnetic field. Intensity of magnetisation, magnetic field. Magnetic susceptbility magnetic permeability, relation between different magnetic quantities, paramagnetic, diamagnetic and ferromagnetic materials, magnetic Hysteresis's and B-H curve (hysteresis loop), Selection of magnetic materials for different uses. Curie law and Curie temperature, comparative study of different magnetic materials.

Unit-5 Electromagnetic induction and alternating current

1. **Electromagnetic induction :-** Magnetic flux, electromagnetic induction, Faraday's law for electromagnetic induction, Lenz's law, induced current and induced charges. Fleming's Right hand Rule, a moving rod in uniform magnetic field. Motion of a rectangular loop in non-uniform magnet field and conservation of energy. Rotation of rod, a disc and coil with uniform angular velocity, in uniform magnetic field and induced EMF. Eddy currents, self and mutual induction.

2. **Alternating Current :-** Direct Current, Alternating current, instantaneous, peak and average value of alternating current, phase relationship between voltage and current in different circuits (i) Pure resistive (ii) Pure inductive (iii) Pure capacitive (iv) Series L-R circuit (v) Series R-C circuit (vi) Series L-C-R circuit series LCR resosance circuit AC circuit, Band width in LCR series ac circuit, Quality factor. Average power in ac circuit, power factor, wattless current and transformer.

Unit-6 Optics :-

1. **Ray Optics :-** Reflection of light, spherical mirror, mirror formula, refraction of light, total internal reflection and its applications, optical fibre. Refraction at a spherical surface, thin lens formula, lens maker's formula, magnification, power of lens combination of thin lenses in contact and resultant power. Refraction through prism, dispersion by a prism, scattering of light, Rainbow, optical instruments, human-eye, refractive errors of vision and theirs correction. Simple and compound microscopes. Astronomical telescope (refracting and reflecting type) and magnifying power.

2. **Wave Optics :-** Nature of light, Huygens's principal, wave front reflection and refraction at a plane surface. Coherent sources and interference of light, conditions for interference. Young's double slit experiment Mathematical analysis of interference, fringe width and its expression, interference by white light. Diffraction, comparison of sound and light diffraction. Diffraction by a single slit and width of central maxima. Difference between interference and diffraction, resolving power of telescope and microscope. Polarized and un polarized light, plane of vibration and plane of polarization. Methods for obtaining polarised light by reflection and Brewster's law polarizetion by scattering, by double refraction – Nicol prism, Dichroism- Polaroid, detection of polarised, un polarized and partially polarised light Malus's law.

Unit-7 Photo-Electric Effect and Matter Waves

Photo electric effect, results of experiment on photoelectric effect and their explanation. Concept of photon. Photo-electric equation of Einstein and explanation of photo electric effect. Dual Nature of light de Broglie hypothesis, matter wave, wave length associated to different particles, Davisson and Germer Experiment and its conclusion, Heisenberg's uncertainity principal.

Unit-8 Atomic and Nuclear Physics

1. Thomson atomic model, Rutherford's atomic model and Bohr Atomic model, line spectrum of hydrogen atom and it explanation. Shortcomings of Bohr's Model explanation of Bohr 2nd postulate using deBroglie wave principle.

2. **Nuclear Physics :-** Structure of Nucleus, size of Nucleus, atomic mass unit, mass defect and Binding energy. Nuclear forces, Radioactivity, Rutherford and Soddy law for radioactivity, half life and mean life. α,

β and γ-rays/particle and their properties, Nuclear energy, Nuclear fission, controlled and uncontrolled Nuclear chain reactions, Nuclear reactor, nuclear fussion.

Unit-9 Electronics

Energy bands in solids, classification as conductor, insulator and semiconductor, Intrinsic and extrinsic semiconductors, majority and minority charge carriers P-N junction diode, forward and reverse bias characteristics, curve Avalanch and Zener diode, P-N junction diode as half wave and full wave rectifier. Special purpose P-N junction diodes. Transistor, transistor working principal, transistor circuit configuration, common base, common emitter, common collector, transistor characteristics in common base and common emitter configuration. Relation between α and β, Transistor as an amplifier (CE configuration) logic gates OR, AND, NOT, NAND, NOR and XOR gates.

UNIT-10 Electromagnetic waves, communication and contemporary Physics

Displacement current, Maxwells equation (Qualitative Analysis) EM waves and their characteristics, EM spectrum, Propagation of EM waves groundwaves, sky waves, space waves, elements of communication, Need of modulation types of modulation, Amplitude modulation.

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Chapter - 1
Electric Field

Electrostatical phenomena can be observed in many ways. It is a common experience for us that when a glass rod is rubbed by a silk cloth it acquires a property of attracting tiny bits of paper. Also, if an air filled baloon after being rubbed by cloth put into contact with a wall it remains cling to the wall for quite a long time. All such phenomena result from the forces between charges at rest. This chapter and next two chapters are devoted to electrostatics which is the study of effects of interactions between charges at rest.

In this chapter we are going to study about electric charges and their properities, force between two charged objects and concepts related with electric field and electric dipole. The study of electrostatics is important not only from concenptual aspects but it has a number of applications which includes, photocopying mechine, computer printer, electrostatic memory and seismograph.

1.1 Electric charge

According to history, Thales of Miletus in Greece is said to have discovered around 600 BC that when amber was rubbed with woolen cloth it would attract tiny pieces of straw, feathers etc. The greek word for amber is electron and from this root word comes the word electricity . Similar effects were observed on rubbing a glass rod with silk or an eboniterod with cat skin. Substances in such states are said to be electrified or electrically charged. In examples cited above the objects were charged (electrified) due to friction and thus the effect is termed as frictional electricity. However, as we will see shortly that there are other ways also to charge a given object.

An object when electrified beheves somewhat different than when it is uncharged, and it can be said that the object has acquired a characteristic property (charge). This characteristic property of an electrified object is termed as electric charge.

Charge is an intrisic property of elementary particles which constitutes the matter, i.e. it is a property that comes automatically with such particles wherever they exist. Although a formal definition of charge can not be given and it can be understood in terms of it effects. However it can be said that "charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects."

1.1.1 Types of Charges

From a number of experiments it was found that there are two kinds of elcetric charges, which are given names positive and negative charges. To determine the type of charge we perfrom the following experiment, experimental setup for which is depicted in Figure 1.1.

Consider a glass rod that has been rubbed on silk is suspended by a thread. If we bring a second, similarly charged glass rod near by, the two rods repel each other; that is, each rod experiences a force directed away from the other rod. Like wise if we suspend an ebonite rod that has been rubbed on catskin and bring a second similarly charged ebonite rod near by, again the two rods repel each other. However if we rub an ebonite rod with catskin and suspend it using a thread and then bring a glass rod rubbed on silk near by, the two rods attract each other.

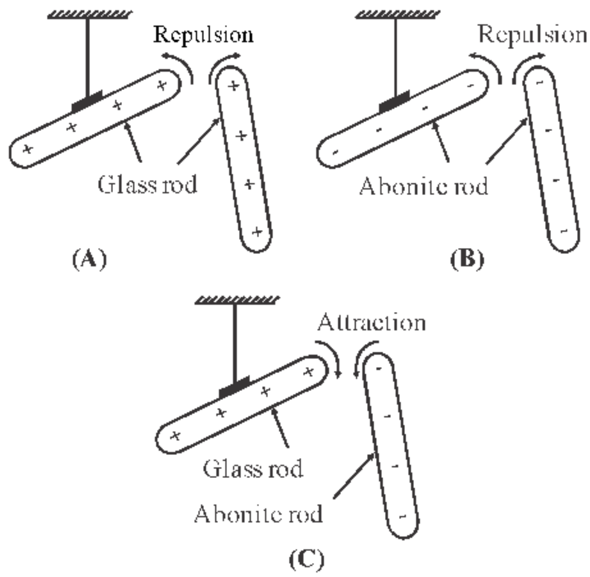


Fig. 1.1 Experimental setup for determining the types of charges

From above experimental observations we can conclude that two glass rods which have been rubed on silk have same type (sign) of charge and hence the charge of same type (sign) repel each other. Likewise, the two ebonite rods which have been rubbed on cat skin have same type of charge and repel each other. However, the type of charge on glass rod and that on ebonite rod are of opposite signs indicated by the fact that there is attraction between them.

The 'positive' and 'negative' names and signs for electric charges were given by Benjamin Franklin. Franklin arbitrarily chosed the type of charge on glass rod rubbed on silk as positive. From experimental observations the conclusion is that "charges with same sign repel each other and the charges with opposite signs attract each other." Thus all other charged objects which are repelled by such a positively charged glass rod must have positive charge and all such charged objects which are attracted to a positively charged glass rod must have negative charge.

According to modern view all matter is composed of atoms. Every atom consits of a nucleus (composed of neutrons and proptons) and electrons. Protons are positively charged, electrons are negatively charged and neutrons are electrically neutral. In atoms, the number of electrons is equal to the number of protons and atoms are neutral or uncharged. As the matter is composed of atoms the same is also true for matter. If in an object there is excess of electrons over it neutral configuration it is said to be negatively charged and if there is a deficiecny of electrons it is said to be positively charged.

Materials throught which charge (generally electrons) can flow freely are called conductors e.g. copper. Materials through which charge can not flow are called insulators or dielectrics e.g. glass, plastic and ebonite. Now let us discuss in brief the methods of charging various objects.

1.1.2 (a) Charging by friction

We have seen the process of charging by frication in experiment described earlier (Fig 1.1). When the two bodies are rubbed together the electricity so produced is called as frictional electricity. In this process a transfer of electrons take place from one body to another. The body from which electrons have been transferred is left with a deficiency of electrons so it gets positively charged and the body which recieves electrons becomes negatively charged. For example, when a glass rod is rubed with silk its gets positively charged while the piece of silk gets equal negative charge. This happens due to transfer of electrons from glass to silk piece at the point of contact. In the process of rubbing though the number of contact points increases, there by amount of charge transferred increases, however it is worth noting that amount of charge transferred in the process is quite small.

In the table presented below on rubbing objects mentioned in column I with objects mentioned in column II, object mentioned in column I gets positively charged and the object belonging to column II gets negatively charged.

Table 1.1

I (+)	II (-)
Glass rod	Silk cloth
Catskin	(i) plastic rod
	(ii) ebonite rod
Woolen cloth	(i) amber
	(ii) plastic
	(iii) ebonite
	(iv) rubber

If in place of a glass rod we take a copper rod in hand and rub it with some woolen cloth then the charge tranferred from woolen cloth to the rod flows through our body to the ground and the conducting rod does not get charged. However if we hold the conducting rod using an insulating handle and then rub it with woolen cloth the conducting rod can be charged. Here the insulating handle does not allow charge to flow through the body to the ground.

1.1.2 (b) Charging by Conduction (contact)

As we here mentioned, conductors are materials in which electric charge moves quite freely. When some charge is given to a conductor, it quickly redistributes it self over the entire outer surface of the conductor, however it is not so for insulators. If some charge is given to an insulator it remains at the place where it was given. This difference in beheviour of conductor and insulator will be explained in next chapter.

The direct transfer of charge from one object to another object in contact is called charging by contact. Conduction from a charged object involves transfer of like charges. Consider two conductors, one charged and another uncharged as shown in Fig. 1.2 . Bring the conductors in contact with each other. The charge (whether positive or negative) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with same sign. This is called as charging by conduction (through contact).

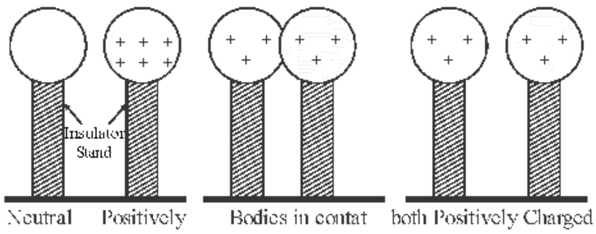


Fig. 1.2 Charging by Conduction

1.1.2 (c) Charging by Induction

The process under which a charged object induces an opposite type of charge on another object without coming into contact with it is called charging by electrostatic induction.

Fig. 1.3 shows an example of charging by induction. An uncharged metal ball is supported on an insulating stand [Fig. 1.3 (a)]. When we bring a negatively charged rod near it, without actually touching it [Fig. 1.3 (b)], the free electrons in the metal balls are repelled by the negative charge on the rod and they shift toward the right, away from the rod. They can not escape the ball because the supporting stand is insulator. So we get excess negative charge at the right surface of the ball and a deficiency of negative charge (electrons) i.e. a net positive charge at the left surface. These excess charges are called induced charges. However, the ball is still electrically neutral.

When you contact one end of a conducting wire to the right surface of the ball and the other end to the earth [Fig 1.3 (a)] the negative charge (electrons) flows through the wire to the earth. Now suppose we disconnect the wire [Fig 1.3 (d)] and then remove the rod [Fig 1.3 (e)] a net positive charge is left on the ball. The charge on the negatively charged rod has not changed in this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

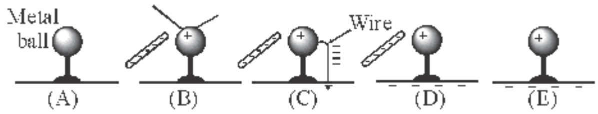


Fig. 1.3 Charging a metal ball by induction

On bringing a positively charged rod near the metal ball and repeating above steps the ball can be negatively charged.

How a charged object (positive or negative) attracts an uncharged object:

When an uncharged object is brought near the charged object electrostatic induction takes place. As a result, the near end of the uncharged object acquires opposite type of charge and hence attraction takes place between two unlike charged objects. The other (far) end of uncharged object acquires similar charge, hence there is force of repulsion between two like charges, but this force is weak (compared to the attractive force) because of larger distance. Thus the net force between a charged and uncharged objects is attractive. As an example we can note that, after combing dry hair with a plastic comb and if we take it near the tiny bits of paper they are attracted by it.

Following points are worth noting regarding charging an object

(1) In charging, the mass of body changes. Consider two identical metallic spheres of excetly the same mass. One is given a positive and the other an equal negative charge. Their masses after charging are different with negative charged sphere having greater mass (in principle). This is because the negatively charged sphere has gained additional electrons so its mass is increased while the positively charged sphere has lost some electrons causing a decrease in mass. However, this increase or decrease in mass is negligibly small owning to the very small mass of electrons.

(2) The true test of electrification is repulsion and not attraction as attraction may also takes place between a charged and an uncharged object.

(3) Charge can be detected or measured with the help of gold leaf electroscape, elecrometer, or ballastic

golvameter.

(4) When X-rays (electromagnetic waves having wavelength between 0.1A to 10A) are incident on a metal surface eletrons are ejected. Thus the surface becomes positively charged.

1.1.3 Electroscape

A simple apparatus to detect charge on an object is the gold leaf electroscape. It is a very sensitive apparatus.

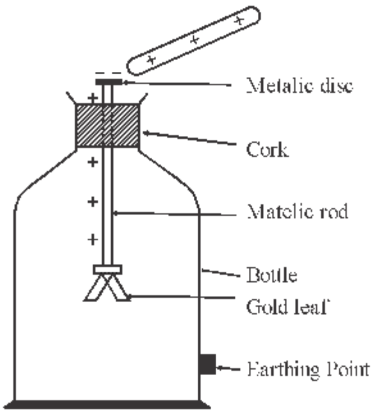


Fig. 1.4 Gold-leaf electroscape

As depicted in Fig. 1.4 in a gold-leaf electroscape a vertical metal rod is enclosed in a glass jar with two thin gold leaves attached to its lower end. The upper end of the rod is connected to a conducting disc. When a charged object touches the metal disc charge flows on to the gold leaves which then spread apart because of electrical repulsion between their charges. The degree of divergence is an indicator of the amount of charge.

If a charged object is brought near a charged electroscape the leaves will further diverge if the charge on the object is similar to that on the electroscape and will usually converge if opposite. In this menner we can determine the nature of charge on an object.

1.1.4 Unit of charge

In SI units current (I) is assumed to be a fundamental quantity with ampere (A) as unit. Since I = charge/time so charge is a derived quantity. In SI unit, the unit for charge is coulomb and is denoted by C.

1 C = 1 As

and dimensions $[Q] = M^0L^0T^1A^1$

Since coulomb is a relatively large unit so follwoing units are also used for charge.

$1\ \mu C = 10^{-6}\ C$

$1\ nC = 10^{-9}\ C$

$1\ pC = 10^{-12}\ C$

In CGS unit the charge is expressed in stat coulomb (esu) also called franklin.

$1\ C = 3 \times 10^9$ esu

A practical unit of charge is Faraday (and not farad)

1 Faraday = 96500 C

1.2 Properties of Charge

We have seen that the charges are of two types positive and negative and they tend to cancel each other. Here, we are discussing some other important properities of electric charge.

1.2.1 Additivity of Electric charges

Charge is a scalar quantity. Electric charge is additive and the net charge in a system is given by the algebraic sum of the charges present within. Special care must be taken regarding the sign of charges while adding. For exapmle if three chages +3q, -4q and +5q are given to an object the net charge on the object is +4q. If the sum of the charges on a object is zero the object is said to be electrically neutral. Here it is worth noting that mass is a scalar quantity but it can have positive values only.

1.2.2 Invarience of Electric Charge

Electric charge is independent of the choice of the frame of reference. In other words the charge on an object is independent of the speed of the object or the observer. i.e. the value of charge (q) on a particle is independent of the velocity of object. Charge on object at rest = charge on this object in motion i.e.

$$q_{rest} = q_{motion}$$

This property is worth mentioning as in contrast to charge, mass of a body depends on its speed. According to Einstein's special theory of realtivity (about which you will learn in higher classes) at speeds comparable to the speed of light ($v \approx c$) the mass of a particle becomes many times larger than its rest mass but charge does not change. The ratio q/m of charge q and mass m of a particle is called it specific charge, this depends on speed and at high speeds its value decreases.

1.2.3 Conservation of Electric charge :

According to this "the net charge on an isolated system is always conserved and it does not change even if some interaction or process is being completed in the system. In other words" charge can neither be created nor destroyed, it can only be transferred.

Illustrations

(i) In the example of frictional electricity, both the glass rod and silk cloth are uncharged (neutral) before rubbing them together. When they are rubbed together a positive charge appears on the rod and a negative charge of equal magenitude appears on the silk. In this process few electrons are transferred from the glass rod to the silk so silk cloth gets negatively charged while glass rod is positively charged by the same amount. Here the glass rod and the silk cloth forms a composite uncharged (neutral) system. Intially both are neutral, after they are rubbed together the charges of equal magnitudes but opposite sign appear on these two objects so the net charge on the system is still zero.

(ii) When an electron (whose charge is -e) and its antiparticle, the position (whose charge is +e), under go an **annihiliation** process they transform into two **gamma rays** (high frequecny electromagnetic wave) which are neutral. In this process total charge before annihiliation was (-e) + e = 0 and after the process is zero again. Thus the charge is conserved. This process is written as -

$$\underset{(\text{Electron})}{e} + \underset{(\text{Positron})}{e} \longrightarrow \gamma(1.02\,\text{MeV})$$

In pair production, the converse of annihilation charge is also conserved. In this process a gamma ray transforms into a positive and an electron.

$$\gamma(1.02\,\text{MeV}) \longrightarrow \underset{(\text{Electron})}{e} + \underset{(\text{Positron})}{e'}$$

(iii) Charge is also conserved in radioactive decay and nuclear reactions. You will learn more about these in a later chapter. Few examples are cited below

Radioactive decay

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}(\alpha\text{-particle})$$

$$Q_i = 92e \qquad Q_f = 90e + 2e = 92e$$

Nuclear reaction:

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{18}_{9}\text{F} \rightarrow {}^{17}_{8}O + {}^{1}_{1}\text{H}$$

$$7e + 2e \rightarrow 9e \rightarrow 8e + e$$

$$\text{Qi} = 9e \qquad Qs = 9e$$

The hypothesis of conservation of charge first put forward by Benjamin Frankilin is empirical with no known exceptions so far.

1.2.4 Quantization of charge

When a physical quantity can have only distrete values rather than any value, the quantity is said to be quantised. The minimum value that this quantity can have is called as the quantum of that quantity.

When two insulators are rubbed together these get charged due to exchange of electrons. The exchage of electrons always takes place in whole numbers. The minimum number of electrons that can be exchanged is unity therefore the total charge on an object must be an integral multiple of electronic charge. This was established experimentally by Millikan by his famous oil drop experiment.

From the observations of Millikan oil drop experiments that the smallest charge that can exists in nature is the charge of an electron which is equal to 1.6021×10^{19} C. It is common to consider its value to be 1.6×10^{19} C for the purpose of calculations. If the charge on an electron e is taken as the elementary unit i.e. quantum of charge than charge on any object can be expressed as -

$$q = \pm ne \text{ with } n = 1, 2, \text{-------}$$

and charge on an object can never be $\pm 1.2\,e$, $\pm 1.6e$, $\pm 2.3\,e$, etc.

The quantum of charge is so small that when electricity is studied on a macroscopic scale the grainness of electricity does not show up and charges appears to be continuous. For explanation of internal structures of protons and neutrons these are assumed to be composed of particles (called quarks) having charges $\pm 2/3$ (e) and $\pm 1/3$ (e). However quarks do not exist in free state, the quantum of charge is still e.

Some important facts regarding the charge are as follows -

(i) Charge is always associated with mass, i.e, charge cannot exist without mass though mass can exist without charge. Photon is a particle which is both massless and chargeless while neutron has no charge but a finite mass. In general each charged particle has some mass.

(ii) A stationary charge produces only electric field in its surrounding space. If a charge particle is moving at a uniform velocity it produces both electric and magnetic fields but does not radiate evergy. An accelerated charge not only produces electric and magentic fields but also radiate energy in the form of electromagentic waves in spece surrounding it.

Example 1.1 - How many electrons are to be removed from a metallic sphere in order to positively charge it with 1 C charge.

Solution : Use q = ne

Here $q = 1$ C

$$n = \frac{q}{e}$$

$$n = \frac{1}{1.6\times10^{-19}} = 6.25\times10^{18} \text{ electrons}$$

Example 1.2 : A body is charged such that its mass increases by 9.1 ng then

(i) How many electrons were given to the body

(ii) Determine the value of charge and its nature

Solution : Here, the change in mass

$$\Delta M = 9.1\times10^{-9} g = 9.1\times10^{-12} kg$$

and mass of electron

$$m_e = 9.1\times10^{31} kg$$

(i) As $\Delta M = nm_e$

$$n = \frac{\Delta M}{m_e}$$

$$n = \frac{9.1\times10^{-12}}{9.1\times10^{31}} = 10^{19} \text{ electrons}$$

(ii) Value of charge $q = ne$

$$q = 10^{19}\times1.6\times10^{-19}$$

$$q = 1.6C$$

As the body is receving electrons it must be negatively charged.

Example 1.3 - Calculate the amount of positive and negative charges present in a cup of water (250gm)

Solution -Here the mass of water m = 250 gm

Molecular mass of water M = 18 gm

Number of Molecular in one cup of water

$$N = \frac{m}{M}\times N_A$$

(Here N_A is Avagaro Number)

$$N = \frac{250}{18}\times6.023\times10^{23}$$

As one molecule water consits of two hydrogen and one oxygen atom so one molecule of water contains 10 protons and 10 electrons. Electrons and protons have equal but opposite charge.

So amount of positive (or negative) charge in one cup of water -

$$q = N\times10e$$

$$q = \frac{250}{18}\times6.023\times10^{23}\times10\times1.6\times10^{-19}$$

$$q = 1.337\times10^{7} C$$

1.3 Coulomb's law

Based on experiments in 1875 Coulomb put forward a law regarding the electric forces acting betweem two point charges at rest. This law is known as Coulomb's law and according to it " the magnitude of the electric force (of repulsion or attraction) acting between two point charges at rest is directly proportional to the product of mageniudes of the charges and inversely proporional to the square of the distance between them. This force acts along the line joining the two charges and depends on the nature of medium between the charges. This law is also termed as Coulomb's inverse square law.

If two poitns charges q_1 and q_2 are at a distance r apart then.

$$F \propto q_1 q_2 \qquad \ldots (1.1)$$

and $$F \propto \frac{1}{r^2} \qquad \ldots (1.2)$$

i.e. $$F \propto \frac{q_1 q_2}{r^2} \qquad \ldots$$

or $$F = k\frac{q_1 q_2}{r^2} \qquad \ldots (1.3)$$

Where k is a proportionality constant whose numerical value depends on the system of units used and the nature of the medium present between the charges.

For vacuum (free space) or air medium in SI units.

$$k = \frac{1}{4\pi \in_0} = 9\times 10^9 \frac{Nm^2}{C^2}$$

Where $\in_0$ (epsilon - knot or epsilon zero) called the permitivity of free space.

$$\in_0 = 8.854\times 10^{-12}\, C^2 / Nm^2$$

Hence for free space or air

$$F = \frac{1}{4\pi \in_0}\cdot\frac{q_1 q_2}{r^2} \qquad \ldots (1.4)$$

Dimensions of $\in_0$

$$\in_0 = \frac{[Q]^2}{[F][Length]^2} = \frac{T^2A^2}{MLT^{-2}L^2}$$

$\therefore \quad [\in_0] = M^{-1}L^{-3}T^{4}A^2$

If $q_1 = q_2 = 1$ C

and $r = 1$ m

Then $F = \frac{1}{4\pi \in_0} = 9\times 10^9$ N

Thus, if the force acting between two equal charges placed in free space or air at a distance of 1m apart is 9×10^9 N then the magnitude of each charge is 1 Coulomb.

For a medium other than freespace (or air)

$$F_{medium} = \frac{1}{4\pi \in}\cdot\frac{q_1 q_2}{r^2} \qquad \ldots (1.5)$$

Here $\in$ is called permitivity of the medium.

1.3.1 Dielectric Constant (Relative permitivity)

It has been found from experiments that if two charges are kept in different media at a given separation then the force acting between them changes with the change in medium. The force is maximum for free space or air as medium while for insulator medium it is relatively small, in presence of a conducting medium the force reduces to zero.

Hence in presence of a medium the factor by which the force is reduced compared to its value in free space is termed as relative permitivity, dielectric constant or specific inductive capacity of medium. It is denated by $\in_r$.

$$\in_r = \frac{\text{Force between the charges in vacuum } (F)}{\text{Force between the charges in that medium } (F_m)}$$

$$\in_r = \frac{\frac{1}{4\pi \in_0}\cdot\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi \in}\cdot\frac{q_1 q_2}{r^2}} = \frac{\in}{\in_0}$$

$\in_r = \frac{\in}{\in_0}$ is a dimensionless quantity

Many times symbol K is used in place of $\in_r$ Insulators are also called dielectrics.

Values of relative permitivity for some common dielectric materials are shown in table 1.2

Table 1.2 : Realtive permitivities of some dielectrics (at 20° C)

Medium	Dielectric Constent	Medium	Dielectric Constent
Air	1.00059	Glycerene	42.5
Glass	5 to 10	Rubber	7
Mica	3 to 6	Oxygen	1.00053
Parefinn wax	2 to 2.5	Conductor	-----(∞)
Distilledwater	80		
Free space	1		

1.3.2 Vector Form of Coulomb's law

As force is a vector quantity it is useful to write Coulomb's law in vector form. For this, let us consider that realtive to some arbitarily chosen origin the position vectors of point charges q_1 and q_2 placed in free space are r_1 and r_2 respectively. According to Fig (1.5) the position vectorof charge q_2 relative to charge q_1 is then.

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

and $|\vec{r}_{21}| = |\vec{r}_2 - \vec{r}_1|$

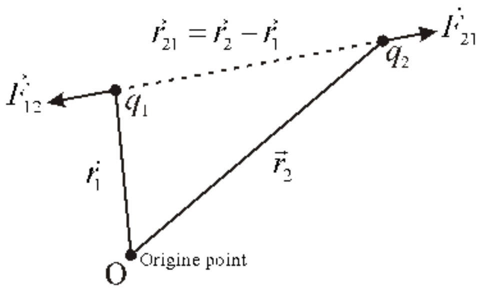


Fig. 1.5 Vector representation of Coulomb's law

From Coulomb's law the electric force on charge q_2 due to charge q_1

$$\vec{F}_{21} = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

Where $\hat{r}_{21}$ is a unit vector directed from q_1 to q_2.

Accordingly $\vec{F}_{21} = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$

or $\vec{F}_{21} = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$ ...(1.6)

Likewise the force on charge q_1 due to charge q_2

$$\vec{F}_{12} = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{|\vec{r}_{12}|^3}\hat{r}_{12}$$

Where $\hat{r}_{12}$ is a unit vector directed from q_2 to q_1.

Accordingly

$$\vec{F}_{12} = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_1 - \vec{r}_2) \quad \text{...(1.7)}$$

as $\hat{r}_{12}$ and $\hat{r}_{21}$ are directed opposite to each other,

therefore $\vec{F}_{21} = -\vec{F}_{12}$

Thus it is clear that for two point charges the force which one charge exerts on the other is equal and opposite to the force which the other charge exerts on the first (whatever the signs of charges may be). Thus the force is an action-reaction pair or Coulomb's law is consistent with Newton's third law. This force acts along the line joining the two charges i.e. electrostatic force is a central force in nature.

Important facts

1. Strictly specking Coulomb's law is valid for point charges at rest. If the point charges are in motion the Coulomb's law can not account for the force acting between them as now in addition to electric force, magnetic force also acts between the charges.
2. When the charges are sperated by 10^{-15} m or less the Coulomb's law is not appicable as now nuclear force also acts between the charges.
3. The force between two charges is not affected by the presence of other charges, therefore the Coulomb force is a two body interaction. Therefore the principle of linear superposition is applicable for Coulomb forces (see section 1.4).
4. Coulomb's law is inverse square law and Coulomb force is conservative in nature.

Example 1.4 - In hydrogen atom the separation between the electron and proton is 5.3×10^{-11} m. Calculate the force of attraction between them. Compare this force with the gravitational force acting between them $G = 6.67\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, electronic charge $e = 1.6\times10^{-19}\ C$ mass of electron $m_e = 9.1\times10^{-31}$ kg, mass proton $m_p = 1.67\times10^{-27}$ kg.

Solution : Electrostatic force of attraction between electron and proton

$$F_e = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

$$F_e = 9\times10^9 \times \frac{\left(1.6\times10^{-19}\right)\left(1.6\times10^{-19}\right)}{\left(5.3\times10^{-11}\right)^2}$$

$$F_e = 8.2\times10^{-8} N$$

Gravitational force between electron and proton

$$F_g = G\frac{m_e m_p}{r^2}$$

$$F_g = \frac{6.67\times10^{-11}\times\left(9.1\times10^{-31}\right)\left(1.67\times10^{-27}\right)}{\left(5.3\times10^{-11}\right)^2}$$

$$F_g = \frac{101.36}{28.09}\times10^{-47} = 3.6\times10^{-47} N$$

$$\frac{F_e}{F_g} = \frac{8.2\times10^{-8}}{3.6\times10^{-47}} = 2.27\times10^{39}$$

Therefore the electrostatic force is 2.27×10^{39} time large than the gravitational force.

Example 1.5 - Two positive ions of same charge repel each other by a force of 3.7×10^{-9} N when they are 5Å apart. How many electrons are less on each ion compared to their neutral atom state.

Solution : Let the charge on each ion = q

Here $r = 5\mathring{A} = 5\times10^{-10} m$

Force $F = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r^2}$ Here $q_1 = q_2 = q$

$$F = \frac{1}{4\pi \in_0}\frac{q^2}{r^2}$$

or $3.7\times10^{-9} = 9\times10^9 \frac{q^2}{\left(5\times10^{-10}\right)^2}$

$$q = \sqrt{\frac{25\times3.7}{9}\times10^{-38}}$$

$$q = \frac{5}{3}\times1.92\times10^{-19}$$

$$= \frac{9.6}{3}\times10^{-19} = 3.2\times10^{-19} C$$

$q = ne$ (n = number of electron)

Example 1.6 - The force between two point charges placed in free space is 18N. Keeping the same seperation between them if these charge are placed in glass medium of dielectric constant 6. Calculate the force acting between them.

Solution : $F_m = \frac{F}{\in_r}$

Here $F = 18 N$ $\in_r = 6$

$$F_m = \frac{18}{6} = 3N$$

Example 1.7 - A point charge $q_1 = 2\mu C, (2m, 1m)$ is located at (2m, 1m) and another point charge $q_2 = -5\mu C, (-2m, 4m)$ is located at (-2m, 4m). Determine the force on q_2 due to q_1

Solution : As per question

$q_1 = 2\mu C,\ q_2 = -5\mu C,\ \vec{r}_1 = 2\hat{i} + 1\hat{j}\, m$ and

$\vec{r}_2 = -2\hat{i} + 4\hat{j}\, m$.

As the charges are of opposite signs so force on q_2 due to q_1 is of attractive nature and acts towards q_1 i.e. from point (-2, 4) to (2, 1) along $\hat{r}_{12}$

$$\vec{F}_{21} = \frac{k|q_1||q_2|}{r^2}\hat{r}_{12} = \frac{k|q_1||q_2|}{|\vec{r}_1 - \vec{r}_2|^2}(\hat{r}_{12})$$

$$= \frac{9\times10^9\times2\times10^{-6}\times5\times10^{-6}}{\left|\left(2\hat{i}+1\hat{j}\right)-\left(-2\hat{i}+4\hat{j}\right)\right|^3}\left[\left(2\hat{i}+1\hat{j}\right)-\left(-2\hat{i}+4\hat{j}\right)\right]$$

$$= \frac{90\times10^{\ 3}}{\left|4\hat{i}-3\hat{j}\right|^{3}}\left(4\hat{i}-3\hat{j}\right) = \frac{90\times10^{\ 3}}{125}\left(4\hat{i}-3\hat{j}\right)$$

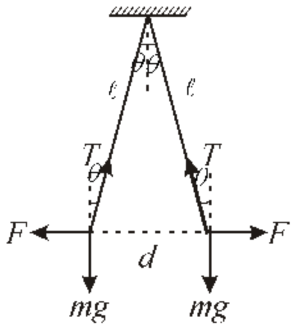
$$= 7.2\times10^{\ 4}\left(4\hat{i}-3\hat{j}\right)\text{N}$$

Example 1.8 - Two small point like spheres, each having a mass of 200 g are suspended from a common point by insulating strings of length 40 cm each. The spheres are identically charged and the separation between them at equilibrium is found to be 4 cm. Find the charge on each sphere.

Soultion : The forces acting on two spheres are shown in figure. As each sphere is in equilibrium the net force on each is zero. So

$$T\cos\theta = mg \qquad \ldots\text{(i)}$$

and $$T\sin\theta = F = \frac{kq^2}{d^2} \qquad \ldots\text{(ii)}$$



From equations (i) and (ii)

$$\tan\theta = \frac{kq^2}{mgd^2}$$

As $d\left(=4cm\right),\ \ell\left(=40cm\right)$

$$\tan\theta \approx \sin\theta = \frac{d/2}{\ell} = \frac{2}{40} = \frac{1}{20}$$

hus $$q = \sqrt{\frac{mgd^2\sin\theta}{k}} = \sqrt{\frac{0.2\times10\times16\times10^{\ 4}}{9\times10^{9}}\times\frac{1}{20}}$$

$$= \frac{4}{3}\times10^{-7}\text{ C}$$

1.4 Force among Many Charges and Superposition Principle

The electric force acting between two point charges does not affect by presence of other charges near by, therefore the force on a point charge at rest due to two or more stationary point charges is obtained by the supper position principle. According to this pricniple "the force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to other charges taken one at a time.

If the force acting on charge under question due to other n charges are $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ respectively then the net force acting on it is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

Consider a system of n charges at rest placed in free space. Let the charges be $q_1, q_2 \ldots q_n$ respectively and we wish to determine the net force due to this system of charges on a charge q_o. Let the position vector of $q_1, q_2 \ldots q_n$ etc and relative distance from q_o to are $\vec{r}_{01}, \vec{r}_{02}, \vec{r}_{02} \ldots \vec{r}_{0n}$ respectively (see Fig. 1.6). If the force on q_0 due to q_1 is represented by $\vec{F}_{01}$ then

$$\vec{F}_{01} = \frac{1}{4\pi\in_0}\frac{q_1q_0}{\left|\vec{r}_{01}\right|^2}\hat{r}_{01}$$

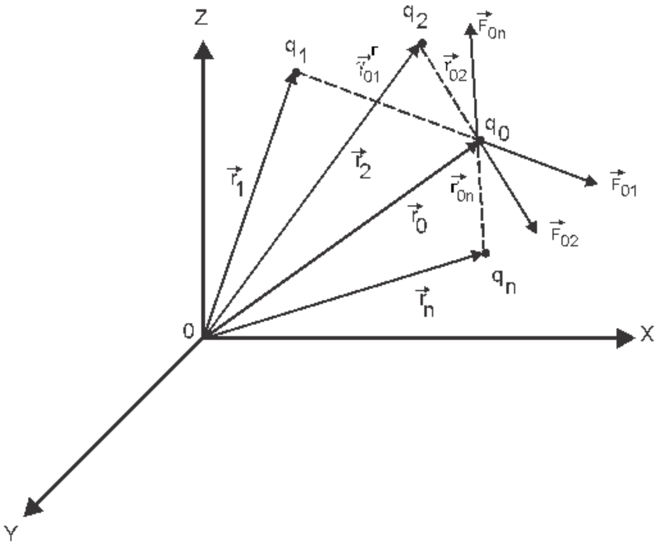


Fig 1.6 : Force on q_0 due to a number of point charges

Where $\hat{r}_{01}$ is a unit vector directed from q_1 to q_0. Similary if the forces acting on q_0 due to other charges are $\vec{F}_{02}$, $\vec{F}_{03}...\vec{F}_{0n}$ respectively then from the principle of superposition, the net force acting on q_0

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ... + \vec{F}_{0n}$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} \frac{q_1 q_0}{|\vec{r}_{01}|^2} \hat{r}_{01} + \frac{1}{4\pi \in_0} \frac{q_2 q_0}{|\vec{r}_{02}|^2} \hat{r}_{02} + ... + \frac{1}{4\pi \in_0} \frac{q_n q_0}{|\vec{r}_{0n}|^2} \hat{r}_0$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} q_0 \left[\frac{q_1}{|\vec{r}_{01}|^2} \hat{r}_{01} + \frac{q_2}{|\vec{r}_{02}|^2} \hat{r}_{02} + ... + \frac{q_n}{|\vec{r}_{0n}|^2} \hat{r}_{0n} \right]$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} q_0 \sum_{i=1}^{n} \frac{q_i}{r_{0i}^{\ 2}} \hat{r}_{0i} \qquad ...(1.8)$$

To determine the net force, paralleogram law of forces or polygon law of forces may be used according to situation.

Example 1.9 Two point charges of 9e and 4e are at a distance r apart. Where on the line joining the two charges another charge q is placed such that it ramins in equilibrium?

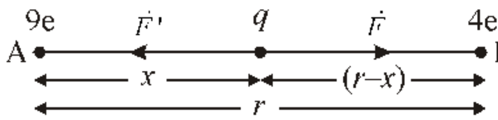
Solution : The pictorial representation of the problem is shown in figure shown below. For equilibrium of q, net force on it must be zero.

$$\vec{F} + \vec{F}^1 = 0$$

$$\vec{F} = -\vec{F}^1$$

However in magnitude

$$F = F'$$



$$\frac{kq9e}{x^2} = \frac{kq4e}{(r-x)^2} \Rightarrow \frac{9}{x^2} = \frac{4}{(r-x)^2}$$

on taking square root

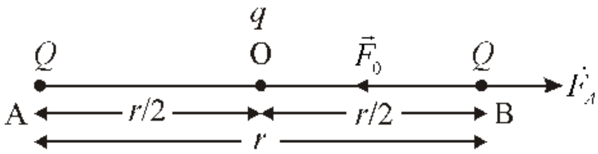
$$\frac{3}{x} = \frac{2}{(r-x)} \quad \text{or} \quad 2x = 3r - 3x$$

$$5x = 3r \Rightarrow x = \frac{3}{5} r$$

It should be placed at $3/5\, r$ from charge $9e$.

Example 1.10 Two identical charges Q are placed r distance apart. At the mid point on the line joining them another charge q is placed. What should be its magnitude and sign so that the entire system is in equilibrium?

Solution :



As the charge q is on the mid point on the line joining the two charges Q each, due to symmetry force on q is always zero. For the equilibrium of entire system it is essential that the force on remining charges a each at A and B must be zero for charge Q at B to be in equilibrium to be a equilibrium

$$\vec{F}_A + \vec{F}_0 = 0$$

$$\vec{F}_A = -\vec{F}_0$$

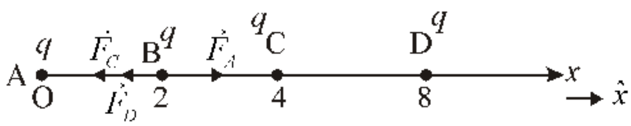
$$\text{or} \quad \frac{kQQ}{r^2} = -\frac{kqQ}{(r/2)^2}$$

$$\frac{Q}{r^2} = -\frac{4q}{r^2} \Rightarrow Q = -4q$$

$$\Rightarrow \quad q = -Q/4$$

Example 1.11 Four identical point charges each 2 μC are placed on axis at positions $x = 0, 2, 4, 8$ cm respectively. Determine the resultant force acting on the charge placed at $x = 2$ cm.

Solution : According to question



Force on charge at B due to charge at A

$$\vec{F}_A = \frac{kqq}{(2\times10^{-2})^2}\hat{i} = \frac{9\times10^9 \times 2\times10^{-6} \times 2\times10^{-6}}{4\times10^{-4}}\hat{i}$$

$$\vec{F}_A = 90\hat{i}\text{ N}$$

force on charge at B due to charge at C

$$\vec{F}_C = \frac{9\times10^9\times2\times10^{-6}\times2\times10^{-6}}{4\times10^{-4}}\left(-\hat{i}\right) = -90\hat{i}\text{ N}$$

force on charge at B due to charge at D

$$\vec{F}_D = \frac{9\times10^9\times2\times10^{-6}\times2\times10^{-6}}{\left(6\times10^{-2}\right)^2}\left(-\hat{i}\right) = -10\hat{i}\text{ N}$$

Therefore the net force on charge at B

$$\vec{F}_B = \vec{F}_A + \vec{F}_C + \vec{F}_D$$

$$= 90\hat{i} + \left(-90\hat{i}\right) + \left(-10\hat{i}\right) = -10\hat{i}\text{ N}$$

$$\vec{F}_B = 10\ (-\hat{i}\ )\text{ N}$$

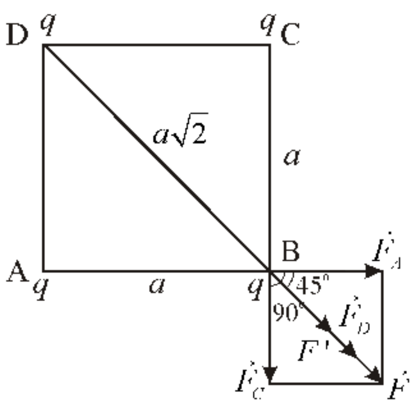
So the net force is 10N in negative x direction.

Note : [In this problem from symmetry it is obvious that force on B due to charges at A and C cancel out so the net force on B is due to charge at D only]

Example 1.12 : Four identical charges q each are placed at the four verticles of a square of side a. Determine the magnitude of net force on each charge due to remaining charges.

Soultion : The situation pertaining to the question is depicted in adjoining figure. Here we determine the force on charge at B due to remaining charges. From symmetry forces acting on charges placed at other points due to remaining charges will be equal in magnitude but differ in directions. From fig.

$$BD = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$



also $\quad \left|\vec{F}_A\right| = \left|\vec{F}_C\right| = \dfrac{kq^2}{a^2}$

and as shown in fig $\vec{F}_A$ and $\vec{F}_C$ are mutually perpendicular. So their resultant $F\,'$ makes angle of 45° with both $\vec{F}_A$ and $\vec{F}_C$

$$\vec{F}\,' = \sqrt{F_A^2 + F_C^2} = \sqrt{2F_A^2} = F_A\sqrt{2}$$

$$\vec{F}\,' = \frac{kq^2}{a^2}\sqrt{2}$$

Next, force on charge at B due to charge at D

$$F_D = \frac{kq^2}{\left(a\sqrt{2}\right)^2} = \frac{kq^2}{2a^2}$$

as $\vec{F}\,'$ and $\vec{F}_D$ are in same direction (fig) so net force on charge at B

$$\vec{F} = \vec{F}\,' + \vec{F}_D$$

$$F = \frac{kq^2}{a^2}\sqrt{2} + \frac{kq^2}{2a^2}$$

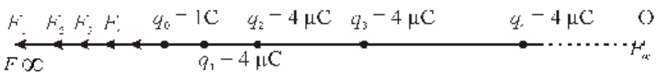
$$F = \frac{kq^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

and is directed along DB

Example 1.13 : Infinite number of point charges 4 μC each are placed on x axis at 1m, 2m, 4m, 8m.......... respectively. Determine the force due to these at a 1C charge placed at origin.

Solution : Here

$$q_1 = q_2 = q_3 = q_4 = 4\mu C = 4\times10^{-6}\ C$$



$$q_0 = 1C$$

Net force on q_0

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \ldots + \vec{F}_\infty$$

$$\vec{F}_0 = \frac{kqq_0}{r_1^2}\left(-\hat{i}\right) + \frac{kqq_0}{r_2^2}\left(-\hat{i}\right) + \frac{kqq_0}{r_3^2}\left(-\hat{i}\right) + \ldots\infty$$

$$\vec{F}_0 = kqq_0\left[\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+...\infty\right]\left(-\hat{i}\right)$$

$$\vec{F}_0 = 9\times10^9\times4\times10^{-6}\times1\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+...\infty\right]\left(-\hat{i}\right)$$

The term within the bracket forms a geometrical progression with first term a = 1 and common ratio r = 1/4. The sum of such a series is given by S =

$$\vec{F}_0 = 36\times10^3\left[\frac{a}{1-r}\right]\left(-\hat{i}\right)$$

$$= 36\times10^3\left[\frac{1}{(1-1/4)}\right]\left(-\hat{i}\right)$$

$$\vec{F}_0 = 36\times10^3\times\frac{4}{3}\left(-\hat{i}\right)$$

$$= 48\times10^3\left(-\hat{i}\right)\text{N}$$

1.5 Electric Field :

When a point charge is brought near another charge, it experiences a force of attraction or repulsion. If we are interested only in determining the force acting between these charges then Coulomb's law is sufficient. Similarly for a system of point charges to determine force acting on a charge due to the remaining charges we use principle of superposition along with Coulomb's law. However a nagging question remains: how does a charged particle interacts with another charge kept at a distance as the charges are not touching each other? In other words how does one charge know about the presence of the other charge? To answer such a question the concept of electric field is very important. To explain the interaction between two charges, it can be imagined that a charge creates an electric field in its surrounding space. When another charge is placed in this eletric field then due to this electric field first charge does some action on the second charge whereby the second charge experieneces the presence of the first. Thus the space surrounding a charge or system of charges in which some other charged particle experiences a force of attraction or repulsion depending upon its nature is called electric field. A particle is considered to be in an electric field if it experiences electric force. The concept of electric field was but forward first of all by scientist Michael Faraday. Electric field is a vector field which is described mathematically in terms of the intensity of electric field.

1.5.1 Intensity of Electric Field

By definition intensity of electric field at some point in an electric field is equal to the force acting on a unit positive test charge placed at that point and its direction is same as the direction of force acting on this unit test charge. The test charge is assumed to be sufficiently small positive point charge such that it does not disturb the charge (or charges) that create the electric field. Thus the presence of test charge does not modify the original electric field. Electric field intensity is a vectory quantity. It is denoted by $\vec{E}$. Many times electric field intensity is referred as electric field.

If $\vec{F}$ is the force acting on some positive test charge q_0 placed at some point in a given electric field, then the intensity of electric field at that point is

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad ...(1.9)$$

Since q_0 should be sufficiently small so as not to disturb the electric field it is more proper to write

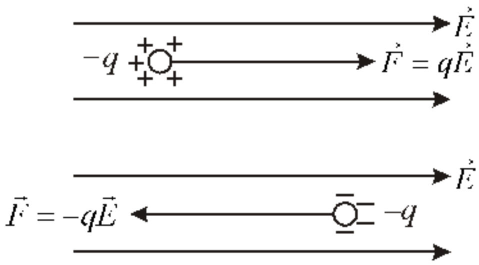
$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0} \qquad ...(1.9(a))$$

The electric field intensity is expressed in units of N/C in SI system. In next chapter we will see that another unit for $\vec{E}$ is V/m. The dimensional formula for electric field intensity is

$$E = \frac{F}{q_0} = \frac{\left[\text{M}^1\text{L}^1\text{T}^{-2}\right]}{\left[\text{A}^1\text{T}^1\right]} = \left[\text{M}^1\text{L}^1\text{T}^{-3}\text{A}^{-1}\right]$$

$$= \left[\text{MLT}^{\ 3}\text{A}^{\ 1}\right]$$

If a charged particle having a charge of magnitude q is placed in electric field of intensity $\vec{E}$ then the force acting on the particle is given by



$\vec{F} = q\vec{E}$

If the particle is positively charged the direction of force $\vec{F}$ is same as that of $\vec{E}$ if the particle is negatively charged, the force $\vec{F}$ on it is directed opposite to field. (Fig 1.7)

Fig 1.7 : Force on a charged particle in electric field

Electric field due to a positive point charge or uniform spherical positive charge distribution is directed radially outward from it. If the point source charge is negative the electric field is directed radially toward it fig (1.8)

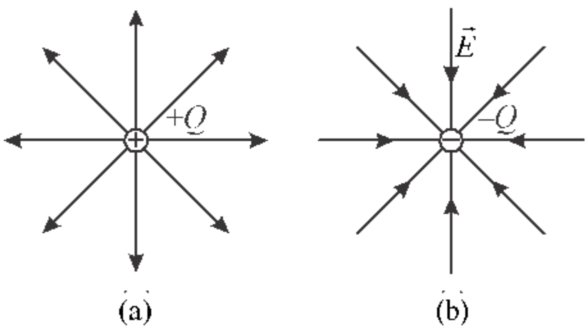


Fig 1.8 : Electric field due to
(a) positive (b) negative point charge

1.6 Electric field due to a point charge

Consider a point charge + Q situated at a point O (Fig 1.9) we are interested in electric field $\vec{E}$ at a point P at a distance r from O. Let us consider a test charge + q_0 imagined to be placed at P.

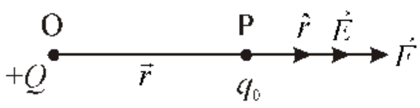


Fig 1.9 : Electric field due to a point charge

From Coulomb's law force exerted on test q_0 at P due to charge + Q is

$$\vec{F} = \frac{kQq_0}{r^2}\hat{r}$$

By definition $\vec{E} = \dfrac{\vec{F}}{q_0}$

so $\vec{E} = \dfrac{1}{q_0} \cdot \dfrac{kQq_0}{r^2}\hat{r}$

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

Where is a unit vector directed from Q towards q_0. Electric field at point **P** is in direction $\overrightarrow{OP}$

If instead of + Q, a point charge - Q is placed at O then electric field at P is given as

$$\vec{E} = \frac{1}{4\pi\in_0} \cdot \frac{Q}{r^2}\hat{r} \qquad \text{. . . (1.10)}$$

$$\vec{E} = \frac{1}{4\pi\in_0} \cdot \frac{-Q}{r^2}(-\hat{r}) \qquad \text{. . . (1.11)}$$

Thus it is obvious that for a point charge i.e. the intensity of electric field is inversely proportional to the square of distance. For a point charge this variation is shown graphically in Fig 1.10

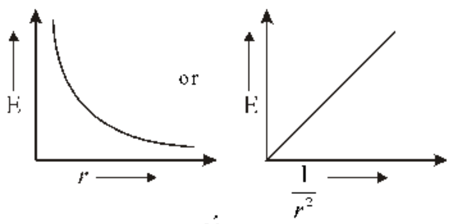


Fig 1.10 : Variation of $\vec{E}$ with r for a point charge

If the point charge is situated in a medium of dielectric constant $\in_r$ then

$$E_m = \frac{1}{4\pi\in} \cdot \frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\in_r\in_0} \cdot \frac{Q}{r^2}\hat{r}$$

$$E_m = \frac{E}{\in_r} \Rightarrow E_m < E \qquad (\because \in_r > 1)$$

Thus for a given distance in a dielectric medium the electric field intensity decreases by $\in_r$ compared to that in free space.

Table 1.3 : Typical values of some electric fields present in various cases

System	Electric field
X-ray tube	$5\times10^6\ N/C$
Dielectric strength of air	$3\times10^6\ N/C$
Van-de Graft Generator	$2\times10^6\ N/C$
Atomsphere	100 N/C
Arround domestic electric wires	300 N/C

1.7 Electric Field due to a System of Charges:

To calculate the electric field intensity in a point due to a system of charges, the principle of super position of electric fields is employed.

According to this principle "the electric field at a point due to a system of point charges is equal to the vector sum of electric fields at that point due to each of the charges of the system".

If for a system of n point charges $q_1, q_2, q_3, ..., q_n$ the electric field at point P due to these charges are $\vec{E}_1, \vec{E}_2, \vec{E}_3, ..., \vec{E}_n$ respectively (Fig 1.11) then net electric field at P.

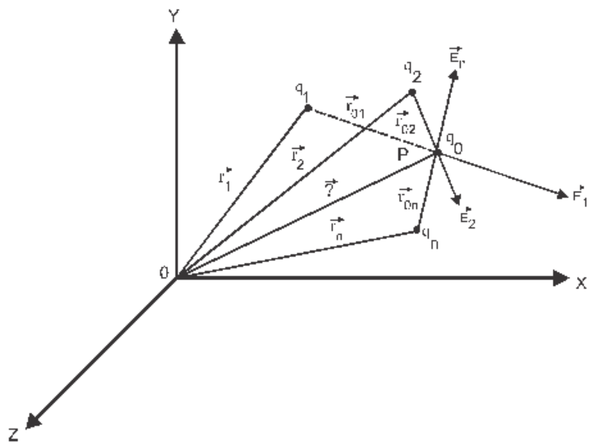


Fig 1.11 : Electric field due to system of point charges

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{q_1}{r_{01}^2}\hat{r}_{01} + \frac{1}{4\pi\in_0}\frac{q_2}{r_{02}^2}\hat{r}_{02} + ... + \frac{1}{4\pi\in_0}\frac{q_n}{r_{0n}^2}\hat{r}_{0n}$$

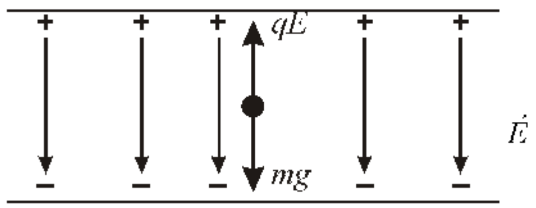
$$\because \quad \hat{r}_{01} = \frac{\vec{r}_{01}}{|\vec{r}_{01}|} = \frac{\vec{r}_{01}}{r_{01}}$$

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{q_1}{r_{01}^3}\vec{r}_{01} + \frac{1}{4\pi\in_0}\frac{q_2}{r_{02}^3}\vec{r}_{02} + ... + \frac{1}{4\pi\in_0}\frac{q_n}{r_{0n}^3}\vec{r}_{0n}$$

$$\vec{E} = \frac{1}{4\pi\in_0}\sum_{i=1}^{n}\frac{q_i}{r_{0i}^3}\vec{r}_{0i} \qquad ...(1.12)$$

Example 1.14 An oil drop has a charge equal to that of 12 electrons and it remains in equilibrium in a constant electric field of 2.55×10^4 N/C . It the density of oil is 1.26×10^3 kg/m^3 then determine the radius of the drop.

Soultion : For equilibrium of charged drop in electric field its weight must be balanced by electric force.



i.e $\quad mg = qE$

but $\quad m = V\rho$

or $\quad m = \frac{4}{3}\pi r^3 \rho$

Where and r is its radius, V is its volume and p is density of oil

then $\quad \frac{4}{3}\pi r^3 \rho g = neE$

$$r = \left[\frac{3neE}{4\pi pg}\right]^{1/3}$$

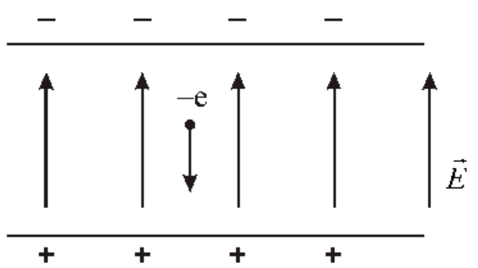
On substituting the values of relevent quantities

$$r = \left[\frac{3\times12\times1.6\times10^{19}\times2.55\times10^{4}}{4\times3.14\times1.26\times10^{3}\times9.8}\right]^{1/3}$$

$$r = 9.8\times10^{7}\,m$$

Example 1.15 An electron falls from rest in a constant electric field of $2.0\times10^{4}\,N/C$ through 1.5 cm. Keeping the magnitude of electric field same now its direction is reversed and now a proton falls from rest in this field through the same distance. Determine the time of fall in both these cases. Compare this situation with "free fall under gravity".

Soultion : First case- As shown in Fig. (a) the electric field is acting vartically upward so force on negatively charged electron F_e = eE is directed downwards.



So the acceleration of electron

$$a_e = \frac{F_e}{m_e} = \frac{eE}{m_e}$$

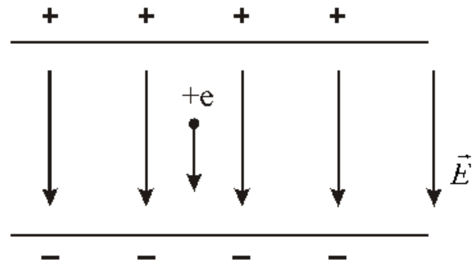
If starting from rest the electrons falls through a distance h in time t then

$$h = \frac{1}{2}a_e t_e^2$$

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

$$t_e = \sqrt{\frac{2\times1.5\times10^{2}\times9.1\times10^{31}}{1.6\times10^{-19}\times2.0\times10^{4}}} = 2.9\times10^{-9}\,s$$

Second case : As shown in Fig (b) the electric field acts vertically downward so force on proton (positive charge) also acts downward, hence acceleration of proton.



$$a_p = \frac{F_p}{m_p} = \frac{eE}{m_p}$$

For proton, time to fall through a distance h

$$t_p = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}}$$

$$t_p = \sqrt{\frac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{19}\times2.0\times10^{4}}} = 1.3\times10^{7}\,s$$

For both the cases accelerations a_e and a_p are much greater compared to acceleration due to gravity g hence gravitional effect is taken as negligible.

e.g. $$a_p = \frac{eE}{m_p} = \frac{\left(1.6\times10^{19}\right)\times\left(2.0\times10^{4}\right)}{1.67\times10^{-27}}$$

$$= 1.9\times10^{12}\ \text{m/s}^2$$

$$g\left(\sim 10\ \text{m/s}^2\right)\ \text{to}\ \sim 10^{11}$$

Which is nearly $\sim 10^{11}$ times more than $g\left(\sim 10\ \text{m/s}^2\right)$, acceleration of electron a_e is 183 times more than a_p

Time to fell freely under gravity is $t_g = \sqrt{\dfrac{2h}{g}}$ is independent of mass so it is same for both electron and proton.

Example 1.16 At some point a force of 2.25 N acts an a charge of $5\times10^{-4}\,C$. Determine the electric field intensity at that point.

Soultion : Here $q_0 = 5\times10^{-4}\,C$

$F = 2.25\,N$

$$E = \frac{F}{q_0} = \frac{2.25}{5\times10^{-4}} = 4.5\times10^{3}\,N/C$$

Example 1.17 In a rectangular coordinate system, two positive point charges 10^{-4} C each are fixed at points $x = +0.1m$, $y = 0$ and $x = -0.1m,\ y = 0$. Find the magnitude and direction of electric field at the following points.

(a) the origin (b) $x = 0.2m, y = 0$ (c) $x = 0,\ y = 0.1m$

Solution : For the system of charges, placed as shown in Fig (a)

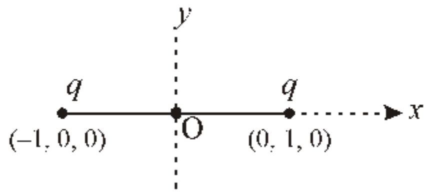


Fig (a)

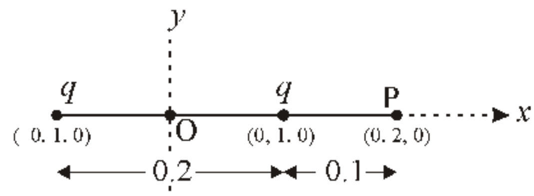
(a) electric field at origin must be zero as fields produced by individual charges at this location are equal and opposite.

(b) For a point such as P shown in Fig (b), electric field due to individual charges are in same direction (along + ve x axis) so net electric field at P

$$E = \frac{kq}{(0.1+0.2)^2} + \frac{kq}{(0.2-0.1)^2}$$

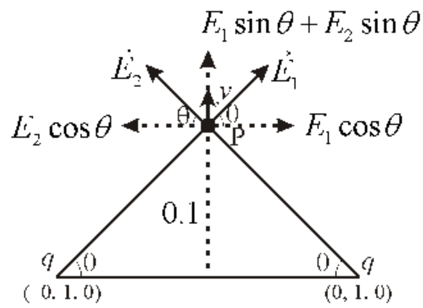
$$= \frac{kq}{0.09} + \frac{kq}{0.01} = \frac{kq}{0.09}[10]$$

$$= \frac{9\times10^{9}\times10^{-8}\times10}{0.09} = 1.0\times10^{4}\ \text{N}$$



(c) At point P (0, 0.1) the electric fields due to the given charges have equal magnitudes i.e. $E_1 = E_2$

In this case, the x components of $\vec{E}_1$ and $\vec{E}_2$ cancel while y components add [fig (c)] net electric field at P is then E



$$E = E_1 \sin\theta + E_2 \sin\theta \ \ \text{(along Y axis)}$$

$$\text{E} = 2E_1 \sin\theta = 2E_1 \sin 45^\circ$$

[From geometry of figure it can be seen that $\theta = 45^\circ$]

$$\therefore\ E = \frac{2kq}{r_1^2}\sin 45^\circ = \frac{2\times9\times10^{9}}{\left[(0.1)^2+(0.1)^2\right]}\times10^{-8}\times\frac{1}{\sqrt{2}}$$

$$= \frac{2\times9\times10}{\sqrt{2}\,[0.02]} = 6.36\times10^{3}\ \text{N/C}$$

1.8 Electric Field Lines

The concept of electric field lines is very useful in visulalizing electric field arround charge configurations graphically. An electric field line is an imaginary line or curve drawn through a region of space (where electric field exists) so that its tangent at any point is in the direction of electric field vector at that point. Fig. 1.12 depicts the basic idea.

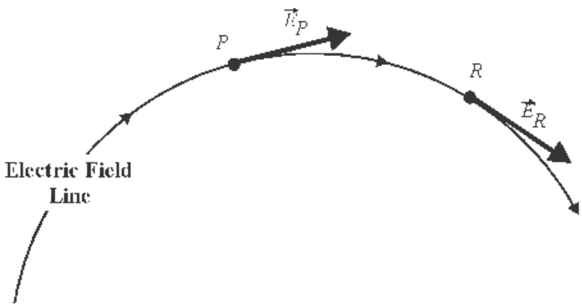
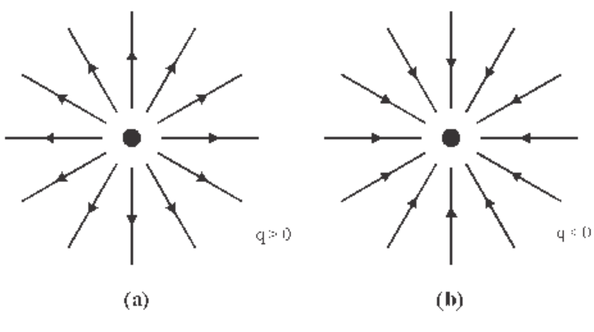


Fig 1.12 : The direction of electric field at any point is tangent to the field line through that point

The concept of field lines was given by British Scientist Michal Faraday (in the first half of 19th century) to develop an intutive non mathematical way of visualizing electric field arround charge configurations. Faraday called them "line of force" but the term "field line" is more appropriate.

Fig 1.13 depicts the field lines arround some simple charge configruations. The field lines are in three dimensional space, however the figure shown here exhibit them only in a plane. For a single isolated positive charge the field lines are radially outward while for a single negative charge field lines are radially inwards. The field lines surrounding a system of two point charges (q,q) depicts a vivid graphical representation of their mutual repulsion, while those surround a dipole (two equal and opposite charges) (q, -q) depicts clearly the mutual attraction between the charges.



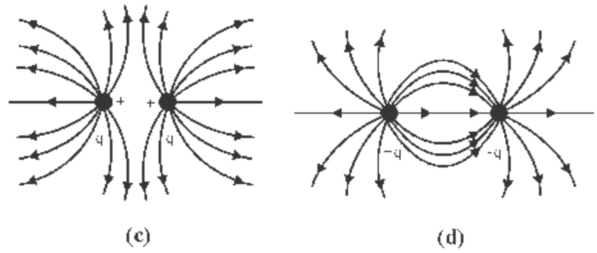


Fig 1.13 : Electric Field lines due to some simple change distributions

The electric field lines follow some general properties:

(i) Field lines always orignates from positive charges and terminates at negative charges. In case of a single charge the may start or terminates at infinity. For an isolated positive charge field lines are radially outwards while for a negative charge these are radially inwards [as in Fig 1.12 (a) and (b)]

(ii) In a charge free region field lines can be considered as continuous curves without any breaks. Tangent drawn at any point on an electric field line gives the direction of electric field at that point, thus it indicates the force acting on a unit positive charge placed at that point.

(iii) The number of field lines that starts from or end on, a charge is proportional to the magnitude of that charge.

(iv) Number of field lines per unit area normal to the area at a point is proporitional to the intensity of electric field at that point. Thus the electric field is strong when the field lines are crowded and weak when they are far apart. In fig. 1.14 field is maximum at A and minimum at C.

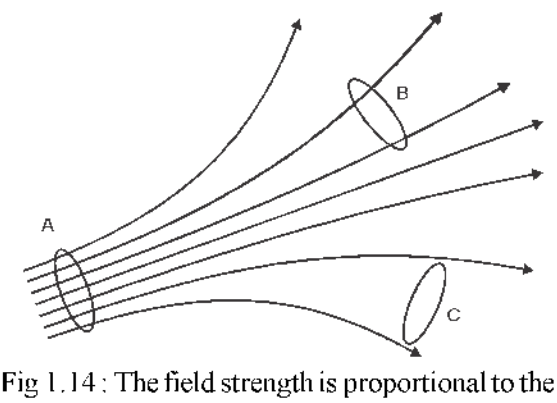


Fig 1.14 : The field strength is proportional to the number of lines that crosses unit area normal to the field

(v) Two electric field lines can never cross each other since it they cross at a point, intensity at that point will have two directions (corresponding to two tangents) which is meaningless.

(vi) In electrostatics, electric field lines can never be closed loops as a line cannot start and end at the same charge. This follows from the conservative nature of electrostatic field.

(vii) There exists a longitudinal tension in the field lines which explains attraction between two unlike charges. The field lines exert a lateral pressure on each other which explains for the repulsion between two like charges. (see figs 1.13 (c) and (d).

(viii) The field lines are perpendicular to an equipotential surface. (You will learn about equilpotential surface in next chapter). Since a charged conductor is an equipotential surface hence field lines are always normal to the conductor surface-

In Fig. 1.15, electric field lines are shown for different types of electric field. For uniform electric field, field lines are equispaced parallel lines as in fig. 1.15 (c).

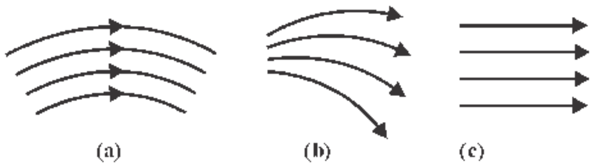


Fig 1.15 : (a) Direction is not constant
(b) magnitude and direction both are not constant
(c) both magnitude and direction are constant.

Electric field lines are not same as trajectories of charged particle. It is a common misconception that a charge particle of charge q in some electric field must move along an electric field line. As electric field $\vec{E}$ at any point is tangent to the field line that passes through that point, it is correct that the net force $\vec{F} = q\vec{E}$ and hence acceleration of the particle are tangent to the field line, however from our study of kinematics we know that when a particle moves on a curve its acceleration can not be tangent to the path. Thus, in general the path of a particle is not same as a field line.

A charged particle will move along a field line only if field line is straight and initially either it is at rest or its velocity is parallel or antiparallel to the field line.

1.9 Electric Dipole and Dipole Moment

"The arrangement of two equal and opposite charges separated by a small distance is called an electric dipole". In fig 1.16 an electric dipole is shown where the magnitude of each charge is q and their separation is 2a.

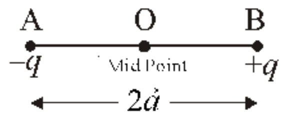


Fig 1.16 : Electric Dipole

The mid point between the charges is called 'centre' of dipole and the line joining the charges is called its axis. The line through centre and perpendicular to the axis is called equatorial line. The electrical behaviour of a dipole is described in terms of its dipole moment. It is a vector quantity denoted by $\vec{p}$.

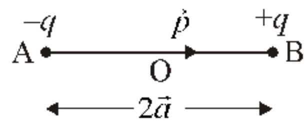


Fig 1.17

The magnitude of electric dipole moment is defined as the product of the magnitude of one of the charges and their separation. If the separation 2a between the charges is considered as a vector directed from the negative charge to the positive charge then by definition.

$$\vec{p} = 2\vec{a}q \qquad \text{. . . (1.13)}$$

The SI unit for dipole moment is Coulomb x meter = C.m and it has dimensions of $M^0L^1T^1A^1$

There exists some molecules in nature where there is a finite separation between centre of positive and centre of negative charges. Such molecules are termed as polar molecules. Few examples are NaCl, H_2O, HCl etc.

There are also some molecules in which normally the centres of positive and negative charges coincides. However, in presence of external electric field the centre of negative charge gets shifted by a small amount relative to the centre of positive charge, there by creating a dipole moment. Such dipoles are termed as induced electric dipoles.

Example 1.18 In NaCl molecule the separation between Na^+ and Cl^- ions is 1.28 Å. Find the electric dipole moment of the molecule.

Solution : Here $q = 1.6\times10^{-19}\,C$

$$2a = 1.28\mathring{A} = 1.28\times10^{-10}\,m$$

$$p = q2a$$

$$p = 1.6\times10^{-19}\times1.28\times10^{-10}$$

$$= 2.048\times10^{-29}\,Cm$$

1.10 Electric Field due to a Dipole

The electric field due to a dipole at some point, is the vector sum of the electric field produced by the individual charges of the dipole at that point i.e. the principle of super position is used to calculate the electric field of a dipole. For the sake of simplicity, here, we will determine the electric field at axial and equatorial points of the dipole.

1.10.1 Electric field at a point on the axial line of an Electric Dipole

In fig 1.18 AB is an electric dipole consisting of charges a q and -q. We wish to determine electric field at a point P on its axis at a distance r from the centre.

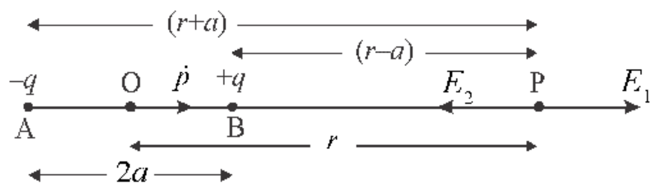


Fig 1.18 : Electric field at an axial point of a dipole

Electric field at point P due to charge +q at B

$$\vec{E}_1 = \frac{1}{4\pi\in_0}\frac{q}{(r-a)^2}\hat{p} \text{ (in direction BP)} \quad \ldots (1.14)$$

here, $\hat{p}$ is a unit vector in direction of dipole moment $\vec{p}$ Electric field at point P due to charge -q at A.

$$\vec{E}_2 = \frac{1}{4\pi\in_0}\frac{q}{(r+a)^2}(-\hat{p}) \text{ (in direction PB)} \ldots (1.15)$$

Hence net electric field at point P

$$\vec{E}_{axial} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}q\left[\frac{1}{(r-a)^2}-\frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}q\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\cdot q\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\frac{2pr}{(r^2-a^2)^2}\hat{p} \quad \ldots (1.16)$$

(Here $q\cdot 2a = p$)

If a is very small compared to r ($a \ll r$) then a^2 can be assumed negligible compared to r^2. Then

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\frac{2p}{r^3}\hat{p} \quad \ldots (1.17)$$

its magnitude is $E_{axial} = \frac{1}{4\pi\in_0}\frac{2p}{r^3} \quad \ldots (1.18)$

Fram the above result it is clear that the field intensity at axial point does not very as r^{-2} as for a single point charge rather it varies as r^{-3}. Thus electric field intensity decreases realtively more rapidly with distance compared to a single point charge.

The direction of electric field at axial line is in direction of dipole moment ($\vec{p}$).

1.10.2 Electric field at a point on Equatorial line of an Electric Dipole

Fig. 1.19 shows an electric dipole AB with charges +q and -q at B and A respectively, the displacement AB $= 2\vec{a}$. We wish to determine electric field at a point P at distance r from the centre O on equatorial line.

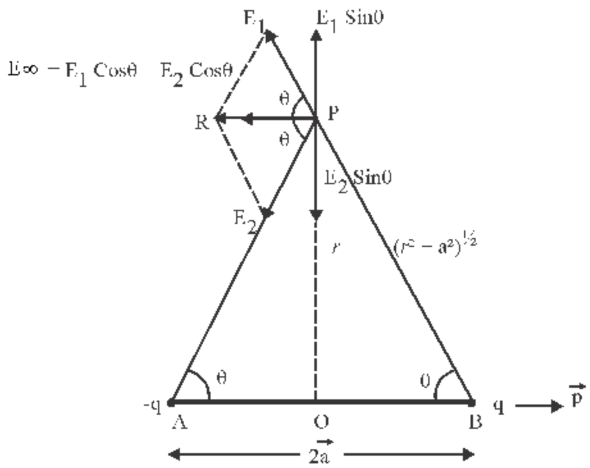


Fig. 1.19 : Electric field at a point on equatorial line of dipole

From Δ AOP and Δ BOP be

$$PA = PB = \left(r^2 + a^2\right)^{1/2}$$

$$(PA)^2 = (PB)^2 = \left(r^2 + a^2\right)$$

Electric field at P due to charge +q

$$\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)}, \text{ (along direction BP)} \quad ...(1.19)$$

Electric field at P due to charge -q

$$\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)}, \text{ (directed along PA)} \quad ...(1.20)$$

Thus $\vec{E}_1$ and $\vec{E}_2$ are of equal magnitudes but differ in directions.

$$\text{i.e. } \left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)} \quad ...(1.21)$$

It we resolve E_1 and E_2 along axial and equatorial lines then equatorial components $E_1 \sin\theta$ and $E_2 \sin\theta$ cancel out being equal and opposite. Axial components $E_1 \cos\theta$ and $E_2 \cos\theta$ add as their directions are same. Therefore

$$\vec{E}_{\text{equater}} = \left(E_1 \cos\theta + E_2 \cos\theta\right)\left(-\hat{p}\right)$$

Here, $-\hat{p}$ indicates that the electric field is opposite to the direction of dipole moment as can be seen from fig 1.19

$$\because \quad \left|\vec{E}_1\right| = \left|\vec{E}_2\right|$$

$$\vec{E}_{\text{equater}} = 2E_1 \cos\theta\left(-\hat{p}\right)$$

$$\vec{E}_{\text{equater}} = 2\frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)} \cdot \frac{a}{\left(r^2 + a^2\right)^{1/2}}\left(-\hat{p}\right)$$

$$\because \quad \cos\theta = \frac{a}{\left(r^2 + a^2\right)^{1/2}}$$

$$\vec{E}_{\text{equater}} = \frac{1}{4\pi \in_0} \frac{p}{\left(r^2 + a^2\right)^{3/2}}\left(-\hat{p}\right) \quad ...(1.22)$$

$$\vec{E}_{\text{equater}} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{\left(r^2 + a^2\right)^{3/2}} \quad ...(1.23)$$

If a is much smaller than r $(a << r)$ then $a^2 << r^2$

$$\text{so } \vec{E}_{\text{equater}} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{r^3} \quad ...(1.24)$$

$$\left|\vec{E}_{\text{equater}}\right| = \frac{1}{4\pi \in_0} \cdot \frac{p}{r^3} \quad ...(1.25)$$

It is clear that for same distance r from centre O.

Therefore (i) the electric field at axial point is twice as that on eqautorial point at the same distance.

(ii) At axial points, the electric field is along the direction of dipole moment, whereas at equatorial points the direction of electric field is opposite to the dipole moment. From the above discussion it is clear that for both axial and equatorial positions the dipole electric field at large distances $(r >> 2a)$ varies as $E \propto 1/r^3$ (and not as $\left(E \propto 1/r^2\right)$ as in case of a single point charge) and it falls off more repidly compared to the electric field due to a single point charge. The physical reason for this rapid decreases in this electric field for a dipole is that from distant points a dipole look like two equal and opposite charges that almost (but not exactly) coincide. Thus their electric fields at distant points almost but not quite cancel each other.

Example 1.19 Two point charges 5 μC and –5 μC are 1cm apart. Calculate the electric field at a distance of 0.3 4 m from their centre at a point.

(i) on the axis (ii) on equatorial line

Solution : Here $q = 5\mu C = 5\times10^{-6}\,C$

$$2a = 1cm = 10^{-2}m$$

$r = 0.30\,m$ (thus r >> a)

Electric dipole moment $p = q.2a$

$$p = 5\times10^{-6}\times10^{-2} = 5\times10^{-8}\,Cm$$

(i) At axial position $E_{axial} = \dfrac{1}{4\pi\in_0}\dfrac{2p}{r^3}$

$$E_{axial} = \frac{9\times10^9\times2\times5\times10^{-8}}{(0.30)^3}$$

$$= 3.33\times10^4\,N/C$$

(ii) At equatorial point $E_{eq} = \dfrac{1}{4\pi\in_0}\dfrac{p}{r^3}$

$$E_{eq} = \frac{9\times10^9\times5\times10^{-8}}{(0.30)^3} = 1.67\times10^4\,N/C$$

1.11 Torque on a Dipole in a Uniform Electric Field

Fig 1.20 (a) shows an electric dipole AB placed in a uniform electric field with its dipole moment oriented at angle θ with E. The force on charge +q, of the dipole is $F = q\vec{E}$, in direction of $\vec{E}$ and on -q is $F = q\vec{E}$, in direction opposite to electric field E. Hence the net force on dipole

$$\vec{F}_{net} = q\vec{E} + (-q\vec{E}) = 0$$

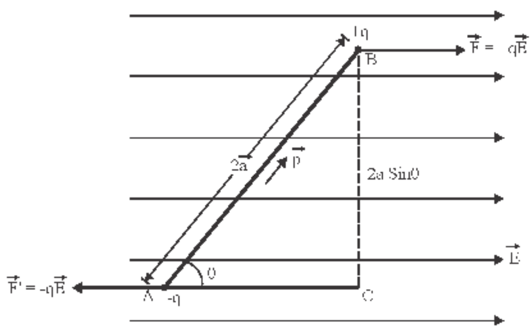


Fig 1.20 (a) : Dipole in Uniform Electric field

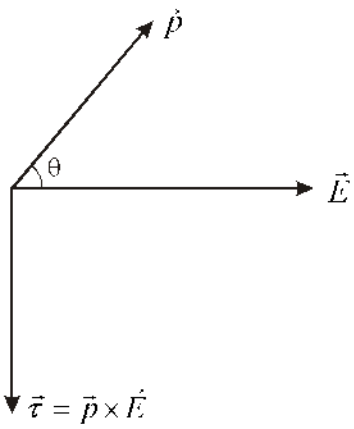


Fig 1.20 (b) : Direction of torque

Thus the net force on the dipole is zero so it will not have a translational motion. However as the two forces are not colinear they constitute a couple producing a net torque on the dipole. This torque tends to align the dipole in direction of electric field.

Magnitude of torque = (force on any of the charge) x perpendicular distance between lines of action of the forces

$$\tau = qE(BC)$$

From fig $\sin\theta = \dfrac{BC}{2a}$

$$BC = 2a\sin\theta$$

So $\tau = qE(2a\sin\theta)$ $\quad\because\; q\cdot2a = p$

$$\tau = pE\sin\theta \qquad \ldots(1.26)$$

In vector notations $\vec{\tau} = \vec{p}\times\vec{E}$ (N m) . . . (1.27)

Direction of $\vec{\tau}$ is perpendicular to plane containing $\vec{p}$ and $\vec{E}$ in accordence with right hand screw rule.

Special cases

(i) (a) When $\theta = 0^o$

$\sin\theta = 0$

dipole is in stable equilibrium

(b) When $\theta = 180^o$

$\tau = pE\sin 180^o = 0$

dipole is in unstable equilibrium

(ii) When $\theta = 90^o$

then $\tau_{max} = pE\sin 90^o$

$\tau_{max} = pE$

Example 1.20 Two charges $\pm$ 1000 μC , 2 mm apart constitute an electric dipole. This dipole is placed in a uniform electric field of $15\times10^4\ N/C$ at 30° with field. Find the torque scting on the dipole.

Solution : Here $q = 1000\,\mu C = 10^{-3}C$

$E = 15\times10^4 N/C$

$2a = 2mm = 2\times10^{-3}m$

$\theta = 30^o$

Torque $\tau = pE\sin\theta$

$\tau = q(2a)E\sin\theta$

$\tau = 10^{-3}\times2\times10^{-3}\times15\times10^4\sin 30^o$

$\tau = 15\times10^{-2}\ Nm$

Important Points

1. An object can be charged in three ways (i) by friction (ii) by conduction (contact) (iii) by electrostatic induction.
2. Charge is not created in process of friction. Actually in process of friction transfer of a few electrons takes place from one object to another as a result of which one object gets positively charged and another negatively charged.
3. Like charges repel and unlike charges attract each other.
4. Electric charge is quantized Quantum of charge = electronic charge e = $1.602\times10^{-19}C$. Any charge can be written as $q = \pm ne \quad n = 1, 2, 3...$
5. For two points charges in free space, electrostatic force between them is given by Coulomb's law as

$$F = \frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2}N$$

$\in_0$ is the permitivity of free space or air

6. In presence of some medium the force between two charges is smaller than the force between them in free space (for same separation) by a factor called relative permitivity or dielectric constant of medium.
7. The electric field intensity at any point is given by

$\vec{E} = \lim_{q_0\to0}\frac{\vec{F}}{q_0}N/C$ where $\vec{F}$, is the force acting on test charge q_0 due to electric field.

8. Electric field due to a point charge is given by

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r}$$

9. Electric field lines can never cross each other since if they cross at a point intensity at that point will have two directions which is absurd.

10. Net electric field due to a number of charges is the vector sum of electric fields due to individual charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

11. A system consiting of two equal but opposite charges kept at a small separation is called an electric dipole.

12. Electric field intensity at an axial point of a dipole

$$\vec{E}_{axial} = \frac{1}{4\pi \in_0} \frac{2p}{r^3} \hat{p} \quad (\text{for } a << r)$$

13. Electric fields intensity at an equatorial point of a dipole

$$\vec{E}_{equater} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{r^3} \quad (\text{for } a << r)$$

14. Torque on a dipole placed in a uniform electric field

$$\tau = pE \sin\theta$$

Questions For Practice

Multiple Choice Questions -

1. Two identical charges separated by a distance of 3 m experience a force of repulsion of 16 N, the magnitude of each charge is

(a) 2 μC (b) 4 μC

(c) 40 μC (d) 80 μC

2. The force acting between two charges is 8N. If the separation between them is trippled then the force acting them will be

(a) F (b) F/3

(c) F/9 (d) F/27

3. To give a charge of 5×10^{-19} C to some object, how many electrona are to be removed from ir?

(a) 3 (b) 5

(c) 7 (d) 9

4. Two point charges + 9e and +e are at a separation of 16 cm. Where on the line joining them another charge q must be put so it remains in equilibrium.

(a) 24 cm away from +9e

(b) 12 cm away from + 9e

(c) 24 cm away from +e

(d) 12 cm away from +e

5. Two identical spheres having unequal and opposite charges are 90 cm apart. They are now made to contant and then again separated by the same distance. Now they repel each other by a force of 0.025 N. Final charge on each sphere is

(a) 1.5 μC (b) 1.5 C

(c) 3 C (d) 3 μC

6. On putting a glass plate in between the two charges the electrostatic force between them as compared to earlier is

(a) more (b) less

(c) zero (d) infinite

7. The dipolemoment of HCl molecule is 3.4×10^{-30} cm the separation between its ions is -

(a) 2.12×10^{-11} m (b) zero

(c) 2 mm (d) 2 cm

8. For an electron and a proton kept in same uniform electric field the ratio of their accelerations is

(a) Zero (b) m_p / m_e

(c) 1 (d) m_e / m_p

9. Four equal and like charges are placed on the four vertices of a square. It the field intensity due to any of the charge at the centre is E then the net electric field intensity at the centre of square will be

(a) Zero (b) E

(c) E/4 (d) 4E

10. On placing a dipole in a uniform electric field it is acted upon by a

(a) Torque only

(b) Force only

(c) Both force and torque

(d) Neither force nor torque

11. For the torque to be maximum on a dipole placed in an electric field angle between $\vec{p}$ and $\vec{E}$ must be

(a) 0° (b) 180°

(c) 45° (d) 90°

12. An electron and a proton are apart. The dipole moment of the system is

(a) 3.2×10^{-29} Cm (b) 1.6×10^{-19} Cm

(c) 1.6×10^{-29} Cm (d) 3.2×10^{-19} Cm

13. For the same distance from centre of dipole the ratio of electric fields at longitudinal and transverse position is

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1

14. The force of attraction between +5 μC and –5 μ charges kept at some distance apart is 9N. When the two charges are made to contact and then separted again by the same distance the force acting between them becomes.

(a) Infinite (b) $9 \times 10^9\ N$

(c) 1 N (d) Zero

15. Two equal but unlike charged objects are kept at some distance apart with a force F acting between them. If 75% charge of one of them is some how transferred to the other then the new force between them is -

(a) $\frac{F}{16}$ (b) $\frac{7F}{16}$

(c) $\frac{9F}{16}$ (d) $\frac{15}{16}F$

Very Short Answer Questions

1. Write the value of one quantum of a charge.

2. The electrostatic force between two protons separated by a distance r is F. It the protons are replaced by electrons then what the force is going to be?

3. The force exerted by one charge on other is F. In presence of a third charge what will be the force on second charge by the first charge.

4. If the dielectric constant of a medium is unity then what is its absolute permitivity.

5. For two point charges q_1 and q_2 product $q_1q_2 < 0$. What is the nature of the force between them.

6. For two point charges q_1 and q_2 the product $q_1q_2 > 0$. What the nature of force acting between them?

7. What is the force acting on a charge placed in electric field E.

8. What is the effect of speed of a charged particle on its charge and mass.

9. What is the magnitude of the intensity of electric field that can balance the weight of an electron? Given

$e = 1.6 \times 10^{-19}\, C$, $m_e = 9.1 \times 10^{31}\, kg$

10. The force acting between two charges placed in free space is F. It a brass plate is now put in the region between the charges then what is the value of force?

11. Name the experiment with which the quantum nature of charge was established?

12. Give definition of electric dipole moment.

13. Write the condition for an ideal electric dipole.

14. Give the example of a particle which has zero he rest mass and zero charge.

15. On what the value of k depends in the expression $k = \frac{1}{4\pi \in_0}$ for Coulomb's law?

16. Write the charge on nucleus $_7N^{14}$ in coulomb.

17. On rubbing an ebonite rod with furr it gets negatively charged, why?

18. Write the CGS and SI unit of charge. What is the relation between them.

19. When an electric dipole is in stable equilibrium in a uniform electric field.

20. What is the net force on an electric dipole in a uniform electric field?

Short Answer Questions

1. What is meant by frictional electricity? Describe its origin.

2. State Coulomb's law for electrostatic force between two point charges at rest?

3. Explain quantisation of charge.

4. Write the law of superposition for forces?

5. The electric field at the mid point of the line joining the two charges is zero. What conculsion you can draw from it regarding the nature of charges.

6. A singley charge negative ion and an electron are allowed to move from rest in a uniform electric field E. Which of them will move faster and why?

7. What is meant by electric field lines? Write its two properties.

8. Explain law of conservation of charge.

9. Define the relative permitivity of a medium.

10. How can a metallic sphere be charged without touching it?

11. How will you show that electric charges are of two types?

12. What does $q_1 + q_2 = 0$ in reference to charges.

13. A dipole is kept in a uniform electric field. Show that it will not have a translatory motion.

14. A charged rod P attracts another charged rod R while the it repels another charged rod Q. What will be the nature of force developed between Q and R.

15. For determining the electric field due to a point charge, the test charge employed should be infintesimal. Why, describe.

16. A copper sphere of 2 gram contains 2×10^{22} atoms. Nucleus of each atom has a charge 29 e. What fraction of electrons should be removed from the sphere to give it a charge of 2 μC.

17. Consider two identical metallic spheres of exactly the same mass. One of them is given some negative charge and other is charged positively by the same amount, will there be any difference in the masses of sphere after charging? If yes, why?

18. On moving away from a point charge the electric field due to charge decreases. The same is true for an electric dipole. Does the electric field for both these cases decreases at the same rate?

19. Use conservation of charge to identify elements X in following nuclear reactions

(a) $_1H^1 + _4Be^9 \rightarrow X + _0n^1$

(b) $_6C^{12} + _1H^1 \rightarrow X$

(c) $_8N^{15} + _1H^1 \rightarrow X + _2He^4$

Essay Type Questions

1. Define Coulomb's law for the electrostatic force between two charges, and discuss its limitations. Using this law define 1 coulomb of charge.

2. Give definition of electric field. Derive expression for electric field due to a point charge. If another charge q_0 is brought in this field what will be the electric force acting on it?

3. What is meant by an electric dipole. Define electric dipole moment. Derive an expression for intensity of electric field at an axial point of an electric dipole.

4. Derive an expression for the intensity of electric field due to an electric dipole on a point situated on its equatorial line.

5. Derive expression for the torque acting on a dipole placed in a uniform electric field. When will its value be maximum?

Answer

Mutliple Choice Questions -

1. (c) 2. (c) 3. (A) 4. (B) 5. (A) 6. (B) 7. (A)
8. (B) 9. (A) 10. (A) 11. (D) 12. (C) 13. (B)
14. (D) 15. (A)

Very Short Answer Questions

1. One quantum of charge = e $= 1.6 \times 10^{\;19}\, C$

2. F

3. F

4. $\in = \in_r \in_0 = 1 \times 8.85 \times 10^{\;12}$

   $= 8.85 \times 10^{\;12}\, C^2 / Nm^2$

5. If $q_1 q_2 < 0$ then one of the charge must be positive and other negative so there will be an attractive force acting between them.

6. If $q_1 q_2 > 0$ then both charges must have same sign (either both positive or both negative) and a force of repulsion acts between them.

7. $\vec{F} = q\vec{E}$

8. If speed is of the order of the speed of light then mass increases with increase in speed however charge remains invariant (constant).

9. $eE = mg$

   $$E = \frac{mg}{e} = \frac{9.1 \times 10^{\;31} \times 9.8}{1.6 \times 10^{-19}}$$

   $= 5.57 \times 10^{-11}\, N/C$

10. $F_m = \frac{F}{\in_r} = \frac{F}{\infty} = 0$

11. Milikens oil drop experiment

12. The product of magnitude of any charge of the dipole and separation between them is called electric dipole moment. It is a vector quanity $\vec{p} = q \cdot 2\vec{a}$ directed from negative to positive charge.

13. The magnitude of charge q should be high and separation (2a) between them should be small such that product q (2a) is finite.

14. Photon

15. On nature of medium and system of units.

16. From $q = Ze$ Is

    $q = 7e = 7 \times 1.6 \times 10^{\;19} C = 11.2 \times 10^{\;19} C$

17. As electrons are more loosely bound in four compared to ebonite, on rubbing them fews electrons are transferred from fur to ebonite.

18. CGS unit is esu or stat coulomb and SI unit is coulomb (C) 1 C = 3×10^9 esu

19. When $\vec{p}$ and $\vec{E}$ parallel i.e. angle between them is 0.

20. Zero (0).

Numerial Problems

1. The charges of $2 \times 10^{-7} C$ and $3 \times 10^{-7} C$ respectively are present on two small spheres kept in air at a distance 30 cm apart. Find the force between them.

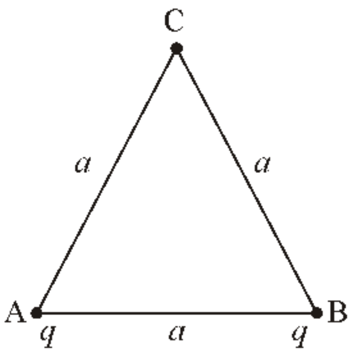
   (Ans: $6 \times 10^{-3} N$)

2. Two identical metallic spheres are charged with $+10\,\mu C$ and $-20\,\mu C$. If they are but into contact and then kept at the same separation as earlier then find the ratio of forces in final and initial situations.

   (Ans : 8:1)

3. Equal charges q each one placed at the vertices A and B of an equilateral triangle. Find the magnitude

of electric field at the vertex C.



(Ans : $E = \frac{1}{4\pi \in_0}\frac{\sqrt{3}q}{a}$)

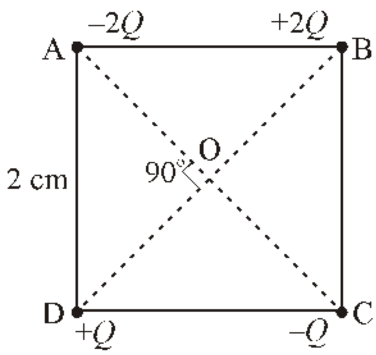
4. Two identical charges spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8\ g\ cm^{-3}$, the angle remains the same. What is the dielectric constant of liquid (density of material of sphere is $1.6\ g\ cm^{-3}$.

(Ans : $\in_r = 2$)

5. Two idensity spherical conductors B and C carry equal like charges and repel each other by a force F when placed at a certain distance apart. Another identical conductor which is uncharged now removed away from B and C with B then with C and removed away from B and C. Find the new force acting between B and C.

(Ans : $\frac{3F}{8}$)

6. In fig four point charges are placed at the four corners of a square of side 2 cm. Determine the magnitude and direction of electric field at the centre O of the square $Q = 0.02\ \mu C$.

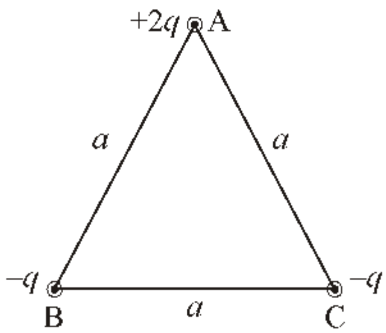


(Ans : $9\sqrt{2}\times 10^5\ N/C$ parallel to $\vec{BA}$)

7. An electric charge Q is divided into two part Q_1 and Q_2 which then are kept at a distance r apart. What will be the condition for on force between them to be maximum.

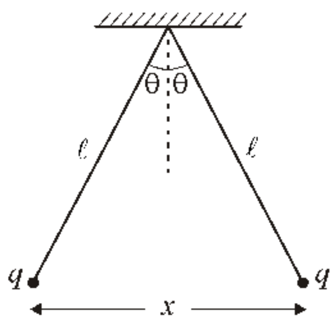
(Ans : $Q_1 = Q_2 = Q/2$)

8. Three charges +2q, $-q$ are $-q$ placed at the vertices A, B and C respectively of a equilateral triangle ABC of side a. Find the magnetude of dipole moment of this system.



(Ans : $\sqrt{3}\, qa$)

9. Two small identical balls each of mass and charge q are suspended same point by silk cords (each card is of length l) as shown in fig. separation between charges is x and angle between cords $(2\theta \approx 10^\circ)$. Calculate the value of x assuming system to be in equilibrium.



10. In a system two charges $q_A = 2.5\times 10^{-7} C$ and $q_B = -2.5\times 10^{-7} C$ are situated at point A(0, 0, -15 cm) and B (0, 0, + 15 cm). find the electric dipole moment of the system.

(Ans : $7.5\times 10^{-8}\ Cm(-\hat{z})$)

11. An electric dipole having dipole moment $4\times10^{-9}\ Cm$ is oriented at 30° from the direction of a uniform electric field of magenitude $5\times10^{4}\ NC^{-1}$. Calculate the magenitude of the torque on the dipole.

(Ans : $10^{-4}\ Nm$)

12. The separation between two point charges q_1 and q_2 is 3 cm. The sum of the two charges is 20 μC and they repel each other by a force of 0.075 N. Find the value of the two chages.

(Ans : 15 μC and 5 μC)

Chapter - 2

Gauss' Law and Its Applications

In previous chapter we have studied about point charge and system of point charges at rest, and concept of electric field. We have also seen, how the principle of superposition is of help in calculating the electric field due to a system of discrete charges. In this chapter our aim is to determine electric field due to a continuous charge distrbution . For such cases, concept of charge density is utilized along with Couloumb's law. What we have to do is to divide the charge distribution into infintesimal elements of charges which may be considered to be a point charge. Electric field due to such an element can then be calculated using Coulomb's law. Accoding to principle of super position the total field is the sum (integral) of all such contributions over the charge distribution.

In principle this method is possible for any continuous charge distribution, however in many cases either it is cumbersome to perform the integration or impossible to solve it exaclty. In situations related to continuous charge distributions where charge distribution is uniform the Gauss's law is very helpful which makes the determination of electric field mathematically very simple. To work with Gauss's law it is essential for us to understand the concept of electric flux, so we start this chapter with the study of electric flux.

2.1 Electric Flux

Electric flux through a flat surface of area S lying in a uniform electric field E is defined by

$$\phi_E = Es\cos\theta \qquad \ldots (2.1)$$

Where θ is the angle between $\vec{E}$ and normal to the surface (Fig 2.1) $\vec{S}$ is area vector

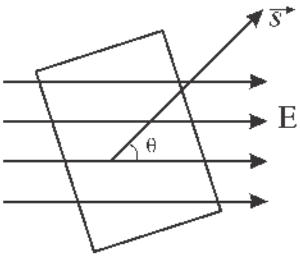


Fig 2.1 : Electric through a flat surface

The area of a flat surface can be represented by vector $\vec{S}$ (called area vector) whose magnitude is equal to the area S and whose direction is normal to the plane of the area. Accordingly we can write

$$\phi_E = \vec{E}\cdot\vec{s} \qquad \ldots (2.2)$$

Thus, electric flux is a scalar quanity value of which depends on electric field, area of surface under consideration and the angle between area vector and the electric field. The electric flux ϕ through a surface is proportional to the net number of electric field lines passing through that surface. ϕ_E is positive (when 90°> θ> 0°), negative 180° > θ> 90° and zero (when θ = 90°). When electric field lines are coming out from the area flux is regarded positive while field lines entering the area corresponds to negative flux. When field lines are parallel to the flat area the flux is considered to be zero.

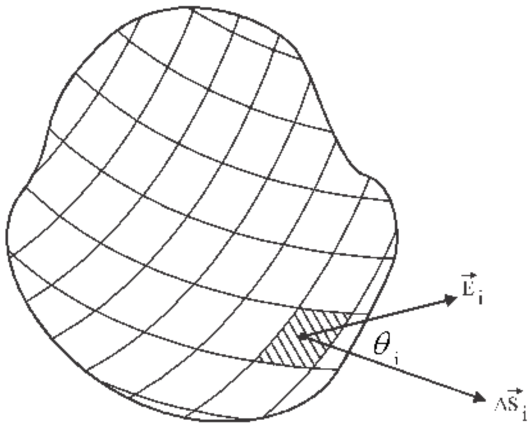


Fig 2.2 : Flux through a closed surface

In a general case where $\vec{E}$ is non uniform and surface is not flat, to calculate electric flux we consider the surface to be divided into a large number 'n' of small area elements Δs_1, $\Delta s_2 \ldots \Delta s_n$ (fig 2.2) where each area element Δs_i is small enough so that

(i) It can be assumed to be flat (planer)

(ii) The variation of electric field over this area element is so small that electric field E_i can be regarded as constant, then using the equation (2.2) the flux linked with such an area element is

$$\Delta\phi_{E_i} = \vec{E}_i \cdot \Delta\vec{s}_i \qquad \ldots (2.3)$$

Summing the contributions of all elements gives an approximation to the total flux through the surface.

$$\phi_E \simeq \Sigma \vec{E}_i \cdot \Delta\vec{s}_i$$

If the area of each element approaches zero the number of elements approaches infinity and the sum Σ is replaced by an intergral. Therefore, the general expression of electric flux is

$$\phi_E = \int \vec{E} \cdot d\vec{s} \qquad \ldots (2.4)$$

The integral in equation (2.4) must be evaluated over the entire surface under question.

We are often interested in evaluating the flux through a **closed surface,** defined as a surface that divides space into an inside and an outside region so that one cannot enter in one region to the other without crossing the surface. For example, the surface of a sphere is a closed surface. Using the symbol $\oint$ to represent an integral over a closed surface the net flux through a closed surface can be written as

$$\phi_E = \oint \vec{E} \cdot d\vec{s} \qquad \ldots (2.5)$$

As we have described above, the vector area element is directed normal to the surface, however, normal can be in two directions. By convention for an area element of a closed surface, area vector $\Delta\vec{S}_i$ always points outward. This convention is used in Fig 2.2. For this fig note that for different area elements, corresponding vector area elements $\Delta\vec{S}_i$ will be pointing in different directions but each such vector will be along outward normal to its corresponding surface element. Also the flux leaving the surface is considered to be positive while that entering into it, is considered to be negative. If the number of field lines leaving the surface is more than those entering the net flux is position. If vice versa flux is negative.

The SI unit of electric flux is N m^{2}C^{-1} or Vm is and it has dimensions of $\left[M^1L^3T^{-3}A^{-1}\right]$

Example 2.1 : Find the electric flux through a vector area $\vec{S} = 5\times10^{-3}\,\hat{j}\,\text{m}^2$ placed in electric field $\vec{E} = 200\hat{i} + 300\hat{j}\,\text{Vm}^{-1}$

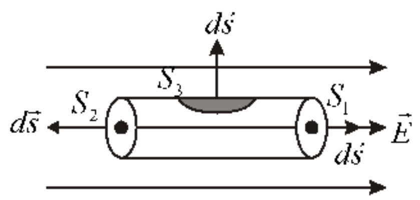
Solution : Electric flux

$$\phi = \vec{E}\cdot\vec{S} = \left(200\hat{i} + 300\hat{j}\right)\cdot\left(5\times10^{-3}\hat{j}\right)$$

$$\phi = 0 + 1500\times10^{-3} = 1.5\,\text{Vm}$$

Example 2.2 : A cylinder is lying in a uniform electric field such that its axis is along the electric field. Show that the net electric flux through the cylinder is zero.

Solution :



As shown in figure we can consider the cylinder to be consisting of three surfaces, two circular faces S_1 and S_2 and curved surfaces S_3, thus the net flux through cylinder.

$$\phi = \int_{S_1} \vec{E}\cdot d\vec{s} + \int_{S_2} \vec{E}\cdot d\vec{s} + \int_{S_3} \vec{E}\cdot d\vec{s}$$

From figure it is clear that for face S_1, $\vec{E}$ and $d\vec{s}$ are parallel $(\theta = 0)$ for face S_2, $\vec{E}$ and $d\vec{s}$ are antiparallel $(\theta = 180^\circ)$ and everywhere on curved surface $\vec{E}$ and $d\vec{s}$ are mutually perpendicular $(\theta = 90^\circ)$, hence

$$\phi = \int_{S_1} Eds\cos0^\circ + \int_{S_2} Eds\cos180^\circ + \int_{S_3} Eds\cos90^\circ$$

or $\quad \phi = ES_1 - ES_2 + 0$

As area $S_1 = S_2 =$ S (say)

$$\phi = ES - ES = 0$$

This result is expected since electric field is uniform so number of field lines entering the cylinder is equal to the number of field lines leaving.

Example 2.3 : A circular sheet of 5 cm radius is situated in a uniform electric field of $5 \times 10^{+5}\ \mathrm{Vm^{-1}}$ such that its plane makes an angle of 30° with field. Determine electric flux through the sheet.

Solution : The angle made by area vector (normal to the plane of sheet) with electric field is $\theta = 90^\circ - 30^\circ$ or $\theta = 60^\circ$

So, electric flux $\phi = ES\cos\theta = E\left(\pi r^2\right)\cos 60^\circ$

$$\phi = 5 \times 10^{-5} \times 3.14 \times \left(5 \times 10^{-2}\right)^2 \times \frac{1}{2}$$

$$\phi = 125 \times 3.14 \times \frac{1}{2} \times 10 = 1.96 \times 10^3\ \mathrm{Vm}$$

2.2 Continuous Charge Distribution

On a microscopic scale, electric charge is quantised. However, there are often situations in which many charges are so close together that they can be conisdered to be continuously distributed. If we consider such a charge distribution to be consiting of point charges the number of such charges is enormously high e.g. a rod containing a small charge of only 1nC it contains 10^{10} point charges. Thus, though it is possible to imagine a charge distribution to be covered by point charges and to calculate the electric field at the desired point using Coulomb's law and then vector sum of electric fields due to all the point charges to give the net electric field, but presence of a very large number of point charges makes such an approach hoplessly complicated. Instead we regard the charge distribution to be continuous, use the concept of charge density and the method of calculus to calculate the electric field. The use of a continuous charge density to describe a large number of closely spaced charges is similar to the use of a continuous mass density to describe air which actually consists of a large number of discrete molecules.

If the net charge on some object is q we divide the charge distribution into many infinitesimal elements dq. Each such element has a length, area or volume dependingon whether considering charges that are respectively distributed in one, two ot three dimensions. We express dq in terms of the size of element and the charge density. Depending upon the number of dimensions over which the charge is distributed we define three types of charge densities as follows -

(i) Linear Charge density

In some situation charges are distributed along a line in space (or along the length of an object) such as charge on a thin rod or wire or on the circumference of a ring. In such cases we express dq in terms of linear charge density (charge per unit length) λ whose SI unit is C/m . If the length of charge element dq is dx by definition.

$$\lambda = \frac{dq}{dx} \qquad \ldots (2.6\ a)$$

or $dq = \lambda\, dx$ . . . (2.6 b)

If a charge q is spreaded uniformly on a rod of length L then we can write $\lambda = q/L$ and it is a constant.

(ii) Surface Charge Density

In some situations charge might be distributed over a two dimensional area such as the surface of a thin disc or sheet or surface of a conductor. In such cases elemental charge dq is expressed in terms of the surface charge density (charge per unit area) σ measured in SI units of $\mathrm{C/m^2}$. If a charge dq is present in an elemental area dS then.

$$\sigma = dq/ds \qquad \ldots (2.7\ a)$$

or $dq = \sigma ds$ . . . (2.7 b)

If a charge q is spreaded uniformly over a surface of area S then $\sigma = q/S$ and is a constant.

(iii) Volume Charge density

The charge also might be spread throughout the volume of a three dimensional object. In this situation, we use volume charge density (charge per unit volume) ρ measured in the SI unit of $\mathrm{C/m^3}$. If charge in a volument dV is dq, then

$$\rho = dq/dV \qquad \ldots (2.8\ a)$$

or $dq = \rho dV$ . . . (2.8 b)

If the charge q is distributed uniformly throughout the volume V then $\rho = q/V$ and is a constant.

2.2.1 Electric Field due to a continuous Charge Distribution

In this subsection we discuss the determination of electric field due to a continuous charge distribution, the general method for which is as follows -

1. Consider the charge distribution to be consist of a large number of infintesimal elements.
2. Choose an arbitrary charge element and express its charge dq in terms of relevent charge density given by equations 2.6, 2.7 and 2.8 depending on whether the charge is distributed over a line, surface or volume.
3. Treating this charge element dq as a point charge the intensity of electric field at the observation point p is given by

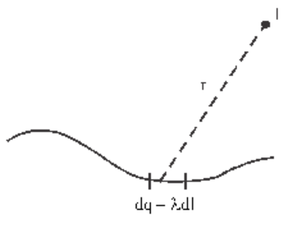
$$d\vec{E} = \frac{1}{4\pi \in_0} \frac{dq}{r^2} \hat{r}$$

here r is the distance between element dq and point P. The direction of vector $d\vec{E}$ is determined by the sign of charge dq according to the force that dq would exert on a unit test charge at P.

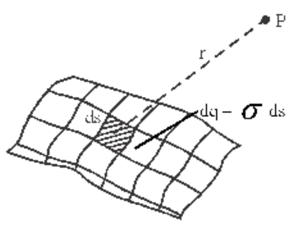
4. The total electric field at P for the entire charge distribution is obtained by taking vector sum of the contributions from all the elements. In the limiting case when the size of element tends to zero

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi \in_0} \int \frac{dq}{r^2} \hat{r} \qquad \ldots (2.10)$$

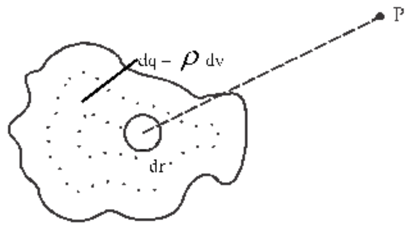
Fig (2.3) shows situations corresponding to linear, surface and volume charge distributions, the corresponding expressions for electric fields are as follows -



(a) linear charge distribution



(b) surface charge distribution



(c) volume charge distribution

Fig 2.3 : Determination of electric field due to (a) linear charge distribution (b) surface charge distribution (c) volume charge distribution

(i) Linear charge distribution : Here $dq = \lambda d\ell$

$$\therefore \quad \vec{E} = \frac{1}{4\pi \in_0} \oint_L \frac{\lambda d\ell}{r^2} \hat{r} \qquad \ldots (2.11)$$

Here the symbol L on integral represents a line integral. If λ is uniform

$$\vec{E} = \frac{1}{4\pi \in_0} \lambda \int_L \frac{d\ell}{r^2} \hat{r} \qquad \ldots (2.12)$$

(ii) Surface Charge Distribution : Here $dq = \sigma ds$

$$\therefore \quad \vec{E} = \frac{1}{4\pi \in_0} \int_S \frac{\sigma ds}{r^2} \hat{r} \qquad \ldots (2.13)$$

here the symbol s on integral suggest that it is a surface integral. If σ is uniform

$$\vec{E} = \frac{1}{4\pi \in_0} \sigma \int_S \frac{ds}{r^2} \hat{r} \qquad \ldots (2.14)$$

(iii) Volume Charge distribution : Here $dq = \rho dV$

$$\therefore \quad \vec{E} = \frac{1}{4\pi \in_0} \int_V \frac{\rho dV}{r^2} \hat{r} \qquad \ldots (2.15)$$

Here the symbol V on integral sign represents a volume integral. If ρ is a constant (uniform)

$$\vec{E} = \frac{1}{4\pi \in_0} \rho \int_V \frac{dV}{r^2} \hat{r} \qquad \ldots (2.16)$$

While solving various integrals it should be taken care of that direction of $d\vec{E}$ due to different elements may be different. Equation (2.10) infact represents a three dimensional vector equation. It can be written is its cartesion components as -

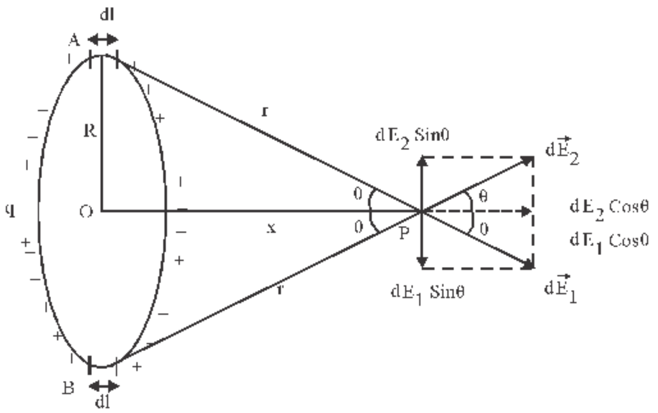
$$E_x = \int dE_x,\ E_y = \int dE_y,\ E_z = \int dE_z \ \ldots (2.17)$$

In many situations one or more of the above integrals may vanish or have identical values owing to symmetry in charge distribution.

If we wish to determine the force on a point charge q due to some continuous charge distribution, we can do so by first determining $\vec{E}$ using equation (2.10) (according to the dimensional situation of the charge distribution) and then use $\vec{F} = q\vec{E}$. In this chapter we shall limit our study to charge distributions for which corresponding charge density (λ, σ or ρ as the case may be) is uniform.

Example 2.4 A thin ring of radius R has a positive charque q uniformly distributed over it. Determine the electric field at a point on the axis of ring at a distance x from the centre of the ring. Discuss the behavior of the result for condition $x >> R$.

Solution : As the ring is uniformly charged, the linear charge density is constant and is given



$$\lambda = \frac{q}{L} = \frac{q}{2\pi R}$$

Now we consider two diametrically opposite elements A and B each of length $d\ell$ on the ring as shown in Fig.

Charge on each element

$$dq = \lambda d\ell$$

If r is the distance of each element from point P then the electric field at P due to element A.

$$d\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{dq}{r^2} \text{ (In direction AP)}$$

and electric field at P due to element B

$$d\vec{E}_2 = \frac{1}{4\pi \in_0} \frac{dq}{r^2} \text{ (In direction BP)}$$

Clearly $|d\vec{E}_1| = |d\vec{E}_2|$

On resolving $d\vec{E}_1$ and $d\vec{E}_2$ as shown in fig, the perpendicular component $dE_1 \sin\theta$ and $dE_2 \sin\theta$ cancel each other being equal and opposite. While component along axis $dE_1 \cos\theta$ and $dE_2 \cos\theta$ add up being in same direction. We can divide the entire ring into pairs of such diametrically opposite elements. For each such pair the axial component is along OP, so electric field due to complete ring, at P.

$$E = \int_L dE \cos\theta = \frac{1}{4\pi \in_0} \int_L \frac{dq}{r^2} \cos\theta$$

From Fig. $\cos\theta = \frac{x}{r}$, and as $dq = \lambda d\ell$ and are R, λ constant and for a given point P, x, > and r are is also treated as constant.

$$E = \frac{\lambda x}{4\pi \in_0 r^3} \int_L d\ell$$

So $\int_L d\ell$ = length of complete ring $= 2\pi R$

So $$E = \frac{\lambda x}{4\pi \in_0 r^3} \cdot 2\pi R$$

$\because$ $\lambda \times 2\pi R = q$ and from Fig. $r = \left(R^2 + x^2\right)^{1/2}$

So $$E = \frac{qx}{4\pi \in_0 \left(R^2 + x^2\right)^{3/2}} = \frac{kqx}{\left(R^2 + x^2\right)^{3/2}}$$

Under condition $x >> R$ the above expression reduces to

$$E = \frac{kqx}{x^3} = \frac{kq}{x^2}$$

Which is identical to expression for the field produced by a point charge q at a point at a diffence x from it. Thus, for distant axial point the ring behaves as if its entire charge is concentrated at its centre.

Example 2.5 A uniformly charged ring and a uniformly charged sphere are both of equal radius R and each has a charge q. The centre of the sphere lies on the axis of ring at a distance of $R\sqrt{3}$ from the centre of ring. Find the electric force acting between sphere and the ring.

Solution : Electric field at an axial point at a distance x from the centre of a uniformly charge ring is given by

$$E = \frac{1}{4\pi \in_0} \frac{qx}{\left(R^2 + x^2\right)^{3/2}}$$

given $x = R\sqrt{3}$, hence the electric field at the location of the centre of sphere, due to ring is

$$E = \frac{1}{4\pi \in_0} \frac{qR\sqrt{3}}{\left(4R^2\right)^{3/2}} = \frac{1}{4\pi \in_0} \frac{\sqrt{3}q}{8R^2} = \frac{\sqrt{3}q}{32\pi \in_0 R^2}$$

From symmerty the total charge q uniformly distributed on the sphere can be considered to be concentrated at its centre, hence the force between sphere and the ring is

$$F = \frac{\sqrt{3}q^2}{32\pi \in_0 R^2}$$

2.3 Gauss's Law

Gauss's law states that the net flux of an electric field through an imaginary closed surface is $1/\in_0$ times the net charge enclosed by the closed surace.

$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\varepsilon_0} \qquad \ldots (2.18)$$

If the closed surface is in some medium other than free space or air, then

$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\varepsilon_0} \qquad \ldots (2.19)$$

The above equation gives Gauss's law for dielectric media. Regarding Gauss's law following points are worth noting

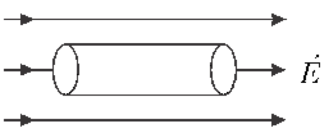
(i) Here Σq, represents the algebraic sum of charges enclosed by the surface.

(ii) The flux entering the surface is considered negative and the flux leaving it is considered positive. The net flux is the algebraic sum of the fluxleaving and flux entering the system.

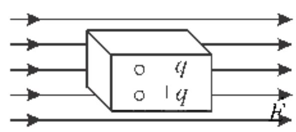
(iii) The closed surface considered for applying Gauss's law is called Gaussian surface. It is an arbitrary imaginary closed surface i.e. it can have sphereical, cylindrical or any other arbitrary shape. It is usually chosen so that the symmetry of charge distribution (if any) gives, on at least part of the surface an electric field of constant magnitude which can then be factored out of the integral of equation (2.18) making calculations easier.

(iv) Gauss's law considers only on the net charge enclosed in the closed surface. The value of flux does not depend on shape and size of the Gaussian surface. It deos not depend on the location or ditribution of charges inside Gaussian surface. It depends on amount of enclosed charges, their nature and medium. For static charge distribution Gauss's law and Coulomb's law are equivalent. However Gauss's law is more general in that it is always valid whether or not the charges are static.

(v) If the net change enclosed by a surface is zero the flux linked with it is always zero whether it is placed in a uniform or nonuniform electric field. For such a surface, flux entering is equal to the flux leaving the surface. [see fig 2.4 (a) and (b)]



(a)



(b)

i.e. $\phi_{net} = \phi_{in} + \phi_{out} = 0$

(vi) Gauss's law is valid only for those vector fields which obeys inverse square law.

(vii) Electric field $\vec{E}$ at any point on the Gaussian surface is the net electric field at that point. This $\vec{E}$ result from all charges both those inside and those outside the Gaussian surface, however term q on the right hand side represent only the net charge enclosed in the Gaussian surface. In some specific cases though the net charge enclosed in the Gaussian surface may be zero but electric field may not. For example if a dipole is enclosed by a Gaussian surface than charge enclosed is zero and ϕ is zero but at some point on the surface $\vec{E}$ is non zero.

(viii) Charges present outside the Gaussian surface do not contribute toward the net flux through the surface.

2.3.1 Gauss's law Derived from Coulomb's law

Considere a positive point charge q at point O enclosed by an arbitrarily shaped Gaussian surface.

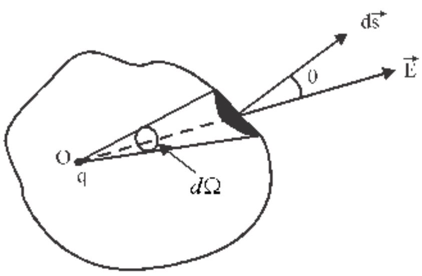


Fig. 2.5 : Solid Angle

Consider an area element $d\vec{s}$, at a distance r from the point charge (Fig 2.5), the electric field at this location is

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} \qquad \ldots (2.21)$$

So flux linked with this area element is

$$d\phi = E ds \cos\theta = \frac{1}{4\pi \in_0} \frac{q}{r^2} ds \cos\theta$$

hence the total flux linked with entire closed surface

$$\phi = \int_S \frac{1}{4\pi \in_0} \frac{q}{r^2} ds \cos\theta = \frac{q}{4\pi \in_0} \int_S \frac{ds \cos\theta}{r^2}$$

By definition $\frac{ds \cos\theta}{r^2} = d\Omega$ solid angle subtended by the area element ds at point O.

$$\phi = \frac{q}{4\pi \in_0} \int_S d\Omega = \frac{4\pi q}{4\pi \in_0} \qquad \left[\because \int_S d\Omega = 4\pi\right]$$

or $\phi = \frac{q}{\in_0}$

which is the mathematical statement of Gauss's law.

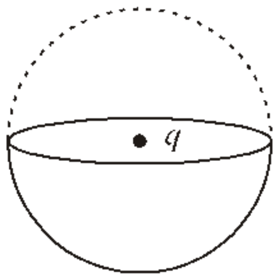
Example 2.6 A point charge of 7.6 μC is situated at the centre of a spherical surface of 0.03 m radius. Find the flux linked with the spherical surface. What will be the change in flux if the radius of surface be doubled.

Solution : As

$$\phi = \frac{q}{\in_0} = \frac{7.6 \times 10^{-6}}{8.85 \times 10^{-12}} = 8.6 \times 10^5 \text{ Nm}^2\text{C}^{-1}$$

As Gauss law does not depend on the size of surface it will not change on doubling the radius of spherical surface.

Example 2.7 A charge q is placed at the centre of a hemispherical surface. Determine the flux of electric field through the surface of hemisphere.



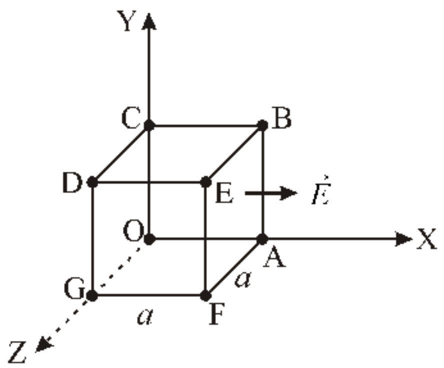
Solution : As Gauss's law deals with the electric flux depends on the charge enclosed by a closed surface, so to enclose the charge and keeping symmetry in view we imagine a complete spherical surface centred at location of q. Flux through this surface.

$$\phi' = \frac{q}{\in_0}$$

Since the charge is placed at the centre, from symmetry considerations we expect that flux through the surface of hemisphere

$$\phi = \frac{\phi'}{2} = \frac{q}{2\in_0}$$

Example 2.8 Figure shows a closed Gaussian surface in the shape of a cube placed in a region where the electric field is given by $\vec{E} = E_0 x\hat{i}$. Each edge of the cube has length a = 1 cm and constant $E_0 = 2.5\times10^5\ \text{NC}^{-1}\text{m}^{-1}$. Find the net electric flux linked with cube and the net charge enclosed by the cube.



Solution : Area of each face of the cube

$$S = a^2$$

Total flux through the cube

$$\phi = \left(\vec{E}\cdot\vec{S}\right)_{ABEF} + \left(\vec{E}\cdot\vec{S}\right)_{OCDG} + \left(\vec{E}\cdot\vec{S}\right)_{BCDE}$$

$$+\left(\vec{E}\cdot\vec{S}\right)_{OAFG} + \left(\vec{E}\cdot\vec{S}\right)_{OABC} + \left(\vec{E}\cdot\vec{S}\right)_{DEFG}$$

As for each of the faces BCDE, OAFG, OABC, and DEFG, $\vec{E}$ is perpendicular to so corresponding flux terms are zero.

$$\phi = \left(E_0 x\hat{i}\cdot a^2\hat{i}\right)_{ABEF} + \left(E_0 x\hat{i}\cdot a^2\left(-\hat{i}\right)\right)_{OCDG}$$

and as for the face OCDG x=0 the contribution of this term toward flux is also zero. As for face ABEF x=a

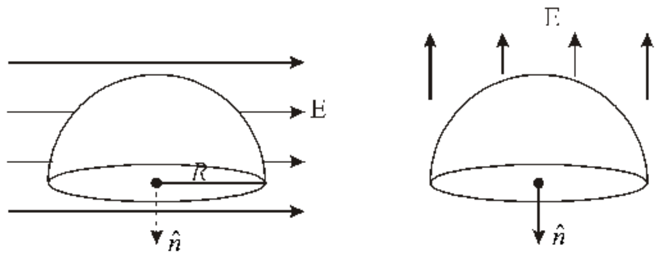
$$\phi = E_0 a^3 - 0 = E_0 a^3$$

$$= \left(2.5\times10^5\right)\times\left(1\times10^{-2}\right)^3 = 0.25\ \text{Nm}^2\text{C}^{-1}$$

and from Gauss's law, the enclosed charge is

$$q = \in_0 \phi = 8.85\times10^{-12}\times0.25 = 2.21\times10^{-12}\ \text{C}$$

Example 2.9 A hemispherical body is placed in a uniform electric field E. What is the electric flux linked with curved surface if the electric field is (a) parallel to its base Fig (a) (b) perpendicular to its base.(Fig. b)



(a) (b)

Solution : We can consider the hemispherical body as a closed body with a curved surface and a flat base (cross section), the flux linked with this body will be zero as it does not enclose any charge. So if ϕ_C and ϕ_b are flux be linked with curved surface and base respectively

$$\phi = \phi_c + \phi_s = 0$$

or $\quad \phi_c = -\phi_b$

(a) For the situation of Fig (a), as the electric field is parallel to the base, vector area of base is perpendicular to $\vec{E}$ thus $\phi_b = 0$ and so $\phi_c = 0$.

(b) For the situation of Fig (b) area vector of base is antiparallel to $\vec{E}$

So $\quad \phi_b = E\cos180^o = -E\pi R^2$

$$\phi_C = -\phi_b = E\cdot\pi R^2$$

(In this case the flux linked with curved surface depends on the radius of cross section (base) and not on the shape of curved surface)

2.4 Application of Gauss's Law

Using Gauss's law $\phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\in_0}$

The electric field due to a highly symmetrical charge distribution can often be easily calculated. The aim in this type of calculation is to determine a surface (Gaussian surface) which satisties one or more of the following conditions -

1. The value of electric field can be argued by symmetry to be constant over the portion of surface.

2. The area vectors of the portions of surface are either parallel or pendicular to $\vec{E}$.

In following subsections we will discuss selection of Gaussian surfaces for some symmetrical charge distributions.

2.4.1 Electric Field Intensity due to an Infinite Line Charge

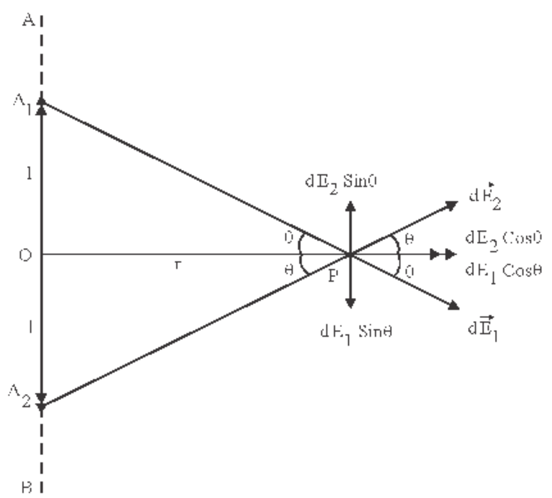


Fig 2.4 : Electric Field due to an infinite line charge

Consider an infinite line charge of constant linear charge density λ. We wish to determine electric field at a point P at as perpendicular distance OP= r from this line charge.

Now consider two small elements A_1 and A_2 of equal lengths situated symmetrically with respect to O on this line charge (Fig 2.6). The electric field inensities at P due to these elements are $d\vec{E}_1$ and $d\vec{E}_2$ respectively with $d\vec{E}_1$ and $d\vec{E}_2$ directed along A_1P and A_2P . On resolving these electric field along OP and perpendicular to OP we note that their perpendicular componenets $dE_1 \sin\theta$ and $dE_2 \sin\theta$ cancel while components along OP, $dE_1 \cos\theta$ and $dE_2 \cos\theta$ add. In this manner the infinite line charge can be divided into such symmetrical pairs. Resulting electric field from each such pair is along OP. Thus we conclude that the electric field due to infinite line charge is directed perpendicular to the line charge i.e. it is in redial direction (which is redially outward or inward depending upon whether the charge is positive or negative). This result is expected on the basis of symmetry. Imagine that while you are watching some one rotates the line charge about its perpendicular axis. When you look again you will not be able to detect any change. From this symetry it can be concluded that the only uniquely specified direction in this situation is along a radial line.

Now consider a Gaussian surface in the form of a closed cylinder of length l coaxial with the line charge such that the point P lies on the its curved surface. (Fig 2.7)

The charge enclosed in this Gaussian surface

$$\Sigma q = \lambda \ell \qquad \ldots (2.24)$$

so from Gauss's law

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\in_0} = \frac{\lambda \ell}{\in_0} \qquad \ldots (2.25)$$

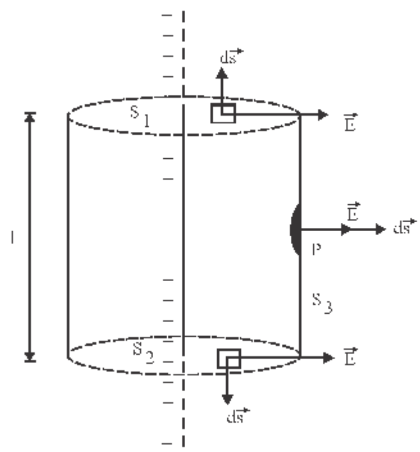


Fig 2.7 : Cylindrical Gaussian surface for a line charge

This closed cylindrical surface can be subdivided into three parts

(i) Upper circular face (cap) S_1

(ii) Lower circular face (cap) S_2

(iii) Curved surface S_3

So the equation (2.25) can be rewritten as

$$\int_{S_1} \vec{E}\cdot d\vec{s} + \int_{S_2} \vec{E}\cdot d\vec{s} + \int_{S_3} \vec{E}\cdot d\vec{s} = \frac{\lambda \ell}{\epsilon_0}$$

or

$$\int_{S_1} Eds\cos 90^\circ + \int_{S_2} Eds\cos 90^\circ + \int_{S_3} Eds\cos 0^\circ = \frac{\lambda \ell}{\epsilon_0}$$

or $$0+0+\int_{S_3} Eds = \frac{\lambda \ell}{\epsilon_0} \qquad \ldots (2.26)$$

Since the magnitude of electric field E is same everywhere on the curved surface so E can be taken out of the integral in equation 2.26, giving

$$E\int_{S} ds = \frac{\lambda \ell}{\epsilon_0}$$

or $$E \times 2\pi r \ell = \frac{\lambda \ell}{\epsilon_0} \qquad \because \int_{S_3} ds = 2\pi r \ell$$

or $$E = \frac{\lambda}{2\pi r \epsilon_0} \qquad \ldots (2.27)$$

In vector form $$\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r}\hat{r} \qquad \ldots (2.28)$$

Clearly the magnitude of electric field due to an infinite line charge is inversely proportional to distance and directly proportional to the charge density and corresponding graphs are as shown in fig 2.8

i.e. $E \propto \frac{1}{r}$ and $E \propto \lambda$ $\qquad \ldots (2.29)$

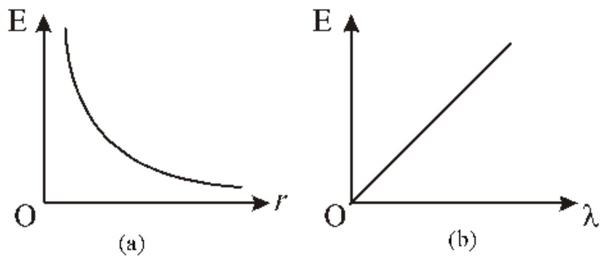


Fig 2.8 : Variation of electric field due to a line charge with (a) distance (b) charge density

Example 2.10 : The linear charge density of a straight infinite wire is $2\ \mu C/m$. Find the magnitude of electric field at a point 20 cm from the wire in air.

Solution :

$$\because \quad E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2\lambda}{4\pi \epsilon_0 r}$$

$$= 9\times 10^9 \times \frac{2\times 2\times 10^{-6}}{20\times 10^{-2}}$$

or $$E = 1.8\times 10^5\ NC^{-1}$$

Example 2.11 : An electron is circulating on a path of radius 0.1 m around a infinite line charge. If the linear charge density is $10^{-6}\ cm^{-1}$ then find the magnitude of the velocity of electron. [Given $m_e = 9.0\times 10^{-31}$ kg, $e = 1.6\times 10^{-19}$ C]

Solution : Force on electron due to infinite line charge

$$F = qE = eE = \frac{1}{4\pi \epsilon_0}\frac{2e\lambda}{r}$$

This force provides the electron necessary centripetal force, so $\frac{m_e v^2}{r} = \frac{2e\lambda}{4\pi \in_0 r}$

or $v = \sqrt{\frac{2e\lambda}{4\pi \in_0 m_e}}$

$= \sqrt{\frac{9\times10^9 \times 2\times1.6\times10^{-19} \times10^{-6}}{9.0\times10^{-31}}}$

$v = \sqrt{2\times16\times10^{14}} = 4\sqrt{2}\times10^7$

$= 5.65\times10^7 \text{ ms}^{-1}$

2.4.2 Electric Field due to an Infinite Uniformly Charged Non Conducting Sheet

Let ABCD is some portion of a uniformly charged non conducting sheet of infinite extension. The surface charge density σ for this surface is uniform. We wish to determine electric field at a point P at a normal distance OP = r from the sheet. (Fig 2.9)

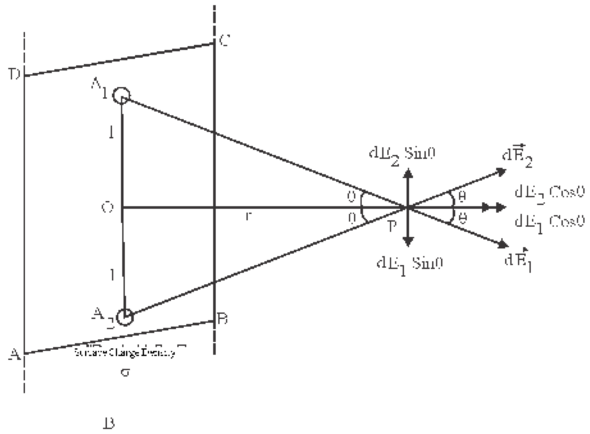


Fig 2.9 : Electric field due to an infinite non conducting charges sheet

Consider two small area elements A_1 and A_2 equidistant from O on its sides at a distance l then electric field $d\vec{E}_1$ and $d\vec{E}_2$ due to these elements have same magnitude and these are directed along A_1P and A_2P respectively. On resolving these electric fields along OP and perpendicular to it, the perpendicular components $dE_1 \sin\theta$ and $dE_2 \sin\theta$ cancel while parallel components $dE_1 \cos\theta$ and $dE_2 \cos\theta$ add.

In this manner we can divide the entire charge sheet into pair of symmetrically placed area elements. For each such pair the net electric field is along OP. Thus we can say that the net electric field due to complete sheet at point P is along the normal joining P to the sheet.

Now imagine a cylindrical closed surface of cross sectional area S and length $2r$ with point P at one of its circular face (cap). The sheet divides this cylinder into two equal parts (Fig 2.10).

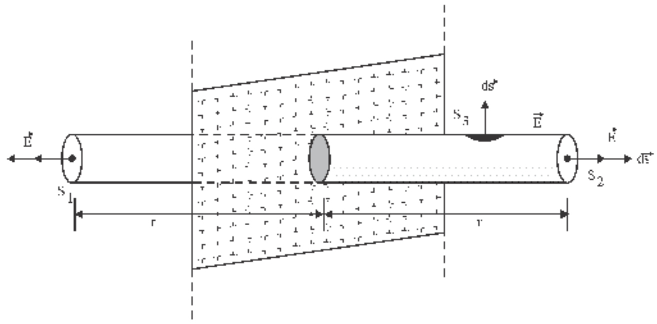


Fig 2.10 : Construction of Gaussion surface for a non conducting uniformly charged infinite sheet.

The net charge enclosed by this surface is

$$\Sigma q = \sigma S \qquad \ldots(2.30)$$

therefore the net electric flux lined with it

$$\phi = \oint_S \vec{E}\cdot d\vec{s} = \frac{\Sigma q}{\in_0} = \frac{\sigma S}{\in_0} \qquad \ldots(2.31)$$

We can consider this closed cylindrical surface to be consisting of three parts (i) circular cap S_1 (ii) circular cap S_2 and (iii) curved surface S_3. Thus the equation (2.31) can be written as.

$$\int_{S_1} \vec{E}\cdot d\vec{s} + \int_{S_2} \vec{E}\cdot d\vec{s} + \int_{S_3} \vec{E}\cdot d\vec{s} = \frac{\sigma S}{\in_0} \qquad \ldots(2.32)$$

For both surfaces S_1 and S_2 $\vec{E}$ and $d\vec{s}$ are parallel so at these surfaces $\vec{E}\cdot d\vec{s} = Eds$, while for

surface S_3, $\vec{E}$ is perpendicular to $d\vec{s}$ so $\vec{E}\cdot d\vec{s} = 0$.
So from equation (2.23)

$$\int_{S_1} Eds + \int_{S_2} Eds + 0 = \frac{\sigma S}{\epsilon_0}$$

As for surfaces S_1 and S_2 E is same at every point, so

$$E\int_{S_1} ds + E\int_{S_2} ds = \frac{\sigma S}{\epsilon_0}$$

$$\because \qquad \int_{S_1} ds = \int_{S_2} ds = S$$

so $$ES + ES = \frac{\sigma S}{\epsilon_0}$$

or $$2ES = \frac{\sigma S}{\epsilon_0}$$

or $$E = \frac{\sigma}{2\epsilon_0} \qquad \ldots (2.33)$$

In vector form $$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n} \qquad \ldots (2.34)$$

Where $\hat{n}$, is a unit vector normal to the sheet. It is obvious that the electric field due to an infinite sheet of charge does not depend on distance i.e. such a charge distribution produces a uniform electric field. This result can be extended for points in the close vicinity if a large but finite sheet of charge provided the points are not near its edges.

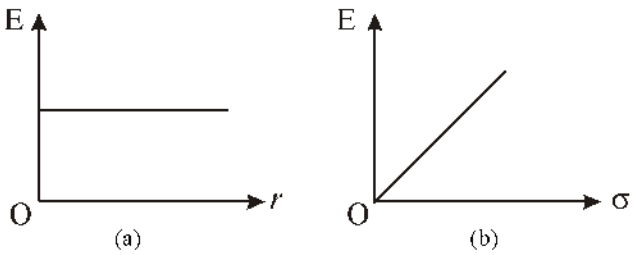


Fig 2.11 : Dependence of electric field for a Uniformly charged infinite sheet

Example 2.12 : For a uniformly charged infinite non conducting sheet, area of 1 cm^2 anywhere on the sheet contains 1 μC charge. Calculate the electric field near the sheet in air.

Solution : Surface charge density

$$\sigma = \frac{q}{A} = \frac{17.70\times10^{-6}C}{10^{-4}m^2}$$

$$\sigma = 17.70\times10^{-2}\ \text{C/m}^2$$

So electric field

$$E = \frac{\sigma}{2\epsilon_0} = \frac{17.70\times10^{-2}}{2\times8.85\times10^{-12}} = 10^{10}\ \text{NC}^{-1}$$

2.4.3 Electric Field due to an Uniformly Charged Infinite Conducting Plate

For a uniformly charged infinite conducting plate the electric field is directed normal to the plate as in case of an uniformly charged infinite non conducting sheet. This can be shown by arguments based on symmetry similar to those used in earlier subsection.

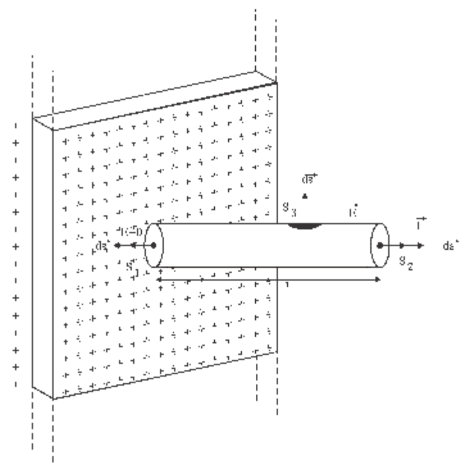


Fig 2.12 : Electric field due to uniformly charged conducting plate

Let the surface charge density for the conductor plate is σ. We wish to determine the magnitude of electric field at a perpendicular distance r from the plate. Imagine a cylindrical Gaussian surface of cross sectional area S and length r as shown in Fig (2.12)

Charged enclosed by this surface $\sum q = \sigma S$ ... (2.35) Form Gauss's law the flux linked with this cylindrical surface

$$\phi = \oint \vec{E}\cdot d\vec{s} = \frac{\sum q}{\epsilon_0} = \frac{\sigma S}{\epsilon_0} \qquad \text{... (2.36)}$$

This Gaussian surface can be considered to be consisting of three parts. (i) Left circular cap S_1 (ii) right circular cap S_2 (iii) curved surface S_3. So the equation (2.36) can be rewritten as

$$\int_{S_1} \vec{E}\cdot d\vec{s} + \int_{S_2} \vec{E}\cdot d\vec{s} + \int_{S_3} \vec{E}\cdot d\vec{s} = \frac{\sigma S}{\epsilon_0} \qquad \text{... (2.37)}$$

As circular surface S_1 is inside conductor so E=0 for it while for S_3 $\vec{E}$ and $d\vec{s}$ mutually perpendicular, thus $\vec{E}\cdot d\vec{s} = 0$. Also for surface S_2 $\vec{E}$ and $d\vec{s}$ are parallel and E is same for every point on S_2, from these considerations equation (2.37) yields

$$0 + E\int_{S_2} ds + 0 = \frac{\sigma S}{\epsilon}$$

or $$ES = \frac{\sigma s}{\epsilon}$$

or $$E = \frac{\sigma}{\epsilon} \qquad \text{... (2.38)}$$

In vector form

$$\vec{E} = \frac{\sigma}{\epsilon}\hat{n} \qquad \text{... (2.39)}$$

Where $\hat{n}$, is a unit vector normal to the surace of conductor. Thus, the electric field due to a uniformly charged infinite conductor plate does not depend on distance i.e. electric field is uniform. This result is approximately true for a uniformly charged conducting plate of finite dimensions for points in close vicinity of it.

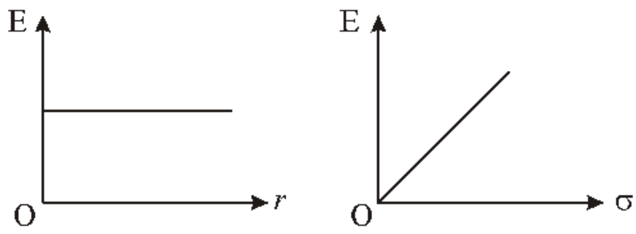


Fig 2.13 : Dependence of electric field for a uniformly charged conducting plate

Example 2.13 : The surface charge density for a uniformly chargedinfinite conductor plate is $4\times10^{-6}\,\text{Cm}^{-2}$. Find the magnitude of force on a charge -2×10^{-6} C placed near it.

Solution : Electric field for the conducting plate

$$E = \frac{\sigma}{\epsilon_0}$$

Magnitude of force on q

$$F = qE = \frac{q\sigma}{\epsilon_0} = \frac{2\times10^{-6}\times4\times10^{-6}}{8.85\times10^{-12}} = \frac{8}{8.85}$$

$$F = 0.903\,\text{N}$$

2.4.4 Electric Field Intensity due to a Uniformly Charged Spherical Shell

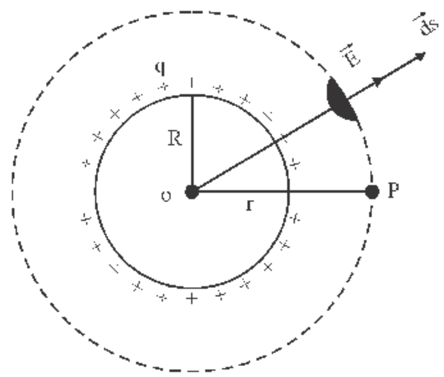


Fig 2.14 : electric field outside the charged spherical shell

Suppose a charge Q is distributed uniformly on the surface of a spherical shell of radius R. The surface charge density for this shell is then.

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2} \quad \ldots (2.40)$$

We wish to determine electric field at a point P from the centre of the shell. The Gaussian surface for such a charge distribution must be spherical. Depending upon location of P three situations are possible.

(a) When P is outside of the sphere ($r > R$)

For this case consider a spherical Gaussian surface of radius r ($r > R$) concentric with spherical shell as shown in fig 2.14. The net charge enclosed by the Gaussian surface is then

$$\Sigma q = Q \quad \ldots (2.41)$$

So the net flux linked with Gaussian surface

$$\phi = \oint_s \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\epsilon_0} = \frac{Q}{\epsilon_0} \quad \ldots (2.42)$$

The intensity of electric field $\vec{E}$ and area element $d\vec{s}$ are both act along the radial line for every point on this Gaussian surface. Also note that as each point on this surface is equidistant from centre the magnitude of $\vec{E}$ is same so from equation (2.42)

$$\phi = \oint_s \vec{E} \cdot d\vec{s} = \oint_s E ds = E \oint_s ds = \frac{Q}{\epsilon} \ldots (2.43)$$

$\because \quad \oint_s ds = 4\pi r^2 =$ Area of spherical Gaussian surface

so $\quad \phi = E \times 4\pi r^2 = \dfrac{Q}{\epsilon_0}$

so $\quad E_{out} = \dfrac{Q}{4\pi \epsilon_0 r^2} = \dfrac{\sigma R^2}{\epsilon_0 r^2} \quad \ldots (2.44)$

$\{ \because$ From equation (2.40) $\dfrac{Q}{4\pi} = \sigma R^2 \}$

From equatin (2.44) it is clear that for external points a uniformly charged spherical shell behaves as if its entire charge is concentrated at its centre. Thus the force due to a uniformly charged sphere having a charge Q on another charge placed outside the shell is same as the force on this charge due to a point charge Q placed at the centre of shell.

(b) When point P is on the surface of shell ($r = R$) : For this case on substituting r = R in equation (2.44) we obtain

$$E_S = \frac{Q}{4\pi \epsilon R^2} = \frac{\sigma}{\epsilon_0} \quad \ldots (2.45)$$

(c) When point P is inside the Shell ($r < R$) In this case Gaussian surface is well within the spherical shell (Fig 2.15) and as the charge is on the outer surface of shell the net charge enclosed by the is Gaussian surface is zero.

$$\Sigma q = 0$$

So form Gauss's law

$$\phi = \oint_s \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\varepsilon_0} = 0$$

$\because \quad ds \neq 0$ and $\vec{E}$ and $d\vec{s}$ are not mutually perpendicular hence at every point inside the spherical shell

$$E_{in} = 0 \quad \ldots (2.46)$$

Also if some charged particle is situated inside a charged spherical shell no force is exerted on it by the charge on the shell.

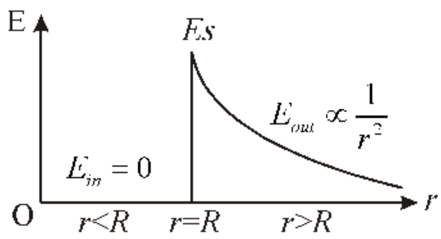


Fig 2.16 Variation with distance of electric field due to a uniformly charged spherical shell

The variation of electric field with distance for a charged sphere is shown in Fig 2.16

2.4.5 Electric Field Intensity due to a Uniformly Charged Conducting Sphere

If an excess charge is placed on an isolated conductor that amount of charge will move entirely to the surface of conductor. None of the excess charge can reside within the body of conductor. This is logical considering that charges of same sign repel each other. We may consider that by moving to the surface the added charges are getting as far away from each other as they can. Under electrostatic condition the electric field inside the conductor must be zero. For metallic conductors this is easy to explain. If it were not so the field would exert forces on free electrons and thus current would always exists with in a conductor of course, there is no such perpetual current in side a conductor and so interal electric field must be zero. An electric field does appear when excess charges is given to the conductor but the added charge distributes very quickly to the outer surface (in a time of the order of nano seconds) so the interal electric field due to all charges both inside and outside is zero. Then the movement of charges cases. As the net electric force on each charge is zero now the conductor is said to be in electronstatic equilibrium.

As the charge resides on the surface a spherical uniformly charged conductor behaves like a uniformly charged spherical shell and expressions for electric fields are exactly same as have been derived in subesection 2.4.4.

2.4.6 Intensity of Electric Field due to a Uniformly Charged Non Conducting Sphere

Consider a non conducting sphere of radius R which is given a charge Q which is distributed uniformly over its entire volume. Therefore the volume charge density is given by

$$\rho = \frac{Q}{V} = \frac{Q}{(4/3)\pi R^3} \qquad \ldots (2.47)$$

We wish to determine electric field at a point P at a distance r from the centre O of the sphere. The Gaussian surface to be considered here is spherical with radius r centred at O. Depending upon the location of P three situations are possible.

(A) When point P lies outside the charged sphere ($r > R$) : In this situation the charged enclosed by the Gaussian surface is same as the charge on the sphere under consideration (Fig 2.17)

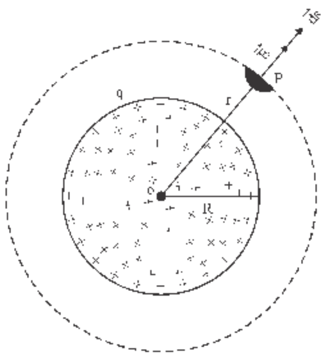


Fig 2.17: Gaussian surface for determination of electric field outside a uniformly charged non conducting sphere

Thus $\Sigma q = Q \qquad \ldots (2.48)$

Hence, from Gauss's law the flux linked with this Gaussian surface is

$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\epsilon_0} = \frac{Q}{\epsilon_0} \qquad \ldots (2.49)$$

or $\qquad \phi = \oint_S E ds = \frac{Q}{\epsilon_0} \qquad$ {$\because$ since for spherical surface $\vec{E}$ and $d\vec{s}$ are parallel}

As from symmetry magnitude of $\vec{E}$ is same at every point on Gaussian surface we heve

or $\qquad \phi = E \oint_S ds = \frac{Q}{\epsilon_0}$

or $\qquad \phi = E \times 4\pi r^2 = \frac{Q}{\epsilon_0} \qquad \ldots (2.50)$

or $\qquad E = \frac{Q}{4\pi \epsilon_0 r^2} \qquad \ldots (2.51)$

or $\qquad E = \frac{\rho}{3\epsilon_0}\left(\frac{R^3}{r^2}\right) \qquad \ldots (2.52)$

{$\because$ From equation 2.47 $Q = \frac{4}{3}\pi R^3 \rho$ }

In vector form

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2}\hat{r} = \frac{\rho}{3 \in_0}\left(\frac{R^3}{r^2}\right)\hat{r} \quad \ldots (2.53)$$

Thus we can say that for points outer to the surface the charged sphere behaves as if the entire charge is concentrated at the centre.

(b) When the point P is on the surface (r=R). In this case on substituting r=R in equation (2.53) yields

$$E = \frac{Q}{4\pi \in_0 r^2} = \frac{\rho R}{3 \in_0} \quad \ldots (2.54)$$

and in vector form

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2}\hat{r} = \frac{\rho R}{3 \in_0}\hat{r} \quad \ldots (2.55)$$

(c) When point P lies inside the sphere $(r < R)$. In this case spherical Gaussian surface is inside the charged sphere (Fig 2.18) and charge enclosed by it say Q^1 is given by

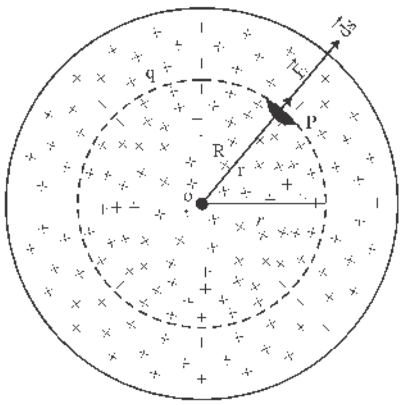


Fig 2.18 : Gaussian surface for Determining E at internal point

$$Q' = \rho \times \frac{4}{3}\pi r^3$$

$$Q' = \frac{Q}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3} \quad \ldots (2.56)$$

So the flux linked with Gaussian surface

$$\phi = \oint_S \vec{E}\cdot d\vec{s} = \frac{Q'}{\varepsilon_0} = \frac{Qr^3}{\in_0 R^3}$$

or

$$\phi = \oint_S E ds = \frac{Qr^3}{\in_0 R^3}$$

{As at each point on Gaussian surface $\vec{E} \parallel d\vec{s}$ }

From symmetry of charge distribution magnitude of E is same for every point on this surface.

So $\phi = \oint_S E ds = \frac{Qr^3}{\in_0 R^3}$

So $E = \frac{q}{4\pi \in_0 R^3} r = \frac{\rho}{3 \in_0} r \quad \ldots (2.57)$

In vector form

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^3}\vec{r} = \frac{\rho}{3\varepsilon_0}\vec{r} \quad \ldots (2.58)$$

as at centre of sphere r = 0, so from equation (2.57)

E centre = O

From above discussion for a spherical charge distribution it is clear that

(i) At the centre E = O

(ii) For the interior of the sphere, electric field is directly proportional to distance (r) from centre $E_{in} \propto r$

(iii) Electric field is maximum at the surface of sphere

(iv) For points outside the sphere electric field is inversely propotional to the square of distance

$$E_{out} \propto \frac{1}{r^2}$$

So for a uniformly charged non conducting sphere the variation of electric field with distance from the centre is as shown in Fig 2.19

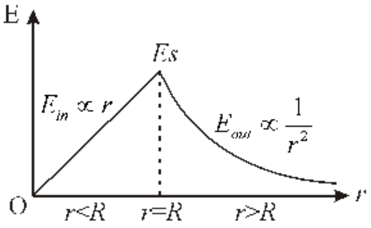


Fig 2.19 : Variation of Electric Field due to a uniformly charged non conducting sphere with distance r

Example 2.14 A conducting sphere of 10m radius is given 1 μC charge. Determine electric field at (a) its centre (b) a point 5 cm from centre (c) at a point 10 cm from centre (d) at a point 15 cm from centre in air.

Solution : (a) The electric field at the centre of a conducting sphere is zero.

(b) r = 5 cm wheres radius of sphere R = 14 cm, so the point is inside the conducting sphere. Electric field inside conductor is zero (c) r = 10 cm, point is on the surface of sphere.

$$E = \frac{Q}{4\pi \in_0 R^2} = 9\times10^9 \times \frac{1\times10^{-6}}{\left(10\times10^{-2}\right)^2}$$

$$= 9\times10^5 \text{ NC}^{-1}$$

(d) r = 15 cm, point is outside the sphere, so

$$E = \frac{Q}{4\pi \in_0 r^2} = \frac{9\times10^9\times1\times10^{-6}}{\left(15\times10^{-2}\right)^2}$$

$$= 4\times10^5 \text{ NC}^{-1}$$

Example 2. 15 : A sphere of diameter 10 cm is uniformly charged so that electric field at its surface is $5\times10^5 \text{ Vm}^{-1}$. Calculate the force on a $5\times10^{-2}\ \mu C$ charge situated at a distance of 25 cm from the centre of sphere.

Solution : Let q be the charge given to the sphere, then at surface

$$E_{\text{सतह}} = \frac{q}{4\pi \in_0 R^2}$$

Electric field outside the sphere at a distance r from centre

$$E = \frac{q}{4\pi \in_0 r^2}$$

so $$\frac{E}{E_{surface}} = \frac{R^2}{r^2}$$

or $$E = \frac{R^2}{r^2}\times E_{surface} = \frac{(5)^2}{(25)^2}\times5\times10^5 = \frac{125\times10^5}{625}$$

$$E = 2\times10^4 \text{ Vm}^{-1}$$

Force on charge $F = q_0E = 5\times10^{-8}\times2\times10^4$

$$= 10\times10^{-4} = 10^{-3} \text{ N}$$

Example 2.16 : A charge of 0.5 μC is distributed uniformly over a non conducting sphere of radius 10 cm. Determine the electric field at a point (a) at the centre of sphere (b) 8 cm from the centre (c) 10 cm from the centre (d) 20 cm from the centre in air.

Solution : At the centre of sphere E = 0

(b) When r = 8 $= 8\times10^{-2}$ m cm the point is internal to sphere so

$$E = \frac{Q}{4\pi \in_0 R^3} r$$

$$= \frac{9\times10^9\times0.5\times10^{-6}\times8\times10^{-2}}{\left(10\times10^{-2}\right)^3} = \frac{360}{10^{-3}}$$

$$E = 3.6\times10^5 \text{ V/m}$$

(c) When r = 10 cm point is on the surface of sphere

$$E = \frac{Q}{4\pi \in_0 R^2}$$

$$= \frac{9\times10^9\times0.5\times10^{-6}}{\left(10\times10^{-2}\right)^2} = 4.5\times10^5 \text{ V/m}$$

(d) When r = 20 cm, point is outside sphere so

$$E = \frac{Q}{4\pi \in_0 r^2}$$

$$= \frac{9\times10^{9}\times0.5\times10^{-6}}{\left(20\times10^{-2}\right)^{2}}$$

$$= 1.125\times10^{5} \text{ V/m}$$

2.5 Force on the surface of a charged conductor :

Excess charge given to a surface gets distributed over its surface. The charge present in any small portion of the conductor is under repulsion from the charge present in the remaining portion of the conductor. Thus a force of repulsion acts on every surface element of the conductor and the net force on the surface is the vector sum of forces acting on all such elements. Thus a charge conductor surface experiences an outward pressure.

Let the surface charge density on a conductor surface be σ. Now consider two points P_1 and P_2 placed symmetrically with respect to the conductor with P_1 just inside and P_2 just outside the conductor (Fig 2.20)

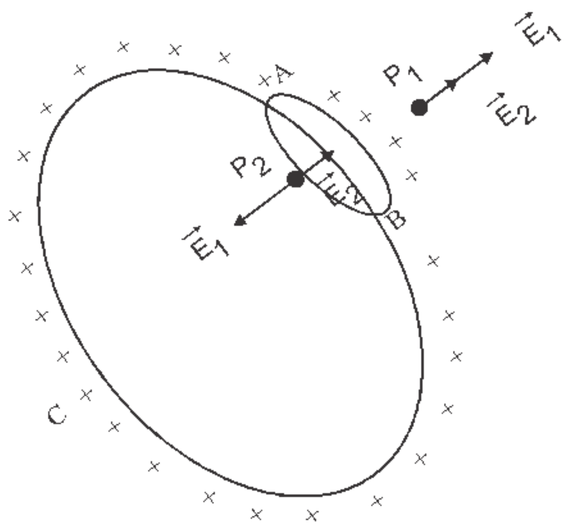


Fig 2.20 : Determination of force on the surface a charged conductor

As the electric field out side the conductor is $\sigma / \in_0$ so the electric field at point P_1

$$E_{P_1} = \frac{\sigma}{\in_0} \qquad \ldots(2.60)$$

and since the electric field inside a conductor is zero so at point P_2

$$E_{P_2} = 0 \qquad \ldots(2.61)$$

Next, we consider this conductor to be consisting of two parts (i) element AB having area ds and (ii) remainder of the conductor, ACB. It $\vec{E}_1$ and $\vec{E}_2$ are the electric field due to AB and ACB respectively at points in their near vicinity then from fig

$$E_{P_1} = E_1 + E_2 \qquad \ldots(2.62)$$

(E_1 and E_2 are in same direction at Point P_1)

$$\text{and } E_{P_2} = E_1 - E_2 \qquad \ldots(2.63)$$

(E_1 and E_2 are directed opposite at point P_2)

From equation (2.16) and (2.63)

$$E_1 - E_2 = 0$$

$$\text{i.e.} \quad E_1 = E_2 \qquad \ldots(2.64)$$

From equation (2.60) (2.62) and (2.64)

$$E_2 + E_2 = \frac{\sigma}{\in_0}$$

$$\text{or} \qquad E_2 = \frac{\sigma}{2\in_0} \qquad \ldots(2.65)$$

So, the electric field due to part ACB at the location of area element AB can be considered as $\frac{\sigma}{2\in_0}$.

If the total charge on element AB is dq then force on it

$dF = E_2 dq = \frac{\sigma}{2\in_0} dq$; As $(\because dq = \sigma ds)$, so

$$dF = \frac{\sigma^2}{2\in_0} ds = \frac{1}{2}\in_0 E^2 ds \qquad \ldots(2.66)$$

$$\left\{ \because E = \frac{\sigma}{\in_0} \text{ so } \sigma = \in_0 E \right\}$$

Force acting on the complete surface is then given by

$$F = \oint_S \frac{\sigma^2}{2\in_0} ds = \oint_S \frac{\in_0 E^2}{2} ds \qquad \ldots(2.67)$$

and the force per unit area i.e. pressure

$$P = \frac{dF}{ds} = \frac{\sigma^2}{2 \in_0} = \frac{1}{2} \in_0 E^2 \quad \text{. . . (2.68)}$$

this pressure is called as electrostatic pressure

2.6 Energy per unit Volume for an electric Field

We have seen that an electric force acts along outward normal on the surface of a charged conductor. For increasing the amount of charge on the conductor or to increase the volume of region in which the electric field is present, the work is required to be done agains this force. This work gets stored in the form of energy in electric field.

For the sake of simplicity we consider a spherical shell of radius r for which surface charge density is σ (Fig 2.21)

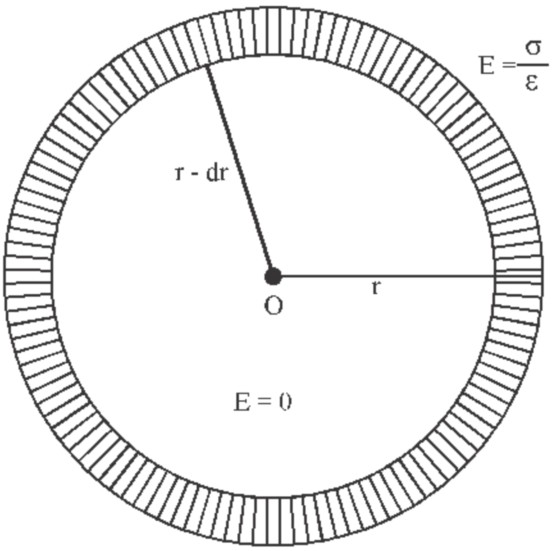


Fig 2.21 : Spherical Charge Distribution

Outward pressure on the surface of shell

$$P = \frac{\sigma^2}{2 \in_0} \quad \text{. . . (2.69)}$$

So the outward force on the surface

$$F = PA = \frac{\sigma^2}{2 \in_0} \times 4\pi r^2 \quad \text{. . . (2.70)}$$

Work done against this force in compressing the shell by a small amount dr is

$$dW = Fdr = \frac{\sigma^2}{2 \in_0} 4\pi r^2 dr$$

Reduction in volume of sphere (or increase in volume of the region where electric field is present)

$dV = 4\pi r^2 dr$

So $$dW = \frac{\sigma^2}{2 \in_0} dV \quad \text{. . . (2.71)}$$

So energy stored in electric field

$$W = U = \int \frac{\sigma^2}{2 \in_0} dV = \int \frac{1}{2} \in_0 E^2 dV \quad \text{. . . (2.72)}$$

and the energy stored per unit voume in electric field or energy density.

$$U_V = \frac{dW}{dV} = \frac{\sigma^2}{2 \in_0} = \frac{1}{2} \in_0 E^2 \quad \text{. . . (2.73)}$$

If some other medium other then free space or air is considered then

$$U_V = \frac{dW}{dV} = \frac{\sigma^2}{2 \in_0} = \frac{1}{2} \in_0 E^2 \quad \text{. . . (2.73)}$$

Although the above relations have been derived by considering a spherical shell but their validity is general.

2.7 Equilibrium of a Charged Soap Bubble

For a soap bubble, the pressure at its internal surface is more than the atmospheric pressure present at its outer surface. This excess pressure is balanced by pressure due to surface tension. If the radius of bubble is r and surface tension is T then excess pressure.

$$P_{ex} = \frac{4T}{r} \quad \text{. . . (2.74)}$$

If now the soap buble is charged with surface charge density σ then an outward electricstatic pressure $\frac{\sigma^2}{2 \in_0}$ also acts on the surface of bubble. In this case

$$P_{ex} + \frac{\sigma^2}{2\in_0} = \frac{4T}{r}$$

or $$P_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2\in_0} \quad \ldots (2.75)$$

On charging bubble in such a manner a situation arise in which excess pressure becomes zero after which the bubble bursts, so for equilibrium,

$$P_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2\in_0} = 0$$

or $$\frac{4T}{r} = \frac{\sigma^2}{2\in_0} \quad \ldots (2.76)$$

Equilibrium radius of bubble $$r = \frac{8T\in_0}{\sigma^2} \quad \ldots (2.77)$$

and surface charge density $$\sigma = \sqrt{\frac{8T\in_0}{r}} \quad \ldots (2.78)$$

and charge $$q = \sigma \times 4\pi r^2 = 4\pi\sqrt{8T\in_0 r^3} \quad \ldots (2.79)$$

Example 2.17 : The surface charge density for a charged soap bubble is 2.96 μC / m². The surface tension of soap solution is 4×10^{-4} N/m . Find the radius of soap bubble so that excess pressure is zero and the bubble is in equilibrium.

Solution :

$$r = \frac{8T\in_0}{\sigma^2} = \frac{8\times4\times10^{-4}\times8.85\times10^{-12}}{\left(2.96\times10^{-6}\right)^2}$$

$$= 3.2\times10^{-1}\ \text{m}$$

$$r = 0.32\ \text{m}$$

Important Points

1. Electric flux through a surface is proportional to the net number of electric field lines passing through that area. electric flux through a flat area in a uniform electric field is

   $\phi = ES\cos\theta$

   Where θ is the angle between direction of E and normal to the area. If electric field vector is denoted by $\vec{E}$ be and area vector by $\vec{S}$ then

   $\phi = \vec{E}\cdot\vec{S}$

2. If the electric field is non uniform and (or) the surface is not flat then electric flux

   $\phi = \int \vec{E}\cdot d\vec{s}$

   Where $d\vec{s}$ is vector area of some surface element and $\vec{E}$ is electric field at this element. If the surface is closed

   $\phi = \oint \vec{E}\cdot d\vec{s}$

3. Depending upon the relative orientation of $\vec{E}$ and $\vec{S}$ (or $d\vec{s}$) ϕ may be positive, negative or zero.

4. For determining electric field due to some continuous charge distribution, it is divided into small charge elements dq and electric field due to each such element is then integrated to obtain total electric field. Electric field at a distance r from charge element dq

$$dE = \frac{1}{4\pi \epsilon_0} \frac{dq}{r^2}$$

For complete continuous distribution

$$E = \int dE = \frac{1}{4\pi \epsilon_0} \int \frac{dq}{r^2}$$

If charge distribution is linear $dq = \lambda d\ell$ where λ is charge linear charge density and is length of element.

If charge distribution is superfecial $dq = \sigma ds$ σ is surface charge density and ds is area of element.

In case of a volume charge distribution $dq = \rho dV$ where p is volume charge density and dV is volume element.

5. Gauss's law is valid only for closed surfaces. According to it the the net flux through a closed surface in an electric field

$$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

here q is the net charge enclosed by the closed surface.

6. Gauss's law is very helpful for finding electric field when charge distributions have a high degree of symmetry.

7. For an infinite line charge of linear charge density λ

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

8. For a uniformly charged infinity non conducting sheet (surface charge density σ)

$$E = \frac{\sigma}{2\varepsilon_0} \text{ (uniform electric Field)}$$

9. For a uniformly charged infinite conducting (surface charge density) $E = \frac{\sigma}{\epsilon_0}$ (uniform electric field)

10. Continuous charge distribution can be classified as

(i) linear charge distribution distribution (ii) surface cahrge distribution (iii) volume charge distribution

11. For a uniformly charged spherical shell or spherical conductor Esurface

$$E_{surface} = \frac{q}{4\pi \epsilon_0 R^2} (r = R); \; E_{in} = 0$$

[Here Q is the charge on sphere and R is its radius]

12. For a uniformly charged non conducting sphere

$$E_{surface} = \frac{Q}{4\pi \in_0 R^2}(r = R)$$

$$E_{in} = \frac{Qr}{4\pi \in_0 R^3}(r < R)$$

13. Pressure on surface of a charged conductor

$P = \frac{\sigma^2}{2 \in_0} = \frac{1}{2} \in_0 E^2$ this is due to mutual repulsion between charges residing on surface.

14. Energy per unit volume in an electric field

$$U_V = \frac{\sigma^2}{2 \in_0} = \frac{1}{2} \in_0 E^2$$

15. For the equilibrium of a charged soap bubble.

$\frac{4T}{r} = \frac{\sigma^2}{2 \in}$ (T is surface Tension, r is radius of soap bubble)

Questions For practice

Multiple Choice Questions

1. The electric field due to a uniformly charged solid non conducting sphere is maximum at -

(a) the centre

(b) the mid point between centre and surface

(c) its surface (d) infinity

2. For free space the energy density in some region where electric field is E, is given by

(a) $\frac{1}{2} \in_0 E$ (b) $\frac{E^2}{2 \in_0}$

(c) $\frac{1}{2} E \in_0^2$ (d) $\frac{1}{2} \in_0 E^2$

3. 1 μC charge is present at the centre of a cube of edge a. The flux through each face of the cube will be (in VM)

(a) 1.12×10^4 (b) 2.2×10^4

(c) 1.88×10^4 (d) 3.14×10^4

4. Two electric dipoles having charges $\pm q$ are placed mutually perpendicular to each other. The net electric flu through the cube is

(a) $\frac{q}{\in_0}$ (b) $\frac{4q}{\in_0}$

(c) Zero (d) $\frac{2q}{\in_0}$

5. On charging a soap bubble with negative charge its radius

(a) decreases

(b) increases

(c) remains unchanged

(d) nothing can be said due to incomplete information

6. A charge q is in a sphere and flux through the sphere is $\frac{q}{\in_0}$. On reducing the radius of sphere by half the change in the flux is

(a) four times its intital value

(b) one fourth of its initial value

(c) half of its initial value

(d) unchanged

7. Complete flux due to a unit charge placed in air is

(a) $\in_0$ (b) $\in_0^{1}$

(c) $(4\pi \in_0)^{-1}$ (d) $4\pi \in_0$

8. The radii of two conducting spheres are a and b. When these are charged with same surface charge density the ratio of electric field intensities at their surfaces is

(a) $b^2 : a^2$ (b) 1 : 1

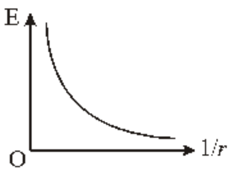
(c) $a^2 : b^2$ (d) b : a

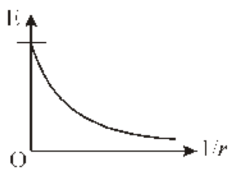
9. The radii of two conducting spheres are a and b. When these are given same charge the ratio of electric field intensities at their surfaces is

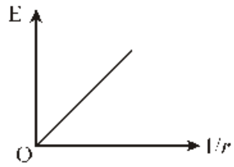
(a) $b^2 : a^2$ (b) 1 : 1

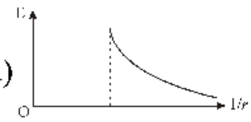
(c) $a^2 : b^2$ (d) $b : a$

10. Electric field intensity due to a long straight charged wire varies with $1/r$ as shown in

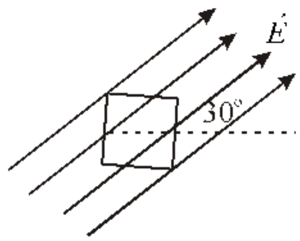
(a)


(b)


(c)


(d)


11. A square is situated in a uniform horizantal electric field such that a line drawn in the plane of square makes angle of 30° with electric field (Fig). If side of square is a the flux through the sphere will be



(a) $\frac{\sqrt{3}Ea^2}{2}$ (b) $\frac{Ea^2}{2}$

(c) Zero (d) none of these

Very Short Answer Questions :

1. When does the electric flux through an area element placed in an electric field $\vec{E}$ is zero?
2. At what positions the electric field intensity due to a uniformly charged sphere is zero?
3. Write the expression for force per unit area of a charged conductor and give its direction.
4. Where does the energy due to a charge is stored?
5. A charge Q is a given to a conducting sphere of diameter d what is the value of electric field inside the sphere?
6. Suppose the Coulomb's law has a $1/r^2$ dependence instead of $1/r^2$ dependence, is the Gauss's law still valid?
7. If the net charge enclosed by a Gaussian surface is positive then what is the nature of flux through the surface?

8. If the net flux through some closed surface in an electric field is zero what can be said about the surface?

9. If net charged enclosed by a Gaussian surface is zero does it mean that electric field at every point on the surface is zero.

10. Define linear charge density.

11. What will be the change in electric field in moving from one side to the other of a charged plane sheet having surface charge density σ.

12. Graph the variation of electric field with distance for a uniformly charged non conducting sheet.

13. What is the value of electric field at the centre of a uniformly charged non conducting sphere?

14. A charge q is at the centre of a sphere. If now this charge is placed at the centre of a cylinder of same volume then what will be the ratio of net flux in the tweo cases?

Short Answer Questions :

1. Explain the term electric flux. Write its SI unit and dimensions.

2. Explain the term linear charge density. Write its SI unit.

3. Explain the term surface charge density. Write its SI unit.

4. Explain the term volume charge density. Write its SI unit.

5. State Gauss's law for electrostatics.

6. The excess charge given to a conductor reside always on its outer surface? Why?

7. Establish expressions for electric force and electrostatic pressure on the surface of a charged conductor?

8. Establish expression for energy stored per unit volume in electric field.

9. Establish expression for maximum charged density for the equilibrium of a charged soap bubble.

10. Verify Gauss's law from Coulomb's law.

11. You are travelling in a car. Lighting is expected what should you do about your safety?

12. Consider two long straight line charges having linear charge densities λ_1 and λ_2 . Derive expression for the force per unit length acting between them.

13. Consider two infinite parallel planes having charge densities + and - respectively. What is the magnitude of electric field at some point in region between them.

Essay type Questions

1. For a spherical conductor of radius R having a charge q determine electric field for following situations

   (A) $r > R$ (B) $r < R$

   (C) at its surface (D) at its centre

   Graph the variation of electric field with distance.

2. Determine the electric field due to a uniformly charged sphere for following cases

   (A) Outside the sphere

   (B) At the surface of sphere

   (C) Inside the sphere

   (D) At the centre of sphere

3. Using Gauss's law determine the intensity of electric field at a point near a uniformly charged infinite wire. Graph the variation of electric field with distance.

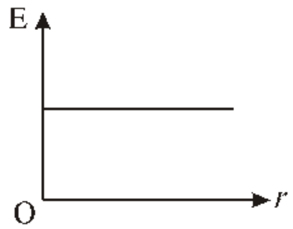
4. Using Gauss's law determine the intensity of electric field at a point near a uniformly charged infinite non conducting plane. Explain the dependence of electric field.

5. Determine the direction of electric field due to a uniformly charged infinite conducting plate for points in its vicinity. Using Gauss's law determine expression for its electric field. Draw necessary diagrams.

Answers (Multiple Choice Questions)

1. (c) 2. (d) 3. (c) 4. (c) 5. (b) 6. (d) 7. (b)
8. (b) 9. (a) 10. (c) 11. (c)

Very Short Answer Questions :

1. When $\vec{E}$ and $d\vec{s}$ are mutually perpendicular.
2. At its centre and infinity
3. $\dfrac{\sigma^2}{2\in_0}$ and is directed normally outward.
4. In the region of elecric field.
5. Zero
6. No, Gauss's law holds only for fields which obeys inverse square law.
7. Positive and outwards
8. Net charged enclosed by the surface is zero and $\phi_{out} = \phi_{in}$
9. No, from $\phi = \oint_S \vec{E}\cdot d\vec{s} = \dfrac{q}{\in_0} = 0$ this situation is possible when $\vec{E}$ but $\vec{E}$ is perpendicular to $d\vec{s}$.
10. Amount of charge per unit length.
11. $\dfrac{\sigma}{2\in_0} - \left(-\dfrac{\sigma}{2\in_0}\right) = \dfrac{\sigma}{\in_0}$
12. 

13. Zero
14. 1 : 1

Numerical Problems

1. The flux entering and leaving a closed surface are400 Nm^2/C and 800 $N\,m^2/C$ respectively. What is the net charge enclosed by this surface. (Ans : 3.54 nC)
2. The surface charge density on a uniformly charged conducting sphere is $80\,\mu C/m^2$. Calculate the charge on sphere and net flux through surface.

   (Ans : 1.45 mC, $1.63 \times 10^8\ Nm^2/C$)
3. Consider a cube of side a, Let a charge q be placed

   (i) at entre (ii) at one corner of cube

   (iii) at one face of the cube

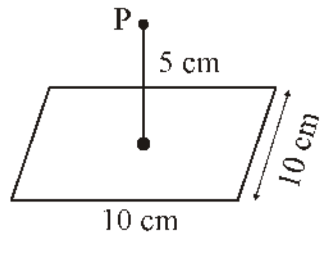
   For each of the above cases calculate the total flux linked with cube and flux linked with each face.

Ans : (i) $\dfrac{q}{6\varepsilon_0}\ \dfrac{q}{6\varepsilon_0}$ (ii) $\dfrac{q}{4\in_0}, \dfrac{q}{16\in_0}$ (iii) $\dfrac{q}{2\in_0}, \dfrac{q}{10\in_0}$

4. The intensity of electric field due to a charged sphere at a point at a distance of 20 cm from in centre is 10 V / m . The radius of sphere is 5 cm. Determine the intensity of electric field at a distance of 8 cm from the centre.

   (Ans : 62.5 V / m)
5. An infinite line charge produces on electric field of 9×10^4 N/C at 2 cm from it. Determine the linear charge density.

   (Ans : 10^{-7} C/m½)
6. A charge of 10 μC is placed directly above the centre of a square of 10 cm side at a height of 5 cm as shown in Fig. Determine the magnitude of electric flux through the square?



   (Ans : $1.88 \times 10^5\ Nm^2/C$)
7. A charge of 10 μC is given to a metallic plate of area $10^{-2} m^2$. Determine the intensity of electric fields at points near by.

   (Ans : 5.65×10^7 V/ m)
8. Two metallic plates each of area are $1m^2$ placed parallel to each other at a separation of 0.05 m. Both have charges of equal magnitude but of opposite nature. If the magnitude of electric field in

space between them is 5.5 V/m then calculate the magnitude of charge on each plate.

(Ans : 4.87×10^{10} C)

9. A particle of mass 9×10^{-5} gm is placed at some height above a uniformly charged horizontal infinite non conducting plate having a surface charge density 5×10^{-5} C/ m². What should be the charge on the particle so that on releasing it will not fall down.

(Ans : 3.12×10^{-13} C)

10. A large uniformly charged sheet having a surface charge density of 5×10^{16} C / m² lies in X – Y plane. Calculate the electric flux through a circular loop of radius 0.1 m, whose axis makes on angle of 60^0 with Z axis.

(Ans : 4.44×10^{7} Nm^2 / C)

11. An electron of 10^3 eV energy is fired from a distance of 5mm perpendicularly towards an infinite charged conducting plate. What should be the minimum charge density on plate so that electron fails to strike the plate.

(Ans : 1.77×10^{-6} C / m^2)

12. The internal and external pressures for a soap bubble are same. The surface tension or soap soultion is 0.04 N/m and its diameter is 4 cm. Determine the charge on soap bubble.

(Ans : 59.8 nC)

Chapter - 3
Electric Potential

In previuos chapter we have studied electric field due to a point charge or group of charges and described it in terms of a vector $\vec{E}$ called intensity of electric field. We have also studied continous charge distribution,force between charges and to calculate $\vec{E}$ using Coulomb's law and Gauss's law. In this chapter we will see that the electric field can also be decribed in terms of a scalar quantity called "electrostatic potenial" V. This is an important concept. It is so because electric field vector $\vec{E}$ and electrostatic potential V can be related to each other. Since V is a scalar quantity and addition of scalars is far easier than addition of vectors, in many problems it is much easier to find V first and then use it to find $\vec{E}$ rather than obtaining $\vec{E}$ from direct calculations. The concept of potential is also important from the point of view that potential is related to potential energy. Thus, by using the law of conservation of energy we can solve many problems in electrostatics without goining into the details of forces involved like we did in study of mechanics.

In this chapter first of all we will define electric potential. After that we will learn to calculate electric potential due to a point charge and system of charges. Subsequently we will study the relation between electric field and potential. After studying about electric potential due to some specific charge configurations, we will study about electrocstatic potential energies of such systems. In the end of this chapter we will learn about the work done in rotating an electric dipole in some external electric field and calculate its potential energy.

3.1 Electrostatic Potential and Potential Difference

From our understanding of conservative forces we know that a potential energy is associated with a conservative force. From experiments it is known that the electric field (force) is conserative and thus has an associated potential energy U called as the electrostatic potential energy. (Unless stated otherwise, in this chapter and chapters to follow the term electric field refers to electrostatic field. In the study related with phenomenon of electro magnetism we shall see that a changing magnetic field also produced an electric field which is not conservative and potential energy can not be associated with such a field).

Consider a positive test charge q_o being brought from some point A to some point B in an electric field. Here we are assuming that in the process the test charge does not distrub source charge(s) which produce electric field i.e. all other charges present in the surrounding remain at their respective places. If in this process the potential energy changes by $U_B - U_A$ then the potential difference between points A and B is defined as

$$V_B - V_A = \frac{U_B - U_A}{q_0} = \frac{\Delta U}{q_0} \qquad \ldots(3.1)$$

The above equation defines the electrostatic potential difference between two points for a given electric field. To define absolute potential (which from now on we call simply electric potential) at a point we can select a reference point at which we consider both potential energy and potential as zero. Generally we take this reference position to be at infinity. Thus if we consider the point A to be at infinity so $V_A = V_\infty = 0$ and $U_A = U_\infty = 0$ and then from equation (3.1)

$$V_B = U_B / q_0$$

Since point B is arbitrary so in general the above equation can be written as

$$V = U / q_0 \qquad \ldots(3.2)$$

Thus electric potential at a point is defined as electric potential energy per unit charge. Clearly electric potential is a scalar quantity and is independent of test charge. It is a characteristic of electric field only.

We know that energy and work are related with each other so we can define potential difference and potential in terms of work. If work done by conservative electric field in moving the system from initial position to final position is denoted by W_e then

$$\Delta U = -W_e$$

therefore the potential difference between points A and B is

$$\Delta V = V_B - V_A = -\frac{W_e}{q_0} \qquad \text{. . . (3.3)}$$

So, the potential difference between two points in an electric field is equal to the negative of the work done by the electric field in bringing a unit positive charge from initial position (A) to final position (B). Depending upon the signs and magnitudes of W_{ba} and q_0 potential difference between two points can be positive, negative or zero. If we assume the potential at infinity (reference point) to be zero then from equation 3.3 we can define potential at a point by

$$V = -\frac{W_{e\infty}}{q_0} \qquad \text{. . . (3.4)}$$

Where $W_{e\infty}$ is the work done by electric field on test charge q_0 in bringing it from infinity (reference point) to the point under consideration. Thus, the electric potential at a point is equal to the negative of the work done by electric field in bringing a unit positive charge from infinity (reference position) to given point.

Suppose we move a particle of charge q_0 from point A to point B in an electric field with the help of some external agent (force). If the motion of the particle does not involve any charge in its kinetic energy i.e $\Delta K = 0$ then by the work-kinetic energy theorm

$$W_{ext} = -W_e \qquad \text{. . . (3.5)}$$

Where W_{ext} refers to the work done by external force during the move, then from equations (3.3) and (3.5)

$$W_{ext} = -W_e \qquad \text{. . . (3.6)}$$

and as assumed earlier if we take point A to be at infinity (reference position) then potential at a point is

$$\Delta V = V_B - V_A = \frac{W_{ext}}{q} \qquad \text{. . . (3.6)}$$

Where $W_{\infty A}$ refers to the work done by external force in bringing the charge q_0 from infinity to point under consideration. Accordingly the electric potential at a point is equal to the work done by the external agent (without changing Kinetic energy) on a unit positive charge in bringing it from infinity (reference position) to the desired point.

From above discussion it is obvious that there are many equivalent definitions for electric potential. (In forth coming subsection we will define electric potential as a line integral of electric field). However , from each definition it is apparent that electric potential is a scalar quantity.

The SI unit of electric potential is volt with

$$1 \text{ volt (V)} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$$

The electric potential at a point is 1 volt; if in moving a unit charge from infinity to that point the work done is 1 joule. Dimensions of electric potential are

$$[V] = \left[\frac{ML^2T^{-2}}{TA}\right] = \left[ML^2T^{-3}A^{-1}\right]$$

It is obvious that unit of potential difference is also volt. This unit of electric potential allows us to adopt a more conventional unit V / m for the electric field E (which till now we have described in unit of N/C) i.e

$$1\, N/C = 1V/m$$

We leave the verification of this expression as an excerise for readers. After defining 'volt' we can now define an energy unit called electron volt (eV) which is convenient for measuring atomic and nuclear energies. One electron volt (eV) is the energy equal to the work needed to move a single elementary charge e (electron or proton) through a potential difference of exactly one volt. From equation

$$W = q(\Delta V)$$

$$1\,eV = e(1V) = \left(1.6\times10^{-19}\,C\right)(J/C)$$

$$= 1.6\times10^{-19}\,J$$

3.1.1 Electric potential Derived From Electric Field

Consider an arbitrary electric field for which electric field lines are as shown in Fig 3.1. Let a positive test charge q_0 move in this field along the curved path shown from point A to B. At any point on this path an electric force $\vec{F}_e = q_0\vec{E}$ acts on it for a differential displacement $d\vec{\ell}$, here $\vec{E}$ is the electric field intensity at the location of differential element. So the work done by electric force during this displacement is given by

$$dW = \vec{F}\cdot d\vec{\ell} = q_0\vec{E}\cdot d\vec{\ell} \qquad ...(3.8)$$

Therefore the total work W done by the electric force as the particle moves from point A to point B is

$$W_e = \int_A^B q_0\vec{E}\cdot d\vec{\ell} = q_0\int_A^B \vec{E}\cdot d\vec{\ell} \qquad ...(3.9)$$

From equations (3.3) and (3.9)then

$$V_B - V_A = -\frac{q_0}{q_0}\int_A^B \vec{E}\cdot d\vec{\ell}$$

$$= -\int_A^B \vec{E}\cdot d\vec{\ell} \qquad ...(3.10)$$

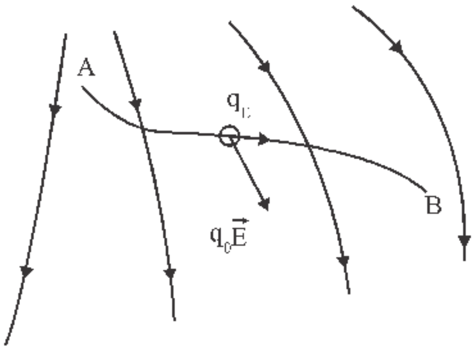


Fig 3.1 : Motion of a test charge from point A to point B in a non uniform field

The integral present in right hand side of equation 3.10 is called line integral (meaning the integral along a particular path) of $\vec{E}$ from A to B. However, since the electric field is consvertive all paths (between A and B) give the same result. Thus the potential difference between any two points A and B in an electric field is equal to the negative of the line intgral of $\vec{E}$ from A to B.

If electric field is along $d\vec{\ell}$ then integral in equation 3.10 is positive and potential difference negative i.e $V_B < V_A$. Electric field tends to move a positive charge from high potential to low potential and tend to move a negative charge from low potential to high potential.

In equation 3.10 if we consider A to be at infinity (reference position) and set $V_A = 0$ then

$$V = -\int_\infty^B \vec{E}\cdot d\vec{\ell} \qquad ...(3.11)$$

Equation 3.11 gives us the potential at any point realtive to zero potential at infinity (reference point).

3.2 Potential Due to a Point Charge

For a point charge Q electric field at a distance r is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad ...(3.12)$$

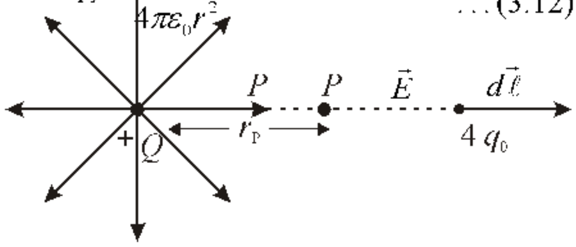


Fig 3.2 : Determinations potential due to a point charge here a test charge q_0 is being moved from point p to infinity

If Q is positive $\vec{E}$ is directed radially outward from the charge. Now we will make use of equations 3.10 for obtaining expression of potential at some point in this field. To do so let us imagine that a test charge q is being moved from a point P to infinity on a radial line along the direction of $\vec{E}$ as shown in fig 3.2. For such a path differntial displacement $d\vec{\ell}$ can be written as $d\vec{\ell} = d\vec{r}$ and as $\vec{E}$ and are in same direction $\vec{E}\cdot d\vec{\ell} = Ed\ell = Edr$. Using equation 3.10 (along with above mentioned charges) between the limits r_P to ∞ we obtain

$$V_\infty - V_P = -\int_{r_P}^{\infty} \vec{E}\cdot d\vec{r} = -\int_{r_P}^{\infty} Edr$$

or $-V_P = -\int_{r_P}^{\infty} \frac{Q}{4\pi \in_0 r^2} dr \quad [\because V_\infty = 0]$

or $V_P = \frac{Q}{4\pi \in_0}\left[\frac{-1}{r}\right]_{r_P}^{\infty} \quad \left[\because \int \frac{1}{x^2} dx = -\frac{1}{x}\right]$

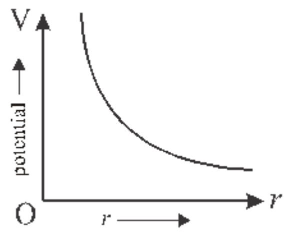
$$= \frac{Q}{4\pi \in_0}\left[\frac{1}{r}\right]_{\infty}^{r_P}$$

$$V_P = \frac{Q}{4\pi \in_0 r_P}$$

as P can be any arbitrary point so in general

$$V = \frac{Q}{4\pi \in_0 r} \quad \ldots (3.13)$$

Therefore the potential due to a point charge is inversely proptional to the distance between point charge and the observation point. It does not depend on the direction of observation point relative to the point charge. This variation is shown graphically in fig 3.3.



r = distance of observation (r) from point charge

Fig 3.3 : Graphical variation of potential due to a point charge with distance

If the source charge is negative

$$V = \frac{1}{4\pi \in_0}\frac{(-Q)}{r} \quad \ldots (3.14)$$

For an isolated positive charge (Q > O) electric potential is positive while for an isolated negative charge (Q < O) it is negative. For a given charge and a given distance potential in some medium is less then the potential in free space and is given by

$$V_m = \frac{V}{\in_0 \in_r}$$

3.3 Potential due to a Group of Point Charges

Electric potential is a scalar quantity so we can find the net potential at a point due to a group of point charges by algebraic sum of potentials due to individual charges at that point. Suppose we wish to determine the potential at a point P due to a group of point charges. Let distance of point P from charges, $q_1, q_2, q_3 \ldots q_n$ be $r_1, r_2, r_3,$ respectively (Fig 3.4. Then as stated above the net potential at P is given by

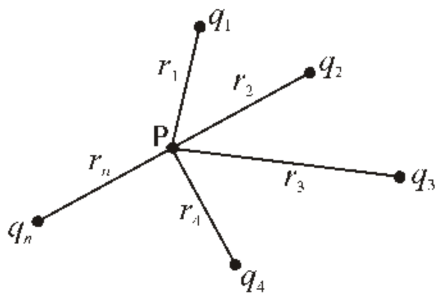


Fig 3.4 : The potential at point P due to a system of point charges

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$V = \frac{1}{4\pi \in_0}\frac{q_1}{r_1} + \frac{1}{4\pi \in_0}\frac{q_2}{r_2} + \ldots + \frac{1}{4\pi \in_0}\frac{q_n}{r_n}$$

$$V = \frac{1}{4\pi \in_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right]$$

$$V = \frac{1}{4\pi \in_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad \ldots (3.15)$$

Example 3.1 : The electric potential at some point is -15 V and at some other point is V(volt). If 150 J of work is needed to move a 6 coulomb charge from the first point to the second then find the value of V.

Solution : Here ; $V_A = -15$ Volt, $W_{exe} = 150$ J

$$V_B = V \;,\; q_0 = 6\,\text{C}$$

$$\because \quad V_B - V_A = \frac{W_{ext}}{q_0}$$

$$V - (-15) = \frac{150}{6} = 25 \text{ volt}$$

$V = 25 - 15 = 10$ volt

Example 3.2 The work done in displacing a 20 C charge through 0.2m for moving it from a point A to a point B is 2 J. Find the potential difference between the two points.

Solution : Here $q_0 = 20\,C$

$W_{ext} = 2$ J

therefore, $V_B - V_A = \dfrac{W_{ext}}{q_0} = \dfrac{2}{20} = 0.1$ V

Example 3.3 Calculate the electric potential at a distance 10 cm from a charge $1.1\times10^{-9}\,C$ in air.

Solution : Here $Q = 1.1\times10^{-9}\,C$

$r = 10\ cm - 0.1\ m$

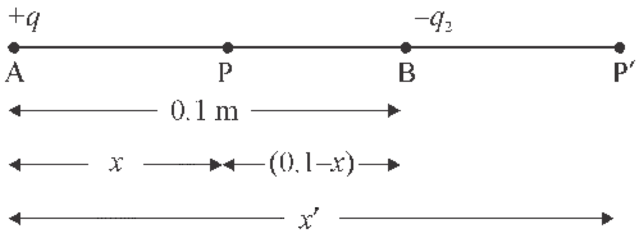
So electric potential

$$V = \frac{1}{4\pi \in_0}\frac{(Q)}{r}$$

$$V = \frac{9\times10^9\times1.1\times10^{-9}}{0.1} = 99\text{ V}$$

Example 3.4 The distance between two charges $4\times10^{-9}\,C$ and $-3\times10^{-9}\,C$ is 0.1 m. Where on the line joining the two charges potential is zero? Assume the potential to be zero at infinity.

Solution : Refer figure shown, here q_1 and q_2 are of opposite nature but $|q_1 > q_2|$. Therefore there will not be any point in the region left of q_1 where the potentials due to q_1 and q_2 are canceling each other. Such points can be in between q_1 and q_2 or to the right of q_2 on the line joining. Such points are shown in Fig by P and P'



If distance of P from q_o is x then for potential at P to be zero

$$V_A + V_B = O$$

$$\frac{1}{4\pi \in_0}.\frac{q_1}{x} = -\frac{1}{4\pi \in_0}\frac{q_2}{(0.1-x)}$$

$$\frac{4\times10^{-9}}{x} = -\frac{-(-3\times10^{-9})}{(0.1-x)}$$

$$3x = 0.4 - 4x$$

$$x = \frac{0.4}{7} = 0.057m$$

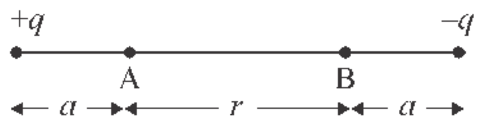
If P'is at distance x' from q_1 then for potential at P' to be zero

$$\frac{4\times10^{-9}}{x'} = \frac{3\times10^{-9}}{(x'-0.1)}$$

$\Rightarrow \quad 3x' = 4x - 0.4$

$\Rightarrow \quad x' = 0.4m$

Example 3.5 Two charges + q and - q are arranged as shown, the potentials at points A and B are V_A and V_B then Calculate $V_A - V_B$



Solution : Potential at A due to +q

$$= \frac{1}{4\pi \in_0}\times\frac{q}{a}$$

potential at A due to - q

$$= \frac{1}{4\pi \in_0} \frac{(-q)}{(a+r)}$$

total potential at A

$$V_A = \frac{1}{4\pi \in_0}\left(\frac{q}{a} - \frac{q}{a+r}\right)...$$

Potential at B due to + q

$$= \frac{1}{4\pi \in_0} \frac{q}{a+r}$$

Potential at B due to - q

$$= \frac{1}{4\pi \in_0}\left(\frac{-q}{a}\right)$$

total potential at B

$$V_B = \frac{1}{4\pi \in_0}\left(\frac{q}{a+r} - \frac{q}{a}\right)...$$

So

$$V_A - V_B = \frac{1}{4\pi \in_0}\left[\left(\frac{q}{a} - \frac{q}{a+r}\right) - \left(\frac{q}{a+r} - \frac{q}{a}\right)\right]$$

$$= \frac{1}{4\pi \in_0} 2q\left(\frac{1}{a} - \frac{1}{a+r}\right)$$

$$= \frac{1}{4\pi \in_0} 2q\left(\frac{(a+r)-a}{a(a+r)}\right)$$

$$V_A - V_B = \frac{1}{4\pi \in_0} \frac{2qr}{a(a+r)}$$

3.4 Electric Potential Due to Electric Dipole

Fig 3.5 depicts an electric dipole AB. Charges at A and B are -q and + q and separation between them is 2a. We wish to calculate potential at point P at a distnace r from the centre O of the dipole. Line OP makes an angle θ with the axis of dipole. Potential at P due to charge - q at A

$$V_1 = \frac{1}{4\pi \in_0} \frac{(-q)}{r_1} \qquad ...(3.16)$$

and potential at P due to charge + q at B

$$V_2 = \frac{1}{4\pi \in_0} \cdot \frac{q}{r_2} \qquad ...(3.17)$$

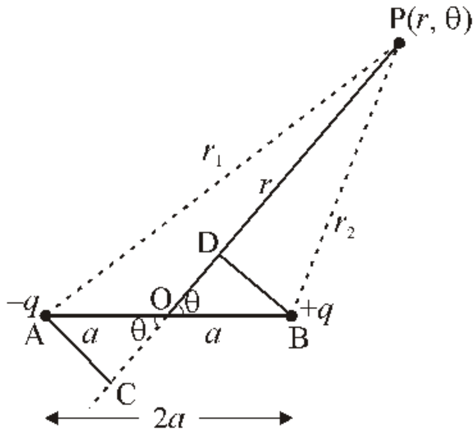


Fig 3.5 : Potential due to dipole at point **P** (r, θ)

So the net electric potential at P

$$V = \frac{q}{4\pi \in_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{q}{4\pi \in_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right] \qquad ...(3.18)$$

To determine $r_1 - r_2$ and $r_1 r_2$ we draw perpendiculars AC and BD respectively on line OP from points A and B. If r > > a $AP \approx PC$ and $PB \approx PD$. From Fig 3.5

$$OP = OD + DP$$

$$r = a\cos\theta + r_2$$

$$\because DP \approx BP = r_2$$

$$r_2 = r - a\cos\theta \qquad \left(\because \cos\theta = \frac{OD}{a}\right)$$

$$AP \approx CP = OP + OC \qquad (OD = a\cos\theta)$$

$$r_1 = r + a\cos\theta$$

$$\cos\theta = \frac{OC}{a} \quad \text{or} \quad OC = a\cos\theta$$

So $\qquad r_1 - r_2 = 2a\cos\theta$

and $\qquad r_1 r_2 = r^2 - a^2\cos^2\theta = r^2$ as $r^2 >> a^2$ substituting these in Egn. (3.18)

$$V = \frac{q}{4\pi\in_0}\frac{2a\cos\theta}{(r^2)}$$

As $\qquad 2aq = p$

So $$V = \frac{1}{4\pi\in_0}\frac{p\cos\theta}{r^2} \qquad \ldots (3.19)$$

Thus the electric dipole potential falls off, at large distance, as r^{2}, not as r^{1}, characteristic of the potential due to a point charge.

Special Cases

(i) For axial points where $\theta = 0^\circ \cos\theta = 1$, from equation (3.19)

$$V = \frac{1}{4\pi\in_0}\frac{p}{r^2}$$

(ii) For equatorial points where $\theta = 90^\circ \cos\theta = 0$, from equation (3.19) $V = 0$

Dipole potential can also be expressed as

$$V = \frac{1}{4\pi\in_0}\frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\in_0}\cdot\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Thus it is clear that

1. For equatorial points the electric potential due to a dipole is zero however field is not zero.
2. Under indential conditions the electric potential due to a dipole is $\frac{2a}{r}\cos\theta$ times of that due to a single charge.
3. For a small dipole (or at large disrances) electic potential is inversely proportional to the square of distance.
4. The potential due to a dipole depends not just on r but also on the angle between the position vector $\vec{r}$ and the dipole moment vector p.

Example 3.6 Two point charges $8\times10^{-19}\,C$ and $-8\times10^{-19}\,C$ are separated by a distance of $2\times10^{-10}\,m$. For such a dipole find electric potential at a point at a distance of $4\times10^{6}\,m$ when the point is (a) on the dipole axis (b) on equatorial line and (c) at an orientation of 60° from dipolemoment.

Solution : Here $q = 8\times10^{19}C$

$2a = 2\times10^{10}\,m$

$r = 4\times10^{6}\,m$

So dipole moment

$$p = q.2a = 8\times10^{19}\times2\times10^{10}$$

$$P = 16\times10^{29}\,C\cdot m$$

(a) For axial position

$$V = \frac{1}{4\pi\in_0}\frac{p}{r^2} = \frac{9\times10^9\times16\times10^{29}}{(4\times10^{6})^2}$$

$$V = 9\times10^{8}\ \text{Volt}$$

(b) For equatorial positions V = O

(c) When $\theta = 60^\circ$

$$V = \frac{1}{4\pi\in_0}\frac{p\cos\theta}{r^2}$$

$$V = \frac{9\times10^9\times16\times10^{29}\cos60^\circ}{(4\times10^{6})^2}$$

$$V = 4.5\times10^{-8}\ \text{Volt}$$

3.5 Equipotential Surface

In some electric field a surface having same potential at all points is called an equipotential surface. As the potential difference between any two points on an equipotential surface is zero, no work is done in moving a charge from one point to other on an equipotential surface. As the work done is zero when electic force (electric field) is perpendicular to displacement, so electric field must be normal to an equipotential surface.

For illustration of equipotential surfaces following examples can be considered.

1. For a uniform electric field $\vec{E}$ equipotential surfaces are flat and perpendicular to field lines. According to fig 3.6, surfaces labelled as I, II, III are equipotentials.

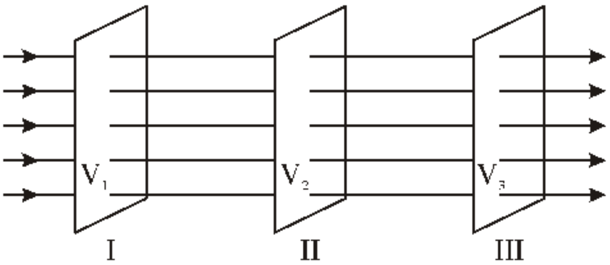


Fig 3.6 : Equipotential surfaces for a uniform electric field

2. For an isolated point charge : For an isolated point charge + q potential at a distance r

$$V = \frac{1}{4\pi \in_0} \frac{q}{r}$$

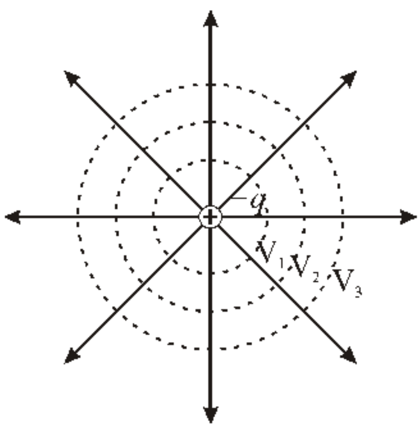


Fig 3.7 : Equipotential surfaces for a point charge

Imagine a spherical surface of radius r the position of point charge +q, is at its center it is obvious that the electric potential at all points on this surface is same. Thus for a point charge equipotential surfaces are spherical around point charge. For a positive point charge with the increases in radius of spherical surface the potential of surface decreases.

3.5.1 Properties of Equipotential Surfaces

1. No net work is done on a charge by an electric field as the charge moves between two points on the same equipotential surface.
2. Electric field is always directed normal to equipotential surface.
3. Two equipotential surfaces can never intersect each other because otherwise the point of interesection will have two potentials which is unacceptable.
4. The surface of a charged conductor is always equipotential. In fact the entire volume of a conductor is equipotential.

3.6 Relation Between Electric Field and Electric Potential

We have already discussed that if we know electric field in some region of space then potential difference between two points can be found using equation (3.14) which is a relation between electric field and electric potential. In this section our aim is to determine electric field for a known potential function V.

In some arbitrary electric field $\vec{E}$ for a differential displacement $d\vec{\ell}$ equation (3.10) can be written in differential form as follows

$$dV = -\vec{E} \cdot d\vec{\ell} = -E d\ell \cos\theta \quad \ldots (3.20)$$

Where θ is the angle between $\vec{E}$ and $d\vec{\ell}$

$$-\frac{dV}{d\ell} = E\cos\theta \quad \ldots (3.21)$$

The quantity $-\frac{dV}{d\ell}$ gives rate of loss (fall) of potential with distance. From above equation it is clear that if angle between $\vec{E}$ and $d\vec{\ell}$, $\theta = 0°$ then the space rate of loss of potential will be maximum. Thus in general $-\frac{dV}{d\ell}$ is a scalar quantity but its maximum value $-\left(\frac{dV}{d\ell}\right)_{max}$ occur for a specific direction $(\theta = 0_i)$ i.e. in direction of $\vec{E}$. Thus the maximum rate of loss of potential with distance can be treated as a vector in direction of $\vec{E}$. In language of mathematics $\left(\frac{dV}{d\ell}\right)_{max}$ is called gradient of V and written as Grad V

$$\left(\frac{dV}{d\ell}\right)_{max} = Grad\, V \qquad \ldots (3.22)$$

Accordingly $\vec{E} = -Grad\, V \qquad \ldots (3.23)$

For an equipotential surface direction of Grad V is along the normal to the surface. This can be explained using Fig 3.8. Here two equipotential surface S_1 and S_2 are shown with potentials V and V – dV respectively. In moving from a point A on surface S_1 to either point B or C on surface S_2 for both the paths AB and AC change in potential is same. However the rate of change of potential with distance i.e. $\frac{dV}{AB}$ and $\frac{dV}{AC}$ are different. As AB < AC so $\frac{dV}{AB} > \frac{dV}{AC}$ and because AB is normal to the surface the rate of loss of potential is maximum along normal.

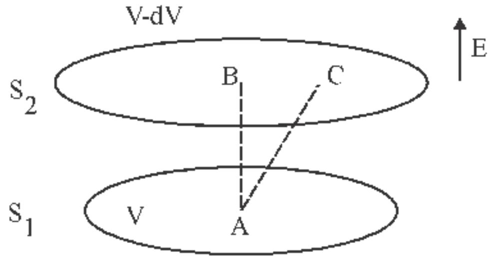


Fig 3.8 : Gradient of potential

Equation (3.21) can be rewritten as

$$-\frac{\delta V}{\delta \ell} = E\cos\theta = E_\ell$$

Here $E_\ell = E\cos\theta$, is the compoment of $\vec{E}$ in direction of $d\vec{\ell}$. Note that here we have used partial derivative which shows that above equation involves only the variation of V along a specified axis (here called l axis) and only the compoment of $\vec{E}$ along that axis. If we take the l axis to be in turn, the x, y and z axes, we find that the x, y and z compoments of $\vec{E}$ at any point are

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z} \ldots (3.24)$$

In cartesian coordinates

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\text{so } \vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$\text{or } \vec{E} = -\nabla V \qquad \ldots (3.25)$$

$$\text{where} \quad \nabla = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \ldots (3.26)$$

is called 'del operator: With the help of equation (3.25) $\vec{E}$ can be determined if potential function V (x, y, z) is known.

If the potential function is spherically symmetric i.e. a function of radial distance r then electric field is given by

$$E_r = -\frac{dV}{dr} \qquad \ldots (3.27)$$

Example 3.7 For some electric field the electric potential is given by the following expression

$$V = \frac{343}{r}\ volt$$

Determine electric field at position given by position vector $\vec{r} = 3\hat{i} + 2\hat{j} - 6\hat{k}$ m

Solution : As $\vec{E} = -\frac{dV}{dr}\hat{r}$

Here $\frac{dV}{dr} = \frac{d}{dr}\left[\frac{343}{r}\right] = -\frac{343}{r^2}$

$$r = |\vec{r}| = \sqrt{(3)^2 + (2)^2 + (-6)^2}$$

$$= \sqrt{49} = 7\,\text{m}$$

$$\text{so } \vec{E} = -\left(\frac{-343}{r^2}\right)\hat{r}$$

$$\text{but} \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

$$\therefore \qquad \vec{E} = \frac{343}{r^3}\vec{r}$$

$$\vec{E} = \frac{343}{(7)^3}\left(3\hat{i} + 2\hat{j} - 6\hat{k}\right)$$

$$\vec{E} = \left(3\hat{i} + 2\hat{j} - 6\hat{k}\right)\text{V/m}$$

Example 3.8 For some electric field represented by potential function

$$V(x, y, z) = 6x - 8xy - 8y + 6yz$$

where V is in volt and x, y, z are in meters. Find magnitude of electric field at point (1, 1, 1) m.

Solution :

$$\vec{E} = -\vec{\nabla}V = \hat{i}\left(-\frac{\partial V}{\partial x}\right) + \hat{j}\left(-\frac{\partial V}{\partial x}\right) + \hat{k}\left(-\frac{\partial V}{\partial z}\right)$$

$$\text{or } \vec{E} = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial Z}\right]$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial x}(6x - 8xy - 8y + 6yz) = (6 - 8y)$$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial y}(6x - 8xy - 8y + 6yz) = (-8x - 8 + 6z)$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(6x - 8xy - 8y + 6yz) = 6y$$

$$\vec{E} = -\left[(6 - 8y)\hat{i} + (-8x - 8 + 6z)\hat{j} + 6y\hat{k}\right]$$

at point (1, 1, 1)

$$\text{or } \vec{E} = -\left[(6 - 8)\hat{i} + (-8 - 8 + 6)\hat{j} + 6\hat{k}\right]$$

$$\text{or } \vec{E} = \left(2\hat{i} + 10\hat{j} - 6\hat{k}\right)\text{V/m}$$

$$|\vec{E}| = \sqrt{(2)^2 + (10)^2 + (-6)^2} = \sqrt{140} = 2\sqrt{35}\ \text{V/m}$$

3.7 Calculation of Electric Potential

3.7.1 Electric Potential due to a charged spherical shell

Consider a spherical shell of radius R and charge q. We wish to calculate electric potential due to such a shell at an internal point, a point on its surface and a point outside the shell. Let the distance of the point of observation from the centre of the shell be r.

(a) For points outside the charged sphere (r > R)

From the definition of potential

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

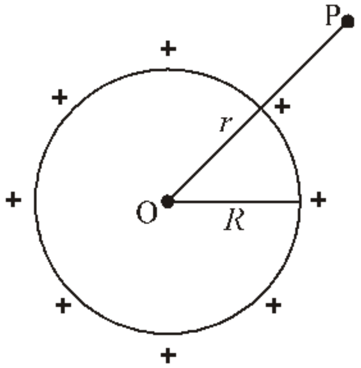


Fig 3.9 : Potential at outer point of shell ($r > R$)

However, for points external to the shell

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r}$$

$$\therefore \quad V = -\int_{\infty}^{r} \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} . d\vec{r}$$

but $\hat{r} \cdot d\vec{r} = dr$ as $\hat{r}$ and $d\vec{r}$ are in some direction

$$\therefore \quad V = -\frac{q}{4\pi \in_0} \left[-\frac{1}{r} \right]_{\infty}^{r}$$

$$V = +\frac{q}{4\pi \in_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$V = \frac{1}{4\pi \in_0} \frac{q}{r} \qquad \text{. . . (3.28)}$$

We see that the potential due to uniformly charged shell is same as that due to a point charge q at the centre, for points external to shell. It is inversely proptional to of distance (r) and tends to zero as r tends to infinity.

(b) Potential at a point on the surface (r = R)

For this case $V = -\int_{\infty}^{R} \vec{E} \cdot d\vec{r}$

can be obtained by substituting r = R in equation (3.28)

$$V_s = \frac{1}{4\pi \in_0} \frac{q}{R} \qquad \text{. . . (3.29)}$$

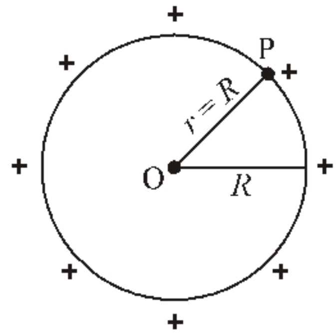


Fig 3.10 : Potential at the surface of shell (r = R)

(c) Potential at an internal point of the shell (r < R)

For determining potential at an internal point of a charged shell we must note that in moving from infinity to an internal point, the dependence of $\vec{E}$ is different for parts of the path outside and inside the charged shell.

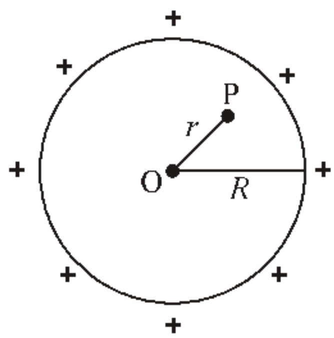


Fig: 3.11 : Determination of potential of an internal point ($r < R$)

Therefore for evaluation of

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

the path of intergration is to be considered to be made of two parts :-

(i) From infinity to distance R from centre (i.e. up to surface) and

(ii) From distance R (surface) to internal point r.

Thus we can write

$$V = -\int_{\infty}^{R} \vec{E} \cdot d\vec{r} - \int_{R}^{r} \vec{E} \cdot d\vec{r}$$

First of these two integral has already been solved and its value can be obtained from equation (3.29) and as for internal points (r < R) electric field is zero for a charged shell the second integral reduces to zero i.e.

$$V_{in} = \frac{1}{4\pi \in_0} \frac{q}{R} + \int_R^r -\vec{0} \cdot d\vec{r}$$

$$V_{in} = \frac{1}{4\pi \in_0} \frac{q}{R} \quad \ldots (3.30)$$

From equation (3.30) it is clear that the potential has a fixed value for all points with in the shell equal to the value of potential at the surface. In fact it is the maximum value of potential due to a uniformly charged spherical shell.

The variation of potential with distance from centre is shown in fig 3.12

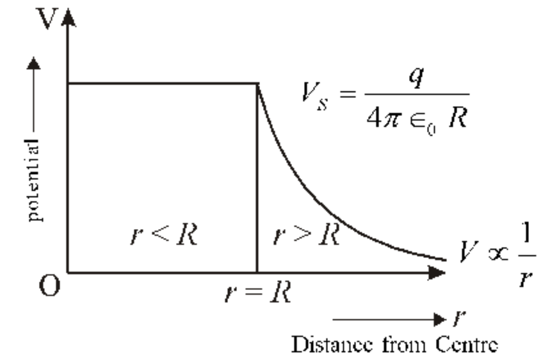


Fig 3.12 : The variation of potential for a charged spherical shell

3.7.2 Electric Potential due to a Charged Conducting Sphere

On charging a conductor as the charge resides on its outer surface, the behaviour of the field intensity due to a charged spherical conductor is same as that of a charged spherical shell. Therefore electric potential due to a charged conducting sphere is same as that due to a charged spherical shell. Thus results derived in section 3.7.1 for spherical shell are applicable in this case.

3.7.3 Electric Potential due to a Uniformly Charged Non Conducting Sphere

Consider a uniformly charged non conducting sphere of radius R having a charge q. For such a sphere expressions of the electric field at external point, point at surface and internal points are as follows.

External points $\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} \qquad (r > R)$

At surface $\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{R^2} \hat{r} \qquad (r = R)$

internal points and $\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{R^3} \vec{r} \qquad (r < R)$

And the general realtion for calculating V from E is

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

Now we will calculate the electric potential for various positions of observation point.

(A) For points outside the charged sphere (r > R)

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

as for such points $\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r}$

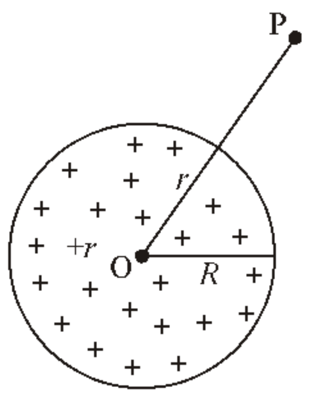


Fig 3.13 : A uniformly charged non conducting sphere point P is external to sphere

So, $$V = -\int_{\infty}^{r} \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} \cdot d\vec{r}$$

or $$V = -\frac{1}{4\pi \in_0} \int_{\infty}^{r} \frac{1}{r^2} dr \qquad \left(\hat{r} \cdot d\vec{r} = dr\right)$$

$$V = -\frac{1}{4\pi \in_0} \left[-\frac{1}{r} \right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi \in_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] \qquad \because \frac{1}{\infty} = o$$

$$V = \frac{q}{4\pi \in_0 r} \qquad \text{. . . (3.31)}$$

Thus for points external to charged sphere $V \propto \frac{1}{r}$

(B) At the surface of Charged Sphere (r = R)

On substituting r = R in equation 3.31, we obtain

$$V_S = \frac{1}{4\pi \in_0} \frac{q}{R} \qquad \text{. . . (3.32)}$$

(C) At a point inside the charged non conducting sphere

As in moving from infinity to a point inside the sphere the variation of electric field with distance is non uniform i.e it varies from infinity to surface in accordance with E 1/2while from surface upto an internal point accodring to E r. Therefore we have to evalute the integral by breaking it into two parts (i) from infinity to R and (ii) from R to r

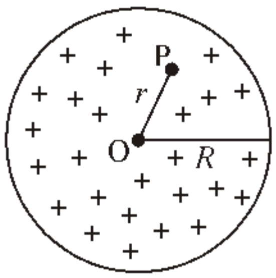


Fig 3.14 : Inside non-conducting sphare r < R

Thus $$V = \left(-\int_{\infty}^{R} \vec{E} \cdot d\vec{r} \right) + \left(-\int_{R}^{r} \vec{E} \cdot d\vec{r} \right)$$

or $$V = \frac{1}{4\pi \in_0} \frac{q}{R} - \int_{R}^{r} \frac{1}{4\pi \in_0} \frac{q}{r^3} r\, \hat{r} \cdot d\vec{r}$$

$$\left(\because \vec{E} = \frac{1}{4\pi \in_0} \frac{q}{R^3} r\, \hat{r} \right)$$

or $$V = \frac{1}{4\pi \in_0} \frac{q}{R} - \frac{1}{4\pi \in_0} \frac{q}{R^3} \left(\frac{r^2}{2} \right)_R^r$$

or $$V = \frac{1}{4\pi \in_0} \frac{q}{R} - \frac{1}{4\pi \in_0} \frac{q}{R^3} \left(\frac{r^2}{2} - \frac{R^2}{2} \right)$$

or $$V = \frac{1}{4\pi \in_0} q \left(\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R} \right)$$

$$V = \frac{1}{4\pi \in_0} \frac{q}{2R} \left(3 - \frac{r^2}{R^2} \right) \quad V = \frac{q}{4\pi \in_0} \left(\frac{3R^2 - r^2}{2R^3} \right)$$

. . . (3.33)

to obtain potential at the centre on putting r = 0 in equation (3.33) we obtain

or

$$V_{centre} = \frac{3}{2} \left(\frac{q}{4\pi \in_0 R} \right) \qquad \text{. . . (3.34)}$$

or $V_{centre} = \frac{3}{2} V_S = 1.5 V_S$

Thus, potential at the centre of uniformly charged solid non conducting sphere is 1.5 times the value of potential at surface .

From above discussion we conclude that inside such a charged sphere potential decreases from. Centre to surface according to r^2 specific dependence and outside surface it decreases with r^{-1} dependence to become zero a infinity. This variation is shown graphically in Fig 3.15

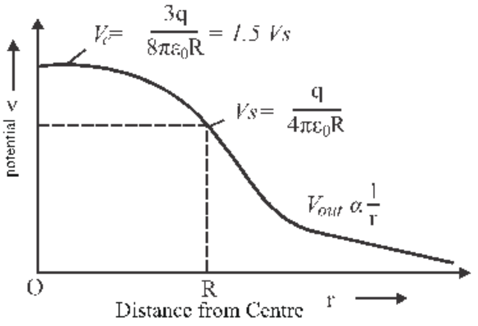


Fig 3.15 : Graph between electric potential versus the distance from centre of a uniformly charged insulating sphere

Example 3.9 A solid insulating sphere of radius 10 cm is given a charge of $3.2\times10^{\ 19}$ C. Determine electric potential at following points from the centre (i) at 14 cm (ii) at 10 cm (iii) at 4 cm.

Solution : Here $R = 10cm = 0.10m$

$q = 3.2\times10^{-19}\,C$

(i) $r = 14cm = 0.14m$

i.e. the observation point is outside the shpere

$$V = \frac{1}{4\pi\in_0}\frac{q}{r}$$

or $V = 9\times10^9\times\frac{3.2\times10^{\ 19}}{0.14} = 2.057\times10^{-8}$ V

(ii) $r = 10\,\text{cm} = 0.10\,\text{m}$ $r = R$ i.e point is on the surface of sphere

$$V = \frac{1}{4\pi\in_0}\frac{q}{R} = \frac{9\times10^9\times3.2\times10^{\ 19}}{0.10}$$

or $V = 2.88\times10^{\ 8}$ Volt

(iii) $r = 4\,\text{cm} = 0.04\,\text{m}$ i.e the point is inside the sphere

$P_3(r_3)$

$$V = \frac{1}{4\pi\in_0}\frac{q}{2R}\left[3-\frac{r^2}{R^2}\right]$$

$$V = \frac{9\times10^9\times3.2\times10^{-19}}{2\times0.10}\left[3-\frac{(0.04)^2}{(0.10)^2}\right]$$

$$V = 1.44\times10^{-8}\left[3-\frac{16}{100}\right]$$

$$V = 1.44\times10^{\ 8}[2.84]$$

$$V = 4.09\times10^{\ 8}\ \text{Volt}$$

3.8 Potential Energy of a System of Charges

In the beginning of this chapter we have seen that a potential energy is associated with a (conservative) electrocstatic field. Now we will discuss about the potential energy for a system of charges. Charges in such system exerts electrostatic force on each other. If positions of one or more charges is changed i.e the configuration of the system is changed then work is done by electrocstatic forces. If the system changes its configuration from an initial state i to a different final state f electrostatic force does work W_e on the particles, then by definition the change in potential energy of the system is

$$\Delta U = U_f - U_i = -W_e$$

i.e the change in potential energy as system changes its configuration is equal to the negative of the work done by the electrostatic force. We can also define U in terms of, W_{ext} the work done by external force. If we assume that the kinetic energy of system in both intitial and final states are zero, then

$$\Delta U = U_f - U_i = W_{ext}$$

For convenience we normally take the reference zero potential energy configuration of a system of charged particles to be that in which the particle are infinitely separated from one another. If this is so then $U_i = U_\infty = 0$ then final potential energy of the system is regarded as its potential energy, i.e.

$$U = -W_{e\infty}$$

Where $W_{e\infty}$ is the work done by electric forces on the particles during the move from infinity. If we wish to define U in terms of work done by external forces a change in kinetic energy, then

$$U = -W_{ex} = +W_{ext}$$

Where W_{ext} is now work done by external force in bringing the charges from infinity to the final configuration.

Based on above definitions let us first determine the potential energy of a two point charge system. Fig 3.16 depicts two point charges q_1 and q_2 of same nature separated by a distance r. First we assume that initially both the charges are infinity (for away) and at rest. When we bring q_1 from infinitely to its present (final) position no work is done either by electrostatic force or external force as no electric field was present. However when we bring q_2 from infinity then work has to be done by external agent because electrostatic force acts on the charge q_2 by q_1 during the move.

The electire potential due to charge q_1 at the location of charge q_2 is

$$V = \frac{1}{4\pi \in_0} \frac{q_1}{r}$$

and from the definition of potential

$$W_{\infty} = -W_e = qV$$

$$U = W_{\infty} = qV$$

On substituting $q = q_2$

$$U = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r} \quad \ldots (3.35)$$

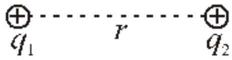


Fig 3.16 : System of two point charges

If the charges have the same sign external agent has to do positive work to push them together against their mutual repulsion. This work gets stored in the form of potential energy of system which is positive in this case. If the system is released then as the charges move apart the potential energy of the system now changes into the kinetic energies of the charges.

If the charges q_1 and q_2 have opposite signs the potential energy of system is negative. Note that depending upon the nature of charges electrostatic potential energy can either be positive or negative on the contrary the gravitational potential energy for a pair of particles is always negative.

3.8.1 Electrostatic Potential Energy of a System of more than Two point Charges

The total potential energy of a system of charged particles can be obtained by calculating potential energy for every pair of charges and summing the terms algebrically (with signs).

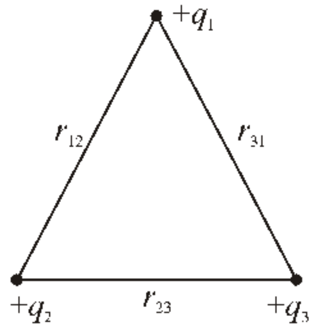


Fig 3.17 : A system of three point charges

Consider three point carges q_1, q_2 and q_3 fixed at points P_1, P_2 and P_3 as shown in fig 3.17, for such a system the potential energy can be determined as follows -

In bringing the charge q_1 from infinity to its position $P_1(\vec{r}_1)$ (while other charges are still at infinity) no work is done as no other charge is present in the region i.e

$$W_1 = 0$$

Then we bring charge q_2 in from infinity to its position P_2 (at a distance r_{12} from P_1) then the work done

$$W_2 = (\text{potential due to q})´\ q_2$$

or $$W_2 = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r_{12}}$$

Like wise the work done in moving charge q_3 from infinity and place it at P_3

$$W_3 = (\textit{Potential due to } q_1 \textit{ and } q_2) \times q_3$$

$$W_3 = \left(\frac{1}{4\pi \in_0} \frac{q_1}{r_{13}} = \frac{1}{4\pi \in_0} \frac{q_2}{r_{23}} \right) \times q_3$$

$$W_3 = \frac{1}{4\pi \in_0} \left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

So the potential energy of this system of charges is

$$U = W_1 + W_2 + W_3$$

$$U = O + \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi \in_0} \left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

$$U = \frac{1}{4\pi \in_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

$$U = \frac{1}{4\pi \in_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \qquad \text{. . . (3.36)}$$

The above equation represents the potential energy of a system of three point charges, note that this expression contains three terms. The process can be extended to a four charge system and expression for potential energy can be determined by

$$U = W_1 + W_2 + W_3 + W_4$$

$$U = \frac{1}{4\pi \in_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}} \right)$$

. . . (3.37)

Which contains six term corresponding to six possible pairs of charges. If there are N charges in a system, the expression for potential energy will contain terms and we can write

$$U = \frac{1}{2} \sum_{j=1}^{j=N} \sum_{k=1}^{k=N} \frac{1}{4\pi \in_0} \frac{q_j q_k}{r_{jk}} \qquad \text{. . . (3.38)}$$

In above equation the factor of 1/2 before the summation sign ensures that although the pairs of charges are appearing twice but their contribution to the sum is effectively considered only once.

Example 3.10 Two protons are separated from each other by a distance of $6 \times 10^{\ 15}$ m . Find the electrostatic potential energy of the system in electron volt units.

Solution : Here $r = 6 \times 10^{-15}\ m$

$$q_1 = q_2 = 1.6 \times 10^{-19} C$$

As $$U = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r}$$

$$U = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6 \times 10^{\ 15}}$$

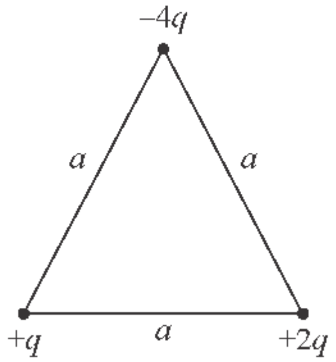
$$U = 3.84 \times 10^{-14} J$$

$$U = \frac{3.84 \times 10^{\ 14}}{1.6 \times 10^{-19}} \text{eV}$$

$$U = 0.24 \times 10^6 \text{eV} = 0.24 \text{MeV}$$

Example 3.11 Three charges are arranged as shown in Fig. Calculate the electrostatic potential energy of the system. Consider

$q = 1.0\times10^{-7}$ C and $a = 0.10$ m .



Solution : Total potential energy

$$U = U_{12} + U_{23} + U_{31}$$

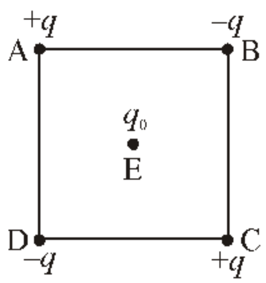
$$= \left(9.0\times10^{9}\right)\left[\frac{(+q)(-4q)}{a} + \frac{(+q)(+2q)}{a} + \frac{(-4q)(+2q)}{a}\right]$$

$$= 9.0\times10^{9}\times\left(-10q^{2}\right)/a$$

$$= -\frac{9.0\times10^{9}\times10\times\left(1\times10^{-7}\right)^{2}}{0.10} = -9.0\times10^{-3}\ \text{J}$$

The negative potential energy means that an external agent would have to do 9×10^{-3} J of work to disassemble this configuration completely ending with three charges infinitely far apart.

Example 3.12 As shown in Fig four charges are placed at the vertices of a square of edge d. (a) calculate the work done in assembling this system (b) If some other charge q_0 is taken from infinity to the centre E of the square and all the four remain fixed at their location, how much additional work is to be done in the process.



Solution : (a) The work done in assembling the system is equal to the potential energy of the system.

Here there will be $\frac{4\times(4-1)}{2} = 6$ pairs of charges for such a four charge system.

$$W = U = k\left[\frac{q(-q)}{AB} + \frac{q(-q)}{AD} + \frac{qq}{AC} + \frac{(-q)q}{BC} + \frac{(-q)(-q)}{BD} + \frac{q(-q)}{CD}\right]$$

$$AB = BC = CD = AD = d$$

$$AC = BD = d\sqrt{2}$$

$$\therefore \quad W = -\frac{4kq^{2}}{d} + \frac{2kq^{2}}{d\sqrt{2}}$$

$$= -\frac{kq^{2}}{d}\left[4-\sqrt{2}\right] \text{ where } k = \frac{1}{4\pi\epsilon_0}$$

(b) Potential at centre E of the square due to charges at four corners

$$V = \frac{+Kq}{(AE)} + -\frac{-Kq}{(BE)} + \frac{+Kq}{(CE)} + \frac{-Kq}{(DE)}$$

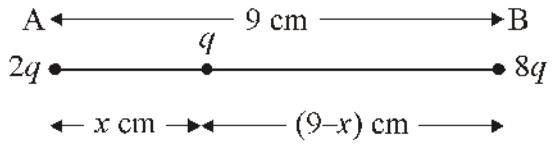
$$V = \frac{Kq}{d/\sqrt{2}} - \frac{Kq}{d/\sqrt{2}} + \frac{Kq}{d/\sqrt{2}} - \frac{Kq}{d/\sqrt{2}} = 0$$

Thus potential at point E is zero so no additional work is needed to be done in moving charge q_0 from infinity to E.

Example 3.13 Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the position where the charge should be placed such that the potential energy of this system is minimum?

Solution : To have minimum potential energy, charges of greater value should be kept farthest thus charges $2q$ and $8q$ should be farthest separated by 9 cm.

Let the charge q is placed at a distance of x cm from $2q$ [Fig] then the potential energy of system



$$U = \frac{1}{4\pi \in_0}\left[\frac{2q\times q}{x\times 10^{-2}} + \frac{8q\times q}{(9-x)\times 10^{-2}} + \frac{2q\times 8q}{9\times 10^{-2}}\right]$$

$$U = \frac{1}{4\pi \in_0}\left[\frac{q^2}{10^{-2}}\right]\left[\frac{2}{x} + \frac{8}{9-x} + \frac{16}{9}\right]$$

For U to be minimum $\frac{dU}{dx} = 0$

$$\frac{dU}{dx} = \frac{1}{4\pi \in_0}\frac{q^2}{10^{-2}}\left[-\frac{2}{x^2} + \frac{8}{(9-x)^2} + 0\right] = 0$$

$$\frac{2}{x^2} = \frac{8}{(9-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(9-x)^2}$$

$$(9-x)^2 = 4x^2$$

$$(9-x) = \pm 2x$$

$$x = 3cm$$

or $x = -9cm$

here x = -9 cm is not possible so charge q should be placed in between $2q$ and $8q$ at a distance of 3 cm from $2q$.

3.9 Work Done in Rotating an Electric Dipole in Electric Field

When a electric dipole is placed in an electric field a torque acts on it. This torque has a tendency to align the dipole along the field. So work has to be done to rotate a dipole if it is in equilibrium under the field.

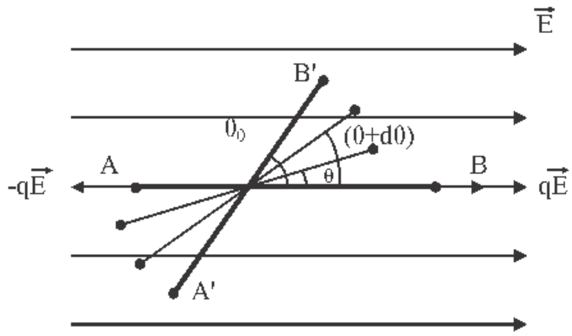


Fig 3.18 : Rotating a dipole in a uniform electric field

Consider a dipole in a uniform electric field $\vec{E}$ as shown in fig 3.13. Initially the dipole is in equilibrium in position AB. Now the dipole is rotated to bring it into position 'AB' making angle θ_0 with direction of $\vec{E}$.

At an angular position θ the torque acting on the dipole is $\tau = pE\sin\theta$

work done is rotating the dipole through a small angle $d\theta$ is then

dW = torque × angular displacement

$$dW = \tau d\theta$$

$$dW = pE\sin\theta d\theta$$

So the work done in rotating the dipole from angular position $\theta = 0^\circ$ to $\theta = \theta_0$ is

$$W = \int_{\theta=0^\circ}^{\theta=\theta_0} pE\sin\theta\, d\theta$$

$$W = pE\left[-\cos\theta\right]_0^{\theta_0}$$

$$W = pE\left(\cos 0^\circ - \cos\theta_0\right)$$

$$W = pE\left(1-\cos\theta_0\right)$$

If $\theta = 0^\circ$ then

$$W = pE\left(1-\cos\theta\right) \qquad \text{. . . (3.39)}$$

Also note that

(i) Work done in rotating a dipole from angular position θ_1 to θ_2 with respect to field

$$W = pE(\cos\theta_1 - \cos\theta_2) \quad \ldots (3.40a)$$

(ii) If $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$ then $W = pE$

$$\ldots (3.40b)$$

(iii) If $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$ then $W = 2pE$

$$\ldots (3.40c)$$

3.10 Potential Energy of an Electric Dipole in Electric Field

The potential energy of a dipole in an electric field is equal to the work done in bringing the dipole from infinity into the field.

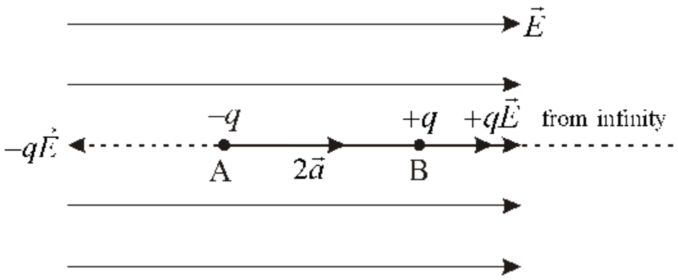


Fig 3.19 : Dipole in external electric field

In fig 3.19 an electric dipole is brought from infinity to its final position in a uniform electric field such that during the move dipole moment $\vec{P}$ always points in direction of $\vec{E}$. Due to electric field force on charge q_1 $\vec{F} = q\vec{E}$ is in direction of $\vec{E}$ and that on -q is $\vec{F} = -q\vec{E}$ is opposite to $\vec{E}$. Thus in bringing the dipole in field $\vec{E}$ external work is to be done on the charge + q while electric field does work on -q. In the move from infinity to their respective final position in field, charge -q covers a distance 2a more than charge +q. Thus work done by electric field is more and negative. This work done is given by

$W =$ Force on $(-q) \times$ additional distance covered by -q

$$W = -qE \times 2a = -2qaE$$

$$W = -pE \qquad \because p = 2qa$$

So, the potential energy of dipole, aligned with with $\vec{E}$ is

$$U_1 = -pE \quad \ldots (3.41)$$

Now, the additional work done in rotating dipole from this position to angular position θ is

$$U_2 = pE(1 - \cos\theta) \quad \ldots (3.42)$$

Thus the potential energy of dipole placed at angle θ with respect to the field is

$$U = U_1 + U_2$$

$$U = -pE + pE(1 - \cos\theta)$$

$$U = -pE\cos\theta$$

which can be rewritten as

$$U = -\vec{p} \cdot \vec{E} \quad \ldots (3.43)$$

Equation (3.4 1) is the expression for the potential energy of electric dipole placed in uniform electric field.

Special Cases

(a) If the dipole is aligned with electric field

$$\theta = 0^\circ$$

$$U = -pE\cos\theta$$

$$U = -pE\cos 0^\circ$$

or $U = -pE \quad \ldots (3.44)$

In this position the electric dipole is in stable equilibrium, as in this case potential energy is minimum.

(b) If the dipole moment is perpendicular to electric field

$$U = -pE\cos 90^\circ$$

$$U = 0 \quad \ldots (3.45)$$

(c) If the dipole moment makes angle $\theta = 180^{o}$ with direction of $\vec{E}$.

$$U = -pE\cos 180^{\circ}$$

$$U = pE \qquad \ldots (3.46)$$

This position is called position of unstable equilibrium as in this situation potential energy of dipole is maximum.

Example 3.14 An electric dipole consits of two point charges $+1.0\times10^{-6}$ and -1.0×10^{-6} at a separation of 2 cm. This dipole is placed in a uniform electric field 1.0×10^{5} V/m . Find

(a) maximum torque on it due to electric field

(b) potential energy of dipole in position of stable equilibrium

(c) potential energy of dipole in angular position of 180^{o} with respect to the position of stable equilibrium.

(d) energy needed to rotate the dipole through 90^{o} with respect to the stable equilibrium position.

Solution : Here $q = 1\times10^{-6}$ C

$$2a = 2cm = 2\times10^{-2}m, E = 1\times10^{5} \text{ V/m}$$

dipole moment $\quad p = q2a = 1\times10^{-6}\times2\times10^{-2}$

$$= 2\times10^{-8}\,C-m$$

(a) maximum torque $\tau = pE = 2\times10^{-8}\times1\times10^{5}$

$$= 2\times10^{-3}\,N-m$$

(b) potential energy in stable equilibrium position

$$U = -pE$$

$$U = -2\times10^{-8}\times1\times10^{5} = -2\times10^{-3}J$$

(c) potential energy in rotated position (relative to stable equilibrium)

$$U = +pE = +2\times10^{-3}J$$

(d) Work done in rotating dipole through angle 90^{o} relative to stable equilibrium position

$$W = pE(1-\cos\theta)$$

$$= 2\times10^{-8}\times1\times10^{5}\left(1-\cos 90^{\circ}\right)$$

$$W = 2\times10^{-3}J$$

Important Points

1. Electric potential : The electric potential at a point in an electric field is equal to the work done against the electric field in bringing a unit positive charge from infinity to the point without changing the kinetic energy of the charge. Its S.I. unit is volt

$$V = -\int_{\infty}^{R} \vec{E}\cdot d\vec{r}$$

2. Potential difference : The potential difference between two points in an electric field is equal to the negative of the work done by electric field or work done by external agent (without change in kinetic energy) in moving a unit positive charge from initial to final point. Its S.I. unit is volt

$$V_A - V_B = \frac{W_{AB}}{q_o} = \int_{B}^{A} -\vec{E}\cdot d\vec{r}$$

3. Potential due to a point charge

$$V = \frac{Q}{4\pi \in_0 r}$$

4. Potential due to a system of point charges

$$V = V_1 + V_2 + V_3 ... + V_n$$

5. Electric potential due to an electric dipole at position (r, θ) [r > > a]

$$V = \frac{1}{4\pi \in_0} \frac{p\cos\theta}{r^2} \qquad \text{if } a^2 << r^2$$

6. Electric potential at equatorial points of a dipole is zero.
7. Equipotential surface : An equipotential surface in an electic field is a surface at all points of which electric potential is same.
8. The electric field $\vec{E}$ is always directed perpendicular to corresponding equipotential surfaces.
9. $\vec{E} = -grad\ v = -\nabla V$
10. Electric potential due to a uniformly charged spherical shell or charged spherical conductor

(i) external points $(r > R)$ $V = \frac{1}{4\pi \in_0} \frac{q}{r}$

(ii) surface $(r = R)$ $V = \frac{1}{4\pi \in_0} \frac{q}{R}$

(iii) internal point $(r < R)$ $V = \frac{1}{4\pi \in_0} \frac{q}{R}$

11. Electric potential due to a uniformly charged insulating solid sphere

(i) external point $(r > R)$ $V = \frac{1}{4\pi \in_0} \frac{q}{r}$

(ii) surface $(r = R)$ $V = \frac{1}{4\pi \in_0} \frac{q}{R}$

(iii) internal point $(r < R)$ $V = \frac{1}{4\pi \in_0} \left[\frac{3R^2 - r^2}{2R^3} \right]$

12. Electric potential energy of a system of two point charges

$$U = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r}$$

13. Potential energy of a system of N point charges

$$U = \frac{1}{2}\sum_{k=1}^{j=N}\sum_{\substack{k=1 \\ j\neq k}}^{k=N} \frac{1}{4\pi \in_0} \frac{q_j q_k}{r_{jk}}$$

14. Work done in rotating an electric dipole in external field

(i) from $\theta = 0$ to θ

$$W = pE(1-\cos\theta)$$

(ii) from $\theta = \theta_1$ to $\theta = \theta_2$

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

15. Potential energy of dipole in external electic field $U = -pE\cos\theta$

Questions For Practice

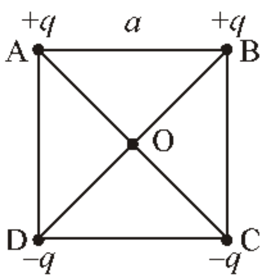
Multiple Choice Questions

1. At certain distance from a point charge electric field and potential are 50 V / m and 300 V respectively, this distance is

(a) 9 m (b) 15 m

(c) 6 m (d) 3 m

2. Four charges are placed on corners of a square as shown in fig. Let electric field and potential at its centre are $\vec{E}$ and V. If the charges at A and B are interchanged with charges placed at C and D. Then



(a) $\vec{E}$ remains the same but V is changed

(b) both $\vec{E}$ and V are changed

(c) both $\vec{E}$ and V are unchanged

(d) $\vec{E}$ is changed but V remains the same

3. The electric potential at some point in an electric field is 200 V. The work done in moving an electron from infinity to that point is -

(a) -3.2×10^{-17} J (b) 200 J

(c) - 200 J (d) 100 J

4. Two charged conducting spheres of radii r_1 and r_2 are at some potential. The ratio of their surface charge densities is

(a) $\frac{r_2}{r_1}$ (b) $\frac{r_1}{r_2}$

(c) $\frac{r_2^2}{r_1^2}$ (d) $\frac{r_1^2}{r_2^2}$

5. A charge of 10 μC is located at the origin of X - Y coordinate system. The potential difference between points (a, 0) and $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ is

(a) 9×10^4 (b) Zero

(c) $\frac{9\times10^4}{a}$ (d) $\frac{9\times10^4}{\sqrt{2}}$

6. The electric potential at the surface of a charged spherical hollow conductor of radius 2 m is 500 V. The potential at a distance 1.5 m from centre is

(a) 375 V (b) 250 V

(c) Zero (d) 500 V

7. An α particle is moved from rest from a point where potential is 70 V to another point having potential 50 V. The kinetic energy of α particle at the second point is

(a) 20 eV (b) 40 eV

(c) 20 MeV (d) 40 MeV

8. In some region where electric field intensity E is zero, the electric potential varies with distance according to

(a) $V \propto \frac{1}{r}$ (b) $V \propto \frac{1}{r^2}$

(c) V = Zero (d) V = Constant

9. Two conducting spheres of radii R_1 and R_2 respectively have the same surface charge density. If the electric potentials at their surface are V_1 and V_2 respectively then V_1/V_2 is equal to

(a) $\frac{r_1}{r_2}$ (b) $\frac{r_2}{r_1}$

(c) $\frac{r_1^2}{r_2^2}$ (d) $\frac{r_2^2}{r_1^2}$

10. The electric potential function for some electric field is defined by $V = -5x + 3y + \sqrt{15}z$. The intensity of electric field (in S.I. units) at point (x, y, z) is

(a) $3\sqrt{2}$ (b) $4\sqrt{2}$

(c) $5\sqrt{2}$ (d) 7

11. A unit charge is moved in a circular path of radius r having a point charge q at the centre. The work done in one complete rotation is

(a) Zero (b) $\frac{1}{4\pi\in_0}\frac{q}{r^2}$

(c) $2\pi r.J$ (d) $2\pi rq.J$

12. For a system of two electrons on bringing one elctron nearer to other, the electrostatic potential energy of system

(a) increases (b) decreases

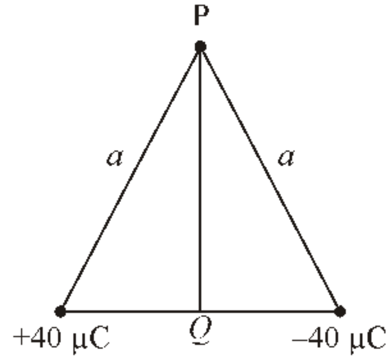
(c) remains the same (d) become zero

13. 1000 tiny water droplets each of radius r and charge q combine to form a big drop. The electric potential of the bigger drop as compared to a tiny droplet is increased by a factor of

(a) 1000 (b) 100

(c) 10 (d) 1

14. For the arrangement of charges as shown in adjoining diagram, the work done in moving a 1 C charge from P to Q (in joule) is



(a) 10 (b) 5

(c) infinite (d) zero

15. 64 identical mercury drops (each having a potential 10 V) are combined to form a bigger drop. The potential at the surface of this bigger drop will be

(a) 80 V (b) 160 V

(c) 640 V (d) 320 V

Very Short Answer Question

1. Mention whether electric potential is a scalar or vector quantity.
2. Give definition of electric potential.
3. Can two equipotential surfaces intersect each other at some point?
4. What will be the electric potential due to a charge, at infinity.
5. Can electric potential at some point in free space be zero though electric field may not be zero at that point? Give exapmle.
6. Can electric field at some point be zero though electric potential is not zero? Give example.
7. What will be the work done in moving a 200 μC charge in moving it form one point to another point at a distance of 10 cm on the same equipotential surface?
8. What is the shape of equipotential surface for the following

   (a) due to a point charge

   (b) for a uniform electric field
9. What is the potential energy of an electric dipole placed parallel to an electric field?
10. The electric potential at the surface of a charged spherical conductor of radius 10 cm is 15 V. What is the electric potential at its centre.
11. The electric potential at the surface of a uniformly charged non conducting sphere of radius 5 cm is 10 V. What is the electric field at its centre.
12. The electric potential at a point (x, y, z) (all in meters) in free space is given by $V = 2x^2$ volt. Determine electric field intensity at point (1m, 2m, 3m).
13. Write the expression for potential energy of a two point charge system.
14. Write the expression for potential energy for a system of three point charges.
15. Write the S.I. unit for potential gradient.
16. How much work is to be done in moving an electron between two points having a potential difference of 20 V.
17. The electric potential due to a point charge at some point in free space is 10V. If the entire system is now placed in a dielectric medium of dielectric constant 2. What will be the potential at the same point?
18. Write the expression for work done in rotating an electric dipole from 0^0 to 180^0 in a uniform field.
19. Write the value of electric potential of earth as has been assumed?
20. If potential function is V = 4x + 3y volt then calculate magnitude of electric field intensity at point (2,1) meter.

Short Answer Question

1. What is meant by electric potential? Write its formula and S.I. unit.
2. Show that the potential inside the potential inside a charged spherical shell is same as that on its surface.
3. What is meant by equipotential surface. Draw equipotential surfaces for a point charge.
4. Determine the expression for potential energy for a system of three point charges.
5. The electric potential in complete volume of a charged conductor is same as that on its surface, why?
6. Derive relation between electric potential and field.
7. Derive expression for work done in rotating an

electric dipole in a uniform electric field.

8. Show that no work is done in moving a test charge from one point to other on an equipotential surface.

9. What is meant by electrostatic potential energy? Derive expression for potential energy of a system of point charges.

10. Determine expression for potential energy of an electric dipole in external electric field.

11. Write expression for electrostatic potential energy for a system of two point charges q_1 and q_2 placed at positions given by position vectors $\vec{r}_1$ and $\vec{r}_2$ in a uniform electric field.

12. Write two properties of equipotential surface.

13. Show that the electric potential due to a point charge when surrounded by some dielectric medium is $1/\in_r$ times of the electric potential when the charge is in free space.

14. Show that the electric potential at the centre of a uniformly charged insulating solid sphere is 1.5 times the value of potential at its surfaces.

15. Two charges 10 μC and 5 μC are 1 m apart. To decreases the separation to 0.5 m how much work has to be done.

16. Define electric potential difference. Distinguish between potential difference and potential.

Essay type Questions

1. Derive an expression for electric potential at a point due to a point charge.

2. Derive an expression for electric potential at a point due to an electric dipole. Show that potential is maximum for axial point while is zero for equatorial line.

3. Derive expression for electric potential due to a charged spherical shell at points (i) outside, (ii) at the surface and (iii) inside the shell. Draw the graph showing variation of potential with distance.

4. Derive expressions for electric potential due to a uniformly charged spherical non conducting solid sphere at points (i) outside (ii) at the surface and (iii) inside the sphere. Draw the graph showing variation of potential with distance.

5. Define electrostatic potential energy. Dervie expression for an electric dipole in a uniform electric field. For what positions; states of stable and unstable equilibrium are obtained?

Answers (Multiple Choice Questions)

1. (c) 2. (d) 3. (a) 4. (a) 5. (b) 6. (d)
7. (b) 8. (d) 9. (a) 10. (d) 11. (a) 12. (a)
13. (b) 14. (d) 15. (b)

Very Short Answer Questions

1. Scalar quantity

2. It is equal to the work done by external agent in brining a unit positive charge from infinity to the point under consideration without changing the kinetic energy of the charge.

3. No, otherwise there would be two values of electric potential which is absured.

4. Zero

5. Yes, electric potential at points on equatorial line of on electric dipole is zero but electric field is not zero.

6. Yes, (a) the electric field at the mid point on the line joining two identical charges is zero but electric potential is not.

   (b) electric field inside a charged spherical shell is zero but electric potential is not.

7. As the potential difference between any two points on a given equipotential surface ΔV = 0

   Work W = q ΔV = 0

8. (a) Spherical surfaces centred at the point charge

   (b) parallel planes oriented normal to the electric field

9. $U = -pE$

10. 15 V

11. 15 V

12. $E_x = -\frac{\partial v}{\partial x} = -\frac{\partial}{\partial x}\left(2x^2\right) = -4x$

$E_y = E_z = 0$

$\vec{E} = -4x\hat{i}$

$\vec{E}_{(\text{at } 1,2,3)} = -4\times 1\hat{i} = -4\hat{i} \quad v/m$

13. $U = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{r}$

14. $U = \frac{1}{4\pi\in_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$

15. V/m

16. $W = qV = 1.6\times 10^{\ 19}\times 20 = 32\times 10^{\ 19} J$

17. $V_m = \frac{V}{E_r} = \frac{10}{2} = 5v$

18. $W = pE\left(\cos\theta_1 - \cos\theta_2\right) = pE\left(\cos 0^\circ - \cos 180^\circ\right)$

$W = 2pE$ joule

19. Zero

20. $\vec{E} = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y}\right]$

$\vec{E} = -4i - 3j$

$\left|\vec{E}\right| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5V/m$

Numberical Problems

1. 6 J of work is needed to move a 3 C point charge between two points. Find the potential difference between the two points. [Ans : 2 V]

2. If the electric potentials at two points A and B are 2 V and 4 V respectively then find the work need to move a 8 μC charge from A to B.

[Ans : $\left(1.6\times 10^{-5} J\right)$]

3. Four charges, 100 μC , -50 μC , 20 μC and -60 μC respectively are placed on four corners of a square of edge $\sqrt{2}$ m. Find the electric potential at the centre of square.

[Ans : $\left(9\times 10^4 V\right)$]

4. Two point charges, $3\times 10^{-8} C$ and $-2\times 10^{-8} C$ are 15 cm apart. At what point(s) on the line joining the charges the electric potential is zero? Assume the electric potential to be zero at infinity.

[Ans : 9 cm away from positive charge, and 45 cm away from it towards negative charge]

5. Four charges, –2 μc, +3 μc, –4 μc and +5 μc respectively are placed on corners of a square of edge 0.9 m. Find the electric potential at the centre of the square. Ans : $\left(2.8\times 10^4 V\right)$

6. A charge of 5 μC is placed on each of the vertices of a regular hexagon of side 10 cm. Find the electric potential at the centre of hexagon.

Ans : $\left(2.7\times 10^6 V\right)$

7. Four charges each 2 μC is placed on four corners of a square of side $2\sqrt{2}$ m. Find the potential at the centre of square. Ans : $\left(36\times 10^5 V\right)$

8. Three charges 1 μC, 2 μC and 3 μC respectively are placed on the vertices of an equilateral triangle of 100m cm side. Calculate the electric potential at the centre of the triangle. [Ans : 93.6 V]

9. Two charges –1 μC and +1 μC at a separation of $4\times 10^{\ 14} m$ forms an electic dipole. Calculate electric potential at an axial point located at a distance $2\times 10^{\ 6} m$ from centre.

Ans : $\left(9\times 10^{\ 2} V\right)$

10. (a) Calculate eletric potential due to a $4\times 10^{-7} C$ point charge at a point at a distance 9 cm from it.

(b) now, Find the work done in bringing another $2\times10^{-9}\,C$ charge from infinity to this point.

11. A 30 μC charge is located at the origin of X - Y coordinate system. Find the potential difference between points $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ and (a, 0).

12. Three charges -q, + q and + q are located in X - Y plane at points (0, -a) (0,0) and (0, a). Show that potential at point at a distance r on a line inclined at angle θ to the axis is given by

$$V = \frac{1}{4\pi \in_0}\left(\frac{q}{r}+\frac{2qa\cos}{r^2}\right), r >> a$$

13. How much work has to be done in putting charges $+q$, $2q$ and $+4q$ respectively at the corners of an equilateral triangle of side 'a'.

Ans : $\left(\frac{1}{4\pi \in_0}\frac{14q^2}{a}\right)$

14. No external electric field is applied on a system of two charges 7 μC and –2 μC located at (–9 cm, 0,0) and (+9 cm, 0, 0). Determine the electrostatic potential energy of this system.

(b) how much work is needed to make the charges separate by infinite distance.

[Ans : = 0.7 J, 0.7 J]

15. For some electric field, potential at a point (x,y) is given as $V = 6xy + y^2 - x^2$. Determine the electric field at this point.

Ans : $\vec{E} = (2x-6y)\hat{i} - (6x+2y)\hat{j}$

16. A hollow metallic sphere of radius 0.2 m is given a charge of + 15μc . Find (i) electric potential at its surface (ii) potential at its centre (iii) electric potential at (iv) potential at 0.3 m from centre.

Ans : (i) 8.75×10^5 V ; (ii) 8.75×10^5 V

(iii) 8.75×10^5 V ; (iv) 4.5×10^5 V

17. Three charges, $+q$, $+2q$ and xq respectively are placed at the vertices of an equilateral triangle of side r. Find the value of x for which the potential energy of system becomes zero.

Ans : $(x = -2/3)$

Chapter - 4

Electrical Capacitance

In chapter third we made ourselves familiar with the electrostatic potential energy of a system of charges. In this chapter our basic aim is to study about "capacitor" a device in which electrical energy can be stored. The electrical energy so stored can be recovered in other forms of energy. For example the flash attachment in a camera uses electrical energy stored by a capacitor. Once the capaciter is charged it can supply energy at a much greater rate by discharing through the associated circuit to provide a sudden bright flash of light, in cameras. Capacitors have many uses as a circuit element in various electric and electronic circuits.

4.1 Conductors and Insulators

On the basis of their ability to conduct electricity materials found in nature can be classified into two broad categories.

(a) Conductors and (b) Insulators

Conductors - The materials in which electric charges and electric current can flow easily for exanple silver, copper, aluminium, iron, mercury, common salt solution, human body and earth etc. are conductors of electricity. Among these silver is the best conductor of electricity.

Insulators - Ideally in such materials electrical current can not flow. Such materials are called insulators or dielectrics. For example rubber, glass, plastic, ebonite, dry wood etc are all insulators.

In addition to these, solid insulators materials can also be semiconductors. We will learn more about semiconductors in chapter 16. In that chapter we will also learn why different solid materials have different behaviour towards electric conduction.

4.2 Free and Bound Charges

Every material is composed of atoms. Each atom contains a positively charged nucleus and several electrons revoling around it. The electrons belonging to inner orbits are subjected to a greater attractive force from the nucleus and are tightly bound to their respective parent atoms.

In metallic solids outer electrons (valence electrons) of each atom are only weakly bound to nucleus. These electrons are almost free to move with in the bulk of the material and are called free electrons. However such free electrons can not escape through the surface of metal. Atoms after loosing valence electron(s) called as positive ions are rigidly bound at their respective locations in the solid. In such materials when an external electric field is applied, these free electrons move in direction opposite to field, consitituting an electric current. Inner electrons are bound to nucleus and can not contribute to electric current. The same is true for positive ions of metals, both these are bound charges. There is no contribution from bound charges in current flow in solids. In electrolytic solutions current flow is due to positive and negative ions.

4.3 Dielectric materials and Polarization

Dielectric materials do not conduct electricity but exhibit electrical effect when subjected to external electric field. In dielectric materials, effectively there are no free electrons hence no electrical conduction. The effect of external field results in a slight rearrangement of charges in atoms or molecules of the dielectric material, however, this rearrangement is enough to modify the electric field inside the material. The dielectric materials can be divided into two categories (a) polar dielectrics and (b) non olar dielectrics.

(a) Polar dielectric : For polar dielectric material, centre of negative charge distribution do not coincide with centre of positive charge distribution. The separation between centres of negative and positive charge distribution causes molecules to have a permanent dipole moment. Such molecules are known as polar molecules. An ionic molecule like HCl (Fig 4.1a) or a molecule of water (H_2O) (Fig 4.1 b) are examples of polar molecule.

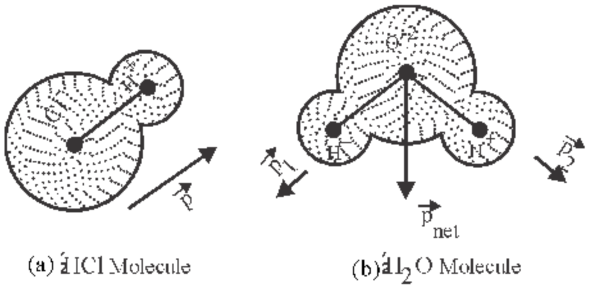


Fig 4.1 : Polar molecules

In the absence of external electric field, the different permanent dipole moments (molecules) are randomly oriented due to their random thermal motion. Thus in any volume containing a large number of atoms the net dipole moment is zero (Fig 4.2 a). When such a dielectric material is subjected to an external electric field, the individual dipoles experience torque due to the electric field and tend to align with the field as shown in Fig 4.2 (b) on increasing the magnitude of the applied field the alignment becomes more complete and net dipole moment is developed in the material.

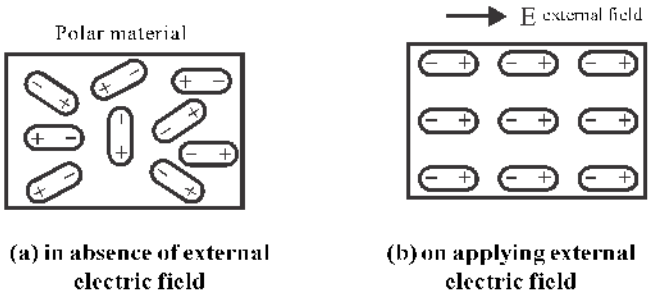


Fig 4.2 : Effect of external electric field (a) external electric field on polar molecules (b) on applying external field

(b) Non Polar dielectrics : Non polar dielectrics are composed of such atoms or molecules whose centre of the negative charges coincides with centre of distribution of positive charges. Such molecules have no permanent dipole moment and are called non polar molecules. Examples of non polar molecules are H_2, CO_2, N_2 and O_2. In Fig 4.2 H_2 and CO_2 molecules are shown. In the absence of external electric field the net dipole moment of a non polar dielectric material is zero [Fig 4.4a]

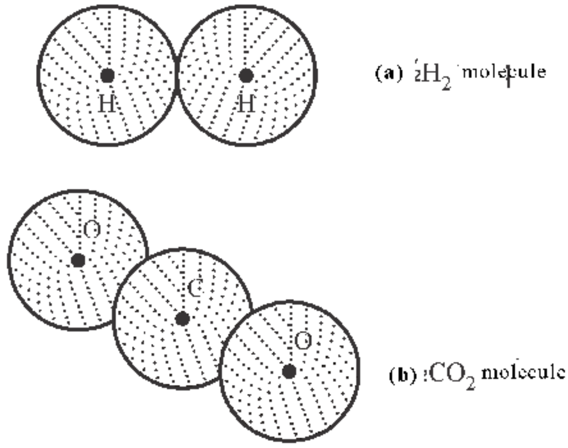


Fig 4.3 : Non polar molecules

If a non polar dielectric material is placed in an external electric field, the centre of negative charge distribution is slightly shifted opposite to electric field while that of positive charge in direction electic field. [Fig 4.4 (b)]. Thus atoms or molecules acquire a dipole moment by induction. The dipole moments of different molecules now tend to align along the electric field and we get a net dipole moment in the material.

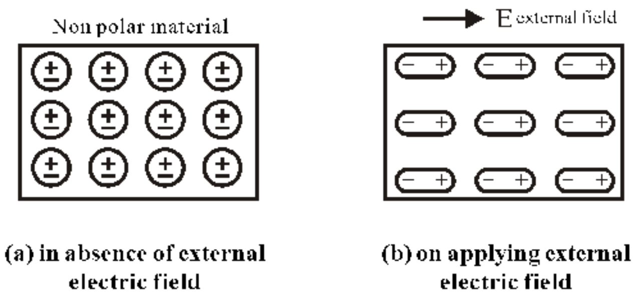


Fig 4.4 : Effect of external electric field on non polar material

Thus, in either case, whether polar or non polar, a dielectric acquires a net dipome moment in electric field. This phenomena is called dielectric polarization.

The materials in which the dipole moment is proportional to the external electric field and is along the field are called linear isotropic dielectrics.

The dipole moment per unit volume of a substance is called polarisation vector and is denoted by $\vec{P}$. For linear isotropic dielectrics the polarisation vector $\vec{P}$ is proportional to applied electric field $(\vec{E})$ i.e $\vec{P} \propto \vec{E}$

or $\quad \vec{P} = \chi_e \vec{E} \qquad \ldots (4.1)$

Where χ_e is a constant, characteristic of the dielectric and is known as electric susceptibility of dielectric. It is a measure of polarisation of the material and is a dimension less quantity.

4.4 Capacitance of a Conductor

The charge on a body is due to transfer of electrons i.e the body either gains or looses electrons. When some charge is given to a body, its potential rises. This is expected, to explain this in simple terms we may assume that a body is given some charge Q in a number of steps involving a transfer of charge dq in each step. When first such installment dq is given to the body no work is involved, but once this charge has been transferred a small potential develops on the body. Therefore the work must be done to move next incremental charge dq through this potential difference, Equivalently we may say that work must be done in putting this additional charge against the repulsion due to charge already present on the body whereby increasing its potential energy or electric potential. Thus if we go on giving charge to a conductor its potential rises in the same ratio.

If on giving a charge Q to an isolated conductor its potential rises by V then

$$Q \propto V \quad \text{or} \quad Q = CV \qquad \ldots (4.2)$$

Here C is a constant of proportionality and is called capacitance of the conductor. Further if we let V = 1 then from equation (4.2) we have C = Q

i.e capacitance of a conductor is numerically equal to the charge required to raise its potential through unity. The capacitance of a capacitor determines its ability to store electric charge. A conductor can store charge upto a certain maximum value of electric potential. The graph between the charge given to a conductor and rise in its potential is a straight line as shown in Fig 4.3 and the slope of this line represents the capacitance of the conductor.

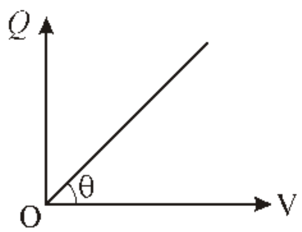


Fig 4.5 : Graph between Q and V for a conductor

The capacitance of a conductor depends on the shape and size of conductor, the nature of medium surrounding it and on presence of other conductor in its neighbourhood. However, it does not depend on the charge given to it or its potenial.

The SI unit of capacitance is coulomb/volt and is called farad (symbol F) Thus

$$1F = \frac{1C}{1V}$$

Capacitance of a conductor is thus said to be one farad if its potential rises through one volt when a charge of one coulomb is given to it.

The farad is a rather large unit so its submultiples like mili farad (mF) = 10^{-3} F, micro farad (μF) = 10^{-6} F, nano farad (nF) = 10^{-9} F and picofarad (pF) = 10^{-12} F are more commonly employed.

As C = Q/V the dimesional formula for C is

$$= \frac{[TA]}{[ML^2T^{-2}/TA]}$$

$$= [M^{-1}L^{-2}T^4A^2]$$

4.5 Capacitance of an Isolated Spherical conductor

Consider an isolated spherical conductor of radius R placed in free space. If a charge Q is given to it the electric potential V at its surface is given by

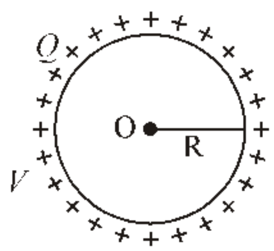


Fig 4.6 : Charged spherical conductor

$$V = \frac{1}{4\pi \in_0} \frac{Q}{R}$$

So its capacitance $C = \frac{Q}{V} = 4\pi \in_0 R \qquad \ldots (4.3a)$

Thus the capacitance of an isolated spherical conductor is directly proportional to its radius (i.e

$C \propto R$)

If the sphere is placed in a medium of dielectric constant $\in_r$ then its capacitance

$$C_m = \frac{Q}{V} = 4\pi \in_0 \in_r R \qquad \text{. . . (4.3b)}$$

or $\quad C_m = C \in_r$

or $\quad \frac{C_m}{C} = \in_r$

Thus the dielectric constant of the medium is equal to the ratio of the capactiance of conductor in the medium and its capactiance in free space.

Example 4.1 : Considering earth to be a spherical conductor calculate its capacitance (Radius of earth $= 6.4 \times 10^6\ m$)

Solution : Capacitance $C = 4\pi \in_0 R = \frac{R}{1/4\pi \in_0}$

$$= \frac{6.4 \times 10^6}{9 \times 10^9} = 0.711 \times 10^{-3} F$$

$$C = 0.711 mF = 711 \mu F$$

It is obvious that the capacitance of a conductor is not very large.

Example 4.2 : What is the radius of a spherical capacitance of 1 F capacitance? Can you put it your cupboard?

Solution : As $C = 4\pi \in_0 R$

So $\quad R = \frac{C}{4\pi \in_0} = \frac{1}{4\pi \in_0} = 9 \times 10^9\ m$

This radius is nearly 1400 times larges than the radius of earth so it is not possible to put this capcitor in a cupboard. (The example also illustrate that Farad is a rather large unit of capacitance).

Example 4.3 : If the capacitances of some spherical capacitor in air and some medium are 2pF and 12pF respectively, what is the value of dielectric constant of medium.

Solution : Dielectric constant of medium

$$C_m = 4\pi \in_0 \in_r R = \in_r (4\pi \in_0 R) = \in_r C_0$$

Here $C_0 = 2pF$,

$$C_m = 12 pF$$

or $\quad 12 = \in_r \times 2 \quad \in_r = 6$

Example 4.4 : On giving same amount of charge to two spheres of different radii the ratio of potential at their surfaces is 1 : 2. What is the ratio of their capacitances?

Solution : $C = \frac{Q}{V}$ or $\frac{C_1}{C_2} = \frac{Q_1 V_2}{Q_2 V_1}$

Here $\qquad Q_1 = Q_2$ and $V_1 : V_2 = 1 : 2 \quad \frac{V_2}{V_1} = \frac{2}{1}$

therefore $\qquad \frac{C_1}{C_2} = \frac{2}{1}$ or $C_1 : C_2 = 2 : 1$

4.6 Capacitor

The capacitance of a conductor can be increased by increasing its size but from practical considertion this is not convinient. Thus capcitance of a conductor is small and limited.

A capacitor is a device whose function is to increase the ability of a conductor to take up charge without increasing its size. This is done by reducing the electric potential of the given conductor for a given amount of charge.

A capacitor is a combination of two conductors called plates placed close to each other. One of the two plates is given a positive cahrge and other on equal amount of negative charge. For this these plates may be connected to the terminals of a battery (Fig 4.7). After charging if the battery is removed the plates retain their charge. So capaciator is a device to store charge.

Note that the net charge on a capacitor is $Q + (-Q) = 0$. Thus the term charge on a capacitor does not mean the net charge on capacitor. In our discussion of capacitor we let Q represent the magnitude of charge on either plate. Likewise the potential difference between plates is called the potential of capacitor.

For a given capacitor, the magintude of charge Q stored on either plate is directly proportional to the potential difference between plates.

$$Q \propto V$$

or $$Q = CV \qquad \ldots (4.4)$$

Where the constant of proportionality is called capacitance of the capacitor.

The shape of capacitor plates may be rectangular cylinderical, spherical or of any arbitrary shape. (No matter what their geometry, the two conductors farming a capacitor are called plates). The capacitance of a capacitor depends on the shape size, relative positions and medium between plates.

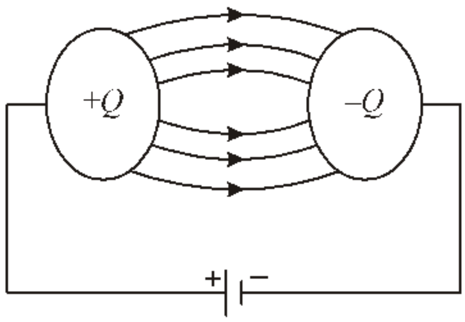


Fig 4.7 : Charging a capacitor

In Fig 4.8 the circuit symbols for capacitor used in electric circuits are shown

(a) Capacitor with fixed capacitance

(b) Capacitor with variable capacitnce

Fig 4.8 : Circuit symbols for a capcitor

4.6.1 Principle of Capacitor

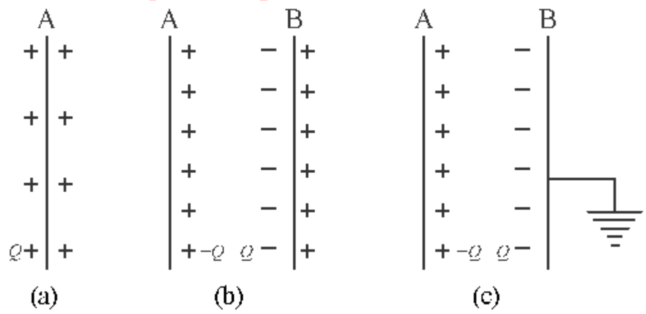


Fig 4.9 : Illustration of principle of capacitor

To understand the principle of a capacitor we consider various situations shown in fig 4.9 in sequence. In fig 4.9 (a) an insulated metallic plate A is shown to which a positive charge +Q has been given. Now if an identical uncharged metallic plate B is placed near plate A, by induction a charge -Q develops on the inner side of B and A positive charge + Q develops on the outer side of B (Fig 4.9 b). The negative charge on B tends to reduce the potential of A whereas the positive charge on it ends to enchance the potential of A. However, as negative charge on B is relatively closer to A than positive charge on B, the potential of plate A is slightly reduced. To make the potential of A at its original value we can give additional charge to plate A. Thus capacitance of A has increased. If we now connect the plate to ground the flow of electrons from earth neutralize the positive charge on B and the negative charge remains on plate B as it is bound (Fig 4.9 c). Under these condition there is nothing to increase potential of A. Thus the potential of the plate is considerably reduced due to induced negative charge on inner side of the plate B. Therefore capacitance of system increases considerably.

Thus "the capacitance of an insulated conductor is increased appreciably by bringing a grounded (earth connected) uncharged conductor near it".

Depending upon the shape of conductors we come across mainly three types of capacitors (a) parallel plate capacitor (b) shperical capacitors and (c) cylindrical capacitors. In this chapter our study is limited to parallel plate and spherical capacitors.

4.7 Parallel Plate Capacitance

A parallel plate cpapcitor consists of two equal plane parallel conductors separated by a small distance see fig 4.10 (a) here we are assuming that the space between plates contain vaccum or air.

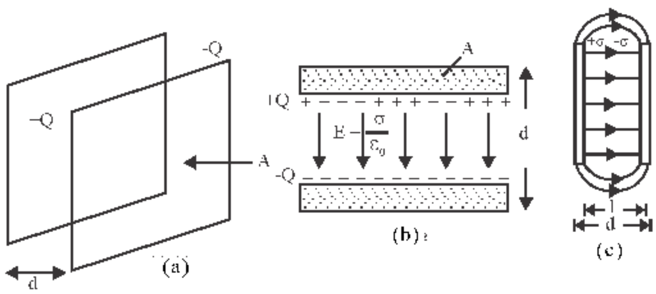


Fig 4.10 : Parallel plate capacitor

To charge a capacitor its plates are connected to the terminals of a battery. The plate connected to the positive terminal of the battery looses electron and the plate connected to the negative terminals recieves as many electrons. Thus equal and opposite charges + Q and - Q appears on the respective plates. As the plates are of equal area the magnitude of surface charge density for both the plates is same. Let $+\sigma$ and $-\sigma$ be the charge densities on the positive and negative plates.

Since the separation between plates is much smaller than the linear dimension of the plates, the electric field is uniform everywhere between the plates (fig 4.10 (b). The electric field in this region due to each plate is $\sigma / 2 \in_0$ and in same direction, perpendicular to the plates. Thus the two fields add up, giving

$$E = \frac{\sigma}{2 \in_0} + \frac{\sigma}{2 \in_0} = \frac{\sigma}{\in_0}$$

$$\because \quad \sigma = \frac{Q}{A}$$

where A = area of either plate

So $$E = \frac{Q}{A \in_0} \qquad \ldots (4.5)$$

Fringing - The electric field due to charged plates is uniform in the central region between the plates as indicated by parallel field lines in Fig 4.10 (b) and Fig 4.10 (c). As at the edges the surface charge density is relatively large field lines repel each other and becomes curved near the edges as in fig 4.10 (c). Thus near the edges field is non uniform. This effect is called fringing. For sufficiently large plates fringing can be ignored and electric field between the plates can be assumed uniform throughout this region.

If the separation between capacitor plates is d then the potential difference between the plates is

$$V = Ed = \frac{Qd}{A \in_0} \qquad \ldots (4.6)$$

therefore, the capacity of parallel plate capacitor

$$C = \frac{Q}{V} = \frac{Q}{Qd / A \in_0}$$

or $$C = \frac{A \in_0}{d} \Rightarrow C = \frac{\in_0 A}{d} \qquad \ldots (4.7)$$

Thus, the capacitance of a parallel plate capacitor is proportional to the area of plates and inversily proportional to the separation between them, i.e

$$C \propto A \text{ and } C \propto 1/d$$

Example 4.5 : A capacitor of capacity 20μF is charged to a potential difference of 10 kV. What is the magnitude of charge on its each plate?

Solution :

$$\because \quad Q = CV = 20 \times 10^{-6} \times 10 \times 10^{3}$$

$$= 20 \times 10^{-2} = 0.2C$$

Example 4.6 : A parallel plate capacitor has plates of area A and the separation between the plates d. If the area of plates is doubled while the separation is halved, the new capacitance is how many times of its orginal value.

Solution : Initial capacitance $C = \frac{A \in_0}{d}$

and new capacitance

$$C' = \frac{(2A) \in_0}{d/2} = \frac{4A \in_0}{d} = 4C$$

Hence new capacity is four times the initial capacity

Example 4.7 : On connecting a capacitor of capacitance C to a battery of potential difference V, charges on its plates are ± 360 μC. On decreasing the potential difference by 120V charges become ± 120 μC. Find

(a) potential difference V across plates

(b) capacitance of capacitor

(c) magnitude of charge if the applied potential difference is increased by 120 V.

Solution :

(a) $\because q = CV = 360 \times 10^{-6} C$ coulomb

Given $q' = C(V - 120) = 120 \times 10^{-6} C$ coulomb

So, $$\frac{CV}{C(V-120)} = \frac{360 \times 10^{-6}}{120 \times 10^{-6}} = 3$$

or $3V - 360 = V$ or $V = 180$ V

(b) As $\because CV = 360\times 10^{-6}$

$$C = \frac{360\times 10^{-6}}{V} = \frac{360\times 10^{-6}}{180}$$

$$= 2\times 10^{-6} F = 2\mu F$$

(c) On increasing the potential differnce by 120 V, new charge

$$q'' = C(V+120) = 2\times 10^{-6}(180+120)$$

$$q'' = 2\times 300\times 10^{-6} = 600\times 10^{-6} C = 600\mu C$$

Example 4.8 Calculate the capacitance of the capacitor formed by two circular discs of radius 5 cm each at a separation of 1 mm.

Solution : Here, plate area

$$A = \pi r^2 = 3.14\times\left(5\times 10^{-2}\right)^2$$

$$= 78.5\times 10^{-4} m^2$$

So, $$C = \frac{A\in_0}{d} = \frac{78.5\times 10^{-4}\times 8.85\times 10^{-12}}{1\times 10^{-3}}$$

$$C = 69.47\times 10^{-12} F = 69.5\,pF$$

4.6 Effect of Dielectric medium filled between plates of Capacitor

When the space between the capacitor plates is filled with some dielectric material (say wax, paper, mica etc) the molecular dipoles (polar or induced) tend to align along the direction of electric field $\left(E = \frac{\sigma}{\in_0}\right)$ due to plates (Fig 4.11). In this process of polarzation, in the interior of dielectric, charges on near by molecular dipole tends to neutralize each other, however, this is not so for dipoles near the edges of dielectric medium. As a result a layer of induced negative charge density $-\sigma_p$ is formed on one edge (near positive plate) and an equal magnitude positive charge $+\sigma_p$ on other edge (near negative plate).

Because of this an induced electric field $E_p = \frac{\sigma_p}{\in_0}$ is established within the dielectric medium. The direction of this induced electric field is opposite to electric field due to capacitor plates.

So the net electric field in the dielectric medium

$$E_1 = E - E_P$$

or $$E_1 = \frac{\sigma}{\in_0} - \frac{\sigma_p}{\in_0} = \frac{\sigma - \sigma_p}{\in_0} \qquad \ldots (4.8)$$

By definition of dielectric constant

$$E_1 = \frac{E}{\in} = \frac{\sigma}{\in_0\in_r}$$

hence equation (4.8) can be rewritten as

$$E_1 = \frac{\sigma - \sigma_p}{\in_0} = \frac{\sigma}{\in_r\in_0} \qquad \ldots (4.9)$$

Therefore the electric field between capacitor plates in presence of a dielectric is now smaller than the field in absence of dielectric. Consequently the potential difference between plates decrease whereby capacitance increases.

From equation (4.9)

$$\sigma - \sigma_p = \frac{\sigma}{\in_r}$$

So, induced charge density

$$\sigma_p = \sigma\left(1 - \frac{1}{\in_r}\right) \qquad \ldots (4.10)$$

Magnitude of Polarsation vector is equal to the magnitude of induced surface charge density, so

$$|\vec{P}| = \chi_e E_1 = \chi_e \frac{\sigma}{\in_0\in_r} = \sigma_p \qquad \ldots (4.11)$$

From equations (4.10) and (4.11) following relation is obtained between electric succesptiblity and dielectric constant of the material.

$$\chi_e = \in_0 (\in_r - 1) \qquad \ldots (4.12)$$

4.8.1 Capacitance of a parallel plate capacitor completly filled with a dielectric

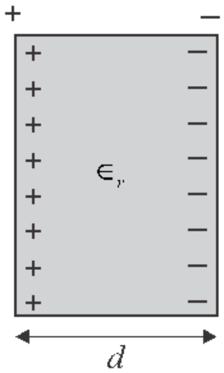


Fig 4.12 : A parallel plate capacitor filled with dielectric

As shown in fig 4.12, consider a parallel plate capacitor with space between plates completely filled with a dielectric of dielectric constant $\in_r$ let A and d are respectively the plate area and separation between plates. In presence of dielectric the net electric field between the plates

$$E = \frac{\sigma}{\in_0 \in_r} = \frac{Q}{A \in_0 \in_r} \qquad \ldots (4.13)$$

So the potential difference between plates

$$V = Ed = \frac{Qd}{A \in_0 \in_r}$$

and capacitance $C = \frac{Q}{V} = \frac{Q}{Qd / A \in_0 \in_r}$

$$C = \frac{A \in_0 \in_r}{d} = \in_r C_0 \qquad \ldots (4.14)$$

where $C_0 = \frac{\in_0 A}{d}$ is capacitance with free space or air as medium between the capacitor plates. Thus the capacitance of a capacitor is increased by a factor of $\in_r$ when the space between the plates is filled with a dielectric of dielectric constant $\in_r$. Although the result has been derived for a parallel capacitor but is valid for any capacitor.

4.8.2 Capacitance of a parallel plate capacitor partilly filled with a dielectric

This situation is shown in fig 4.13 where a dielectric slab of thickness t (< d) is placed in space between the capacitor plates. In this situation the thickness of region between plates where free space or air is present is d-t.

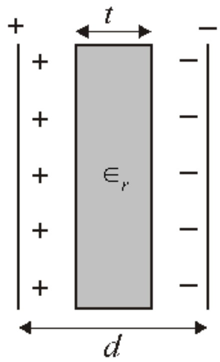


Fig 4.13 : Capacitor partially filled with dielectric

If the electric field in free space or air portion of the space is $E_0 = \frac{\sigma}{\in_0} = \frac{Q}{A \in_0}$ the electric field in the dielectric portion will be $E_0 / \in_r$ and the effective potential difference between the plates is then given by

V = potential difference across portion containing air + potential difference across portion containing dielectric

$$V = E_0 (d - t) + \frac{E_0}{\in_r} t$$

or $V = E_0 \left[d - t + \frac{t}{\in_r} \right] = \frac{Q}{A \in_0} \left[d - t + \frac{t}{\in_r} \right]$

So capacitance $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A \in_0} \left[d - t + \frac{t}{\in_r} \right]}$

or $C = \frac{A \in_0}{(d - t + t / \in_r)} = \frac{A \in_0}{\left[d - t (1 - t / \in_r) \right]} \qquad \ldots (4.15)$

If t = d i.e the dielectric fills the space between plates completely then we obtain $C = \frac{A \in_0 \in_r}{d} = \in_r C_0$ which is same is in equation (4.14). Also if t = 0 we obtain $C = \frac{A \in_0}{d}$ as expected in the absence of dielectric medium.

4.8.3 Capaitance of a parallel plate capaitor filled with different dielectric materials of different thicknesses

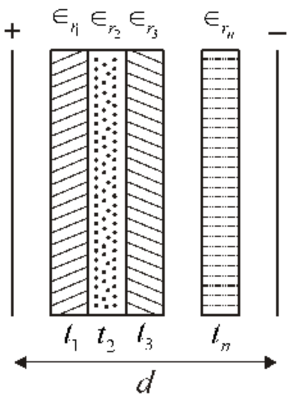


Fig 4.14 : Capacitor filled with different dielectrics

As shown in fig 4.14 if a number of dielectric media of dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}, \ldots \in_{r_n}$ having thicknesses $t_1, t_2, t_3, \ldots t_n$ respectively are present in region between capacitor plates then the thickness of the portion having free sapce or air as medium will be $\left[d-(t_1+t_2+t_3+\ldots+t_n)\right]$. In this case the potential difference across the capacitor is given by

V = potential difference across air portion + sum of potential across different media

$$V = E_0\left[d-(t_1+t_2+t_3+\ldots+t_n)\right]$$

$$+\frac{E_0}{\in_{r_1}}t_1+\frac{E_0}{\in_{r_2}}t_2+\frac{E_0}{\in_{r_3}}t_3+\ldots+\frac{E_0}{\in_{r_n}}t_n$$

where $E_0 = \frac{\sigma}{\in_0} = \frac{Q}{A \in_0}$ is electric field in region of capacitor with air or free space.

or $V = E_0\left[d-(t_1+t_2+\ldots+t_n)\right.$

$$\left.+\frac{t_1}{\in_{r_1}}+\frac{t_2}{\in_{r_2}}+\frac{t_3}{\in_{r_3}}+\ldots+\frac{t_n}{\in_{r_n}}\right]$$

$$V = \frac{Q}{A \in_0}\left[d-(t_1+t_2+\ldots+t_n)\right.$$

$$\left.+\frac{t_1}{\in_{r_1}}+\frac{t_2}{\in_{r_2}}+\frac{t_3}{\in_{r_3}}+\ldots+\frac{t_n}{\in_{r_n}}\right]$$

So, capacitance

$$C = \frac{Q}{V} = \frac{A \in_0}{\left[d-(t_1+t_2+t_3+\ldots+t_n)+\frac{t_1}{\in_{r_1}}+\frac{t_2}{\in_{r_2}}+\frac{t_3}{\in_{r_3}}+\ldots+\frac{t_n}{\in_{r_n}}\right]} \quad \ldots(4.16)$$

If $(t_1+t_2+t_3+\ldots+t_n) = d$ i.e there is no part of the space between capacitor plates where air or free space is present, then

$$C = \frac{A \in_0}{\frac{t_1}{\in_{r_1}}+\frac{t_2}{\in_{r_2}}+\frac{t_3}{\in_{r_3}}+\ldots+\frac{t_n}{\in_{r_n}}} \quad \ldots(4.17)$$

Example 4.9 : The plate area of a parallel plate capacitor is 100 cm^2 and separation between plates is 1 mm. On connecting this capacitor across a 120 volt battery a charge of 0.12 μC accumulates on the plates. Find the dielectric constant of material present between the capacitor plates.

Solution : Capacitance with dielectric

$$C = \frac{Q}{V} = \frac{A \in_0 \in_r}{d}$$

So $\quad \in_r = \frac{Qd}{A \in_0 V} = \frac{0.12\times10^{-6}\times1\times10^{-3}}{100\times10^{-4}\times8.85\times10^{-12}\times120}$

$$= 0.001129\times10^4$$

or $\quad \in_r = 11.3$

Example 4.10 : A slab of ebonite (dielectric constant = 3) of 6 mm thickness is placed in space between capacitor plates of a parallel plate capacitor having plate area $2\times10^{-2}\,m^2$ and plate separation 0.01 m. What is the capacitance of this capacitor.

Solution : Use $C=\dfrac{A\in_0}{d-t+\dfrac{t}{\in_r}}$

$$=\frac{2\times10^{2}\times8.85\times10^{12}}{0.01-6\times10^{3}+\dfrac{6\times10^{3}}{3}}$$

$$C=\frac{17.70\times10^{14}}{0.01-4\times10^{-3}}=\frac{17.70\times10^{14}}{0.01-0.004}=\frac{17.70\times10^{14}}{0.006}$$

$$C=2.95\times10^{11}F=29.5\times10^{12}F=29.5\,pF$$

Example 4.11 The capacitance of a capacitor is C. The separation between the plates of this capacitor is d. If the space between the plates is filled with a dielectric material of dielectric constant $\in_r$ upto a distance of 3d/4 then calculate the new capacitance.

Solution : Let new capacity be C[1] then

$$C'=\frac{A\in_0}{d-t+\dfrac{t}{\in_r}}$$

Given $t=\dfrac{3}{4}d$

$$\therefore\quad C'=\frac{A\in_0}{d-\dfrac{3d}{4}+\dfrac{3d}{4\in_r}}=\frac{A\in_0}{\dfrac{d}{4}+\dfrac{3d}{4\in_r}}=\frac{A\in_0}{\dfrac{d}{4\in_r}(\in_r+3)}$$

So $\quad C'=\dfrac{4\in_r A\in_0}{d(\in_r+3)}=\dfrac{4\in_r}{\in_r+3}C$

$$\because\quad C=\frac{A\in_0}{d}$$

Hence capacity of capacitor will be $\dfrac{4\in_r}{(\in_r+3)}$ times.

4.9 Capacitance of a spherical capacitors

A spherical capacitor consists of two concentric shperical conductors of nearly equal size, one of the conductor is given a positive charge Q and other an equal and opposite charge -Q.

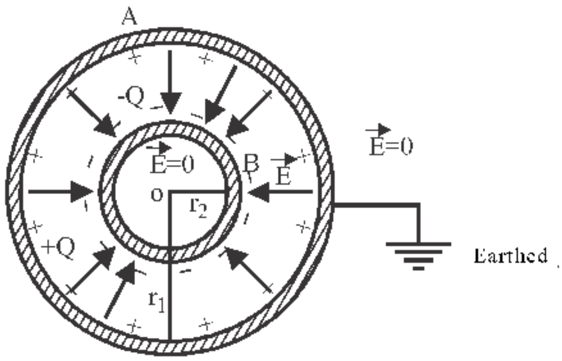


Fig 4.15 : A spherical capacitor

Fig. 4.15 shows a spherical capacitor. Inner conducting sphere can be solid or hollow and is surrounded by a concentric conducting shell A. The radii of A and B are r_1 and r_2 respectively. Note that the outer sphere B is grounded (earhted). Let a negative charge -Q is given to the inner sphere, induced charges +Q and -Q appears on the inner and outer surfaces of shell A. Since shell A is grounded the charge - Q on its outersurface flows to earth.

Potential on sphere B due to its own charge - Q is

$$V_B=\frac{1}{4\pi\in_0}\cdot\frac{(-Q)}{r_2}\qquad\ldots(4.18)$$

However, sphere B is surrounded by shell A which has a positive charge Q and as for all internal points of a charged shell the potential is same as that on its surface, so the potential at surface of B due to charge on sphere A

$$V'_B=\frac{1}{4\pi\in_0}\cdot\frac{(+Q)}{r_1}\qquad\ldots(4.19)$$

So the net potential on sphere B is

$$V=V_B+V'_B$$

$$V = \frac{Q}{4\pi \in_0 r_1} - \frac{Q}{4\pi \in_0 r_2}$$

$$V = \frac{Q}{4\pi \in_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V = -\frac{Q}{4\pi \in_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right] \qquad \ldots (4.20)$$

As the outer sphere A is earthed its potential is zero. Thus the potential difference between A and B

$$V_{AB} = 0 - V$$

$$V_{AB} = 0 - \left[\frac{-Q}{4\pi \in_0}\frac{(r_1 - r_2)}{r_1 r_2}\right]$$

or $$V_{AB} = \frac{Q}{4\pi \in_0}\frac{(r_1 - r_2)}{r_1 r_2} \qquad \ldots (4.21)$$

So if for free space between A and B if the capacitance of this capacitor is C_0 then

$$C_0 = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{4\pi \in_0}\left(\frac{r_1 - r_2}{r_1 r_2}\right)}$$

or $$C_0 = 4\pi \in_0 \left(\frac{r_1 r_2}{r_1 - r_2}\right) \qquad \ldots (4.22)$$

Thus, the capacitance of a spherical capciator depends on the size of spheres and $(r_1 - r_2)$. To have a large capacity, two spheres of sufficient size with nearly equal radii must be taken.

If a dielectric medium of dielectric constant $\in_r$ is filled between the spheres, then capcitance is given by

$$C_m = 4\pi \in \frac{r_1 r_2}{r_1 - r_2}$$

$$C_m = \in_r 4\pi \in_0 \left(\frac{r_1 r_2}{r_1 - r_2}\right) \Rightarrow C_m = \in_r C_0$$

From equation 4.22 it can be seen that]

(i) It $r_1 = R$ and $r_2 = \infty$ than $C_0 = 4\pi \in_0 R$

Which is the capacitance of an isolated spherical conductor i.e a spherical conductor can be treated as a spherical capacitor with outer sphere at infinity.

(ii) If both r_1 and r_2 are made large but $r_1 - r_2 = d$ is kept fixed we can write $r_1 r_2 \approx 4\pi r^2 = A$ where r is approximatelythe radius of each sphere and A is its area. Equation 4.22 then becomes which is same as the equation for a parallel plate capacitor.

Example 4.12 : The radii of inner and outer spheres of a spherical capacitors are 2 m and 1 m and a dielectric mediumof dielectric constant $\in_r = 8$ fills the space between them. Calculate the capacitance of the capacitor.

Solution : For a spherical capacitor filled with dielectric

$$C_m = \frac{4\pi \in_0 \in_r r_1 r_2}{(r_1 - r_2)}$$

Here $r_1 = 2\,\text{m}$, $r_2 = 1\,\text{m}$, $\in_r = 8$

$$C_m = \frac{1}{9\times10^9}\frac{8\times2\times1}{(2-1)}$$

$$C_m = \frac{16}{9}\times10^{-9}\ \text{F}$$

$$C_m = 1.78\times10^{-9}\ F = 1.78\,\text{nF}$$

4.10 Combination of Capacitors

There are many situations in electric circuit where two or more capacitors are used. Two special methods of combinations frequently used are series and parallel combinations. Any combination should have two points which may be connected to a battery to apply a potential difference.

4.10.1 Series Combination

In series combination capacitors are wired serially one after the other and a potential difference may be applied across the two ends of the series. In other words in such a combination second plate of first capacitor is connected to the first plate of second capacitor the second plate of the second capacitor is connected to the first plate of the third and so on. The charge delivered to each capacitor of the series combination has the same value.

Figure 4.16 shows the series combination of three capacitors having capacitance C_1, C_2 and C_3 respectively. The points P and N are end points of this series combination and serve as the points through which a potential difference may be applied. In figure point P is connected to the positive terminal and N to the negative terminal of the battery. Second plate of capacitor C_1 is connected to the first plate of capacitor C_2 and second plate of C_2 is connected the first plate of capacitor C_3. In this manner any number of capacitors can be connected in series.

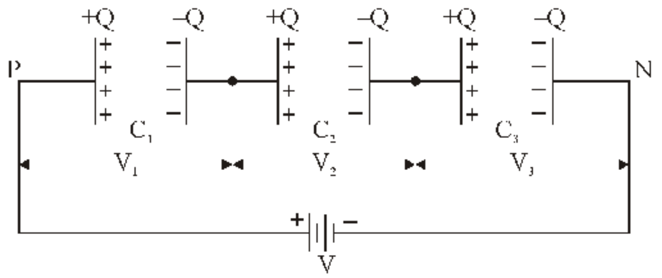


Fig 4.16 : Series combination of capacitors

We can understand how the capacitors end up with identical charge by following reasoning. Let the battery supply a charge + Q to the first plate of the first capacitor (C_1). By electrostatic induction a charge - Q appears on the inner face of second plate of C_1 and a charge + Q an its outer face. This + Q charge on the second plate of C_1 flows to the first plate of second capacitor C_2 inducing charge + Q on its outer face and charge - Q on inner face. The process is repeated for the third capacitor and the charge + Q on outer plate of C_3 flows to the negaive terminal of the battery. Thus we have a positive charge + Q on the inner side of the first plate of capacitor C_1 and a negative charge - Q on the inner side of the second plate of capacitor C_3. This compeletes the charge distribution with each capacitor ending with same charge. Since their capacitances are different the potential differences across the plates of each capacitor are different. Let the potential differences across C_1, C_2 and C_3 are V_1, V_2 and V_3 respectively then

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

If the potential difference across P and N is V then

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \text{. . . (4.23)}$$

If the total (equivalent) capacity of the combination is C_s, then

$$V = \frac{Q}{C_S} \qquad \text{. . . (4.24)}$$

So
$$\frac{Q}{C_S} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \qquad \text{. . . (4.24)}$$

or
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \text{. . . (4.25)}$$

Like wise the equivalent capacitance of n capacitors C_1, C_2..........C_n connected in series is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \qquad \text{. . . (4.26)}$$

If the combination is replaced by a single capacitor of this capacitance (C_s) the single capacitor will store the same amount of charge for a given potential difference as the combination does.

It is obvious that

(i) The reciprocal of the equivalent capacitance of the series combination is equal to the sum of the reciprocals of individual capacitances of capacitors in series combination.

(ii) The equivalent capacitance is smaller then the smallest capcitance present in the series combination.

Some points worth noting regarding series combination are

(i) Series combination of the capacitors is used in situations where the applied voltage is high and a single capacitor can not sustain it or when we require a capacitor of smaller capacitance than that of capacitors available.

(ii) In series combination amount of charge is same for all capacitors irrespective of their capacitances

$Q_1 : Q_2 : Q_3 : ... = 1 : 1 : 1 : ...$

(iii) If n capacitors of equal capacitance C are connected in series then their equivalnt capacitance is $C_S = \frac{C}{n}$ and potential difference across each is V/n.

(iv) If a number of dielectric slabs having dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}$ ... are placed between the plates of a capacitor parallel to plates as shown in Fig 4.17, then this capacitor can be regraded as a series combination of as many capacitors, however, the distance between plates of each such capacitor is equal to the thickness of corresponding dielectric slab.

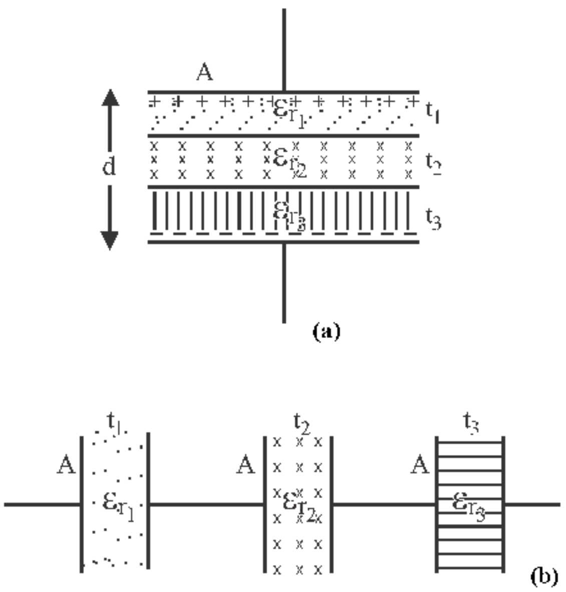


Fig 4.17 : Capacitor containing different dielectric slab parallel to plates

For series combination 4.17(b)

$$C_1 = \frac{\in_{r_1} \in_0 A}{t_1}, \; C_2 = \frac{\in_{r_2} \in_0 A}{t_2}$$

$$C_3 = \frac{\in_{r_3} \in_0 A}{t_3}$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{t_1}{\in_{r_1} \in_0 A} + \frac{t_2}{\in_{r_2} \in_0 A} + \frac{t_3}{\in_{r_3} \in_0 A}$$

$$\frac{1}{C_S} = \frac{1}{\in_0 A}\left[\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}}\right]$$

$$C_S = \frac{\in_0 A}{\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}}}$$

4.10.2 Parallel Combination of Capacitors

A number of capacitors are said to be connected in parallel if potential difference across each is the same (and is equal to the applied voltage). In such a combinations capacitors are arranged (wired) in such a manner that all right hand (first) plates of all capacitor are connected at one common point and left hand (second) plates are connected at another common point. One of these common points is at a higher potential than other when a battery is connected between these points.

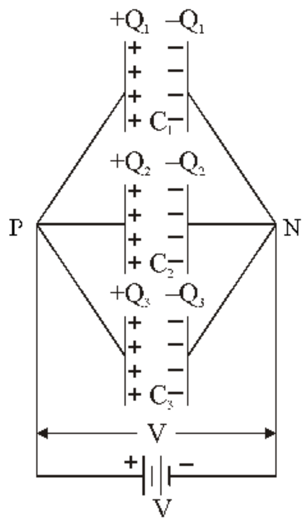


Fig 4.18 : Parallel combination of capacitors

Figure 4.18 shows parallel combination of three capacitors of capacitances C_1, C_2 and C_3 respectively. Note that first plates of each capacitor is connected to higher potential point P (positive terminal of battery) while negative plates are connected to lower potential point N (negative terminal of battery). Thus potential difference across each capacitor is same (equal to the potential difference across the terminals of battery).

Let the battery supplies a charge + Q which is distributed on the first plates of the three capacitors according to the cpapacitance of each capacitor. Let the charges on these plates be Q_1, Q_2 and Q_3. Due to electrostatie induction the charges -Q_1, -Q_2 and -Q_3 appears on inner faces of second plates of respective capacitors. Positive induced charges appearing on outer faces of these plates flows to the negative terminal of battery. So

$$Q_1 = C_1V,\ Q_2 = C_2V \text{ and } Q_3 = C_3V$$

and total charge $Q = Q_1 + Q_2 + Q_3$

or $Q = C_1V + C_2V + C_3V$

or $Q = (C_1 + C_2 + C_3)V$ . . . (4.27)

If the equivalent capacitance of the parallel combination is denoted by C_P then

$$Q = C_PV \quad \text{. . . (4.28)}$$

From equation (4.27)m and (4.28)

$$C_PV = (C_1 + C_2 + C_3)V$$

or $C_P = C_1 + C_2 + C_3$ . . . (4.29)

Like wise for a parallel combination of n different capacitors

$$C_P = C_1 + C_2 + C_3 + ... + C_n \quad \text{. . . (4.30)}$$

It is obvious that

(a) For parallel combination of capacitors, equivalent capacitance is equal to the sum of he capacitances of individual capacitors.

(b) In a parallel combination the equivalent capacitance is always greater then the largest capacitance present in the combination.

Following points are worth noting regarding the parallel combination of capacitors.

(i) Such a combination is used when a large capacitance required for a given working voltage or a capacitor of large capacitance is to be obtained from given capacitors having smaller capacitances.

(ii) In parallel combination potential difference across each capacitor is some

$$V_1 : V_2 : V_3 : ... = 1 : 1 : 1 : ...$$

but the charge on each capacitor is proportional to its capacitance

$$Q_1 : Q_2 : Q_3 \text{ ....} = C_1 : C_2 : C_3$$

(iii) If n identical capacitors each of capacitance C are connected in parallel the equivalent capacitance of the combination is n times the individual capacitance

$$C_P = nC$$

(iv) If n a number of dielectric slabs each of some thickness d having dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}$ ... are placed in the space between a capacitor (of plate area A) as shown in fig 4.19 then these can be considered equivalent to a parallel combination of as may capators for the capacitor with three dielectric placed as in fig 4.19 (a), the equivlalent parallel combination is shown in fig 4.19 (B)

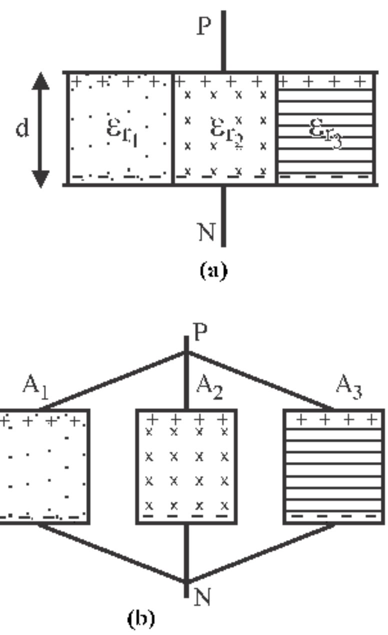


Fig. 4.19 : Parallel combination for different media

$$C_1 = \frac{\in_{r_1} \in_0 A_1}{d},\ C_2 = \frac{\in_{r_2} \in_0 A_2}{d}$$

$$C_3 = \frac{\in_{r_3} \in_0 A_3}{d}$$

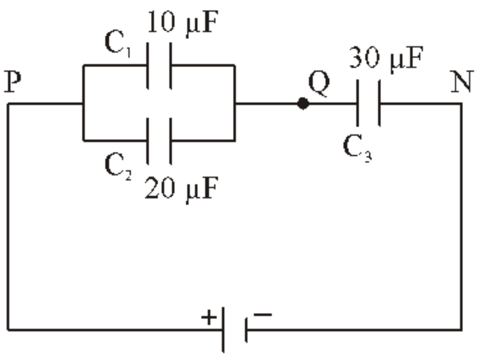
here A_1, A_2 and A_3 are areas corresponding dielectric slabs respectively.

Capacitance of parallel combination

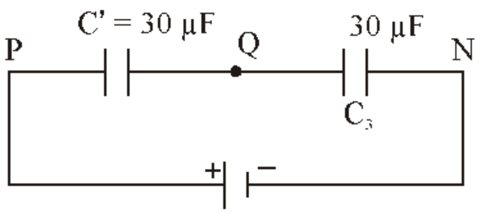
$$C_P = C_1 + C_2 + C_3$$

or $$C_P = \frac{\in_0}{d}\left[A_1 \in_{r_1} + A_2 \in_{r_2} + A_3 \in_{r_3}\right]$$

Example 4.13 : For the capacitor combination shown in Fig find the equivalent capacitance between points P and N



Solution : In the circuit shown capacitors of 10 μF and 20 μF are connected in parallel and their equivalent capacitance $C' = 10 + 20 = 30\,\mu F$ so these can be replaced by a 30 μF capacitor, which is now in series with another 30 μF (C_3) capacitor already present in the circuit so the capacitance of the given combination.

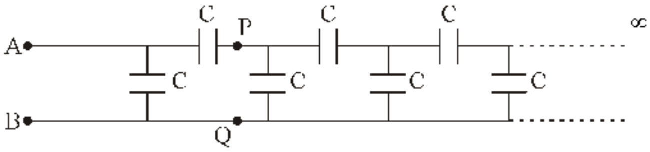


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_3}$$

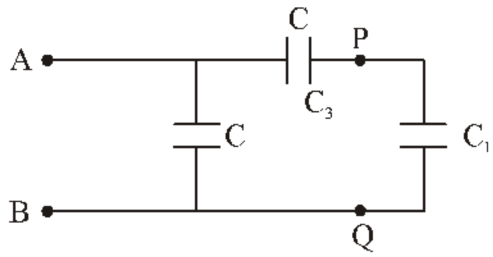
$$\frac{1}{C} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30}$$

$$C = 15\,\mu F$$

Example 4.14 An infinite circuit is formed by the repetition of same link consisting of two identical capacitors each of capacitance C. Find the effective capacitance between points A and B.



Solution : As the ladder shown in fig of question is infinity long the capacitance of ladder to the right of points P and Q (say C) is same as that between A and B. The ladder shown above can then be replaced by the circuit shown in Fig below



Fom this figure equivalent capacitance between A and B

$$C_1 = C + \frac{CC_1}{C + C_1}$$

$$C_1 = \frac{C(C + C_1) + CC_1}{(C + C_1)}$$

$$C_1^2 - CC_1 - C^2 = 0$$

$$C_1 = \frac{C \pm \sqrt{C^2 + 4C^2}}{2} = \frac{C \pm \sqrt{5C^2}}{2}$$

$$C_1 = \frac{C\left(1 \pm \sqrt{5}\right)}{2}$$

$$C_1 = \frac{\left(1 + \sqrt{5}\right)C}{2} \text{ or } C_1 = \frac{\left(1 - \sqrt{5}\right)C}{2}$$

but as C_1 can not negative we have to choose

$$C_1 = \frac{\left(1+\sqrt{5}\right)C}{2}$$

as correct value out of the two values calculated above.

4.11 Energy stored in a capacitor

As has been mentioned earlier a capacitor is a device to store electrical energy. In this section we discuss about this aspect.

We have seen in previous chapter that electrostatic potential energy is associated with any charge configuration which is equal to the work done by extenral force in assembling this configuration by bringing the constituent charges from infinity (where charges are assumed at rest) to their respective locations in configuration under condsideration. As work must be done by an external agent in charging a capacitor which is stored in the form of electrostatic potnetial energy in the electric field between capacitor plates. To understand the process of charging we imagine that some external agent is transfering electrons from one plate to other plate of a capacitor. The plate from which electrons are removed is becoming positively cahrged while the plate recieving electrons is becoming negative, thus a charge separation takes plate in the process. As the charge accumlates on the capacitor plates this external agent has to do increasingly large amount of work to transfer more electrons. In practice this work is done by a battery at the cost of chemical energy stored in it.

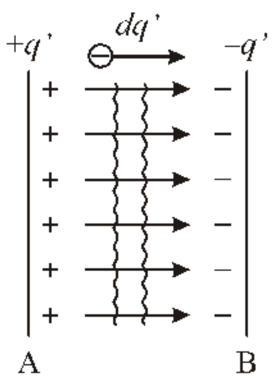


Fig. 4.20 : Imagination of charge transfer from one plate of a capacitor to other

Suppose that a certain instant a charge q^1 is already present one plate of capacitor during the charging process. At that instant the potential difference V between capacitor plates is q^1/C if now an increment of charge dq is to be transferred to this plate the increment of work needed is given by

$$dW = V'\,dq \qquad \ldots (4.31)$$

the total work required to charge the capcitor from

$$W = \int_0^Q V'\,dq$$

$$W = \int_0^Q \frac{q'}{C}\,dq$$

$$W = \frac{1}{C}\int_0^Q q'\,dq$$

$$\Rightarrow \quad W = \frac{1}{C}\left[\frac{q'^2}{2}\right]_0^Q$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

This work is stored as potential energy U in the capcitor

$$\text{So } U = \frac{1}{2}\frac{Q^2}{C} \qquad \ldots (4.32)$$

On substituting Q = CV in equation (4.32) we obtain

$$U = \frac{1}{2}CV^2 \qquad \ldots (4.33)$$

and if we substitute V = Q / C in equation (4.32)

$$U = \frac{1}{2}QV \qquad \ldots (4.34)$$

thus we can express the energy stored in a charged capacitor as

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This result applies to any capcitor regardless of its geometry. These equations, however, do not tell us where this energy is stored. By following argument we can explain that this energy is stored in capacitor in electric field between the plates. Consider two parallel plate capacitors 1 and 2 having same plate area A but the plate separation for capacitor 1 is double of that for capacitor 2. Thus the capacitance of capacitor 1 is half the capacitance of capacitor 2. If both the capacitors are given same amount of charge q then electric field between the plates given by $E = \frac{q}{\in_0 A}$ is same for both the capacitors. Because of the (above mentioned) difference in capacitance, from equation 4.33 we note that the energy stored in capacitor 1 is twice that in capacitor 2. Also note that for the volume between capacitor plates capacitor 1 has twice the volume than capacitor 2. So for same value of q for both capacitors, one with twice volume has twice the stored energy. However as the electric field is present in the entire volume between the capacitor plates it is logical to associate electrostatic potential energy with electric field present in this region.

4.11.1 Energy Density of Electric Field between Plates for a parallel Capacitor

For a parallel plate capaciotor having plate area A and charge Q the electric field between capacitor plates is given by

$$E = \frac{\sigma}{\in_0} = \frac{Q}{\in_0 A}$$

$$Q = \in_0 EA$$

The electrostatic energy of a charged capacitor

$$U = \frac{1}{2}\frac{Q^2}{C}$$

but $C = \frac{\in_0 d}{d}$

So $$U = \frac{1}{2}\frac{(\in_0 EA)^2}{\left(\frac{\in_0 A}{d}\right)}$$

$$U = \frac{1}{2}\in_0 E^2 Ad$$

but Ad = V (Volume between capacitor plates)

not to confuse with potential difference.

So energy density, u, i.e energy per unit volume

$$u = \frac{U}{V} = \frac{1}{2}\hat{I}_0 E^2 \qquad \ldots (4.35)$$

Although the equation 4.35 is derived for the special case of a parallel plate capacitor, it has a general validity. Thus wherever electric field E is present the energy per unit volume u is given by 1/2 $\hat{I}_0$ E^2. In general E varies with position, so u is a function of coordinates. For the special case of the parallel plate capacitor E and u do not vary with position in the region between capacitor plates.

Example 4.15 : The area of each plate for a parallel plate capacitor is 90 cm^2 and plate separation is 2.5 mm. It is charged by connecting across a 400 V supply. Find the electrostatic energy stored in the capacitor.

Solution : Here $A = 90\,cm^2 = 90\times10^{-4}\,m^2$

$$d = 2.5\,mm = 2.5\times10^{-3}\,m$$

$$V = 400\,V$$

Use $$U = \frac{1}{2}CV^2$$

Here $$C = \frac{\in_0 A}{d} = \frac{8.85\times10^{12}\times90\times10^{4}}{2.5\times10^{-3}}$$

$$C = 3.18\times10^{-11}F = 31.8\,pF$$

$\therefore$

$$U = \frac{1}{2}\times3.18\times10^{11}\times(400)^2 = 2.54\times10^{6}J$$

4.11.2 Energy stored in combination of capacitors

(a) Series Combination : Consider n capacitances $C_1, C_2, C_3, \ldots, C_n$ connected in series, the equivalent capacitance of the combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Here charge Q on each capacitor is same, Energy stored in equivalent capacitor $U = \frac{1}{2}\frac{Q^2}{C_S}$

$$U = \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}\right]$$

or $U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \ldots + \frac{Q^2}{2C_n}$

or $U = U_1 + U_2 + U_3 + \ldots + U_n$

(b) Parallel combination : For n capacitors having capacitances $C_1, C_2, C_3, \ldots, C_n$ connected in parallel the equivalent capacitance is given by

$$C_P = C_1 + C_2 + C_3 + \ldots + C_n$$

Here potential difference V across each capacitor is same energy stored in equivalent capacitor

$$U = \frac{1}{2}C_P V^2$$

or $U = \frac{1}{2}[C_1 + C_2 + C_3 + \ldots + C_n]V^2$

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + \ldots + \frac{1}{2}C_nV^2$$

or $U = U_1 + U_2 + U_3 + \ldots + U_n$

Therefore, for both series and parallel combination of capacitor the total energy stored in the combination is equal to the sum of energies stored in individual capacitors.

4.12 Redistribution of Charge and Loss of energy on sharing of Charges

When two conductors charged to different potential are connected through, a conducting wire or brought in contact with each other, charge flows from the conductor at higher potential to the conductor at lower potential till both acquire a common potential. There is a loss in energy in this process of redistribution of charges.

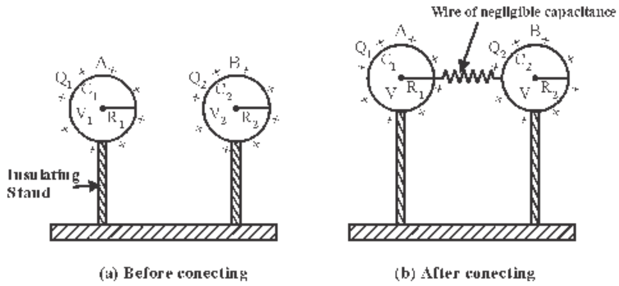


Fig 4.21 : (a) Before conecting (b) After conecting

Consider two isolated spherical conductors A and B of radii R_1 and R_2 charged to potentials V_1 and V_2 [Fig 4.21 a]. If capacitances of the conductors are C_1 and C_2 and charges on them are Q_1 and Q_2 respectively.

Then $Q_1 = C_1V_1$ . . . (4.36)

and $Q_2 = C_2V_2$ . . . (4.37)

So total charge on the system before the two conductor are brought into contact

$$Q = Q_1 + Q_2$$

$Q = C_1V_1 + C_2V_2$ . . . (4.38)

If the two conductors are now connected through a conducting wire of negligble capacitance then the charge will flow from the conductor at higher potential to other at lower potential till both acquire same potential V(say). In this case treating both condcutors as one the capacity of the combined system is $C_1 + C_2$ as the two conductors are at same potential can be treated in a parallel combination.

After the redistribution of charges if Q_1 and Q_2 are the cahrges on C_1 and C_2

$Q_1' = C_1V$ . . . (4.39)

and $Q_2' = C_2V$ . . . (4.40)

From conservation of charge, total charge before redistribution must equal the total charge after redistribution i.e

$$\therefore \quad Q = Q_1' + Q_2' = (C_1 + C_2)V$$

or $\quad C_1V_1 + C_2V_2 = (C_1 + C_2)V$

therefore the common potential is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \qquad \ldots (4.41)$$

and ratio of charges after redistribution

$$\frac{Q_1'}{Q_3'} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2} \qquad \ldots (4.42)$$

thus after redistribution charge is shared in proportion to capacity

If $V_1 > V_2$

then $\quad V_1 > V > V_2$

the change in potential of first conductor is

$$\Delta V_1 = V_1 - V$$

$$\Delta V_1 = V_1 - \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

or
$$\Delta V_1 = \frac{C_2(V_1 - V_2)}{C_1 + C_2} \qquad \ldots (4.43)$$

The change in potential of second conductor

$$\Delta V_2 = V - V_2$$

or
$$\Delta V_2 = \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right) - V_2$$

or
$$\Delta V_2 = \frac{C_1(V_1 - V_2)}{C_1 + C_2} \qquad \ldots (4.44)$$

so
$$\frac{\Delta V_1}{\Delta V_2} = \frac{C_2}{C_1}$$

Thus after redistribution the magnitudes of change in potential of the two conductors are in inverse ratio of their capacitances. The above discussion is also true for sharing of charge between two charged capacitors initially at different potentials.

4.13 Energy loss

Initially before contact the total electrostaic potential energy of the system is

$$U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \qquad \ldots (4.45)$$

After the conductors are joined through a conducting wire, total electrostatic potential energy of the system is

$$U' = \frac{1}{2}(C_1 + C_2)V^2$$

$$U' = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$

$$\because \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

So
$$U^1 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2} \qquad \ldots (4.46)$$

So energy loss $\Delta U = U - U'$

or
$$\Delta U = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

or
$$\Delta U = \frac{1}{2}\left[\frac{C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2}{C_1 + C_2}\right]$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2 \qquad \ldots (4.47)$$

Since $(V_1 - V_2)^2$ is always positive so $\Delta U = 0$

thus there is always a loss of electrostatic potential energy, when two conductors or two capacitors charged at different potentials are connected in parallel. This loss of energy is mainly in the form of heat when charge flows from one conductor to other through the conducting wire having some finite resistance and also in form of light and sound if sparking takes place. In sharing there is no loss of energy

If $V_1 = V_2$ so that $\Delta U = 0$

In process of charging a capacitor by a battery of emf V half of the energy QV supplied by battery gets stored in capacitor ($U=\frac{QV}{2}$) and remaining half (QV/2) is dissipated in form of heat in connecting wires.

Example 4.16 A 600 pF capacitor is charged to 200V using a battery. Now the battery is disconnected and the plates of this capacitor are connected in parallel to an initially uncharged capacitor of 600 pF. Find the loss of electric potential energy in the process.

Solution : Here $C_1=C_2=600\,pF$

$$=600\times10^{12}F=6\times10^{10}F$$

$$V_1=200V,\ V_2=0$$

loss in energy $=\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

$$=\frac{1}{2}\times\frac{6\times10^{10}\times6\times10^{16}}{12\times10^{-10}}(200-0)^2$$

$$=6\times10^{6}\text{ J}$$

Example 4.17 A 900 pF capacitor is charged to 200 V (a) Find the electrostatic potential energy stored in the capacitor (b) It now the capacitor is disconnected from battery and joined in parallel with another uncharged 900 pF capacitor then calculate the electrostatic potential energy stored in the combination.

Solution : (a) Given $C=900\,pF=9\times10^{-10}F$

$$V=100V$$

energy stored

$$U_1=\frac{1}{2}CV^2=\frac{1}{2}(9\times10^{10})(100)^2$$

$$U_1=4.5\times10^{6}J$$

(b) When given charged capacitor is connected in parallel to an identical uncharged capacitor the charge on first is now divided equally on the two. Let the final charge on each capacitor is Q^1 and common potential be V.

$$Q=\frac{Q}{2}\text{ and }V=\frac{V}{2}$$

In this case both capacitors have same value of stored energy so the total energy of system

$$U_2=2\left(\frac{1}{2}QV\right)$$

$$U_2=2\times\frac{1}{2}\times\frac{Q}{2}\times\frac{V}{2}=\frac{1}{4}QV$$

$$U_2=\frac{1}{2}\times4.5\times10^{-6}=2.25\times10^{-6}\text{ J}$$

and rest half of the energy is spent is the form of heat and EM radiations.

Important Points

1. **Dielectric Materials :** These materials are non conductor of electricity they get polarised when placed in external electric field.
2. **Capacitane** The electric potential V of a conductor is proportional to charge Q given to it i.e Q = CV here the constant of proportionality C is called capacitance of conductor. Numberically the capacitance of a conductor is the amount of charge given to a conductor to raise its electric potential by unity.
3. A capacitor is a device that consists of two closely spaced conductors (plates) with charges + Q and - Q. Its capacitance is also given by Q = CV where V is the potential difference between the plates.
4. The capacitance of a capacitor depends on area of plates, plate separation and dielectric constant of medium present between plates.
5. For an isolated spherical conductor radius R^1 placed in air or vacuum the capacitance is given by $C_0=4\pi\in_0R$.

6. The radius of a spherical conductor of 1 F is greater even than the radius of earth.

7. On filling the space between the plates of a capacitor completely with a of medium of dielectric constant $\in_r$ its capacitance increases $\in_r$ times.

8. Capacitance for a parallel plate capacitor $C = \in_r \frac{\in_0 A}{d}$ having a medium between the plates.

with free space as medium $C_0 = \frac{\in_0 A}{d}$

with medium of dielectric constance $\in_r$

9. For a capacitor partially filled with a dielectric (dielectric constant $\in_r$, and thickness t)

$$C = \frac{\in_0 A}{d - t + \frac{t}{\in_r}}$$

10. Capacitance for a spherical capacitor with free space medium $C_0 = \frac{4\pi \in_0 r_1 r_2}{(r_1 - r_2)}$; with medium of dielectric constant $\in_r$, $C_m = \in_r \frac{4\pi \in_0 r_1 r_2}{(r_1 - r_2)}$

11. In series combination of capacitor charge on each capacitor is same and equivalent capacitance of the combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

12. For parallel combination of capacitor potential difference across each capacitor is same and equivalent capacitance is given by $C_P = C_1 + C_2 + C_3 + \dots + C_n$

13. In charging a capacitor some work has to be done. This work is stored in capacitor in electric field between the plates in the form of electrostatic potential energy . This energy is given by

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

14. When two conductors charged to different potentials are connected, the redistribution of charge takes place in ratio of their capacitance i.e $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$ and their common potential (after charge redistribution) is given by

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

15. The energy loss in above redistribution of charges is given by

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2 \ J$$

Questions for Practice

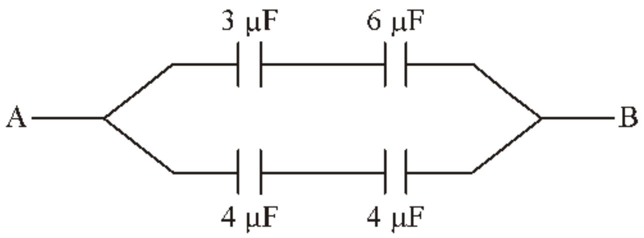
Multiple Choice Questions

1. The correct relation for the capacitance of a parallel plate capacitor is

   (a) $C \propto R$ (b) $C \propto R^2$

   (c) $C \propto R^{-2}$ (d) $C \propto R^{-1}$

2. For the figure shown equivalent capaitance between points A and B is



   (a) 2 μF (b) 4 μF

   (c) 25 μF (d) 3 μF

3. On connecting the two plates of a charged cpacitor by a conducting wire

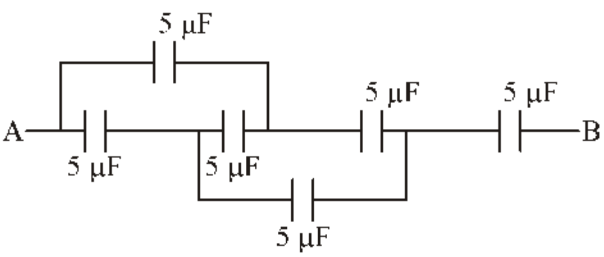
   (a) potential difference across the plates will become infinite

   (b) charge on capacitor will become infinite

   (c) charge on capacitor will become double of its initial value

   (d) Capacitor will be discharged

4. For the figure shown the equivalent capacitance between position A and B



   (a) 5 μF (b) 2.5 μF

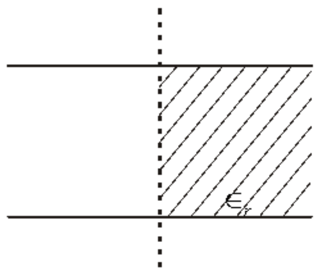
   (c) 10 μF (d) 20 μF

5. The radii of two spherical conductors are in ratio 1 : 2 the ratio of their capacitanes is

   (a) 4 : 1 (b) 1 : 4

   (c) 1 : 2 (d) 2 : 1

6. As shown in Fig. a dielectric slab of dielectric constant $\in_r$ is slided in half the space between capacitor plates. If the initial capacitance of the capacitor is C its new value will be



   (a) $\frac{C}{2}(\in_r+1)$ (b) $\frac{1}{2}\cdot\frac{C}{(\in_r+1)}$

   (c) $\frac{(1+\in_r)}{2C}$ (d) $C(1+\in_r)$

7. Eight drops of mercury of equal radii and each possessing the same charge combine to form a big drop. The capacitance of this big drop as compared to that of each smaller drop is

   (a) 2 times (b) 8 times

   (c) 4 times (d) 16 times

8. A capacitor has capacitance C. It is charged to a potential difference V. It now be is connected across a resitor R then amount of heat dissippated will be

(a) CV^2 (b) $\frac{1}{2}CV^2$

(c) $\frac{1}{3}CV^2$ (d) $\frac{1}{2}QV^2$

9. On giving a charge Q to a capacitor its stored energy is W. On doubling the charge stored energy will be

(a) 2 W (b) 4 W

(c) 8 W (d) W/2

10. Two spherical conductors of 3 μF and 5 μF are charged to 300 V and 500 V and then connected together. The common potential will be

(a) 400 V (b) 375 V

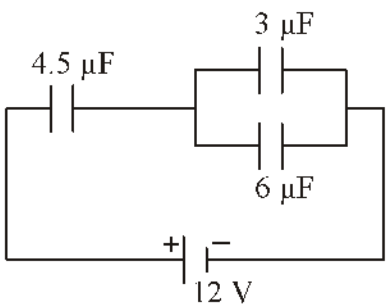
(c) 425 V (d) 350 V

11. The potential energy stored in the region between plates of a capacitor is U_0. If a dielectric slab of $\in_r$ now fills the space between the plates completely then new potential energy will be

(a) $\frac{U_0}{\in_r}$ (b) $U_0 \in_r^2$

(c) $\frac{U_0}{\in_r^2}$ (d) U_0

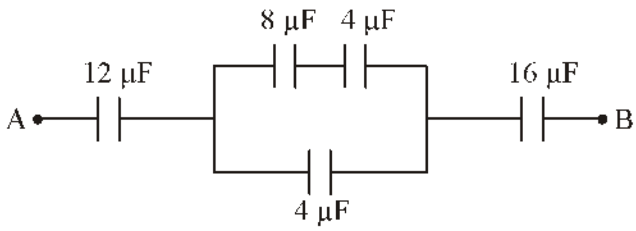
12. For the circuit shown in fig the potential difference across 4.5 μF capcitor is



(a) $\frac{8}{3}V$ (b) 4 V

(c) 6 V (d) 8 V

13. For the circuit shown in fig the equivalent capacitance between A and B will be



(a) 1μF (b) 9 μF

(c) 1.5 μF (d) $1/3\mu F$

Very Short Answer Questions

1. If area of one plate of a parallel plate capacitor is halved then can the device be called a capacitor.
2. What will be the maximum and minimum capacitance obtainable with three capacitors each of 6 μF capacitance.
3. Mention the factors affecting capacitance of a conductor.
4. On assuming earth as a spherical conductor what will be its capacitance?
5. What is the electric field between the plates of a charged parallel plate capacitor with surface charge density σ_on plates.
6. If n capacitors of equal capacitance C are concepted in series what will be the equivalent capacitance.
7. Write expression for energy density for electric field between plates of a parallel plate capacitor.
8. Write unit of energy density.
9. Two capacitors of capacitance C_1 and C_2 are given equal charge write the ratio of electrostatic potential energy stored in them.
10. Mention a conductor which can be given nearly infinite amount of charge.
11. Where and in what form does the energy is stored in a capacitor.
12. What is the net charge on a charged capcitor?
13. On filling the space between the plates of a parallel

plate capacitor with some dielectric its capacitance increases five times. What is th dielectric constant of this materials?

14. What is the basic use of a capacitor?

15. The plate separation of a parallel plate capacitor is d. If a metalic plate of thickness d/2 is placed in the space between plates not touching any plate, then what will be effect on its capacitance?

16. How much work has to be done in charging a 25 μF capacitor if potential differece across it is 500 V.

17. How will you obtain a capacitance of higher value if you have been provided with capacitors of relaively low capacitance.

18. What is the equivalent capacitance of two 2 μF capacitors connected in series.

19. On immersing a parllel plate capacitor in oil what will be the effect on its capactiance. The dielectric constant of oil is 2.

20. The radius of a circular plate for a circular parallel plate capactor is r. If its capacitance is equal to that of a spherical conductor of radius R then find the plate separation.

Short Answer Questions

1. Explain terms conductor and insulator with examples.

2. Distinguish between polar and non polar dielectrics.

3. Derive expression for capacitance of a spherical conductor.

4. What will be the effect on the potential difference across the plates of a capacitor if its plates are brought nearer keeping the charge constant? Explain.

5. A parallel plate capacitor with air as medium is charged to a potential difference V using a source (battery). Without disconnecting the battery air medium is replaced by a dielectric medium of dielectric constant $\in_r$. Explain with reasons the changes observed in the following -

   (i) potential difference

   (ii) electric field between plates

   (iii) capacitance

   (iv) charge

   (v) energy

6. A parallel plate capacitor with air as medium is charged to a potential difference V_0 using a voltage supply. Ater it is disconnected from the voltage supply the space between plates is completely filled by a dielectric. Explain with reasons the changes observed in the following

   (i) charge (ii) potential difference

   (iii) capacitance (iv) electric field

   (v) energy

7. Derive expression for energy stored in a charged capacitor.

8. Three capacitors of capcitance C each are connected first in series and then in parallel. Calculate the ratio of equivalent capacitances in these two situations.

9. n capactors of capaitance C each when connected in series yields an equivalent capacitance C_S and when in parallel yields equivalent capacitance C_p.

   Find the value of $\frac{C_p}{C_s}$.

10. Define electric capacitance and mentioned its SI unit.

11. What will happen to the capacitance of a spherical conductor on trippling charge on it? Give reason.

12. On filling the space between a 2 μF air capacitor by mica its capacitance becomes 5μF. Calculate the dielectric constant of mica.

13. Two spherical conductors of radii R_1 and R_2 have capacitance C_1 and C_2 and charges Q_1 and Q_2 and potential difference V_1 and V_2 respectively $(V_1 > V_2)$. These conductors are now connected by a conducting wire of negligilbe capacitance then show that the ratio of chages in their potential is

$$\frac{\Delta V_1}{\Delta V_2} = \frac{C_2}{C_1}$$

14. What is a capacitor? Explain.
15. Three capacitor having capacitance C_1, C_2 and C_3 are connected in series. Derive expression for equivalent capacitance.
16. Three capacitors having capacitances C_1, C_2 and C_3 are connected in parallel. Derive expression for equivalent capacitance.

Essay Type Questions

1. Discuss the principal of a capacitor and derive the expression for the capacitance of a parallel plate capacitor.
2. Obtain an expression for the capacitance of a partially filled capacitor.
3. Derive an expression for energy density of electric field between the plates of a parallel plate capacitor.
4. What is a spherical capacitor? Derive an expression for the capcitance of spherical capacitor.
5. Explain the charge redistribution when two charged conductors are connected together. Determine the ratio of charges after redistribution and expression for energy loss.

Answer (Multiple Choice Questions)

1. (A) 2. (B) 3. (D) 4. (B) 5. (C) 6. (A)
7. (A) 8. (B) 9. (B) 10. (C) 11. (A)
12. (D) 13. (A)

Short Answer Question

1. No
2. $C_{max} = 18\,\mu F$ and $C_{min} = 2\,\mu F$
3. The capacitance of a conductor depends on its geometry and nature of surrounding medium.
4. $C_0 = 4\pi \in_0 R$ here

   $R = 6400 km = 6400 \times 10^3 m$, $C_0 = 711\mu F$
5. $E = \frac{\sigma}{\in_0}$
6. $C_s = \frac{C}{n}$
7. $\frac{U}{V}$; or $u = \frac{1}{2} \in_0 E^2$
8. J/m^3
9. $$\frac{U_1}{U_2} = \frac{\frac{1}{2}\frac{Q^2}{C_1}}{\frac{1}{2}\frac{Q^2}{C_2}} = \frac{C_2}{C_1}$$
10. Earth, because its capacitance is very large.
11. In form of electrostatic potential energy in electric field between capacitor plates.
12. Zero as plates have equal and opposite charges.
13. $\in_r = \frac{C}{C_0} = \frac{5C_0}{C_0} = 5$
14. To store electric charge and electric energy.
15. $$C' = \frac{\in_0 A}{(d-t)} = \frac{\in_0 A}{(d-d/2)} = 2\left(\frac{\in_0 A}{d}\right) = 2C$$

    i.e capacitance is doubled
16. $W = QV = (CV)V = CV^2$

    $= 24 \times 10^{-6} \times 500 \times 500 = 6\,J$
17. By joining given capacitors is parallel.
18. From $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ this gives $C_s = 1\mu F$
19. $C = \in_r C_0 = 2C_0$ i.e doubled
20. $C = 4\pi \in_0 R \Rightarrow \frac{\in_0 \pi r^2}{d} = 4\pi \in_0 R \Rightarrow d = \frac{r^2}{4R}$

Numberical Problems

1. Calculate the radius of a spherical conductor of 1 pF capacitance. [Ans : 9 mm]

2. The area of each plate of a parallel capacitor is 100 cm^2 and the intensity of electric field in between the plates is 100 N/C. What is the charge on each plate.

   [Ans :+8.85 × 10^{-12} C, –8.85 × 10^{-12} C]

3. A parallel plate capacitor is kept at a certain potential difference. Keeping this potential difference constant to put a dielectric slab of 3 mm thickness between the plates the plate separation is to be increased by 2.4 mm. Calculate the dielectric constant of slab. [Ans : $\in_r = 5$]

4. Two capacitors are of 2μF and 4μF capacitance respectively. Compare the ratio of equivalent capacitances of their series and parallel combinations.

   [Ans : 2 : 9]

5. Consider two metallic spheres of radii 0.05 m and 0.10 m, each having a 75 μF charge. On connecting these spheres by a conducting wire find (i) common potential (ii) amount of charge flown.

   [Ans : 9 × 10^6 V, 25 μC]

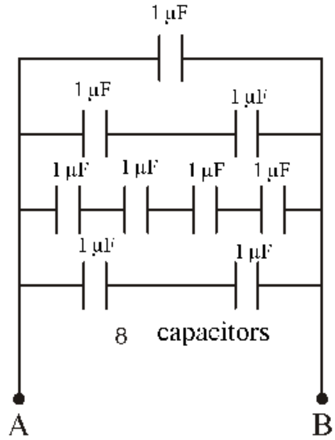
6. A spherical conductor of capacitance 2μF, charged at 150 volt is connected to an 1μF uncharged and conducting sphere. Find common potential, charge on each capacitor after joining.

   [Ans. : V = 100 V, Q_1' = 200μC, Q_2' = 100μC]

7. 125 droplets are charged to 200 V potential. These droplets are combined to make a single drop. Calculate the potential of big drop and change in potential energy.

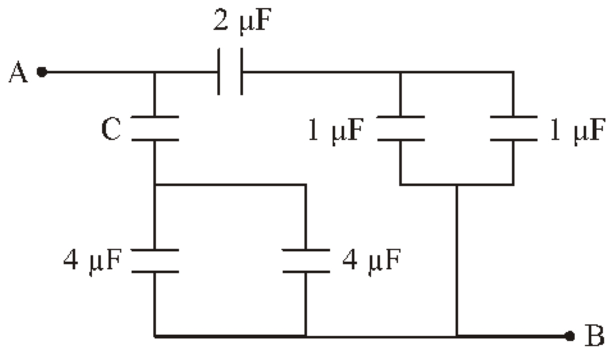
   [Ans : V_b = 500 V, U_b = 25 U_s]

8. In fig shown each capacitor is of 1 μF capacitance. Find equivalent capacitance between points A and B.



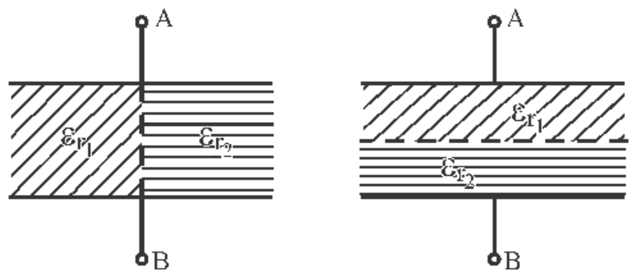
[Ans : 2 μF]

9. In fig shown the equivalent capacitance between points A and B is 5. Calculate the capacitance of capacitor C.



[Ans : C = 8 μF]

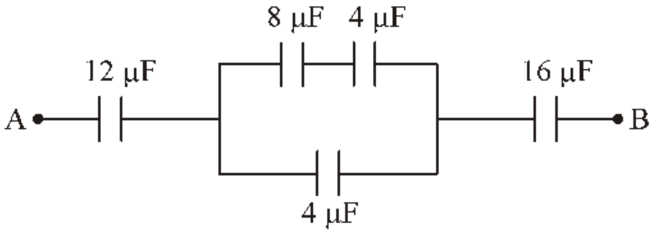
10. For capacitors shown in fig (a) and (b) find capacitances Area of each plate is A and the plate separation is d



(a) (b)

[Ans : (a) $\left[\frac{\in_0 A}{2d}\left(\in_{r_1} + \in_{r_2}\right)\right]$ (b) $\left[\frac{2 \in_{r_1} \in_{r_2} \in_0 A}{d\left(\in_{r_1} + \in_{r_2}\right)}\right]$]

11. Determine equivalent capacitance between points A and B for the Fig shown



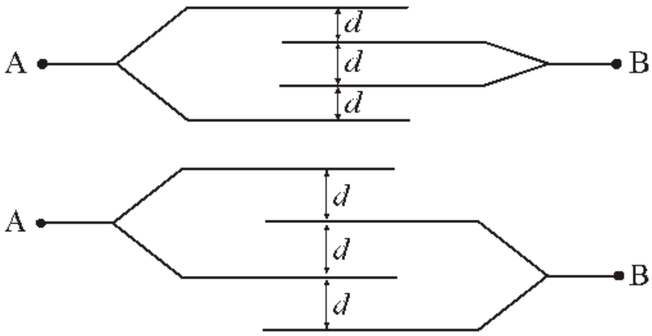
[Ans : $C = \frac{1}{31}\mu F$]

12. An isolated spherical conductor is surrounded by another concentric conducting sphere whose outer surface is earthed. the ratio of the radii of these two spheres is $\frac{n}{n-1}$. Prove that due to this arrangment the capacitance of spherical conductor increases n times.

13. The energy density of a parallel plate condensor is $4.43\times10^{-10}\ J/m^3$. Find electric field intensity between the plates. $\in_0 = 8.86\times10^{\ 12}\ F/m$

[Ans : E = 10 N/C]

14. Determine the equivalent capacitance of the systems shown in fig (a) and (b) between A and B. Area of each plate is A and the separation between adjancent plates is d.



[Ans : (a) $\frac{2\in_0 A}{d}$ (b) $\frac{3\in_0 A}{d}$]

15. Let equivalent capacitances for n indentical capacitors each of capacitance C in series and parallel arrangements are Cs and Cp . Prove that

$$C_p - C_s = \frac{(n^2-1)}{n}C$$

16. The radius of plate used in a parallel plate capaitor is 10 cm. If the plate separation is 10 cm, determine the capacitance of this capacitor for air medium.

[Ans : C = 2.78 pF]

Chapter - 5

Electric Current

Up till now, our discussion regarding the electrical phenomenon was concentrated only on electrostatics, i.e. charges are at rest. Now, we will discuss the situations in which the charges are in motion. The word electric current is used to explain the flow of charges in space. A large number of electrical applications are based on electric current. For example the electric equipment like bulbs, fans in houses. In study of electric current is also important in other areas of science. For example geophysists have interest in charge in atomsphere whereas biolosists study the neuro current in humans that controls muscles .

In this chapter, we define electric current, after that we will discuss its principles. For flow of electric current in a conductor, potential difference is necessary and the device which is used in it is called a cell or battery. So, in this chapter, we will also study regarding cells.

5.1 Electric Current

Net charge flowing per second from any area of cross section is called electric current. If ΔQ is the net charge flowing through the area of cross section in time Δt, then average current, is given by -

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad \ldots (5.1)$$

If the rate of flow of charge does not remain constant over interval of time, then, we define the instantaneous current I as the limit of the preceding equations (5.1) as Δt tends to zero.

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \ldots (5.2)$$

For defining the current, we must use the word net charge. Even though the current is due to the flow of charges, but all the moving charges do not produce electric current. In the absence of external field, there is no net current, because the motion of free electron is random and net effect in any particular direction is zero.

SI unit of electric current is ampere (A), which is a fundamental unit.

$$1\text{A} = \frac{1 \text{ coulomb}}{1 \text{ second}} = 1\text{C/s}$$

(From 20 May 2019 in SI system of unit 1 ampere is redefined and given in terms of a the fundamental constant *e* (the electronic charge). It can be searched at http://physics.nist.gov.

According to convention, the direction of current is given by the direction of flow of +ve charge. Hence, it is assumed that the direction of current is in the opposite direction, i.e. in the direction of flow of -ve charge. Moving charges are called charge carriers. In different cases, the current will be due to flow of different type of charges. Hence,

(i) In conductors, the current is due to the flow of free electrons.

(ii) In electrolysis (electrolytes), current flow is due to the flow of +ve and -ve ions.

(iii) In semi-conductors, the current is due to the flow of electrons and holes.

(iv) In discharge tube, the current is due to the flow of +ve ions of the gas and electrons.

The current can also flow in vacuum, e.g. in picture tube of TV, electrons flow in vacuum and hence the current flows. Although, direction is considered in the flow of electric current, yet it is a scalar quantity, because it is defined in terms of charge and time which are scalar quantities. Current does not follow the law of vector addition, which is explained in the figure 5.1. Here the junction is shown by three wires. The flow does not depend on the shape and direction of wires. Hence, the current is not a vector quantity.

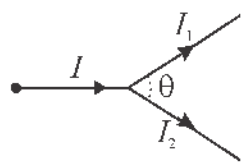


Fig 5.1 The current is represented

by $I = I_1 + I_2$, whatever be the value of θ .

In this chapter we will confine ourselves to the flow of current in conductors. Simultaneously, we will concentrate our study on steady current, which does not depend on time.

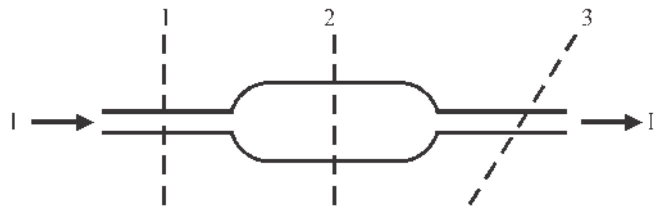


Fig 5.2 The current I flowing through all planes 1,2 and 3 is same.

In steady state, the current in any section of the conductor will remain same, whatever be the position of cross section as shown in the fig (5.2). The current I flowing through all planes 1, 2 and 3 is same. It is also the outcomes of principle of conservation of charge.

5.2 Current Density

In certain cases we are interested in studying the flow of charge from any point on the cross section of the conductor. In this situation we use current density, a vector quantity. At any point P, to define this quantity we consider a small area dS at point P. Which is perpendicular to the direction of flow of current. Fig (5.3A). If ΔI is the current passes through a small area ΔS. Then average current density

$$J_{av} = \frac{\Delta I}{\Delta S} \quad \text{. . . 5.3 (a)}$$

and the current density at point

$$J = \lim_{\Delta S \to 0} \frac{\Delta I}{\Delta S} = \frac{dI}{dS} \quad \text{. . . 5.3 (b)}$$

Here, J = Current density at point P. It is a vector quantity.

If current flows due to positive charges, then direction of $\vec{J}$ will be same as that of +ve charge. If the current flow is due to negative charges then direction of $\vec{J}$ will be opposite. Hence, direction of $\vec{J}$ at that point or area will be in the direction of current. If current is uniformly distributed and perpendicular to point, then

$$J = \frac{I}{S} \quad \text{. . . 5.3(c)}$$

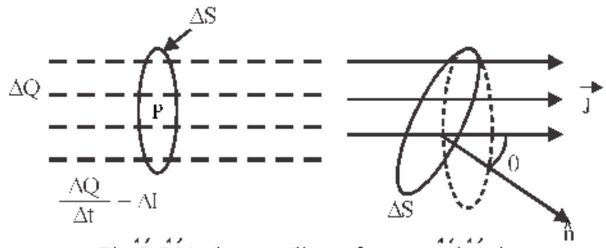


Fig 5.3 Understanding of current density

If area ΔS is not normal, we can use a unit vector $\hat{n}$ which is perpendicular to it and makes an angle θ with the direction of current. Then the current density will be,

$$J = \frac{\Delta I}{\Delta S \cos\theta} \quad \text{. . . (5.4a)}$$

$$I = J \Delta S \cos\theta$$

$$\Delta I = \vec{J} \cdot \Delta\vec{S} \quad \text{. . . (5.4b)}$$

Unit of current density is $\frac{\text{ampere}}{m^2} = \frac{\text{A}}{m^2}$

Here, we have to note that current density is a vector quantity whereas electric current is a scalar quantity.

In a conductor of non-uniform area, current flowing will be same but current density will be different. For a finite area

$$I = \int \vec{J} \cdot d\vec{s} \quad \text{. . . (5.4c)}$$

5.3 Flow of Electric Charge in Metallic Conductors:

In atoms of metallic conductor, attractive force between nucleus and valance electrons is very weak due to which, the valance electrons are weakly bound with nucleus and a few of them get free to move randomly within metal. These electrons are called free electrons or conduction electrons. In conductors, number of free electrons are very large. e.g. in copper where every atom release one free electron, their free electron density is nearly 8.49×10^{28} / m^3. In conductors free electrons move randomly in whole volume just as gas molecules move in gas chamber. In the absence of electric field, these electrons move randomly due to collisions of electrons with ions of conductor the direction of electron will suddenly change. As in molecular theory of gases this random motion of electrons can be related to free path λ

which mean free path between two succesive collisions. The time between two succesive collisions is τ (τ is also called relaxation time this is average time between two succesive collision. Due to collisions, the directions of electron will change as zig-zag path). In metals, the electrons are in random motion at large. So within a given time interval and given area ΔS, number of electrons through transverse section is equal to the number of electrons crossing opposite to the direction. Therefore, resultant current is zero. When external electric field is applied on the conductor. A force ($\vec{F} = -e\vec{E}$) acts in opposite direction to the field. Due to this electric field, electrons drift in direction opposite to the direction of electric field. The electron gain a velocity. This type of velocity is called drift velocity.

5.4 Drift Velocity and Mobility

5.4.1 Drift Velocity

In the presence of electric field, random motion of electrons is modified such that these electrons move with average slow speed (drift) in opposite direction of electric field. This speed is known as drift speed. To represent in form of vector the drift of electron is expressed in form of drifts velocity $\vec{v}_d$. The drift velocity. $\vec{v}_d$ is always opposite in direction to applied electric field $\vec{E}$.

The value of this drift speed is very less then the random average speed between the collisions of electrons. (approximately less than 10^{10} times). Figure 5.4 helps us in understanding the drift velocity. In this figure, the path followed by the electrons is shown in the presence of electric field and without electric field. The continuous lines represent the path of electrons in the absence of electric field and the dotted line represent the path of electrons in the presence of electric field.

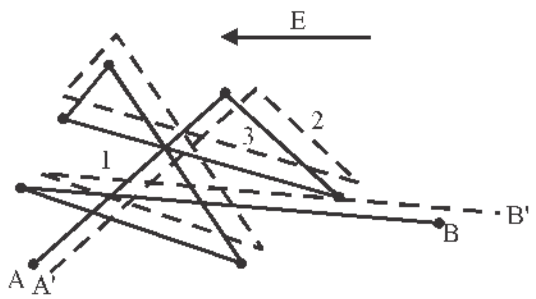


Fig 5.4: The continuous lines represent the path of electrons in the absence of electric field and the dotted line represent the path of electrons in the presence of electric field.

In the absence of electric field, electrons start from A, after some collision it reaches point B. Whereas in the presence of electric field, after same number of collisions the electron reaches B' instead of B. In this way after drift, electron reaches B' in place of B in the direction opposite to the direction of electric field. (It can be compared with still air and slow-motion wind or air). In still air every molecule will move randomly with thermal velocity and then will be no velocity in particular direction. In case of slow-motion in air every molecule has random motion as well as small velocity in the direction of air flow.

Now we will find the relation between the drift velocity and electric field. In the presence of electric field the force on every free electron of metal will be eE. Due to this the acceleration of electron will be,

$$a = \frac{eE}{m}$$

In vector representation,

$$\vec{a} = -\frac{e\vec{E}}{m}$$

For given value of $\vec{E}$, $\vec{a}$ will be constant.

Here it is important that electrons are accelerated in time interval between two successive collisions. Its reason is that due to collisions with vibration ions its drifting nature stops momentarily and its velocity becomes random in any direction.

If the velocity of n electron is $\vec{u}$ and after time t its velocity just before the next collision is $\vec{v}$, then

$$\vec{v} = \vec{u} + \vec{a}t$$

or

$$\vec{v} = \vec{u} - \frac{e\vec{E}t}{m} \qquad \text{. . . (5.5)}$$

If we take the average over the all free electrons of metal, then,

$$\langle \vec{v} \rangle = \langle \vec{u} \rangle - \frac{e\vec{E}}{m}\langle t \rangle \qquad \text{. . . (5.6)}$$

Here, the quantity $\langle \vec{u} \rangle = 0$ because for electrons average of random velocities is zero and just immediately after collision electron will not acquire any energy from electric field.

Simultaneously, t is the time of travel between two successive collisions. Hence $\langle t \rangle$ is the average of time intervals between two successive collisions of all the electrons and it is equal to τ.

In this situation, if $\langle \vec{v} \rangle = v_d$. Here $\vec{v}_d$ is the average drift velocity,

Then, $$\vec{v}_d = \frac{-e\vec{E}}{m}\tau \qquad \ldots (5.7)$$

And average drift speed

$$v_d = \frac{eE}{m}\tau \qquad \ldots (5.8)$$

In general, if the charge is q which is moving in electric field. Then drift velocity of this charge will be

$$v_d = \frac{q\tau}{m}E$$

If the charge is positive then $\vec{v}_d$ will be in the direction of $\vec{E}$, if the charge is negative then $\vec{v}_d$ will be in the direction opposite of $\vec{E}$.

5.4.2 Mobility

In the equation for drift velocity, $v_d = (eE/m)\,\tau$, in the right-hand term eE/m, e and m are constants and the relaxation time (τ) is the characteristic property of material of the conductor. Therefore, the drift velocity is constant for the material of the conductor. This constant is called the mobility of the conductor (μ), and it is represented by the following equations,

$$\vec{v}_d = -\mu\vec{E} \qquad \ldots (5.9A)$$

$$v_d = \mu E \qquad \ldots (5.9B)$$

From these relations it is clear that for any conductor $v_d \propto E$, hence drift velocity (v_d) is proportional to applied electric field (E).

$$\mu = \frac{e\tau}{m} = \frac{v_d}{E} \qquad \ldots (5.10)$$

(In general for any charge q, $\mu = \frac{q\tau}{m}$)

Hence, according to equation (5.10), mobility can be defined as drift velocity per unit electric field. It is a positive quantity having the unit

$$\frac{m/s}{V/m} = m^2 s^{-1} V^{-1}$$

For two given metals, when the electric field is same,

$$\frac{\mu_1}{\mu_2} = \frac{v_{d1}}{v_{d2}}$$

From this relation, it is clear that mobility of electron in a conductor is more if the drift velocity is more. We shall study the mobility in semiconductors in chapter 16 of this book.

5.4.3 Relation between drift velocity and electric current

After understanding the concept of drift velocity, now we can use it to find the electric current in a conductor. Let n be the free electrons per unit volume (free electron density) inside a metallic conductor. A is the area of cross section of this conductor. If electric field is applied across a conductor, the free electrons inside the conductor will move with drift velocity, opposite to the direction of electric field. All the free electrons in the conductor can be assumed to be moving with the same drift velocity v_d. Now, let us think of a small element ΔL of this conductor as shown in the figure (5.5). The number of charge carriers in this small element will be $nA\Delta L$ and the net charge on these charge carriers will be $(nA\Delta L)e$.

Therefore, $\Delta Q = (nA\Delta L)e$

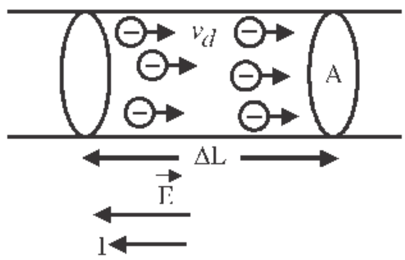


Fig 5.5 Drift velocity in a conductor

Because, all the charge carriers (free electrons) in the wire are moving with the same drift velocity v_d. Then, the time taken by these charge carriers in crossing this element of conductor is,

$$\Delta t = \frac{\Delta L}{v_d}$$

By the definition, the current is the free charge passing through any area of cross section of the conductor per second.

$$I = \frac{\Delta Q}{\Delta t} = \frac{(nA\Delta L)e}{\Delta L / v_d}$$

or $$I = nAev_d \qquad \ldots(5.11)$$

Equation (5.11) represents the relation between the current and drift velocity. By equation (5.9B), $v_d = \mu E$, Therefore Eq. (5.11) can be written as,

$$I = nAe\mu E \qquad \ldots(5.12)$$

The equation (5.12) shows the relation between the current and mobility of the charge carriers. From equation (5.11) for a constant electric current,

$$nAev_d = \text{constant.}$$

Here, e is constant and for a given metallic conductor, n is also constant. Hence for a given conductor, for constant current,

$$Av_d = \text{constant}$$

or $$A_1 v_{d1} = A_2 v_{d2} = \text{constant} \qquad \ldots(5.13)$$

Thus, us for a given conducting wire of non-uniform cross section area, the drift velocity (v_d) of electron will be more where the area of cross section is small and the drift velocity (v_d) of electron will be small where the area of cross section is more.

5.4.4 Relation between drift velocity and potential difference

According to section 5.4.3, electric field inside the conductor of length l will be,

$$E = \frac{V}{l} \qquad \ldots(5.14)$$

So, the drift velocity of electron according to the equation (5.7) will be,

$$v_d = \frac{eE}{m}\tau$$

By putting the value of E from the Eq. (5.14),

$$v_d = \frac{eV}{ml}\tau = \frac{e\tau}{ml}V \qquad \ldots(5.15)$$

From the equation (5.15) it is clear that the drift velocity (v_d) of the free electron in a conductor is proportional to potential difference (V).

Hence, $v_d \propto V$

The drift velocity (v_d) of the free electron in a conductor does not depend on the length of the conductor.

Example 5.1 : The dependence of charge Q crossing a surface in time t is given as,

$$Q = 4t^3 + 5t + 6$$

Then calculate the instantaneous current from the surface at $t = 1\ sec$.

Solution : Instantanous current,

$$I = \frac{dQ}{dt} = \frac{d}{dt}(4t^3 + 5t + 6) = (12t^2 + 5)\text{ A}$$

At $t = 1\ sec$,

$$I = 12(1)^2 + 5 = 17A$$

Example 5.2 : Estimate the average drift speed of electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}\text{m}^2$ carrying a current of 1.5 A. Assuming that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3 and its atomic mass is 63.5 a.m.u.

Solution : The formula for drift speed is given by,

$$v_d = \frac{I}{neA}$$

Given, $I = 1.5$ A

$e = 1.6 \times 10^{-19}$ C.

$A = 1.0 \times 10^{-7}\text{m}^2$

$n =$ the number of electrons per unit volume in copper

$= N_A = 6.0 \times 10^{23}$ atoms per mole

Given that atomic mass of copper is 63.5 amu and each copper atom contributes roughly one conduction electron to the current flow.

Number of atoms in 63.5 amu copper = ($N_A = 6.0 \times 10^{23}$ Avogadro number).

number of atoms in 1 gram copper

$$= \frac{6.02 \times 10^{23}}{63.5}$$

number of free electrons per unit volume

$$n = \frac{6.02 \times 10^{23}}{63.5 \times 10^{-3}} \times 9 \times 10^{3} = 8.5 \times 10^{28}\, m^{-3}$$

Hence, drift speed will be,

$$v_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.10 \times 10^{-3}\, m/s$$

Note : If this speed is compared with the speed of electric field (electromagnetic wave, i.e. 3×10^8 m/sec, then it was found that drift velocity is very less than the velocity of e.m. wave. Simultaneously the drift velocity is also very much less than the thermal velocity ($\sim 10^6$ m/s) of electrons. The time taken by an electron in travelling the length of 1m with drift velocity 10^{-3} m/s in a conductor is about 15 minutes. Now the question arises how an electric bulb lights up instantaneously after the circuit is switched on even though the bulb is few meters from the electric switch. Here one simple example can simplify our problem. If one end of a very long pipe is connected to tap and the other end to the tank, then water will take some time to reach the other end. But, if the pipe is already filled with water, then water will not take much time to reach the tank.

Example 5.3 : The electron revolves in an orbit of radius 5.3×10^{-11} m of hydrogen atom with speed of 2.2×10^6 m/s. Calculate the average electric current.

Solution : Given,

Radius of orbit, $r = 5.3 \times 10^{-11}$ m

Speed of electron, $v = 2.2 \times 10^6$ m/s

Charge on electron, $e = 1.6 \times 10^{-19}$ C

Therefore, the time period of orbital motion of electron

$$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 5.3 \times 10^{-11}}{2.2 \times 10^{6}}$$

$= 15.13 \times 10^{-17}$ sec

So, the average current produced due to orbital motion of electron is,

$$I = \frac{q}{T} = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{15.13 \times 10^{-17}}$$

$= 1.06 \times 10^{-3}$ A $= 1.06$ mA

5.5 Ohm's Law:

In the year 1828, German scientist G.S. Ohm, after performing so many experiments, enunciated a law regarding the flow of current in a conductor. To honour him, the law is named after him.

According to this law, when the physical conditions of the conductor remains same (length, temperature, nature of material of the conductor and area of cross section), then the current flowing through the conductor will be proportional to the potential drop across its ends.

$$V \propto I$$

$$V = RI \qquad \ldots (5.16)$$

Where, the constant of proportionality is called the resistance of the conductor.

$$R = \frac{V}{I} \qquad \ldots (5.17)$$

The S.I. unit of resistance is ohm and is represented by the Greek symbol Ω. If on applying a potential difference of one volt across the ends of a conductor a current of 1 ampere flows through the conductor, then resistance of the conductor is said to be one ohm (Ω).

By knowing the values of electric current (I) for the potential difference (V) applied across the terminals of the conducting wire a graph can be plotted between V and I. This graph will be a straight line as shown in the fig 5.6.

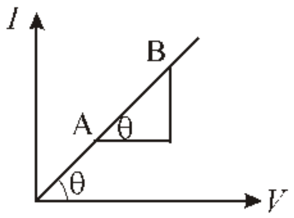


Fig: 5.6 Graph between V and I according to Ohm's Law

Slope of this straight line curve will be equal to the reciprocal of resistance of the wire.

slope of the curve =

$$(\tan q) = \frac{I}{V} = \frac{1}{R} \qquad \ldots(5.18)$$

5.5.1 Deduction of Ohm's Law

In section 5.4.3 of this chapter regarding free electron model, we have seen expression for electron current related to drift speed. According to equation (5.11), we have -

$$I = nAev_d$$

Thus, the current density will be,

$$J = \frac{I}{A} = nev_d \qquad \ldots(5.19)$$

But we have seen in equation (5.8)

$$v_d = \frac{e\tau}{m}E$$

Thus, $J = \frac{ne^2\tau}{m}E \qquad \ldots(5.20)$

In this equation, the terms of right side $\frac{ne^2\tau}{m}$, m and e are constant and terms n and τ are the characteristics of the conductor. For anisotropic homogeneous conductor, the quantity $\frac{ne^2\tau}{m}$, can be considered as a constant. It is called as conductivity and represented by the symbol σ.

Hence the conductivity

$$\sigma = \frac{ne^2\tau}{m} \qquad \ldots(5.21)$$

Now equation (5.20) can be written as,

$$J = \sigma E$$

Since $\vec{J}$ and $\vec{E}$ are vector quantities, therefore in vector notation the above equation can be rewritten as,

$$\vec{J} = \sigma\vec{E} \qquad \ldots(5.22)$$

According to these equations, the conductivity (σ) of any conductor does not depend on electric field (E). Whereas the current density ($\vec{J}$) in the conductor is proportional to the electric field ($\vec{E}$). Equation (5.22) is called Ohm's Law in microscopic form. For so many conductors it is applicable for long range of electric field. Now we see the formal definition of ohm's law which is equivalent to equation (5.16)

Now let us consider a conductor of length l and area of cross section A connected to a battery of potential difference V as shown in the figure 5.7. The field produced inside the conductor is $E = \frac{V}{\ell}$ and current density is $J = I/A$. Now put these values of E and J in equation (5.22), we get,

$$\frac{I}{A} = \sigma\frac{V}{\ell}$$

or $$V = \frac{1}{\sigma}\frac{\ell}{A}I$$

$$= \left(\frac{\rho\ell}{A}\right)I \qquad \ldots(5.23)$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad \ldots(5.24)$$

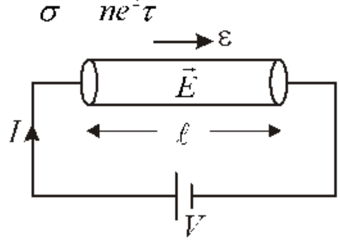


Fig: 5.7 Current flow in a conductor

ρ is called the resistivity of the material of the conductor. For any given material of length l and area of cross section A are constant and the resistivity (ρ) is the characteristic property of the material of the conductor.

Thus, the quantity $\frac{\rho \ell}{A}$ is a constant known as the resistance of the conductor (R).

$$V = IR$$

where $R = \frac{\rho \ell}{A}$ . . . (5.25)

We see that the equation (5.24) is same as that of Ohm's Law. Equation, $V=IR$ is sometimes also called macroscopic form of Ohm's Law. The macroscopic quantities V, I and R can be measured directly with the help of meters. When we are interested to know the fundamental electrical properties, then quantities E, σ and J are useful.

5.5.2 Resistivity

After performing so many experiments it was found that resistance of isotropic and homogenous conductor is directly proportional to length l and inversely proportional to area of cross section A of the conductor. i.e.

$$R \propto \frac{\ell}{A}$$

or $R = \frac{\rho \ell}{A}$

Here, the constant of proportionality ρ is called the resistivity of the material of conductor. Here, it is essential to note the difference between resistivity and resistance. The resistivity is a property of the material of the substance whereas the resistance of the material is the property related to the material and the geometric parameters such as length and area of cross section. Two wires of same material can have different resistance but their resistivity will be same. Similarly, two wires of different material will have different resistivity but may have same resistance. This we have already seen in the derivation of Ohm's law. There we have also seen the formula for resistivity as, in equation (5.24).

$$\rho = \frac{m}{ne^2\tau} \quad . . . (5.26)$$

Because the quantities n and τ are the characteristics of the substance, therefore resistivity depends on the nature of the substance as well as the temperature of the substance. Resistivity of the material is also known as specific resistance. From equation (5.25),

$$\rho = \frac{RA}{\ell} \quad . . . (5.27)$$

Unit of resistivity is $\frac{\Omega m^2}{m} = \Omega \text{m}$ and the dimensions are $M^1L^3T^{-3}A^{-2}$.

If the conductor is in the form of a cylinder of radius r, then $A = \pi r^2$. Then,

$$\rho = \frac{RA}{\ell} = \frac{R\left(\pi r^2\right)}{\ell}$$

In the above equation, if we take, $A = 1\,\text{m}^2$ and $\ell = 1\,\text{m}$, then, $\rho = R$.

Thus, the resistivity of a material will be numerically equal to the resistance of a sample whose length is unity and area of cross section is also unity.

Reciprocal of resistivity of any substance is known as conductivity.

The unit of conductivity is ohm ^{1} meter ^{1} and the dimensions of conductivity are $M\ ^{1}L\ ^{3}T^{3}A^{2}$.

The table 5.1 shows the resistivity of some common materials. It is clear from the table that the resistivity of a conductor lies between $10\ ^{8}\Omega \times m$ to $10\ ^{6}\Omega \times m$. At the other end are insulators like ceramic, rubber and plastic having resistivity of the order of $10^{16}\Omega \times m$. In between the two are semiconductors like Germanium and Silicon, which behave as an insulators at 0 K. On increasing the temperature the resistivity of semiconductors decreases.

Table 5.1 Resistivity of some common materials

Material	Resistivity ρ at 0^0 (In $\Omega \times m$ units)
Conductor	
Silver	1.6×10^{-8}
Copper	1.7×10^{8}
Aluminium	2.7×10^{8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}
Platinum	11×10^{8}
Mangnin	48×10^{8}
Mercury	98×10^{-8}
Nichrome	100×10^{-8}
Semiconductor	
Carbon (Graphite)	3.5×10^{5}
Germanium	0.60
Silicon	2300
Insulators	
Pure water	2.5×10^{5}
Glass	$10^{10} - 10^{14}$
Hard rubber	10^{14}
Dry wood	10^{8} to 10^{14}

5.6 Electric Resistance

The property of conductor which creates obstacle in the flow of electric current is called electric resistance. We know that in every conductor there are free electrons. On applying potential across its ends, electrons flow from one end to the other by doing random motion. This causes the flow of electric current in the conductor. During the motion electrons keep on colliding with ions and atoms. In this way obstacle is created in the flow of electric current and it is called electric resistance. Resistance of a specimen of a substance not only depends on the nature of the substance but also on the length and area of cross section of the wire. In addition to this the resistance of any conductor also depends on the faces of the conductor across which the potential difference is applied. The resistance also depends on the temperature of the conductor. The resistances which are prepared for a particular value are called resisters.

5.6.1 Ohmic and Non Ohmic Resistance

The conductors (or devices) obeying Ohm's law, i.e. for which, the graph between potential difference V and the current I is a straight line and passing from origin are called Ohmic conductors (or devices). In such devices the current flow does not depend on the polarity of the applied potential difference.

There are several devices or substances used in electric circuits which do not obey ohm's law. i.e. the graph between potential difference V and the current I is not a straight line but a curve. These types of devices or substances are known as non-ohmic.

In addition to this there are certain substances in which the flow of electric current depends on the polarity of the applied voltage. The examples of such devices are vacuum tube, semi-conductor diode, liquid electrolyte, transistor etc.

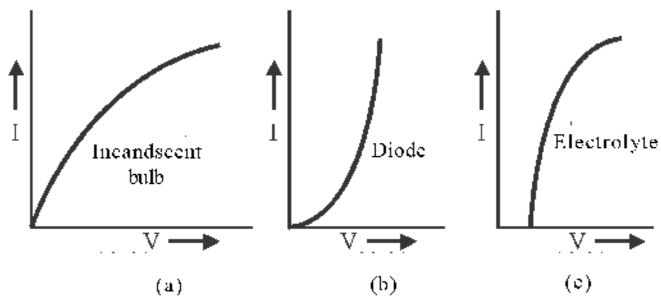


Fig 5.8 Non-ohmic behaviour

Figure (5.8A) is a *V-I* curve for a torch bulb. It is clear from the graph that Ohm's law is not obeyed. The reason is that when current flow increases in the filament of the bulb, the temperature of the filament increases. The resistance of the filament will increase. Therefore, the ratio of V and I will not remain constant. Graph (5.8B) is drawn for semi-conductor diode device and graph (5.8C) is drawn for a liquid electrolyte. From the graphs it is evident that these devices do not obey Ohm's law.

5.7 Carbon Resistance and colour codes for Carbon Resistance

Commercially produced resistances for domestic use or laboratory use are of two major types: wire wound resistors and carbon resisters. Wire wound resisters are made by winding the wires of an alloy, viz. magainin, constantan, nichrome etc. The temperature coefficient of resistance of these alloys is very small due to this the conductivity is least affected by temperature. Resisters of

(b)

very high value cannot be made using these materials because it will need very long length of wire which is inconvenient. On the other hand, the resisters of very high value are made up of Carbon resisters, which are inexpensive and compact, therefore they are extensively used in electronic circuits. Carbon resisters are moulded into the cylindrical shape by using a binding agent and wire leads (for connecting the carbon resister in any electric circuit) are attached to the ends.

The value of carbon resister in indicated by four coloured bands. Every coloured band (strip) has a special coded meaning. Every colour has a special colour code. Key to these colour codes are is given in the table 5.2.

Table 5.2 Colour code for resistances

Colour	digit	multiple	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Golden		10^{-1}	5
Silver		10^{-2}	10
No colour			20

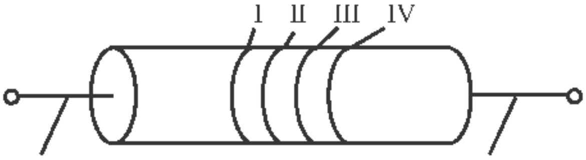


Fig 5.9 Carbon resistance colour code

Identification of the value of a carbon resister:

(i) Hold the Carbon resister such that the tolerance ring (Silver or Golden Colour) is on the right side.

(ii) The first two coloured rings indicate the first two significant digits of the value of carbon resistance as per colour code. The colour of the third ring indicates the decimal multiplier and the fourth ring indicates the tolerance percentage. Sometimes the fourth ring is absent, and it indicates that the tolerance is 20%. It indicates ±% in the value of resistor.

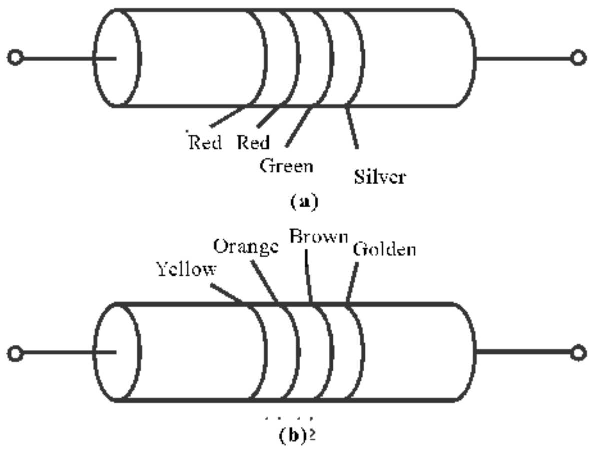


Fig 5.10 Carbon resistance

For carbon resistance shown in the Fig 5.10 (a) and (b),

the value of resistance (a) is $22 \times 10^5 \, \Omega \pm 10\%$ and that of (b) is $(43 \times 10^1 \, \Omega) \pm 5\%$ respectively.

Example 5.4 : A wire of length *l* and cross sectional area of A has a resistance R. Find the percentage change in the resistance, when it is stretched to double of its length.

Solution : Given, the length of the wire is l and its cross sectional area is A, then the resistance of wire will be,

$$R = \rho \frac{\ell}{A}$$

On stretching the wire to double the length, its cross sectional area decreases, because the mass and volume of the substance remains constant.

$$A\ell d = A'(2\ell)d$$

$$\Rightarrow \quad A' = \frac{A}{2}$$

Hence the new resistance of the substance,

$$R' = \frac{\rho(2\ell)}{A'} = \frac{\rho(2\ell)}{A/2} = 4\frac{\rho\ell}{A} = 4R$$

Therefore, percentage change in resistance,

$$= \frac{R' - R}{R} \times 100\%$$

$$= \frac{4R - R}{R} \times 100\% \ = 300\%$$

Example 5.5 : The value of a carbon resistance is $62 \times 10^3 \Omega$. The percentage tolerance is 5%.Write down the colour code in sequence.

Solution : Given,

$$R = (62 \times 10^3 \Omega) \pm 5\%$$

According to the colour code, the colour of strips on the carbon resistance are Blue, Red, Orange and Golden.

Example 5.6 : If the length of two conductors,

$X = 4\Omega$ and $Y = 48 \times 10^{\ 8} \Omega \times m$ reduced to half, then write down the corresponding value of X and Y.

Solution : In the question, the X is resistance of the wire and Y is resistivity of the material of wire. Therefore, by changing the length the resistance will change whereas the resistivity remains unaltered.

Therefore $X' = 2\Omega$ and Y remains same, i.e. $Y' = 48 \times 10^{\ 8} \Omega m$.

Example 5.7 : A potential difference of 0.9V is applied across the ends of a tungsten wire of length 1.5 m and cross-sectional area $0.60 \times 10^{-6} m^2$. Find the current flowing through the wire. Specific resistance (r) of Tungsten is $5.6 \times 10^{-8} \Omega \times m$.

Solution : Given,

$$\ell = 1.5 m$$

$$A = 0.60 \times 10^{\ 6} m^2$$

$$r = 5.6 ' \ 10^{-8} W ' \ m$$

Therefore, resistance of the wire,

$$R = \rho\frac{\ell}{A}$$

$$R = \frac{5.6 \times 10^{-8} \times 1.5}{0.60 \times 10^{\ 6}} \Omega$$

$$R = 0.14 \Omega$$

Thus, the current through the wire,

$$I = \frac{V}{R} = \frac{0.90}{0.14} = 6.43 \text{ A}$$

5.8 Effect of temperature on Resistance and Resistivity

The resistivity of various materials changes with temperature in different manner according to their nature.

(A) For conductors

Previously we have seen that the resistivity of a conductor is given as,

$$\rho = \frac{m}{ne^2\tau}$$

Where, m is the mass and e is the charge of an electron and n is the free electron number density of the conductor. All these quantities are constant. On increasing temperature, the amplitude of vibration in the conductor and the frequency of collision of free electrons increase due to which the relaxation time t decreases. Thus, the resistivity of the conductor increases and thereby the conductivity decreases. If ρ_0 and ρ_t are the resistivity of a conductor at 0^0C and t^0C, then a close relation between these quantities is,

$$\rho_t = \rho_0(1 + \alpha t) \qquad \ldots(5.28)$$

Where, α is a constant known as temperature coefficient of resistivity. α depends on the nature of material. The value of some common materials is given in the table (5.3). For some substances is positive, whereas for others it is negative.

Table 5.3 Temperature coefficient of Resistivity

Material	Temperature coefficient of Resistivity $(^{\circ}C)^{-1}$
A. Conductor	
(A) Metals	
Silver	4.1×10^{-3}
Copper	3.9×10^{-3}
Aluminium	4.3×10^{-3}
Tungsten	4.5×10^{-3}
Iron	6.5×10^{-3}
Platinum	3.9×10^{-3}
Mercury	0.9×10^{-3}
(b) ***Alloys***	
Nichrome	0.4×10^{-3}
Mangnin	0.002×10^{-3}
Constantan	0.001×10^{-3}
B. ***Semiconductor***	
Carbon	− 0.0005
Germanium	− 0.05
Silicon	− 0.07

According to the above equation (5.28), the temperature coefficient of resistivity will be given by,

$$\alpha = \frac{\rho_t - \rho_0}{\rho_0 t} = \frac{\Delta\rho}{\rho_0 t} \qquad \ldots (5.29)$$

The dimensions of α in the equation (5.29) will be θ^{-1}. Thus, for a unit change in temperature ($\Delta t=1$), the ratio of change in resistivity to the resistivity at 0°C, i.e. $\left(\frac{\Delta\rho}{\rho_0}\right)$ is equal to the temperature coefficient of resistivity. The unit of temperature coefficient of resistivity is $^{\circ}C^{-1}$.

Similarly, the dependence of resistance on temperature can be written as,

$$R_t = R_0(1+\alpha t) \qquad \ldots (5.30)$$

If the change in temperature of a conductor is Δt, then, equivalent value of resistivity and resistance will be given by,

$$\rho_{t_2} = \rho_{t_1}(1+\alpha\Delta t) \qquad \ldots (5.31)$$

$$R_{t_2} = R_{t_1}(1+\alpha\Delta t) \qquad \ldots (5.32)$$

Here, t_1 and t_2 are the initial and final temperature of the conductor.

Here $\Delta t = t_2 - t_1$

If a graph is plotted between the resistivity and temperature for metals, then it will be more or less a straight line. But at low temperature it is curved, i.e deviation from the straight line.

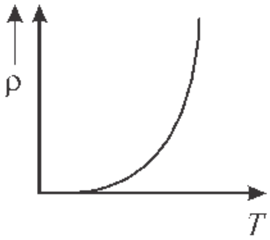


Fig 5.11 Resistivity of a conductor at low temperature

Some materials like Nichrome, which is an alloy (Nickel 80 % and Chromium 20%) exhibits very weak dependence of resistivity on temperature. Such materials are widely used in wire wound standard resisters. Figure (5.12) shows a graph between ρ_T and T.

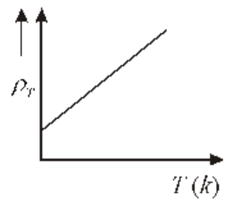


Fig 5.12 Resistivity of a conductor at high temperature

(B) For insulators

The resistivity of insulating material decreases exponentially on increasing temperature and increases on decreasing temperature. At absolute zero temperature, the resistivity becomes very high and tends to infinite value. Thus, the conductivity of insulators at absolute zero temperature becomes zero. The variation of resistivity of insulator with temperature is given in the following relation,

$$\rho = \rho_0 e^{E_g/2K_BT} \qquad \ldots (5.33)$$

Where,

K_B = Boltzmann constant

T = Temperature of the material in kelvin

E_g = Energy gap between valance band and conduction band.

(C) For semi-conductors:

On increasing the temperature of semi-conductors, the bonds of semi-conductors break up rapidly. Then the number of electron-hole pairs increase exponentially. Thus, the resistivity of semi-conductors decreases exponentially as temperature increases. The temperature coefficient of resistivity in this case will be $-ve$. Figure (5.13) shows a graph between r and T.

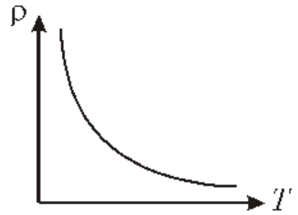


Fig 5.13 Conductivity of semi-conductors

Example 5.8 : The resistance of a platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.23 Ω. When the thermometer is inserted in hot bath, the resistance of platinum wire is 5.795 Ω. Calculate the temperature of the hot bath.

Solution : Given,

$$R_0 = 5\Omega, \qquad R_{100} = 5.23\,\Omega$$

and $\quad R_t = 5.795\,\Omega$

We know the formula for the resistance of a wire at temperature t,

$$R_t = R_0(1+\alpha t)$$

Therefore, the resistance $R_{100} = R_0[1+\alpha 100]$ is,

$$R_{100} - R_0 = R_0\alpha \times 100 \qquad \text{. . . (i)}$$

$$R_t - R_0 = R_0\alpha t \qquad \text{. . . (ii)}$$

Diving eq (2) by eqn (1),

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

$$t = \frac{5.795-5}{5.23-5} \times 100$$

$$t = \frac{0.795}{0.23} \times 100 = 345.65°C$$

Example 5.9 : A platinum resistance thermometer, a device used to measure the change in temperature, has a resistance of 50 Ω at a temperature of 20°C. When this thermometer is placed inside a container filled with silver at its melting point the resistance increases to 80 Ω. Assuming that there is linear change in the resistance of platinum in this temperature range, calculate the melting point of silver. Given $\alpha = 3.8\times10^{-3}\,°C^{-1}$ for silver.

Solution : For limited temperature range, the resistance at t_2°C is given by,

$$R_{t_2} = R_{t_1}(1+\alpha\Delta t)$$

Where,

Δt = initial temperature, t_2°C = final temperature

and Δt = increase in temperature

Given $\quad \alpha = 3.8\times10^{-3}\,°C^{-1}$,

$$R_t = 80\Omega, \quad R_0 = 50\Omega$$

$$\Delta t = \frac{R_t - R_0}{\alpha R_0} = \frac{80-50}{3.8\times10^{-3}\times50}$$

$$= \frac{3\times10^3}{19}$$

or $\quad t_2 - t_1 = 157.9°C$

or $\quad t_2 = t_1 + 157.9°C$

or $\quad t_2 = 20°C + 157.9°C = 177.9°c$

$\therefore$ melting point of silver = t_2 = 177.9°C

5.8.1 Super conductivity

It was found in some metals or composite metals that at particular very low temperature their resistivity decreases abnormally or rapidly and becomes zero. Such substances are called Super conductors and this property is called Superconductivity. The temperature at which this phenomenon occurs is called critical temperature.

The phenomenon of Superconductivity was first observed by the physicist Heike Kamerlingh Onnes in the year 1911 by cooling mercury to 4.2 K. In the state of superconductivity, magnetic field inside the conductor will also be zero. This effect is called Meissner Effect.

The phenomenon of superconductivity is exhibited at very low temperature (10 K to 0.1 K). Although now some materials have been found to exhibit the property of superconductivity at higher temperature of the order of 90K. Now a days scientists are putting great efforts to search superconductors at normal temperature so to get rid of the energy lossproblem during transmission.

Uses of Superconductors: To construct magnets known as Superconducting magnets which can create high magnetic fields (of the order of 10 tesla), small and very efficient transformers, motor electric generators, energy transmission and super computers etc.

5.9 Series and Parallel Combination of Resistances:

According to the requirement of certain electric current in any electric circuit, we must have definite resistance in that circuit. Suppose this resistance is not available with us. Then we use the combination of the resistances available with us so that the resistance of desired value is obtained. There are two ways in which the combination is carried out.

5.9.1 Series Combination

Two or more than two resistances are said to be connected in series if same current is passed through all the resistances. In series combination, second end of every resistance is connected to first end of the next resistance.

In fig. (5.14),three resistance R_1, R_2 and R_3 are connected between two ends. A & B in series and a current I is flowing in this. The potential difference at resistance are V_1, V_2 and V_3 respectively than according to ohm's law $V_1 = TR_1$, $V_2 = IR_2$, $V_3 = IR_3$

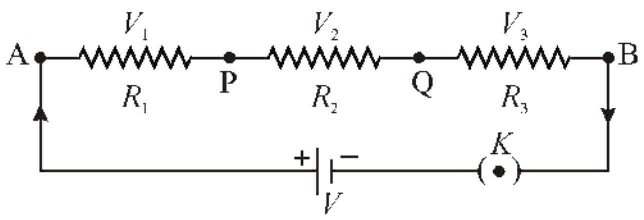


Fig 5.14 Series Combination of resistances

If the potential difference between points A and B is V, then,

$$V = V_1 + V_2 + V_3$$

or $$V = IR_1 + IR_2 + IR_3$$

or $$V = I(R_1 + R_2 + R_3) \quad \ldots (5.34)$$

If the equivalent resistance of the series combination is R_{eq}, Then,

$$V = IR_{eq} \quad \ldots (5.34A)$$

By combining equation (5.34) and (5.34A)

$$IR_{eq} = I(R_1 + R_2 + R_3)$$

or $$R_{eq} = R_1 + R_2 + R_3 \quad \ldots (5.35)$$

i.e. the equivalent resistance will be the sum of all individual resisters. If there are n resisters then,

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots + R_n$$

If all the resistances are identical and equal to R, then,

$$R_{eq} = R + R + \ldots\ldots \text{ up to } n$$

or $$R_{eq} = nR \quad \ldots (5.36)$$

Important points for series combination of resistances :

(a) In all the resistances the current flowing will be same.

(b) Resultant potential difference will be sum of potential difference across each resistance.

(c) Equivalent resistance will be greater than any of the resistance connected in series.

5.9.2 Parallel Combination:

Two or more than two resistances are said to be connected in parallel if same potential difference exists across each of the resisters. In Parallel combination, one end of all resistances is connected at one point and other end of all resistances are connected to another point. In figure 5.15 three resistances of value R_1, R_2 and R_3 connected in parallel.

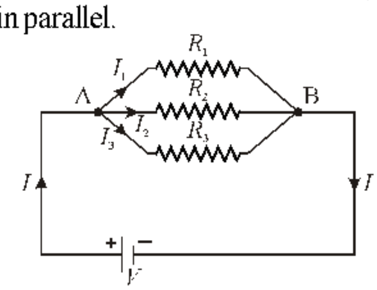


Fig 5.15 Parallel Combination of resistances.

The parallel combination of resistances have two end points A and B. One end of each resistance is connected to A and the other end of each resistance is connected to B. Points A and B of the parallel combination is connected to a battery of e.m.f. V volt.

In this combination, the potential difference across each resistance will be same and equal to V. In this circuit main current, I at point A is divided among the resistances such that the potential difference across each resistance is same and equal to potential difference V of the battery. If I_1, I_2 and I_3 are the currents through the resistances R_1, R_2 and R_3, then total current I is given by,

$$I = I_1 + I_2 + I_3 \qquad \ldots (5.37)$$

According to Ohm's law,

$$V = I_1 R_1 = I_2 R_2 = I_3 R_3$$

Hence, $I_1 = \dfrac{V}{R_1},\ I_2 = \dfrac{V}{R_2},\ I_3 = \dfrac{V}{R_3}$

Put these values in Eq (5.37),

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad \ldots (5.38)$$

If equivalant resistance of combination is R_{eq}

thus, $I = \dfrac{V}{R_{eq}}$

Compare this equation with Eq (5.38)

$$\frac{V}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \ldots (5.39)$$

In this way, the reciprocal of equivalent resistance will be sum of reciprocal of individual resistances.

If n resistances are connected in parallel, then,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n} \qquad \ldots (5.40)$$

If n identical resistances are connected in parallel, then,

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots\ldots + \frac{1}{R} \ \ldots\ldots \ up\ to\ n$$

If all resistances connected in parallel are equal, then,

$$R_{eq} = \frac{R}{n} \qquad \ldots (5.41)$$

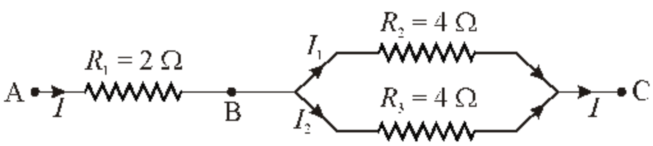
Important points for parallel combination of resistances :

(a) Potential difference across each resistance will be same.

(b) If R_1, R_2 and R_3 are values of resistances connected in series and if $R_1 > R_2 > R_3 > \ldots\ldots > R_n$. The current flowing in these resistances will be I_1, I_2 and I_3, where $I_1 < I_2 < I_3 < \ldots\ldots < I_n$

(c) In parallel combination, equivalent resistance will be lesser than the least.

Example 5.10 : In the given electric circuit. Calculate the equivalent resistance between terminals A and C.



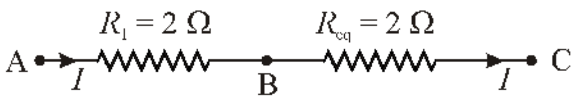
Solution : In the given figure, R_2 and R_3 are in parallel across B and C, therefore this parallel combination can be replaced by,

$$\frac{1}{R_{eq_{23}}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq_{23}} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eq_{23}} = \frac{4 \times 4}{4 + 4} = \frac{16}{8} = 2\Omega$$

Now, the simplified circuit can be redrawn as,

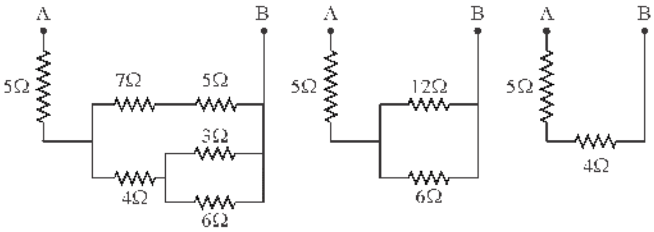


In this simplified circuit, R_1 and Req_{23} are in series.

$$R_{eq_{13}} = R_1 + R_{eq_{23}} = 2\Omega + 2\Omega$$

$$R_{eq_{23}} = 4\Omega$$

Example 5.11: Calculate the equivalent resistance of the given circuit between terminals A and B.



Solution : In such type of problems, it is useful to start with the smallest identifiable series or parallel sub-combination of the given circuit.

Step I: The equivalent resistance of 3 Ω and 6 Ω resisters connected in parallel is,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or $$R_{eq} = \frac{3 \times 6}{3+6} = 2\Omega$$

Step II : Thus, these two resisters are equivalent to 2 Ω. When this 2 Ω resister is connected in series to 4 Ω resistance we obtain 6 Ω resistance.

Step III : This 6Ω is now connected in parallel to 12Ω (a series combination of 7Ω and 5Ω). Combining there resistances in parallel we get,

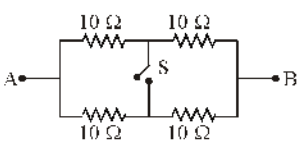
$$R^1_{eq} = \frac{6 \times 12}{6+12} = 4\Omega$$

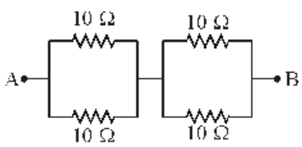
Step IV : Now, this 4Ω is connected in series to 5Ω and the total resistance 9Ω is obtained.

Example 5.12 : Find the equivalent resistance of the electric circuit between terminals A and B in the following two cases.

(a) when switch S is open

(b) when switch S is closed





Solution :

(a) When switch S is open, then the resistances in the upper branch of the circuit will be in series combination for which the equivalent resistance will be 20Ω. Similarly the resistances in the lower branch of the circuit will also be in series combination for which the equivalent resistance will also be 20Ω.

These two resistances (20Ω each) are connected in parallel. Hence the equivalent resistance will be,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{20 \times 20}{20+20} = 10\Omega$$

(b) When switch S is closed, then the resistances in the two branches can be simplified as shown in the figure. The left circuit will be the parallel combination for which the equivalent resistance will be 5Ω. Similarly the resistances in the right circuit will also be in parallel combination for which equivalent resistance will be 5Ω.

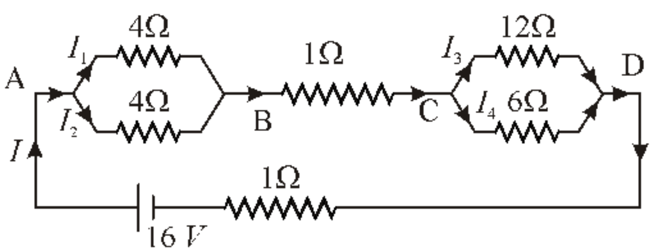
Now left and right circuit are connected in series combination. And equivalent series resistance will be 5Ω+5Ω= 10Ω.

Example 5.13 : A battery of e.m.f. 16 V and internal resistance 1Ω is connected to the network of resistances shown in the figure. Calculate,

(a) Equivalent resistance of network between points A and D.

(b) Current in each resistance.

(c) Potential drop V_{AB}, V_{BC} and V_{CD}.



Solution : Case (a) :

Step I : Two 4Ω resistances are connected in parallel between the terminals A and B. Hence the equivalent resistance R_{AB} will be,

$$R_{AB} = \frac{4 \times 4}{4+4} = 2\Omega$$

Step II : Given $R_{BC} = 1\Omega$.

Step III : Two resistances 12Ω and 6Ω are connected between the terminals C and D. Hence the equivalent resistance R_{CD} will be,

$$R_{CD} = \frac{12 \times 6}{12+6} = 4\Omega$$

Step IV : Now the resistances R_{AB}, R_{BC} and R_{CD} are all connected in series between the points A and D. Therefore,

$$R = R_{AB} + R_{BC} + R_{CD}$$

$$= 2\Omega + 1\Omega + 4\Omega$$

$$= 7\Omega$$

Case (b): Total current in the circuit,

$$I = \frac{Total\ e.m.f.}{total\ resistance}$$

$$= \frac{\varepsilon}{R+r} = \frac{16V}{(7+1)\Omega} = 2A$$

Step I : Potential difference between the points A and B will be $V_{AB} = I_1 R_1 - I_2 R_2$

$$4 \times I_1 = 4 \times I_2$$

Hence, $I_1 = I_2$ But $I_1 + I_2 = 2A$,

or $\quad 2I_1 = 2A$

$$I_1 = 1A \quad I = 2A$$

Step II : Current in the wire between B and C will be $2A$.

Step III : Total current in the wires between C and D will be $2A$.

i.e. $\quad I_3 + I_4 = 2A \quad \ldots(1)$

Now, $\quad V_{CD} = I_3 R_3 = I_4 R_4$

$$I_3 \times 12 = I_4 \times 6$$

or $\quad I_4 = 2I_3 \quad \ldots(2)$

On Solving equation (1) and (2), we get,

$$I_3 = \left(\frac{2}{3}\right)A \text{ and } I_4 = \left(\frac{4}{3}\right)A$$

Case (c): Potential difference between A & B=

$$V_{AB} = I \times R_{AB} = 2 \times 2 = 4\ V$$

Similarly,

$$V_{BC} = I \times R_{BC} = 2 \times 1 = 2V$$

$$V_{CD} = I \times R_{CD} = 2 \times 4 = 8V$$

$$V_r = I \times r = 2 \times 1 = 2V$$

Total e.m.f. in the circuit E,

$$E = V_{AB} - V_{BC} - V_{CD} - V_r$$

$$E = 4 - 2 + 8 - 2 - 16\ V$$

5.10 : Cell, Electro Motive Force, Terminal Voltage and Internal Resistance

Electric Cell is a device which maintains the steady electric current in an electric circuit or it is a simple device which converts chemical energy into electrical energy.

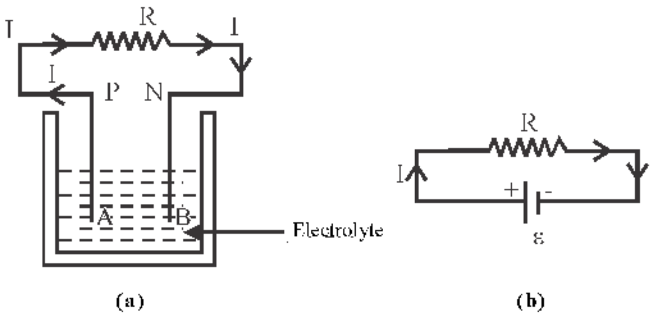


Fig 5.16 (a) & (b) Electric cell

Basically, an electric cell has two electrodes as shown in the fig (5.16), which are immersed in an electrolyte. These electrodes are shown as +*ve* electrode (P) and -*ve* electrode (N). These electrodes, immersed in electrolyte exchange charges with the electrolyte. Due to this reason, +ve electrode P develops a +ve potential V_+ ($V_+ > 0$) with respect to its adjacent electrolyte marked A. Similarly, the -*ve* electrode N develops a -*ve* potential V_- ($V_- < 0$) with respect to its adjacent electrolyte marked B. When no current flows through the cell, the electrolyte has the same potential difference throughout, so that the potential difference between the two electrodes P and N is,

$$V_+ - (-V_-) = V_+ + V_-$$

When cell is in open circuit i.e. no current is drawn from the cell, then the potential difference across its electrodes is known as Electro Motive Force (EMF) and is represented as ε .

It is to be remembered that ε is actually a potential difference and not a force. The term e.m.f. is however used due to historical reasons and this name was given at a time when this phenomenon was not understood properly. When current is flowing through external resistance R from the cell, then electric current will flow in the electrolyte from -ve electrode to +ve electrode.

The electrolyte through which a current flows has a finite resistance r, called internal resistance of the cell, i.e. internal resistance is the hinderance produced by the electrolyte of the cell in the flow of electric current.

Internal resistance of an ideal cell is zero, but practically all cells have a finite internal resistance.

If current is flowing in an external resistance due to a cell, i.e. cell is in closed circuit, then potential difference across two electrodes will be known as terminal voltage. This terminal voltage is represented by V and $V < \varepsilon$. In a closed circuit, the value of $(\varepsilon - V)$ will be equal to voltage drop across the internal resistance of the cell.

$$\varepsilon - V = Ir \qquad \text{. . . (5.42)}$$

here, I is the electric current from the cell.

Hence, terminal voltage,

$$V = \varepsilon - Ir \qquad \text{. . . (5.43)}$$

But, from Ohm's law,

$$V = IR \qquad \text{. . . (5.44)}$$

Combining Eq 5.43 and Eq5.44, we get

$$I(R+r) = \varepsilon$$

$$I = \frac{\varepsilon}{R+r} \qquad \text{. . . (5.45)}$$

$$r = \frac{\varepsilon - V}{I} = \frac{\varepsilon - V}{V/R}$$

$$r = \left(\frac{\varepsilon - V}{V}\right)R \qquad \text{. . . (5.46)}$$

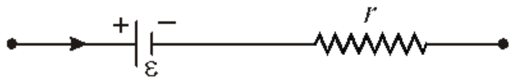


Fig 5.17 Charging of a cell

Note : If a cell is being charged as shown in the figure 5.17, then, the current enters from the +ve electrode. Therefore, the terminal voltage in this case will be,

$$V = \varepsilon + Ir$$

$$V > \varepsilon$$

5.11 Combination of Cells :

In general, cells can be connected in two manners:

(a) Series combination

(b) Parallel combination

5.11.1 Series Combination of Cells :

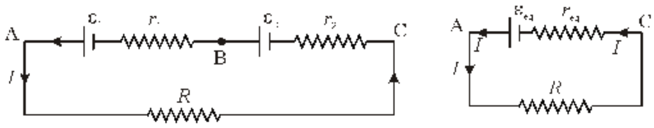


Fig 5.18 Series combination of cells

An arrangement of two or more than two cells, in which cells are connected such that the negative terminal of one cell is connected to the positive terminal of the next cell is called series combination. The end points of the combination are of opposite polarities across which an external resistance is connected.

Suppose two cells having emf ε_1 & ε_2 and internal resistance r_1 & r_2 are connected in series as shown in the figure 5.18. End points of the combination are connected to external resistance R. We want to find the equivalent emf and equivalent internal resistance and current in the circuit.

According to ohm's law, the potential difference across the resistance R.

$$V = IR = V_A - V_C$$

Potential difference between the end points A and B will be,

$$V_{AC} = (V_A - V_B) + (V_B - V_C)$$

$$= (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$

$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) \quad \ldots (5.47)$$

If ε_{eq} and r_{eq} are the equivalent emf and equivalent internal resistance of the combination then,

$$V_{AC} = \varepsilon_{eq} - I\, r_{eq} \quad \ldots (5.48)$$

By comparing Eq 5.47 & 5.48

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \quad \ldots (5.49)$$

$$r_{eq} = r_1 + r_2 \quad \ldots (5.50)$$

The value of terminal voltage =

potential drop across the external resistance R

Therefore, using equation 5.44 and the Ohm's law,

$$V_A - V_C = IR = \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$$

or $$I(R + r_1 + r_2) = \varepsilon_1 + \varepsilon_2$$

or $$I = \frac{\varepsilon_1 + \varepsilon_2}{R + r_1 + r_2} = \frac{\varepsilon_{eq}}{R + r_{eq}} \quad \ldots (5.51)$$

From the above expression it is clear that in series combination,

(i) Net emf of cells will be the sum of the emf of cells.

(ii) If n cells of equal emf and internal resistance are connected in series, then

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots\ldots \varepsilon_n = \varepsilon$$

and $r_1 = r_2 = r_3 = \ldots\ldots = r_n = r$ है तो

$$I = \frac{n\varepsilon}{R + nr} = \frac{\varepsilon}{r + R/n} \quad \ldots (5.52)$$

$$\varepsilon_{eq} = n\varepsilon \quad \ldots (5.53)$$

$$r_{eq} = nr \quad \ldots (5.54)$$

(iii) If the polarity of either of the cells in the combination is reversed, the value of equivalent emf will be $\varepsilon_1 - \varepsilon_2$ or $\varepsilon_2 - \varepsilon_1$,

But $r_{eq} = r_1 + r_2 + r_3 + \ldots r_n$. will remain same.

5.11.2 Parallel Combination of Cells :

An arrangement of two or more cells are connected in such a way that all the terminals of same polarity are joined together is known as the parallel combination of cells.

In figure 5.19 a parallel combination of two cells is shown. Let the emf of two cells are ε_1 and ε_2 and their respective internal resistances r$_1$ and r$_2$. External resistance R is connected to the end terminal A and B as shown in the figure.

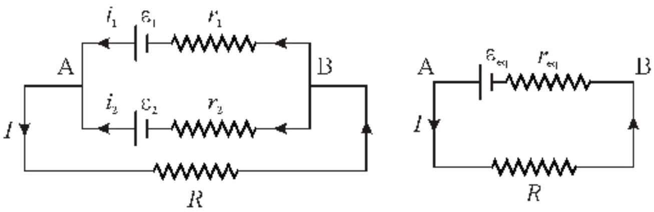


Fig 5.19: Parallel combination of cells.

If I_1 and I_2 are the current through each cell and I is the total current in the external circuit.

$$I = I_1 + I_2 \quad \ldots (5.55)$$

If terminal voltage between the terminals A and B for two cells is V_1 and V_2 each equal to V, then

$$V = \varepsilon_1 - I_1 r_1 \quad \ldots (5.56)$$

$$V = \varepsilon_2 - I_2 r_2 \quad \ldots (5.57)$$

Solving above equations,

$$I_1 = \frac{\varepsilon_1 - V}{r_1}$$

and $$I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Putting these values of I_1 and I_2 in eq. (5.55)

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) - V\left(\frac{r_1 + r_2}{r_1 r_2}\right) \quad \ldots (5.58)$$

On solving

$$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \quad \ldots (5.59)$$

If ε_{eq} and r_{eq} are the equivalent emf and internal resistance respectively of the cell combination, then terminal voltage will be,

$$V = \varepsilon_{eq} - I\, r_{eq} \quad \ldots (5.60)$$

After comparing equation (5.59) and (5.60),

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad \ldots (5.61)$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \ldots (5.62)$$

And the value of current through the external resistance is,

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}} \quad \ldots (5.63)$$

In the above equation, the values of ε_{eq} and r_{eq} are given by eqs (5.61) and (5.62).

It is clear from the parallel combination of cells, that,

(i) If $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $r_1 = r_2 = r$. i.e. two cells of same emf and same internal resistance then $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$.

(ii) n cells of same emf and same internal resistance are connected in parallel then, $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{n}$ and current in the external resistance will be,

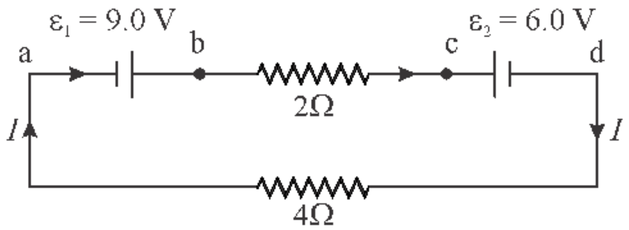
$$I = \frac{\varepsilon}{R + r/n} \quad \ldots (5.64)$$

(iii) Equations (5.61) and (5.62) can also be written as follows,

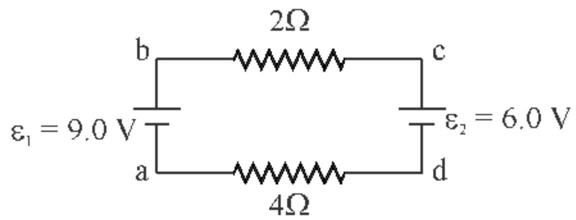
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \ldots (5.65)$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \quad \ldots (5.66)$$

Example 5.14 : The circuit shown in the figure consists of two ideal batteries connected in series through two resistances. Find the value of current in the circuit (a).



(a)



(b)

Solution : The simplified equivalent circuit of the given circuit (a) is redrawn in figure (b). Let I be the current flowing through the circuit, then the terminal voltage across the ends a and d will be,

$$\varepsilon_{aq} = \varepsilon_1 - \varepsilon_2 \qquad (\because \varepsilon_1 < \varepsilon_2)$$

$$\varepsilon_{eq} = 9.0 - 6.0 = 3.0V$$

Effective resistance of the circuit,

$$R = 2\Omega + 4\Omega = 6\Omega$$

Hence Electric current in the circuit,

$$I = \frac{\varepsilon_{eq}}{R} = \frac{3.0V}{6\Omega} = 0.5A$$

5.12 Electric Energy

Total work done (or the energy supplied) by the source of emf (i.e. cell) in maintaining an electric current in a circuit for a given time is called electric energy consumed in the circuit. If I current is flowing in a resistance R for the time t, then charge flowing in time t will be

$$q = I \times t$$

If V potential difference is applied across the ends of a wire then, by the definition of potential difference, work done by electric source in taking a charge q from one end to the other end of wire will be,

$$W = qV = VIt \qquad \ldots (5.67)$$

But according to Ohm's law, $V = IR$

hence, $$W = I^2Rt \qquad \ldots (5.68)$$

or $$W = \frac{V^2}{R}t \qquad \ldots (5.69)$$

SI unit of electric energy is joule.

Here, 1 joule = 1 watt × second

Commercial unit of electric energy is kilo watt hour (kWh). This unit is also called Board of Trade Unit (B.O.T.U.)

Therefore,

1 electric Unit = 1kWh

=1kW×1h

=1000W×3600 second

= 1 kWh = 3.6×10^6 watt second or joule

Important Note :

Using joule's law we can find equivalence between work and heat.

W = JH

$$H = \frac{W}{J} = \frac{VIt}{J} = \frac{I^2Rt}{J} = \frac{V^2}{R}\frac{t}{J}$$

Here, H= heat generated and J= mechanical equivalent of heat.

The value of $J = 4.2 \frac{\text{J}}{\text{cal}}$

5.13 Electric Power

In any electric circuit, work done per second by electric source to the flow of electric current or loss of energy per second is known as power of electric circuit.

It is generally represented by the symbol P.

In electric circuit, loss of energy in time t for current flow i.e. work done is W.

Then power,

$$P = \frac{W}{t} \qquad \ldots (5.70)$$

We have studied in the earlier section that,

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

Therefore, $$P = VI = I^2R = \frac{V^2}{R} \qquad \ldots (5.71)$$

SI unit of electric power is watt.

Here, $1 \text{ watt} = \frac{1 \text{ joule}}{\text{second}}$

In practice electric power is measured in kilo watt or megawatt.

1 kilowatt = 10^3 watt

1 megawatt = 10^6 watt

There is another unit used to measure electric power known as Horse Power (HP).

1 HP = 746 watt

The equations $P = VI$ and $P = \frac{V^2}{R}$ have important role in electric power loss in electric power transmission. The electric power is transmitted from power station to homes / factories through the cable even thousands of miles away from the power station. We want such an arrangement in which the transmission power loss would be minimum. If P power is transmitted to a device of V volt through a cable of resistance R_C, then, power loss in connecting wire will be,

$$P_C = \frac{P^2R_C}{V^2}$$

This power loss is inversely proportional to V^2. Due to this reason, to reduce electric power loss in electric power circuit, current is passed at high voltage. Due to this high voltage danger, the transmission power lines are away from residential areas. Near homes, this high voltage is again converted to low voltage by means of electric transformers.

Example 5.15 : A Bulb of 220 V and 110 Watt is connected with a source of 110 Volt, calculate the value of the power consumed by the bulb.

Solution :

According to the question,

V = 220 V, P = 100 W

So, the resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{220 \times 220}{100}$$

or $R = 484\Omega$

New voltage of the source, $V' = 110V$

Therefore, the power consumed by the bulb, when bulb is connected to new voltage of the source, $V' = 110V$,

$$P^1 = \frac{(V^1)^2}{R} = \frac{(110)^2}{484} = 25W$$

Important Points

1. The flow of charge per unit time is known as electric current.
2. When an electric field is applied across a conductor, then the average velocity by which free electrons of that conductor move is called its drift velocity.

   Drift velocity $= \vec{v}_d = \frac{-e\vec{E}}{m}\tau$
3. Mobility is numerically equal to drift velocity per unit electric field. Hence mobility (μ) is given by :

   $$\mu = \frac{|v_d|}{E}$$

   The S.I.unit of mobility is $\frac{m^2}{V\,s}$
4. The relation between electric current and drift velocity is I = neAv_d. The relation between the current density J = nev_d, drift velocity and potential difference is $v_d = \frac{eV}{ml}\tau$
5. According to ohm's law, when physical conditions of the conductor remains same, the electric current flowing through the conductor is proportional to the applied potential difference across its ends. i.e. $V \propto I$ or $V = RI$, where R is the resistance of the conductor. The unit of R is ohm $\Omega = VA^{-1}$.
6. The resistance of conductor is $R = \frac{m\ell}{ne^2A\tau}$. On increasing the temperature, relaxation time decreases, hence R will increase.
7. The resistance $R \propto \ell$ and $R \propto \frac{1}{A}$, hence $R = \rho\frac{\ell}{A}$, where ρ is the resistivity or specific resistance of the conductor. The resistivity ρ of the conductor depends on the material of the conductor and temperature,

whereas it does not depend on the length and area of cross section.

8. The relation between electric field E and resistivity, $E = \rho J$ and the relation between current density and conductivity, $J = \sigma E$ are the microscopic forms of Ohm's law.

9. Many substances and devices do not obey ohm's law, such devises are called non-ohmic devises. For these devises the graph between V and I is no longer a straight line.

10. Dependence of resistance on temperature is given as $R_t = R_0(1+\alpha t)$. On increasing the temperature of the conductor, its resistivity as well as resistance increases whereas for semiconductors and insulators on increasing the temperature their resistivity as well as resistance decreases.

11. The equivalent resistance in series combination is $R_{eq} = R_1 + R_2 + ... + R_n$ and the equivalent resistance in parallel combination is $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$

12. When the cell is in open circuit, then the potential difference across its electrodes is called its e.m.f.

13. When the cell is in closed circuit, i.e. the current is flowing through the cell and the external circuit then the potential difference across its electrodes is called the terminal voltage.

14. The relation between the terminal voltage (at the time of discharging of the cell) and e.m.f. will be $\varepsilon = V + Ir$.

15. The relation between the terminal voltage (at the time of charging of the cell) and e.m.f. will be V=E+ir. i.e. $V > E$.

Questions for Practice

Multiple Choice Questions

1. The product of resistivity and conductivity of a conductor depends on -

(a) Area of cross section

(b) Temperature

(c) Length

(d) None of the above

2. Two similar wires of same size of resistivity ρ_1 and ρ_2 are connected in series. Equivalent resistivity of the combination will be -

(a) $\sqrt{\rho_1\rho_2}$ (b) $2(\rho_1 + \rho_2)$

(c) $\frac{\rho_1 + \rho_2}{2}$ (d) $\rho_1 + \rho_2$

3. A conducting resistance is connected to a battery. The temperature of the conductor decreases due to cooling. The current flowing through the resistance will -

(a) increase (b) decrease

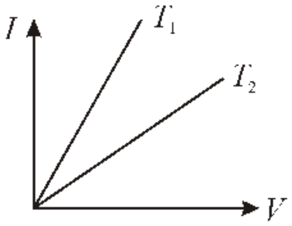
(c) remain constant (d) become zero

4. A cell of emf 2.1 volt gives a current of 0.2 A. This current passes through a 10 Ω resistance. Internal resistance of the cell will be.

(a) 0.2 Ω (b) 0.5 Ω

(c) 0.8 Ω (d) 1.0 Ω

5. The voltage current graph of a conductor at two different temperature are shown in the Figure. If the resistances corresponding to these temperatures are R_1 and R_2, then which of the following statement s is true.



(a) $T_1 = T_2$ (b) $T_1 > T_2$)

(c) $T_1 < T_2$ (d) None of the above

6. Electric power is transmitted from on city to another city through copper wire, situated 150 km apart. The voltage drop per km is 8 V and the resistance per km is 0.5 Ω, then the power loss in the transmission line will be-

(a) 19.2 W (b) 19.2 kW

(c) 19.2 J (d) 12.2 kW

7. There are 5 resistances each of R Ω. First three resistances are connected in parallel, after that remaining two are connected in series, then the equivalent resistance of the combination will be -

(a) $\frac{3}{7}R\Omega$ (b) $\frac{7}{3}R\Omega$

(c) $\frac{7}{8}R\Omega$ (d) $\frac{8}{7}R\Omega$

8 . From which of the following relations between the drift velocity v_d and electric field E, obeys ohm's law.

(a) $v_d \alpha E^2$ (b) $v_d \alpha E$

(c) $v_d \alpha E^{1/2}$ (d) $v_d =$ constant

9 . In a carbon resistance there are 4 rings in order blue, yellow red and silver. The resistance of the resister will be -

(a) $64 \times 10^2 \Omega$

(b) $(64 \times 10^2 \pm 10\%)\Omega$

(c) $642 \times 10^4 \Omega$

(d) $(26 \times 10^3 \pm 5\%)\Omega$

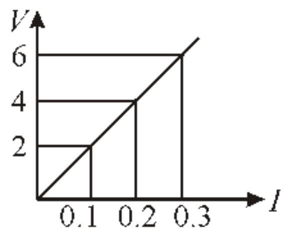
10 . When a wire connected to a battery gets heated, then which of the following quantity will not change -

(a) Drift velocity (b) Resistivity

(c) Resistance (d) No. of free electrons

Very Short Answer Questions

1. From the following graph between V & I, calculate the resistance of the resister.



2. Write down the S.I. unit of current density.

3. Write down the relation between conductivity and current density of a conductor.

4. Give two examples of non-ohmic devices.

5. Give the dependence of resistivity on temperature of a conductor.

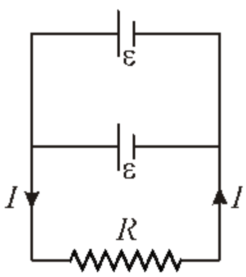
6. Write the names of two materials whose resistivity decreases on increasing the temperature.

7. Write down the value of current flowing through a bulb of 40 W 220 V.

Short Answer Questions

1. How much charge will be there when a current is flowing through a conductor.

2. In the given figure, the resistivity of some conductor are ρ_1 and $\rho_2 \Omega \times m$. What is the ratio of ρ_1 , ρ_2



3. In the given figure, there are two similar cells whose emfs are same and internal resistances are negligible. These cells are connected in parallel. What is the value of current flowing through the resistance R ?

4. Explain the difference between terminal voltage and emf of a cell.

5. Define drift velocity.

6. A resistance wire of 8R is bent in the form of a circle, then what is its equivalent resistance across the ends of diameter.

7. When the shape of a conductor is deformed then what is the effect on its resistance and resistivity?

8. Can the terminal voltage of a cell be greater than the emf of a cell?

Essay Type Questions

1. Define drift velocity. On the basis of drift velocity derive Ohm's law $\vec{j} = \sigma \vec{E}$?.

2. Derive the relation between the drift velocity and electric field. What is mobility? Explain dependence of drift velocity and mobility.

3. Derive relation between resistance and resistivity of any conductor. Explain temperature dependence on the resistance of a material. Explain in reference to a conductor, insulator and semi-conductor.

4. There are two cells of emf ε_1 and v and internal resistance r_1 and r_2 respectively connected in parallel. Then find out the equivalent emf and equivalent internal resistance of this combination. If external resistance R in connected to this combination, then find out the value of electric current flowing through R.

Answer Key (Multiple Choice Questions)

1. (d) 2. (c) 3. (a) 4. (b)

5. (c) 6. (b) 7. (b) 8. (b) 9. (b) 10. (d)

Answer Key (Very Short Answer Questions)

1. 20 Ω

2. A/ m^2

3. $\vec{j} = \sigma \vec{E}$

4. diode, electrolytes

5. $\rho = \rho_0 (1 + \alpha \Delta t)$

6. Germanium and Silicon

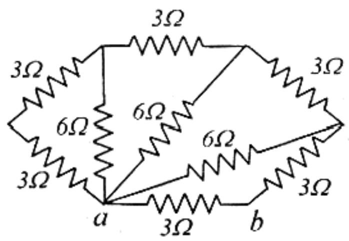
7. 0.18 A

Numerical Questions

1. A potential of 120 V is applied across the ends of a cylindrical copper rod of length 1 cm and radius 2.0 mm. Find the value of the current through the rod. (The resistivity of copper is $1.7 \times 10^{-8}\,\Omega m$.)

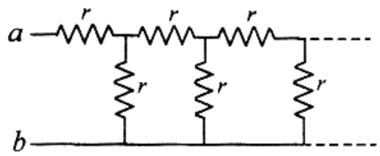
Ans. $\left[6.85 \times 10^{-5}\text{ A}\right]$

2. Find the equivalent resistance between a and b in the given circuit diagram.



[Ans. 2 Ω]

3. Find the equivalent resistance between points of an infinite ladder network circuit as shown in the figure.



Ans. $\left[\left(\frac{1+\sqrt{5}}{2}\right) r\right]$

4. (a) Three resisters 1Ω, 2Ω and 3Ω are connected in series. What is the total resistance of the combination?

(b) If this series combination is connected to a battery of e.m.f. 12 V and negligible internal resistance, obtain the potential drop across each resistance.

[Ans. 6 Ω, 2V, 4 V and 6V]

5. At room temperature (27°C), the resistance of a heating element is $100\,\Omega$. What is the temperature of the element if the resistance is found to be $117\,\Omega$, given that the temperature coefficient of the material of resistance is $1.70\times10^{-4}\,{}^{\circ}C^{-1}$.

[Ans. 1027°C]

6. A negligibly small current passes through a wire of length 15 m and uniform cross section $6.0\times10\ ^{7}m^2$ and its resistance is measured to be $5.0\,\Omega$. What is the resistivity of the material at the temperature of the experiment?

[Ans. $2.0\times10\ ^{7}\,\Omega m$]

7. A Copper wire of cross section area 1.0 mm^2 is carrying a current of 0.5 A. If the density of free electrons is $8.5\times10^{22}\,cm^{-3}$. Calculate the drift velocity of free electrons.

[Ans. 3.7×10^{-5}m/s]

8. Find the temperature at which the resistance of a material is doubled that of the resistance at (0°C). The temperature coefficient of the material of resistance is $4.0\times10^{-3}\,{}^{\circ}C^{-1}$

[Ans. 250°C]

9. The storage battery of a car has an e.m.f. of 12 V. If the internal resistance of the battery is $0.4\,\Omega$. What is the maximum current that can be drawn from the battery?

[Ans. 30 A]

10. A coil of resistance $4.2\,\Omega$ is immersed in water. A current of 2A passes through it for a duration of 10 minutes. How many calories of heat will be produced in the coil? [J = 4.2 J / cal]

[Ans. 2400 cal]

11. A cylindrical tube of length l has inner and outer radii a and b. The resistivity of the material is ρ, then calculate the resistance between the two end of the cylindrical tube.

Ans. $\left[\dfrac{\rho\ell}{\pi(a^2-b^2)}\right]$

12. In a house 4 bulbs of 100 W and 4 bulbs of 40 W glow every day for 4 hours and 6 hours respectively. Two fans of 60 W are also used 8 hours every day. Calculate the electrical energy consumed in a month of 30 days. Also calculate the electricity bill for the month at the rate of Rs. 5 per unit.

[(Ans. Electricy consumed: 105.6 units, Bill amount Rs. 528)]

Chapter - 6
Electric Circuit

In previous Chapter we studied Ohm's law and series and parallel combination of resistors. In simple electric circuit, electric current and potential difference can be calculated by using Ohm's law. In complicated electric circuits (in which so many resistors and cells are connected in a complex way) to calculate electric current and potential difference, German scientist Robert Kirchhoff gave two laws. In this chapter we will study Kirchhoff's Laws and their uses, Wheatstone bridge and potentiometer is a device used to measure potential difference accurately and its applications.

6.1 Kirchhoff's laws

Junction is a point where three or more than three branches of a circuit meet. In a network of electric circuit in which electric current remains constant is called a branch. A closed circuit consisting of different conductors, resistances and other elements is called a loop or mesh. For complex electric circuits, Kirchhoff's laws are as follows.

6.1.1 Kirchhoff's first law or junction law

According to this law, the algebraic sum of electric currents meeting at a junction is zero.

i.e. $\sum I = 0$

Thus, we can say that the sum of electric currents entering at the junction is equal to the sum of electric currents leaving the junction. This law is known as *Kirchhoff's first law or junction law.*

This law is based on the conservation of charge. In electric circuits, at any junction charge can not be accumulated or generated. Thus, at junctions, the rate of entering charge is equal to rate of leaving charge.

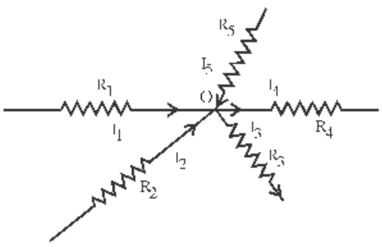


Fig 6.1 Kirchhoff's junction law

In Fig (6.1) at junction O, according to this law,

$$I_1 + I_2 - I_3 - I_4 + I_5 = 0$$

or $\quad I_1 + I_2 + I_5 = I_3 + I_4 \quad \quad \ldots (6.1)$

6.1.2 Kirchhoff's second law or loop law:

Kirchhoff's loop law is applicable for closed electric circuits. Hence, it is called loop rule. According to this rule, for a circuit consisting of resistances and cells. The agebraic sum of voltages in a circuit is zero.

$$\sum V = 0 \quad \quad \ldots (6.2)$$

This law can be represented in a different form as follows.

In any closed loop algebraic sum of potential difference across resistors will be equal to the algebraic sum of emf of cells used.

Thus, $\quad \sum IR = \sum \varepsilon \quad \quad \ldots (6.3)$

While using equation (6.3) following sign conventions are used.

1. In any circuit if we move in the direction of current potential difference across resistance is considered to be positive. If we move in the opposite direction of current, potential differences are considered to be negative.
2. If we move in the circuit in assigned direction of current and move from negative electrode to the positive electrode of a cell, then emf is considered to be positive. Similarly, in the circuit if we move from positive electrode to negative electrode of the cell is considered to be negative.

Kirchhoff's loop rule is based on the law of conservation of energy. Loop rule can be explained by the example given in Fig 6.2

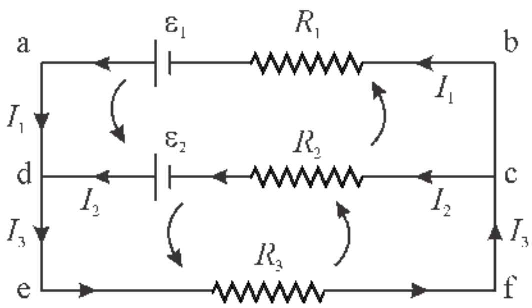


Fig 6.2 A closed circuit

In the given figure, apply junction rule at junction 'd'. Current in resistance R_3 will be,

$$I_3 = I_1 + I_2 \quad \ldots (6.4)$$

Applying loop rule for ' a d c b a' loop,

$$I_1R_1 - I_2R_2 = \varepsilon_1 - \varepsilon_2$$

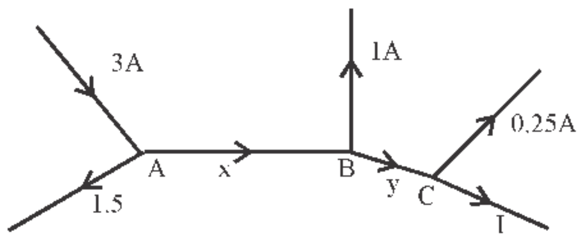
or $$I_2R_2 - I_1R_1 = \varepsilon_2 - \varepsilon_1 \quad \ldots (6.5)$$

Applying loop rule for '$defcd$' loop,

$$I_3R_3 + I_2R_2 = \varepsilon_2 \quad \ldots (6.6)$$

On simplifying equations (6.4), (6.5) and (6.6), we can calculate current in different branches and potential difference across different resistances. We will understand these rules by few solved examples.

Example 6.1 : Find the value of I in the network as shown in the figure,



Solution : Let the current through the branches AB and BC are x and y respectively. Using Kirchhoff's junction law, we have,

At junction $3 - 1.5 - x = 0$

or $x = 1.5\,A$

At junction B, $1.5 - y - 1 = 0$

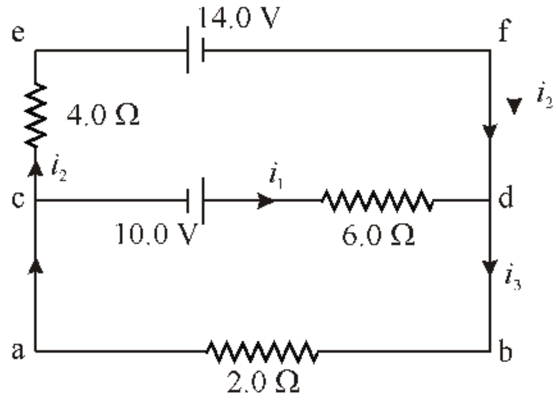
or $y = 0.5\,A$

Similarly, at junction C,

$$0.5 - 0.25 - I = 0$$

or $I = 0.25A$

Example 6.2 : Find the values of currents in the circuit using Kirchhoff's law in the network are given below,



Solution : In the given circuit i_1, i_2 and i_3 are three unknown currents. For calculating these unknowns, we need three equations.

Using junction law at junction C we have,

$$i_3 = i_1 + i_2 \quad \ldots (i)$$

Using loop law in loop '$acdba$' we have,

$$+2i_3 + 6i_1 = +10$$

or $$6i_1 + 2i_3 = 10$$

Putting the value of i_3 from equation (i)

$$8i_1 + 2i_2 = 10 \quad \ldots (ii)$$

Using loop law in loop '$cdfec$' we have,

$$6i_1 - 4i_2 = 10 + 14$$

or $$6i_1 - 4i_2 = 24 \quad \ldots (iii)$$

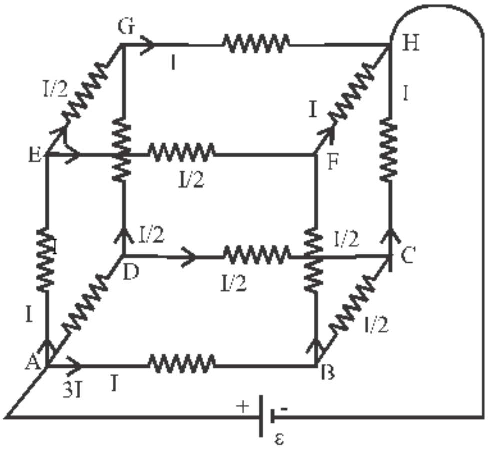
By solving equation (ii) and (iii)

$$i_1 = 2\,A\,,\ i_2 = -3A$$

By putting the value of i_1 and i_2 from equation (i) we get, $i_3 = -1A$

Note : Negative sign of i_2 and i_3 in the solution indicates that their directions will be opposite to the directions shown in the figure 6.2.

Example 6.3 : A battery of emf 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω. Determine the equivalent resistance of the network and the current along each wire of the cube.



Solution : The paths AE, AB and AD are obviously symmetrically placed in the network. Thus, the current in each of these wires must be same. Further, at the corners E, D and B the incoming current I must divide equally into the outgoing branches. In this manner, the current in all the 12 wires of the cube can easily be written in terms of I using Kirchhoff's first law and the symmetry of the problem.

Next, take a closed loop ABCHA and apply Kirchhoff's second law.

$$IR + \frac{I_2R}{2} + IR = \varepsilon$$

$$\text{or } \varepsilon = \frac{5}{2} IR \qquad \ldots \text{(i)}$$

Here, ε is the emf of the cell and R is the resistance of each edge of the cube.

Total current drawn from the battery is 3I, therefore the equivalent resistance of the Cubical network is,

$$R_{eq} = \frac{\varepsilon}{3I} \qquad \ldots \text{(ii)}$$

Put the value of ε from (i) to eq. (ii),

$$R_{eq} = \frac{5}{6} R$$

According to the question $R = 1\Omega$,

$$\text{Therefore,} \quad R_{eq} = \frac{5}{6}\Omega$$

Emf of the cell is given as $E = 10V$, therefore, using equation (ii),

$$3I = \frac{\varepsilon}{R_{eq}} = \frac{10}{3\frac{5}{6}} = 4\ A$$

Therefore, the current through each edge of the cube can be calculated from the given figure.

6.2 Wheatstone Bridge:

In 1942, English scientist Prof. C.F. Wheatstone, joined four resistances, one cell and one galvanometer to make a special type of circuit as shown in the figure 6.3. It is known as Wheatstone Bridge. This circuit is used to determine the value of some unknown resistance.

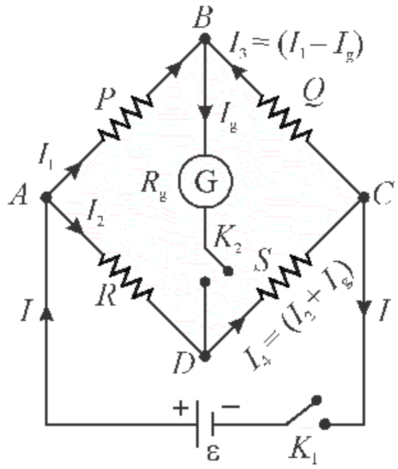


Fig 6.3 Wheatstone Bridge

Construction :

Arrangement of Wheatstone Bridge is shown in the figure (6.3). In this circuit two resistances P and Q are connected in series and remaining two resistances R and S are also connected in series. These two series combinations are connected in parallel. In this way a quadrilateral arrangement is formed between A and C. A cell of emf ε is connected between these terminals along with a key K_1 whereas a galvanometer is connected between the terminals B and D along with key K_2. Keys K_1 and K_2 are used to allow current in the circuit.

Resistance arms P and Q are called ratio arms and the arm AD in which known resistance R is connected is called known resistance arm. The arm CD in which unknown resistances is connected is called unknown resistance arm. Arm AC in which cell is connected is called cell arm. Arm BD in which galvanometer is connected is called galvanometer arm.

6.2.1 Principle of Wheatstone Bridge and condition of Balance :

When key K_1 is closed, current I is flowing and it divides into two parts at junction A. Current through branch AB is I_1 and current through branch AD is I_2. When key K_2 is closed, galvanometer gives deflection. When $V_B > V_D$, then the current in galvanometer flows from B to D. Potentials at points B and D are V_B and V_D. The values of V_B and V_D depends on the values of resistances of the arms. Now we select the resistances in Wheatstone's bridge such that galvanometer gives no deflection and this is called balanced condition. In this condition, potentials at B and D are equal i.e. $I_1 = I_3$ and $I_2 = I_4$. In this condition (refer to figure 6.3)

$$V_B = V_D \quad (I_g = 0) \qquad \ldots (6.7)$$

or $$V_A - V_B = V_A - V_D$$

According to Ohm's law $I_1 P = I_2 R \qquad \ldots (6.8)$

Similarly, $V_B - V_C = V_D - V_C$ (from eq.6.7)

According to Ohm's law, $I_3 Q = I_4 S$

$$I_1 Q = I_2 S \quad (I_1 = I_3 \text{ and } I_4 = I_2) \qquad \ldots (6.9)$$

Dividing equations (6.8) and (6.9)

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$$

or $$\frac{P}{Q} = \frac{R}{S} \qquad \ldots (6.10)$$

Equation (6.10) is the condition of balanced Wheatstone bridge. It is clear from this equation that in the balanced condition the ratio of resistances in ratio arms is same.

From equation (6.10), unknown resistance can be calculated,

$$S = \frac{Q}{P} R \qquad \ldots (6.11)$$

To determine the value of unknown resistance we connect it in 4th arm of the bridge. Known resistances P and Q are connected in 1st and 2nd arm of the bridge. Known resistance R is adjusted such that galvanometer gives no deflection. Thus, balanced condition is obtained for Wheatstone bridge.

For Wheatstone bridge to be sensitive resistances of all the four branches must be of same order.

6.2.2 Balancing Condition of Wheatstone Bridge using Kirchhoff's Laws

Using Fig (6.4) we can derive the condition of balance of Wheatstone bridge using Kirchhoff's laws. Let the current in the galvanometer be I_g and the its resistance be R_g.

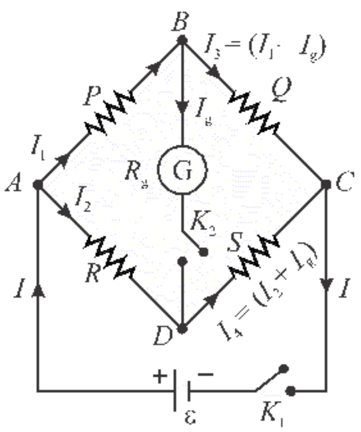


Fig 6.4 Wheatstone Bridge

Apply Kirchhoff's loop rule (Voltage law) in the loop '$abda$'.

$$I_1 P + I_g R_g - I_2 R = 0 \qquad \ldots (6.12)$$

Similarly, apply Kirchhoff's loop rule (Voltage law) in the loop '$bcdb$'.

$$(I_1 - I_g) Q - (I_2 + I_g) S - I_g R_g = 0 \qquad \ldots (6.13)$$

In the balancing condition of the bridge, $I_g = 0$,

Hence from the equations (6.12) and (6.13) we get,

$$I_1 P - I_2 R = 0$$

$$I_1 P = I_2 R \qquad \ldots (6.14)$$

and $$I_1 Q - I_2 S = 0$$

$$I_1 Q = I_2 S \qquad \ldots (6.15)$$

From equations (6.14) and (6.15), we get,

$$\frac{P}{Q} = \frac{R}{S} \qquad \ldots (6.16)$$

Which is the condition of balanced Wheatstone bridge.

6.3 Meter bridge:

Meter bridge is based on the principle of Wheatstone bridge. Meter bridge is a device which consists of a one-meter long resitance wire with uniform cross section. It is used to determine unknown resistance. Outline of the meter bridge is shown in the figure (6.5).

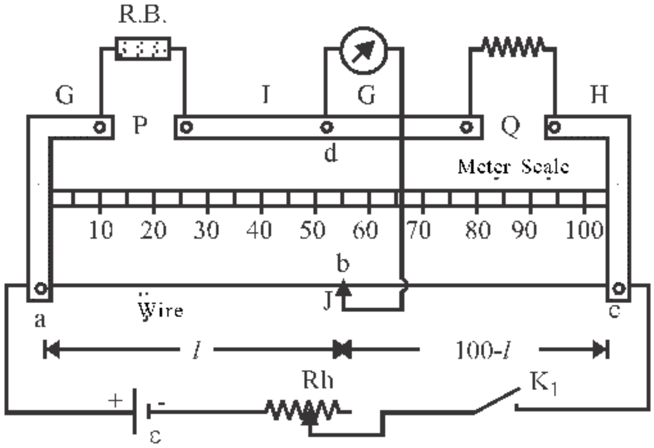


Fig. 6.5 Meter bridge

Construction : A one-meter long constantan or magainin resistance wire is stretched between two screws *a* and *c*. A meter scale is also fixed over the wooden board along the length of wire. Two L shaped copper strips G and H are also joined to the screws a and c. Another copper strip I is also placed between the strips G and H such that a proper gap exists between G and H with I. An unknown resistance S is connected across the gap between I and H strip. A known resistance R is connected across the gap between G and I. J is a sliding key that can be slided on the wire ac, the point of contact of the key, b on the wire divides the wire into arms ab and bc.

Working Principle : In the gaps of meter bridge a resistance box is connected in the left gap and unknown resistance is connected between the right gap with the help of nuts (A nut is a fastened with a threaded hole). In between the points A and C, a leclanche cell, a rheostat and key K are connected. In between d and sliding jockey J, a galvanometer is connected. In this position, meter bridge works as Wheatstone bridge. Now, remove certain plug from the resistance box. To check the correctness of connections, hold the jockey first near the end 'a' of the meter bridge wire and then near the end 'c' of the wire. If the galvanometer shows opposite deflection at the 'a' and 'c' ends of the wire then the connection is said to be correct. Otherwise, choose appropriate resistance R from the resistance box to get this condition. Once we are sure about the correctness of the connections, then slide the jockey over the meter bridge wire and find the null point. Suppose the galvanometer shows the null point when the jockey is at the position b. This is the balanced condition of Wheatstone bridge. At point b if it shows zero deflection, then the part of wire ab works as resistance P and the part 'bc' works as resistance Q .

$$\frac{P}{Q} = \frac{R}{S} \qquad \ldots (6.17)$$

Let point b is at a distance ℓ from the point a. Let R_{cm} be the resistance per unit length of the wire then the resistance of section P of the wire will be $R_{cm}\ \ell$ and that of the section Q of the wire will be $R_{cm}(100 - \ell)$. If the resistance of these sections is P and Q then,

P = resistance of section ab of wire = $R_{cm}\ \ell$ And Q = resistance of section bc of wire = $R_{cm}(100 - \ell)$. By substituting the values of P and Q in equation (6.17) we get,

$$\frac{R}{S} = \frac{R_{cm}\ell}{R_{cm}(100 - \ell)}$$

or $$S = \left(\frac{100 - \ell}{\ell}\right) R \qquad \ldots (6.18)$$

Knowing the values of l and R we can get the value of unknown resistance. Meter bridge will be sensitive if the null point is obtained near the middle point of the wire.

Limitations of meter bridge :

(i) In the derivation of the formula for meter bridge we have considered the resistance of copper plates (G, H and I) as negligible. But actually, these copper plates do have some resistance due to which there will be error in the result. To eliminate this error, we interchange the position of resistance box and unknown resistance S and then calculate the value of unknown resistance and take the mean of these values to minimise the error.

(ii) Due to the resistance of end points of the meter bridge wire, the sensitivity of the experiment is affected. To eliminate this effect we use, Carey Foster's bridge.

(iii) Do not pass electric current in the meter bridge for a long time otherwise the wire will be heated up due to which the resistance of meter bridge wire will get changed.

(iv) We should not slide the jockey by rubbing over the meter bridge wire, otherwise the uniformity of the area of cross section of wire (and there by the resotance per unit length of the meter bridge wire) will be affected.

Example 6.4 : In an experiment of meter bridge, a resistance of 8Ω is taken out from the resistance box to get a null deflection position at 45.5 cm. Calculate (a) value of unknown resistance, (b) New balance length on interchanging the positions of resistance box and unknown resistance.

Solution : Using meter bridge principle,

$$S = R\left(\frac{100-\ell}{\ell}\right)$$

Here, $R = 8\,\Omega,\ \ell = 45.5\,\text{cm}$

Hence, $S = 8\left(\frac{100-45.5}{45.5}\right)$

$$= 8 \times \frac{54.5}{45.5}$$

$$S = 9.58\,\Omega$$

(b) On interchanging the positions of resistance box and unknown resistance, the new balance point will be $(100 - 45.5) = 54.5$ cm

6.4 Potentiometer :

Potentiometer is an ideal experimental device or arrangement which is used to measure the emf of a cell or potential difference between any two points in a circuit. In no deflection condition no current will be drawn from the circuit. Therefore, its measurement will be accurate. Potentiometer works as a voltmeter of infinite resistance (i.e. ideal voltmeter). It can be understood by the following fig (6.6)

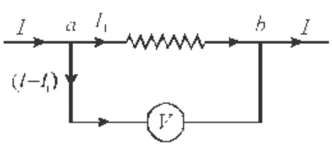


Fig: 6.6 Measurement of potential difference

In figure (6.6), a voltmeter is connected across a current carrying resistance for measuring potential difference. A fraction of current is drawn by the voltmeter due to its own resistance. As a result, the potential difference across the ends a and b of the resistance is observed slightly less than the actual potential difference. Due to this measured potential difference will be less than actual potential difference.

This error in potential difference can be reduced to zero if we use an ideal voltmeter of infinite resistance. But in realistic situation it is not possible at all.

In case of balanced condition, no current is drawn by the potentiometer from the circuit. In state of no deflection, it works as a voltmeter of infinite resistance. So, we can say, that potentiometer is an ideal device to measure potential difference in comparison to voltmeter.

6.4.1 Construction of Potentiometer

Fig 6.7 shows a potentiometer. Mainly potentiometer wire is made up of magainin, eureka, constantan like alloys. The specific resistance of these materials is very high and the temperature coefficient is very low.

It consists of a 10-meter-long resistance wire of uniform cross section area spread over a wooden plank in 10 equal parts each with a length of one meter. The ends of wire are connected across the connecting terminals A and B. A meter scale is also fixed over the wooden plank parallel to the length of the wire. A sliding jockey J is also capable to slide along the wire with the help of a rod which is fixed over the wooden plank. By pressing the jockey, we can establish the electric connection to any wire of the potentiometer wire (Remember, there are 10 wires). The position of the jockey can be determined with the help of meter scale.

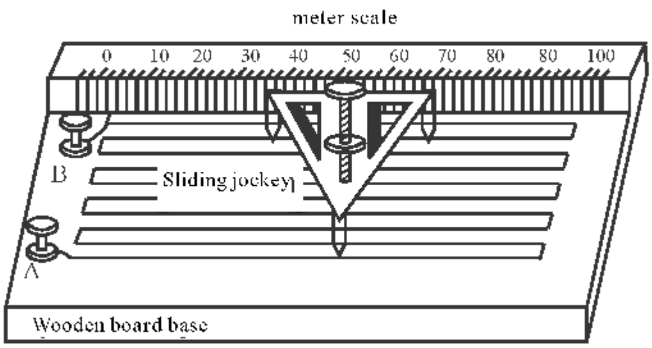


Fig: 6.7 Construction of potentiometer

6.4.2 Principle of Potentiometer

The principle of a potentiometer is that the potential dropped across a segment of a wire of uniform cross-section carrying a constant current is directly proportional to its length. To determine an unknown potential difference (or emf) it is compared with a known potential difference distributed uniformly over the poteniometer wire. In condition of no deflection the unknown potential difference is equal to the known potential difference. This is known as the principle of potentiometer.

To understand this, an electric circuit is made as shown in the figure (6.8). Now we connect a battery of emf ε_P, a key K_1 and a rheostat R_h in series with the potentiometer wire AB. This circuit is called primary circuit of potentiometer. In the secondary circuit, positive terminal of cell ε of emf E is connected to positive terminal A of the potentiometer and negative terminal of the cell is connected to jockey through a galvanometer.

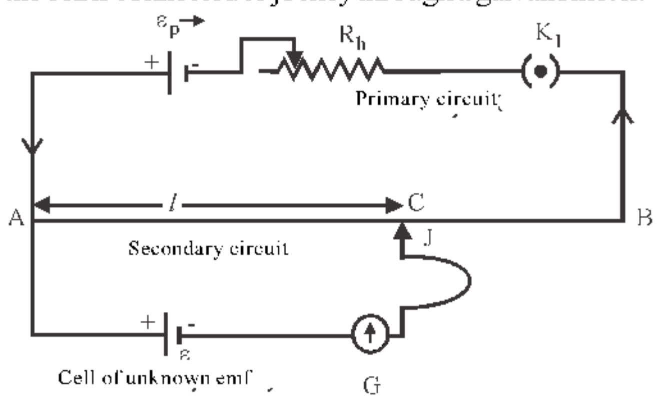


Fig. 6.8 Potentiometer circuit

Here, it is essential that the emf of cell in secondary circuit must be less than the emf of battery in the primary circuit. If in the primary circuit resistance of the rheostat is very low or zero then whole emf ε_P of the battery is uniformly distributed over the potentiometer wire AB. Here, it is considered that potentiometer wire AB has uniform area of cross section. If length of potentiometer wire AB is L, emf of the battery is ε_P then it is uniformly distributed all over the length of potentiometer wire. This fall of potential per unit length is called potential gradient, it is represented by x, thus,

$$x = \frac{\varepsilon_P}{L} \qquad \ldots (6.19)$$

The SI unit of potential gradient is volt/m,

Hence, $\varepsilon_p = V_{AB} = xL$

If the resistance of potentiometer wire is R and the current flowing in the potentiometer wire is I, then,

$$V_{AB} = IR \qquad \ldots (6.20)$$

From Eq (6.19) we get

$$x = I\frac{R}{L} \qquad \ldots (6.21)$$

or $$x = IR_m \qquad \ldots (6.22)$$

Here, $R_m = \frac{R}{L}$ = is the resistance per unit length of the potentiometer wire.

Now we consider some point on the potentiometer wire at a distance l from terminal A, then the potential difference between A and C will be given by.

$$V_{AC} = x\ell \qquad \ldots (6.23)$$

Here we know the value of x and ℓ, therefore, V_{AC} is known potential difference, the value of ℓ is variable, therefore $V_{AC} \propto \ell$. Now by placing jockey at point C on potentiometer wire. If galvanometer gives zero deflection which is called null deflection point and its length from point A, $AC = \ell$ is called balancing length on potentiometer wire for emf ε. In this situation, unknown emf ε will be equal to known potential difference V_{AC} on potentiometer wire.

$$\varepsilon = V_{AC} = x\ell \qquad \ldots (6.24)$$

This is known as principle of potentiometer.

Note : We get two situations if we press jockey J at points C_1 and C_2 shown in the figure (6.9)

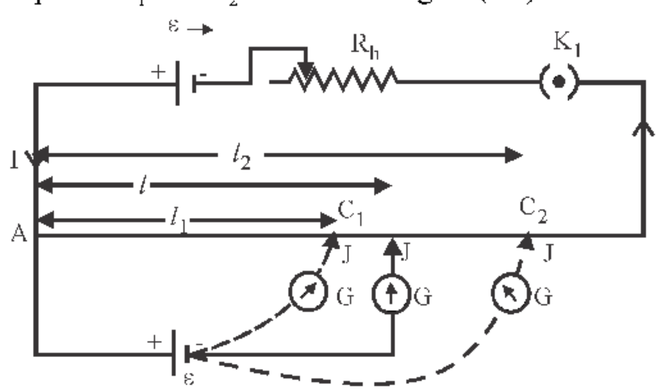


Fig 6.9 Working principle of potentiometer

(i) If, jockey is kept at C_1 and $AC_1 = \ell_1$ in this situation $V_{AC_1} < \varepsilon$, due to which the resultant current flows in sense AC_1, G ε A i.e. clockwise. Thus, the galvanometer shows deflection.

(ii) If we press jockey J at point C_2 then $AC_2 = \ell_2$ at this condition $V_{AC_2} > \varepsilon$. Due to this resultant flow in the secondary circuit is anticlockwise i.e. opposite to what in case (i) ($A\varepsilon GC_2A$). From equation (6.22).

Some important relations for potential gradient,

$$x = IR_m \qquad \ldots(6.25)$$

If in the primary circuit of potentiometer, R' is the external resistance and r is the internal resistance of cell then,

$$I = \frac{\varepsilon_P}{R + R' + r} \qquad \ldots(6.26)$$

Using Eq. (6.25) and (6.26) we get,

$$x = \left(\frac{\varepsilon_P}{R + R' + r}\right)\frac{R}{L}\left(\because R_m = \frac{R}{L}\right) \ldots(6.27)$$

If, $r = 0, R' = 0$, then from eq (6.22) we get the same value as in eq (6.19)

$$x = \frac{\varepsilon_P}{L} \qquad \ldots(6.28)$$

$$x = I\frac{R}{L}$$

If specific resistance of potentiometer wire is ρ and area of cross section is A, then,

$$\left(R = \rho\frac{L}{A}\right)$$

$$x = \frac{I\rho}{A} \qquad \ldots(6.29)$$

If r is the radius of potentiometer wire, then,

$$x = \frac{I\rho}{\pi r^2} \qquad (\because A = \pi r^2) \qquad \ldots(6.30)$$

Thus, we conclude that,

(i) Potential gradient is directly proportional to current I and specific resistance ρ and inversely proportional to area of cross section A of the wire.

(ii) Apparently, the value of potential gradient (x) depends on the emf of the battery connected in the primary circuit, length of the wire and internal resistance of the battery,

(iii) If $r = 0$ and $R' = 0$, then the potential gradient x does not depend on the area of cross section, material of wire and resistance of wire.

6.4.3 Precautions with Potentiometer:

(i) The emf of cell used in primary circuit should be greater than the emf of cell in the secondary circuit otherwise we will not get the situation of null deflection.

(ii) Positive terminals of all the cells must be connected to point A.

(iii) Balanced length is always measured from point A which is at higher potential.

(iv) The potentiometer wire must be of uniform cross section otherwise the value of potential gradient x will not be same at all positions.

(v) In potentiometer wire, current should not be passed over a long-time otherwise wire will get heat up which will change the resistance of the wire and therefore potential gradient will not remain constant.

6.4.4 Standardisation of Potentiometer

In previous section, we studied that potential gradient of potentiometer depends on the emf of cell in primary circuit and its internal resistance and other resistances connected in series with potentiometer wire. In general, the values of these resistances are not known. Hence, potential gradient can be calculated by indirect method. **The procedure of finding the exact potential gradient of potentiometer is called standardisation.**

For standardisation of a potentiometer a standard cell of known emf is connected in the secondary circuit of potentiometer as shown in the fig (6.10). Standard cell is that cell whose emf remains constant for a long time and it can be known precisely. For standardisation we use Cadmium Cell or Danial Cell as standard cell. The values of emf of Cadmium Cell and Danial Cell are 1.0186 V and 1.08 V respectively.

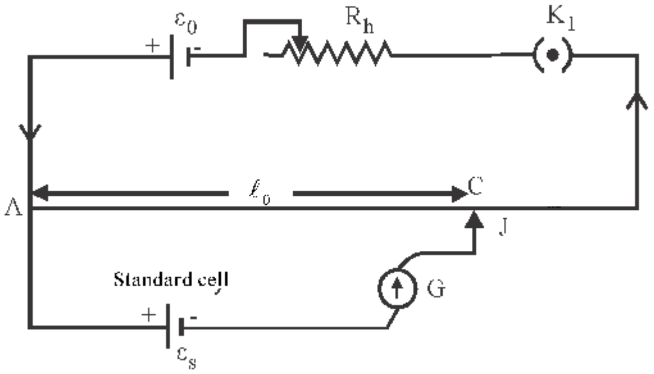


Fig 6.10 Standardisation of Potentiometer

To find the potential gradient of potentiometer with the help of a standard cell by placing a sliding jockey on the potentiometer wire and the balancing length ℓ_0 is determined at this time, (Galvanometer shows zero deflection). If emf of standard cell is ε_s, then, according to the principle of potentiometer

$$\varepsilon_s = x\ell_0$$

or $$x = \frac{\varepsilon_s}{\ell_0} \qquad \ldots (6.31)$$

Here, it is to be kept in mind that after standardisation, there should not be any change in primary circuit otherwise x will change.

6.4.5 Sensitivity of potentiometer

By sensitivity of potentiometer we mean, its ability to measure accurately small value of emf or small value of potential difference. The sensitivity of potentiometer depends on the fall of potential per unit length on the potentiometer wire or potential gradient. Smaller the value of potential gradient, larger will be the sensitivity of potentiometer.

Because balancing length ℓ is measured directly with the help of potentiometer hence. If the value of ℓ is larger then the percentage error in its measurment will be less. Hence, potentiometer is more sensitive if x is small.

$(\because E = x\ell)$

We can increase the sensitivity of potentionmeter (or decrease x).

(i) By increasing the length (L) of potentiometer wire.

(ii) By decreasing electric current in primary circuit.

By decreasing electric current in primary circuit, potential difference across potentiometer wire will also decrease.

Thus, it is better to enhance the sensitivity of potentiometer by increasing the length rather than decreasing the current in primary circuit, Due to this reason the length of potentiometer wire is taken very large.

Example 6.5 : In the primary circuit of a potentiometer experiment, a battery of emf 2.2 V and internal resistance r = 1Ω is connected. If the resistance of rheostat in the primary circuit lies in the range (0-20Ω) and length and resistance of potentiometer wire are 10 m and 20 Ω respectively. Find the minimum and maximum values of the potential gradient.

Solution :

$$x = \frac{\varepsilon_p}{R + r + R'} \times \frac{R}{L}$$

Here,

ε_p = emf of the battery in primary circuit = 2.2 V

R = Resistance of the potentiometer wire = 20Ω

r = internal resistance of battery = 1.0 Ω

L = Length of potentiometer wire = 10m

R' – Range of rheostat (0-20Ω)

For minimum value of x, the value of R' should be maximum. Hence R' = 20Ω

$$x_{min} = \left(\frac{2.2}{20+1+20}\right) \times \frac{20}{10}$$

$$= \frac{2.2}{41} \times \frac{2}{1} = \frac{4.4}{41} = 0.11 \text{ V/m}$$

Similarly,

$$x_{max} = \left(\frac{2.2}{20+1+0}\right) \times \frac{20}{10} = \frac{4.4}{21} = 0.21 \text{ V/m}$$

Example 6.6 : The area of cross section of potentiometer wire is $0.8 \times 10^{-6}\, m^2$ and it has specific resistance $40 \times 10^{-8}\, \Omega m$, If the current through the wire is 0.2 A. Calculate the value of potential gradient.

Solution : The potential gradient is,

$$x = \frac{I\rho}{A}$$

Here, $I = 0.2\, A$,

ρ = specific resitance of wire

$= 40\times10^{-8}\ \Omega m$

$A = 0.8\times10^{-6}\ m^2$

On substituting values

$$x = \frac{0.2\times40\times10^{-8}}{0.8\times10^{-6}} = 0.1\ \text{V/m}$$

6.5 Uses of Potentiometer :

6.5.1 Determination of Internal Resistance of a Primary Cell

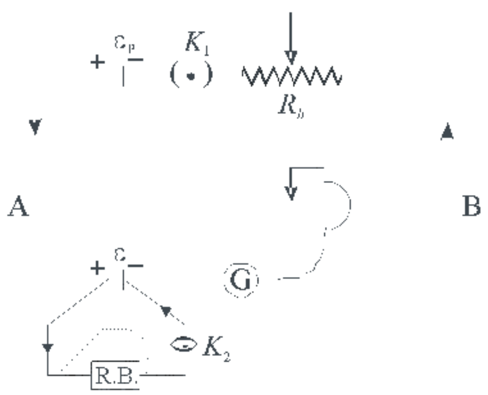


Fig 6.11 Determination of Internal Resistance of a Cell

Circuit Arrangements : A circuit is arranged as shown in the figure 6.11. Primary circuit is made up by joining a battery of emf ε_p, a rheostat R_h and a plug key K_1 in series with the potentiometer wire. To make secondary circuit, positive terminal of cell whose internal resistance is to be measured is connected to the higher potential point A of the potentiometer wire and the negative terminal of the cell is connected to the sliding jockey J through the galvanometer. A resistance box and a plug key K_2 is connected parallel to the cell.

Working Principle : First of all, by keeping primary cell of emf ε in open circuit (i.e. Keeping plug key K_2 open) balance point is obtained by sliding the jockey. Let the balancing length in this case be ℓ_1. If the potential gradient is x, then according to the principle of potentiometer,

$$\varepsilon = x\ell_1 \qquad \ldots (6.32)$$

Next, without changing the configuration of the primary circuit, plug key K_2 is closed, and some resistance R is inserted in the resistance box. The potential drop V across the resistance box is balanced on the potentiometer wire. And balancing length ℓ_2 is obtained with the help of sliding jockey J.

$$V = x\ell_2 \qquad \ldots (6.33)$$

We know, if r is the internal resistance of the cell and I is the current through the resistance R, then,

$$\varepsilon = V + Ir$$

or $$r = \frac{\varepsilon - V}{I}$$

or $$r = \left(\frac{\varepsilon - V}{V}\right)R \quad (\because\ V = IR) \qquad \ldots (6.34)$$

From eq. (6.32), (6.33) m and (6.34) we get,

$$r = \left(\frac{x\ell_1 - x\ell_2}{x\ell_2}\right)R$$

or internal resitance $= r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R \qquad \ldots (6.35)$

By substituting the values of ℓ_1 and ℓ_2, internal resistance of the primary cell can be determined. For varying the values of R, different values of ℓ_2 are recorded and corresponding values of r are calculated. We see that in all observations the value of r is different. i.e. internal resistance depends on the current drawn from the cell. Thus, while measuring the internal resistance, we should not take the average of the values of r. We must state that, the internal resistance varies between minimum value to maximum value.

Example 6.7 : A battery of emf 2.0 V and internal resistance 2.0 Ω is connected in the primary circuit of the potentiometer of wire of length 10 m and resistance 10 Ω. The emf of primary cell is balanced at 5.0 m length of potentiometer wire. When a current of 0.1 A is drawn from the cell then terminal voltage of the cell is balanced at a length of 4.0 m of potentiometer wire. Find the internal resistance of the cell.

Solution :

$$x = \left(\frac{\varepsilon_p}{R + r}\right)\times\frac{R}{L}$$

$R = 10\,\Omega$

$L = 10\,m$

$\varepsilon_p = 2V$

$r = 2\Omega$

$$x = \left(\frac{2}{10+2}\right) \times \frac{10}{10} = 0.17\,V/m$$

Internal resistance of cell,

$$r = \frac{\varepsilon - V}{I} = \frac{x\ell_1 - x\ell_2}{I}$$

or $$r = \frac{x(\ell_1 - \ell_2)}{I}$$

Here, $\ell_1 = 5.0\,m$, $\ell_2 = 4.0\,m$, $I = 0.1\,A$

After putting the values, we get,

$$r = \frac{0.17(5.0-4.0)}{0.1} = \frac{0.17 \times 1}{0.1} = 1.7\,\Omega$$

6.5.2 Comparison of Electro Motive Forces of Two Cells

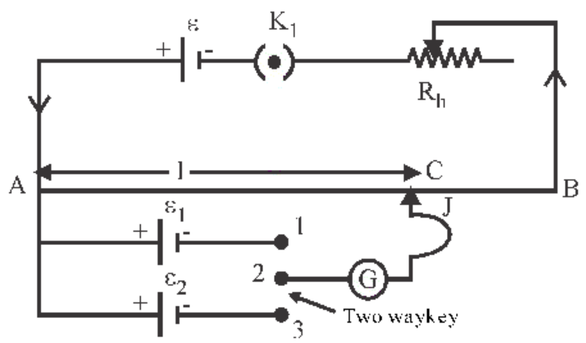


Fig. 6.12 Comparison of emf of two cells

Circuit Arrangements : Circuit is completed as given in the Figure 6.12. Primary circuit is made as explained in previous section. For preparing secondary circuit positive terminal of both cells whose emfs are to be compared are connected to the higher voltage end (i.e. A) of the potentiometer wire. If negative terminals of these cells are connected to terminal 1 and 3 of two-way key. Terminal 2 of the two-way key is connected to sliding jockey through galvanometer.

Working : First of all, switch on the primary circuit by inserting the plug of key K_1. Now insert the plug between terminals 1 and 2 of the two-way key and determine the balancing length ℓ_1 by sliding jockey over the potentiometer wire, for the cell of emf ε_1.

$$\varepsilon_1 = x\ell_1 \qquad \ldots (6.36)$$

Here x is the potential gradient.

Without disturbing the primary circuit remove the plug between 1 and 2 and insert the plug between terminals 2 and 3 of the two-way key and determine the balancing length ℓ_2 by sliding jockey over the potentiometer wire, for the other cell of emf ε_2

$$\varepsilon_2 = x\ell_2 \qquad \ldots (6.37)$$

On dividing eq. (6.36) by eq. (6.37), we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{x\ell_1}{x\ell_2}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2} \qquad \ldots (6.38)$$

Hence, ratio of emf of cells is equal to the ratio of the corresponding balancing lengths. By changing the value of resistance of rheostat in the primary circuit (i.e. changing the value of potential gradient x), various values of balancing lengths ℓ_1 and ℓ_2 are obtained. In this way various ratios $\varepsilon_1/\varepsilon_2$ are calculated and the mean value is reported.

In this experiment standardisation is not necessary. In laboratory, Leclanche cell is taken as first cell and the Denial cell is taken as the second cell.

If one of the cells is standard cell, then the emf of the second cell can be calculated by using the following formula,

$$\varepsilon_2 = \left(\frac{\ell_2}{\ell_1}\right)\varepsilon_1 \qquad \ldots (6.39)$$

6.5.3 Measurement of Small resistance :

The essential circuit for measurement of small resistance is as shown in the figure (6.13). The primary circuit of potentiometer is completed in accordance with the previous section as shown in the figure. An unknown low resistance r is connected in series with known high resistance R, Rheostat R_{h_2} the battery of emf ε' and key K_2 is connected in secondary circuit of the potentiometer. Higher potential point of potential difference across resistance R is connected to higher potential point A of potentiometer wire. The low potential ends of the resistances R and r are connected to the terminals 1 and 3 of the two-way key. The terminal 2 (middle terminal) of the two-way key is connected to jockey through a galvanometer.

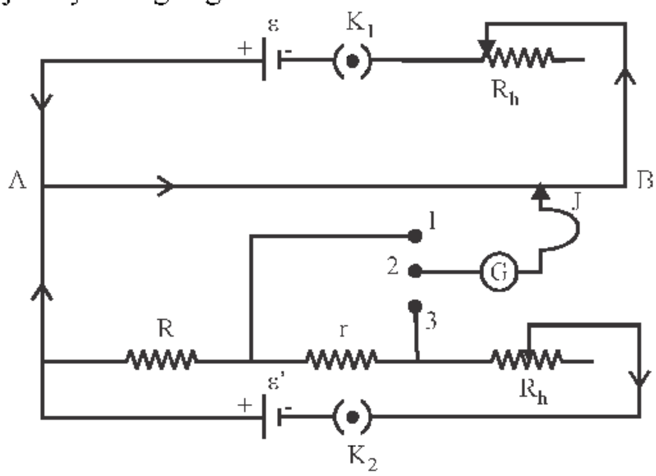


Fig. 6.13 Measurement of small resistance with the help of potentiometer

Working : First of all, we complete the primary circuit by inserting the plug in key K_1 and similarly in secondary circuit by inserting the plug in key K_2. In this condition potentiometer measures potential difference V across the known resistance R. If the current in secondary circuit is I and the balancing length for potential difference V is ℓ_1 according to the principle of potentiometer,

$$V = x\ell_1 \qquad \ldots (6.40)$$

as $\quad V = IR$ (Ohm's law)

so $\quad IR = x\ell_1 \qquad \ldots (6.41)$

Now disconnect the connections between the terminals 1 and 2and join terminals 2 and 3. In this case series combination of resistance R and r will be in the circuit of potentiometer.

Let the potential drop across (R + r) is balanced on the potentiometer wire at a length ℓ_2 and this potential difference is V_1, then

$$V_1 = x\ell_2$$

$$V_1 = I(R+r)$$

$$I(R+r) = x\ell_2 \qquad \ldots (6.42)$$

From equations (6,41) and (6.42)

$$\frac{I(R+r)}{IR} = \frac{x\ell_2}{x\ell_1}$$

$$1+\frac{r}{R} = \frac{\ell_2}{\ell_1}$$

$$\frac{r}{R} = \frac{\ell_2}{\ell_1} - 1$$

$$r = \left(\frac{\ell_2 - \ell_1}{\ell_1}\right)R \qquad \ldots (6.43)$$

By substituting the values of R, r and ℓ_1 and ℓ_2 in equation (6,43) we get the value of r.

Example 6.8 : For finding a low resistance r, it is connected with a high resistance R and constant current is allowed to pass through it. If the balancing lengths for potential drop across the high resistance R and across the two resistances combined in series are 3.2 m and 3.6 m respectively, then find the ratio of R and r.

Solution : Let the current passing through the resistances be I. Balancing length for the potential drop across the resistance R is ℓ_1, then,

$$IR = x\ell_1 \qquad \ldots (1)$$

Balancing length for the potential drop across the resistance (R+r) is ℓ_2, then,

$$I(R+r) = x\ell_2 \qquad \ldots (2)$$

$$\frac{R+r}{R} = \frac{\ell_2}{\ell_1}$$

$$\frac{r}{R} = \frac{\ell_2 - \ell_1}{\ell_1}$$

Here, $\ell_1 = 3.20\,m$ and $\ell_2 = 3.60\,m$

Thus, $\dfrac{r}{R} = \dfrac{3.60-3.20}{3.20} = \dfrac{0.40}{3.20} = \dfrac{1}{8}$

Thus, $R : r = 8 : 1$

6.5.4 Calibration of Voltmeter :

The voltmeter readings are not accurate due to certain reasons like mechanical faults, non-uniformities in the spacing of marking on the scale, in the spring constant etc. The potentiometer gives the correct value of potentioal difference. *A method to check the correctness of voltmeter reading with the help of potentiometer is called calibration of voltmeter.*

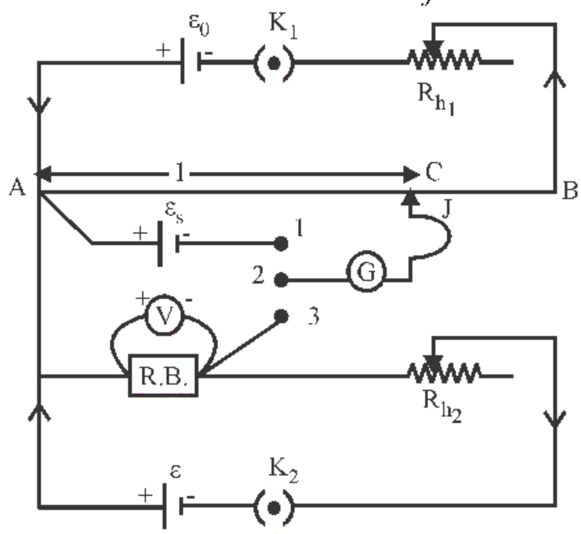


Fig: 6.14 Calibration of voltmeter with the help of potentiometer

The required circuit diagram for voltmeter calibration is shown in the fig 6.14. Primary circuit is completed by joining a battery of emf ε_p, a rheostat R_h and a plug key K_1 in series with the potentiometer wire. In secondary circuit positive terminal of standard cell of emf ε_s is connected to higher potential point (A) of the potentiometer wire AB.

Another cell of emf ε, a rheostat R_{h_2} and a plug key K_2 and resistance box (R.B.) are connected in series. Higher potential point of R.B. is connected to the higher potential point (A) of the potentiometer wire and the low potential point is connected to terminal 3 of the two-way key as shown in the figure. Voltmeter which is to be calibrated is connected in parallel to the resistance box. The middle point (2) of the two-way key is connected to sliding jockey through galvanometer.

Working : First of all, primary circuit is completed as explained in earlier experiments. By inserting plug-in between terminal 1 and 2 of the two-way key, balancing length ℓ_0 is obtained, for the emf of standard cell , then

$$\varepsilon_s = x\ell_0$$

$$x = \frac{\varepsilon_s}{\ell_0} \quad \ldots (6.44)$$

Here x is the potential gradient. This is known as standardisation of potentiometer. Now removing the plug from the gap between 1 and 2 and inserting it into gap between 2 and 3. Now, closing plug key K_2 we take out appropriate resistance from the resistance box. With the help of rheostat by passing current of desired value such that we obtain some deflection in voltmeter. This voltmeter reading is noted down. This reading is called incorrect reading. To obtain correct reading corresponding to voltmeter reading V, balancing length ℓ_2 is obtained on potentiometer. Then, according to the principle of potentiometer, correct reading will be,

$$V' = x\ell_2$$

$$= \varepsilon_s\left(\frac{\ell_2}{\ell_0}\right) \quad \text{use eq. (6.44)} \quad \ldots (6.45)$$

Hence, error in the voltmeter reading will be,

$$\Delta V = V - V'$$

With the help of resistance box and varying the value of rheostat R_{h_2}, and adjusting the reading of voltmeter, we can obtain the corresponding correct readings of potential difference. The difference between the voltmeter reading V and potentiometer reading V',

$$\Delta V = V - V'$$

is called error.

A graph is plotted between the error and the voltmeter reading. It is called calibration curve as shown in the figure (6.15). With the help of this graph we can have correct reading for potential difference as $V' = V - \Delta V$.

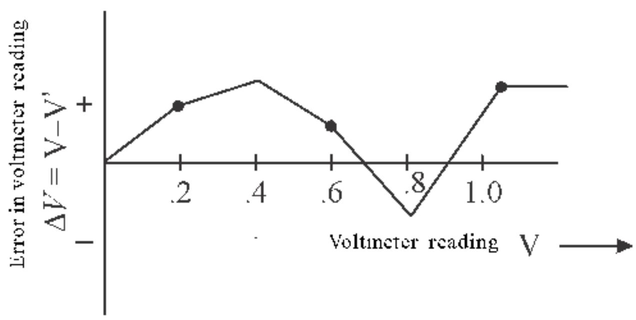


Fig 6.15 Calibration curve of voltmeter

6.5.5 Calibration of ammeter :

A method of checking the correctness of ammeter readings connected in electric circuit with the help of potentiometer is called calibration of ammeter.

The required circuit for calibration of ammeter is shown in the fig 6.16. This circuit is almost similar to the previous circuit used for calibration of voltmeter. Here, the resistance box is replaced by a 1Ω standard resistance coil and in place of voltmeter, an ammeter is connected in series with 1Ω coil. In secondary circuit.

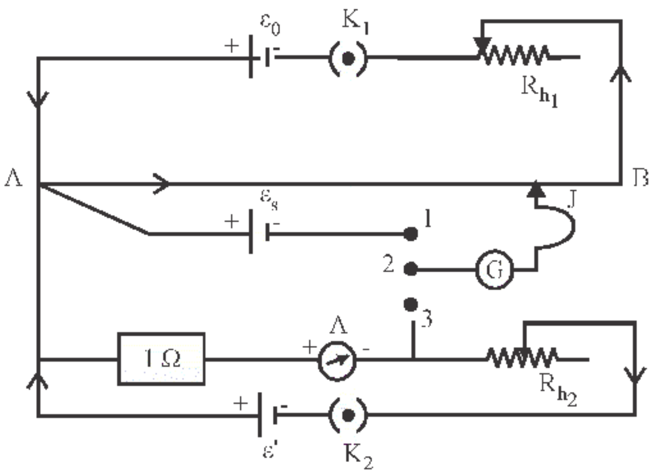


Fig 6.16 Calibration of ammeter

Working : By placing a plug-in key K_2 in the primary circuit and also inserting a plug between the terminals 1 and 2 of the 2-way key, emf ε_s of standard cell is balanced and balancing length ℓ_0 is measured then,

$$\varepsilon_s = x\ell_0$$

$$x = \frac{\varepsilon_s}{\ell_0} \qquad \ldots (6.46)$$

With the help of eq. (6.46), we can determine the value of x. This is called standardisation of potentiometer. Without making any change in the primary circuit (i.e. without disturbing the value of potential gradient, plug is removed from the gap between the terminals 1 and 2, of two-way key and plug is inserted between the terminals 2 and 3. By putting the plug in the plug key K_2, current is made to pass through the secondary circuit. With the help of rheostat R_{h_2}, a desired value of current I is obtained in 1Ω standard resistance coil. This is erroneous value measured by the Ammeter.

According to ohm's law, current flowing through 1 Ω standard resistance coil will be equal to potential difference across its ends. If the balancing length is ℓ_2 and potential difference is V', then,

$$V' = x\ell_2 \text{ but } V' = I'R \text{ or } V' = I' \quad (\because R = 1\Omega)$$

$$I' = x\ell_2$$

or $$I' = \varepsilon_s\left(\frac{\ell_2}{\ell_0}\right) \left(\because x = \frac{\varepsilon_s}{\ell_0}\right) \qquad \ldots (6.47)$$

Here I' is the correct value of current measured with the help of potentiometer. In this way error in the current measured by the ammeter $\Delta = I - I'$ is determined. Next we determined the correct value of ammeter reading with the help of potentiometer for different readings of ammeter and calculate the corresponding errors ($\Delta = I - I'$). A graph is plotted between the error and ammeter reading. It is called calibration curve of ammeter. It may be a Zig-Zag curve (or of any shape).

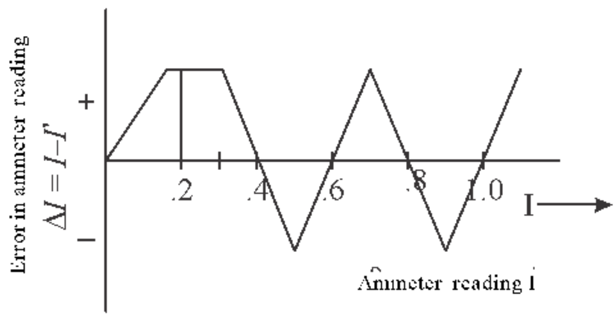


Fig 6.17 Calibration curve.

Now we can determine the correct value of the ammeter with the help of calibration curve as.

$$I' = I - \Delta I \qquad \ldots (6.48)$$

Important Points

1. Kirchhoff's first law is based on the conservation of charge and it is also called as junction rule. According to this law, at any junction algebric sum of the currents is zero. i.e. $\sum I = 0$.
2. Kirchhoff's second law is called voltage law or loop rule. It is based on the law of conservation of energy. According to this law, $\sum V = \sum \varepsilon \Rightarrow \sum IR = \sum \varepsilon$.
3. In balanced condition of Wheatstone bridge, the ratio of ratio arms is equal.
4. Meter bridge is based on Wheatstone bridge. Here of unknown resistance is given by

$$S = \left(\frac{100 - \ell}{\ell}\right)R$$

5. Potentiometer is an experimental device with the help of which we can measure the potential difference between any two points or emf of cell accurately.
6. Potentiometer is based on no deflection method. At no deflection it works as an ideal voltmeter of infinite resistance.
7. Fall of potential per unit length on potentiometer wire is called potential gradient. It's unit is V/m. Potential gradient is equal to $x - I(R_m)$, Here I = Current through the primary circuit and R_m = resistance per unit length of potentiometer wire.
8. To find the potential gradient with the help of a standard cell is called standardisation of potentiometer.
9. The sensitivity of potentiometer is inversely proportional to potential gradient. By increasing the length of potentiometer wire sensitivity can be increased.
10. Formula for measuring the internal resistance with help of potentiometer is,

$$r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R$$

Here, ℓ_1 and ℓ_2 are the balancing lengths in open and closed circuit and R is the resistance taken out from the resistance box.

11. If ε_1 and ε_2 are the emfs of two cells and ℓ_1 and ℓ_2 are corresponding balancing lengths on potentiometer wire, then

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$$

12. Formula for measuring the low resistance with the help of potentiometer,

$$r = \left(\frac{\ell_2 - \ell_1}{\ell_1}\right)R$$

Here, ℓ_2 = Balancing length for the potential difference across the series combination of $R + r$

ℓ_1 = Balancing length for the potential difference across resistance R .

13. Correctness of the measured value of ammeter and voltmeter is carried out by means of potentiometer and it is called calibration of ammeter and voltmeter.

Questions for Practice

Multiple Choice Type Questions

1. Kirchhoff's first law and second law are based on,

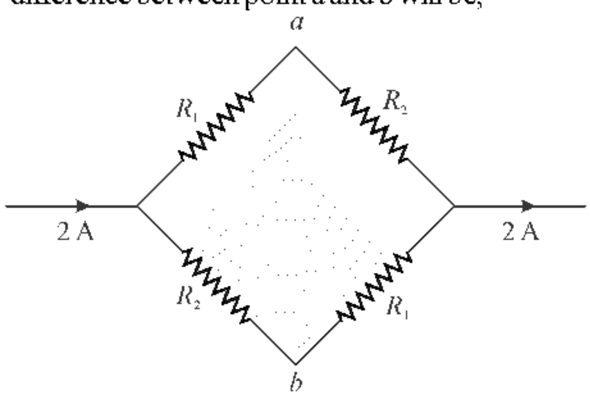
   (a) Law of conservation of charge and energy.

   (b) Law of conservation of current and energy.

   (c) Law of conservation of mass and charge.

   (d) None of the above.

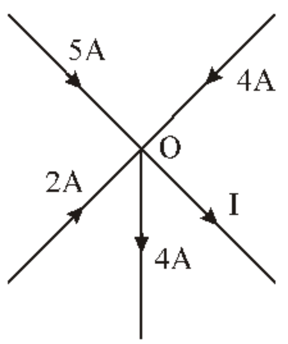
2. For the circuit shown in the figure, the potential difference between point a and b will be,



   (a) $R_1 - R_2$ (b) $R_2 - R_1$

   (c) $\frac{R_1 R_2}{R_1 + R_2}$ (d) Zero

3. In the given figure, the value of I will be,



   (a) 6A (b) 11A

   (c) 7A (d) 5A

4. On inter changing the position of battery and galvanometer in Wheatstone bridge respectively, the new balance point,

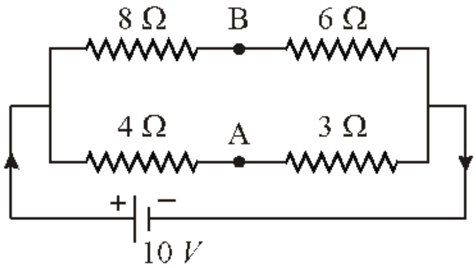
   (a) remains unchanged.

   (b) will change.

   (c) nothing can be said.

   (d) it may change or not will depend on the resistance of battery and galvanometer.

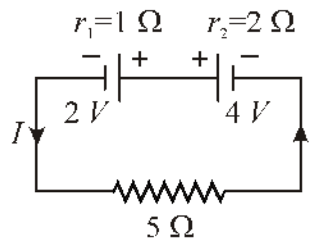
5. In the given figure, the potential difference between the terminals A and B will be,



   (a) $\frac{20}{7}V$ (b) $\frac{40}{7}V$

   (c) $\frac{10}{7}V$ (d) Zero

6. In the given figure, the value of I will be,



   (a) 2.5A (b) 0.75A

   (c) 0.5A (d) 0.25A

7. Potentiometer is such a measuring apparatus to measure the potential difference whose effective resistance is,

   (a) Zero (b) Infinite

   (c) uncertain

   (d) depends on external resistance.

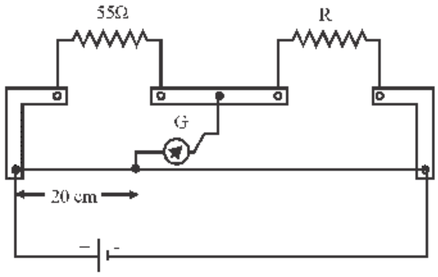
8. Which of the following quantities cannot be measured with the help of Potentiometer,

   (a) emf of a cell

   (b) capacitance and inductance

   (c) resistance (d) current

9. In the given figure, the galvanometer shows no deflection. What is the value of R?



(a) 220 Ω (b) 110 Ω

(c) 55 Ω (d) 13.75 Ω

10. The temperature coefficient of resistance of a potentiometer wire should be,

(a) high (b) low

(c) negligible (d) infinite

11. The formula for internal resistance of a cell will be, (Here ℓ_1 and ℓ_2 are the balancing lengths of cell in open and closed circuit respectively.)

(a) $r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R$ (b) $r = \left(\frac{\ell_2 - \ell_1}{\ell_2}\right)R$

(c) $r = \left(\frac{\ell_1 - \ell_2}{\ell_1}\right)R$ (d) $r = \left(\frac{\ell_2 - \ell_1}{\ell_1}\right)R$

12. In a potentiometer experiment, the emf ε of a cell is balanced at L length. Another cell of same emf ε is connected parallel to it, then, new balancing length will be,

(a) 2L (b) L

(c) L/2 (d) L/4

13. In a potentiometer experiment, a standard cell of emf 1.1 V is balanced at 2.20 m length. Potential difference across a resistance wire is balanced at 95 cm length and voltmeter read this potential difference as 0.5 V. Then, error in the voltmeter will be,

(a) +0.025 V (b) +0.525 V

(c) -0.025 V (d) -0.525 V

Very Short Answer Type Questions

1. Write down the mathematical expression of Kirchhoff's junction law?
2. On what conservation, law Kirchhoff's voltage rule is based?
3. Write down the condition of balanced Wheatstone bridge.
4. On what principle, meter bridge works.
5. Why potential gradient of potentiometer depends on the temperature of wire?
6. What happens if the emf of cell in the primary circuit is less than the emf of cell in the secondary circuit?
7. Write down the definition of potential gradient.
8. Why area of cross section of the potentiometer wire should be uniform?
9. For standardisation of potentiometer which cell is used other than the Danial cell?
10. How the sensitivity of potentiometer can be increased?
11. Length of a potentiometer wire is 10 m. A standard cell of emf 1.1 volt is balanced at length 8.8 m of potentiometer wire. How much potential difference can be measured from it?
12. Why copper wire is not used in potentiometer?
13. Potential gradient of potentiometer wire is 0.3 V/m. In an experiment for calibration of ammeter, potential difference across 1 Ω resistance is balanced across 1.5 meter length of potentiometer wire. If the reading of ammeter connected in circuit 0.28 A, Calculate the error in ammeter reading.

Short Answer Type Questions

1. State Kirchhoff's junction Law and loop law.
2. How is the resistance of a wire determined with the help of a meter bridge? Obtain required formula and draw circuit diagram.
3. What is Wheatstone bridge, derive its balanced condition for balance using Kirchhoff's law.
4. What is potential gradient? On what factors does it depend?
5. What do you mean by the standardisation of potentiometer? Explain it by drawing a circuit diagram.
6. What do you mean by the sensitivity of a potentiometer? How we can increase it?

7. How will you compare the emf's of two cells with the help of potentiometer? Explain with the help of proper circuit diagram and derive its formula.

8. A standard cell of emf 1.2 V is balanced on a 2.4 m length of potentiometer wire. Obtain the balancing length across a resistance 3.5 Ω, if a current of 0.2 A is flowing through it. Also calculate the potential gradient.

[Ans : $x = 0.5\,V/m$ $\ell = 1.40\,m$]

9. Why correct emf of a cell or potential difference cannot be measured with the help of a voltmeter? How it is possible to determine the correct value?

10. Why do we try to obtain the null point near the middle of meter bridge wire?

11. Why the current through the potentiometer wire should not be passed for a long time?

12. Why the current in the primary circuit of potentiometer is kept constant?

13. Write two precaution while using the potentiometer.

14. What do you mean by calibration of a voltmeter. Draw necessary circuit diagram.

15. Draw required circuit diagram for the measurement of low resistance with the help of potentiometer.

Essay Type Questions

1. State Kirchhoff's loop rule and junction rule. With the help of these rules deduce the condition of balanced Wheatstone bridge. Draw necessary diagram?

2. What is meter bridge? On what principal it is based. Explain the construction of meter bridge and derive an expression for unknown resistance of a wire. Draw essential diagram?

3. What do you mean by the internal resistance of a cell? Explain the method to determine the internal resistance of a cell with the help of potentiometer and obtain the required formula with the help of circuit diagram.

4. What do you mean by the calibration of ammeter or voltmeter? Explain the method of calibration of voltmeter with the help of potentiometer. Draw the necessary circuit diagram. Draw the calibration curve.

5. What is potentiometer? Explain its principle. With the help of potentiometer describe the method to determine the value of a low resistance and derive proper formula. Draw necessary circuit diagram.

Answer Key (Multiple Choice Questions)

1. (a) 2. (b) 3. (c) 4. (a) 5. (d)

6. (d) 7. (b) 8. (b) 9. (a) 10. (c)

11. (a) 12. (b) 13. (a)

Short answer Type Questions

1. $\Sigma I = 0$

2. Based on law of conservation of energy.

3. Balanced condition of Wheatstone bridge is that ratio $\frac{P}{Q} = \frac{R}{S}$ of arms will remains same.

4. Meter bridge is based on the principal of Wheatstone's bridge.

5. One increasing the temperature of potentiometer wire, the resistance of wire increases. Hence, potential gradient will be affected.

6. We will not obtain the condition of null point on potentiometer wire.

7. Fall of potential per unit length on the potentiometer wire is called potential gradient.

8. So, the potential gradient remains same at all the points of potentiometer wire.

9. Cadmium cell

10. By increasing the length of potentiometer wire.

11. Potential gradient (x)

$$x = \frac{\varepsilon_s}{\ell_0} = \frac{1.1V}{8.8} = 0.125\,V/m$$

Maximum potential-gradient that can be measured,

$$V_{AB} = xL = 0.125 \times 10 = 1.25 Volt$$

12. Temperature coefficient of resistance of copper wire is very small and specific resistance is very low.

13. 1 x Ammeter reading I =0.28 A

For actual value of current

I' = current in 1Ω resistance,

= Potential difference across 1Ω resistance

$= V = x\ell$

$I' = 0.3 \times 1.5 = 0.45$ A

Error in measurement of current = ΔI

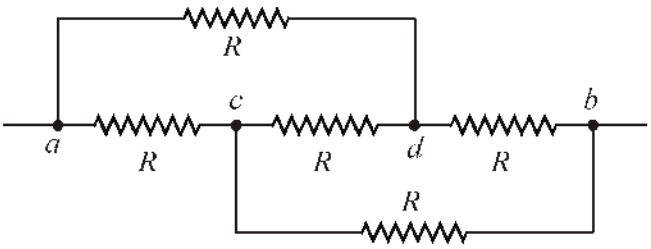
$= I - I'$

$= 0.28 - 0.45$

$= -0.17\,A$

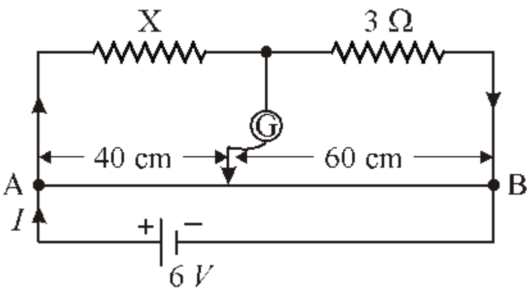
Numerical Questions

1. Find the equivalent resistance between terminal 'a' and 'b' of the network shown in the figure.



[Ans : R Ω]

2. In the following figure, a balanced meter bridge is shown. If the resistance of the wire of meter bridge is 1 Ω/m, then find the value of resistance X and the current passing through the resistance X.



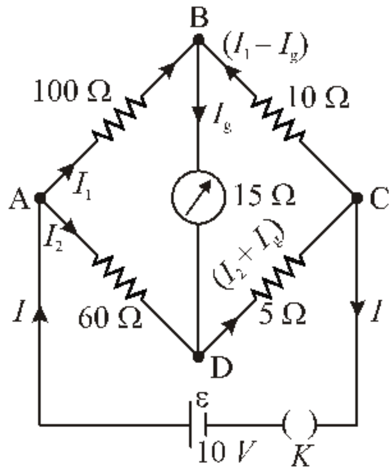
[Ans: X=2 Ω; I=1.26 A]

3. The resistances of four arms of the Wheatstone bridge are given in the circuit given below.

$R_{AB} = 100\,\Omega$, $R_{BC} = 10\,\Omega$,

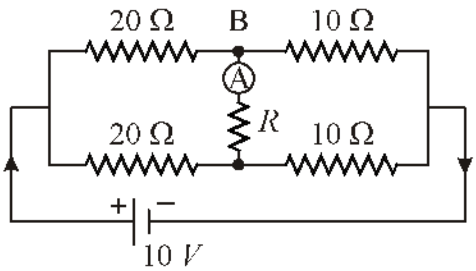
$R_{CD} = 5\,\Omega$ and $R_{DA} = 60\,\Omega$.

A galvanometer of 15Ω is connected between the terminals B and D. Calculate the current flowing through the galvanometer. The potential difference between terminals A and C is given as 10V.



[Ans: 4.87 mA]

4. What will be the value of resistance R for the network shown in the figure so that the current in ammeter may be zero.

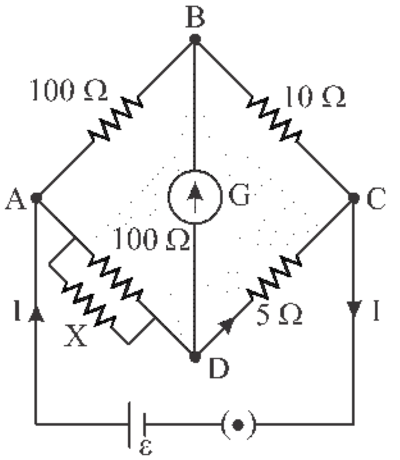


[Ans: Ammeter reading will be zero for all values of R]

5. The length of potentiometer wire is L. The primary circuit consists of a battery of emf 2.5 V and a resistance of 10Ω connected in series. In an experiment the balancing length for emf of 1 V is obtained at L/2. Find the new balancing length if the value of series resistance in the primary circuit is doubled.

[Ans: 0.6L]

6. In a Wheatstone bridge the branch resistances are as shown in the following circuit diagram. What will be the value of X in balancing condition of the Wheatstone bridge?



[Ans: X=100 Ω]

7. In a potentiometer experiment for the calibration of ammeter, the balanced length of a battery of emf 1.1 V is obtained at 0.88 m. The potential difference across one-ohm resistance is balanced at 0.20 m of potentiometer wire. If the reading of the ammeter connected in series is 0.20 A, calculate the error in ammeter.

[Ans: 0.05 A]

8. In a potentiometer experiment, the balancing length for cell of emf 1.25 V is 4.25 m. The balancing length with another cell is obtained at 6.80 m. Determine the emf of cell.

[Ans: 2.00 V]

9. The resistance of a 10 m long potentiometer wire is 1 Ω/m. An accumulator of 2.2 V emf, negligible internal resistance and a high resistance are connected in series. What is the value of high series resistance, if potential gradient on the potentiometer wire is 2.2 mV/m.

[Ans: 900 Ω]

10. In a potentiometer experiment, the balancing length for two cells of emf ε_1 and ε_2 $(\varepsilon_1 > \varepsilon_2)$ connected in series is observed as 60 cm. On reversing the position of terminals of one cell (of smaller emf) in the arrangement of experiment, the new balancing length is observed as 20 cm. Find the ratio of emfs ? ($\varepsilon_1 / \varepsilon_2$) of the cells.

[Ans : $\frac{\varepsilon_1}{\varepsilon_2} = \frac{2}{1}$]

Chapter - 7

Magnetic Effects of Electric Current

Even before 2000 years, people knew about electricity and magnetism, but as two separate subjects. In 1820 a Danish scientist Orested found a close relation between electricity and magnetism. Ampere and Farady found that a moving charge produced magnetic field and a moving magnet produced electric current. Later the Scotish physicist Maxwell and Lorentz from Holland, showed that both elecricity and magnetism depend on each other. From this, a new field of study as electromagnetism came into existance.

The modern technology is based on science of electricity and magnetism. The important devices for our common use, such as electric power, telecommunication, radio, television, mobile etc, are based on it.

In this chapter, we will study the magnetic field produced by current carrying conductor which is also called as magnetic effect of electric current. We will study, the force on a moving charge in a magnetic field, cyclotron and galvanometer etc.

7.1 Orested's Experiments

To study the magnetic field produced by a current carrying wire, Orested performed an experiment whose arrangement is shown in the diagram (7.1). In it a conducting wire AB is connected to a key and a battery with rehostate. A magnetic needle is placed under the wire and parallel to it, in north-south direction.

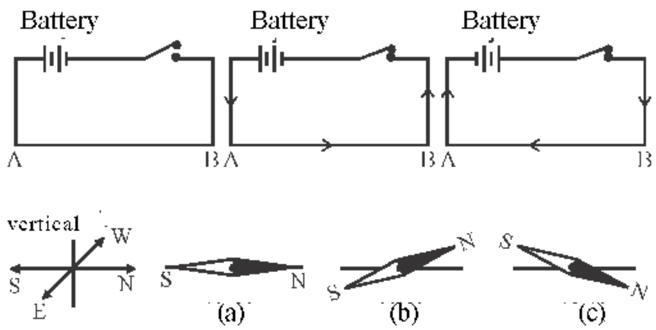


Fig 7.1 Orested's experiment

Orested found from his experiments that -

(i) When there is no current in wire, the magnetic compass needle remain parallel to wire, but as soon as the key is pressed to pass the current, the needle gets deflected.

(ii) The deflection is increased by increasing the current or by bringing the needle close to the wire.

(iii) If the current in the wire is reversed, the deflection is also reversed, to its previous direction.

Similarly, if the compass needle is kept above the wire and the experiment is repeated, the deflection will be opposite to the previous one. Since the magnetic needle is deflected only by external magnetic field, it is clear from Orested's experiment that -

Due to current in a conductor or moving charges, a magnetic field is developed around it, it is called magnetic effect of electric current.

7.1.1 Conculsion from Orested's Experiment

The following conculsions are drawn from Orested experiment -

(i) A magnetic field is developed across a conductor due to electric current in it.

(ii) The magnitude of magnetic field increases with increase in current.

(iii) Magnitude of magnetic field depends on the relative distance from the conductor, it decreases with increase in distance.

(iv) If the current is in S to N direction, the north pole of the magnetic needle placed under the wire deflects towards west directions.

(v) If the current in conductor is in N to S, the deflection will be towards east.

(vi) The direction of magnetic field, above and below the conductor are in opposite direction.

In next discussion we will define magnetic field. It may be a function of space and time.

7.2 Magnetic Field

In chapter 1, we have defined electric field, as the force on a unit positive test charge at that point, $E = F/q$. Had the magnetic mono pole existed, we could have defined magnetic field as simply as above. But since magnetic monopole does not exist, we use another method to define magnetic field. From experiments it is known that a charge at rest in magnetic field does not experience a force. Also if the test charge moves parallel or antiparallel to magnetic field, the force is zero. In the absence of electric field, (neglecting gravitational field), if a moving charge experiences a force in the direction perpendicular to velocity then there must exist a magnetic field B. It is a vector quantity.

The defenition of magnetic field or magnetic induction B, can be given by the force experienced by a moving charge. If a charge q is moving with a velocity $\vec{v}$, at an angle θ with $\vec{B}$, the force on the charge is given by -

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \ldots (7.1)$$

$$\vec{F} = q\, v\, B \sin\theta \hat{n}$$

here θ is the angle between $\vec{B}$ and $\vec{v}$ and $\hat{n}$ is a unit vector in the direction of force $\vec{F}$, which is perpendicular to both $\vec{B}$ and $\vec{v}$.

The magnitude of force is $|\vec{F}| = qvB\sin\theta$.

If $\theta = 90°$

$$F_{max} = q\, v\, B \text{ or } B = \frac{F_{max}}{qv} \quad \ldots (7.2)$$

in equation (7.2), if q = 1C and $v = 1$ m/s

then $B = F_{max}$.

Hence "the magnetic field at any point is equal to the max force experienced by a unit charge moving perpendicular to magnetic field with unit velocity".

Magnetic field is a vector quantity, its S.I. unit is $weber/m^2$ which is also called Tesla T.

$$1\,\text{Tesla} = \frac{1\,Weber}{m^2} = \frac{1N}{A \times m}$$

In CGS system the unit of B is Maxwell/cm^2 or Gauss. Relation between the two units is 1 T = 10^4 G. The dimensional formula for B is $= M^1L^0T^{-2}A^{-1}$.

The magnetic field B is also known as intensity of magnetic field, magnetic flux density and magnetic induction.

Stationary charge produce only electric field where as a moving charge also produce magnetic field along with electric field.

Just as electric field, the magnetic field also obey law of super position.

7.3 Biot-Savart's Law

The French physicsts Biot and Savart proposed a law about the magnetic field produced by current, on experimental basis, which is known by their name.

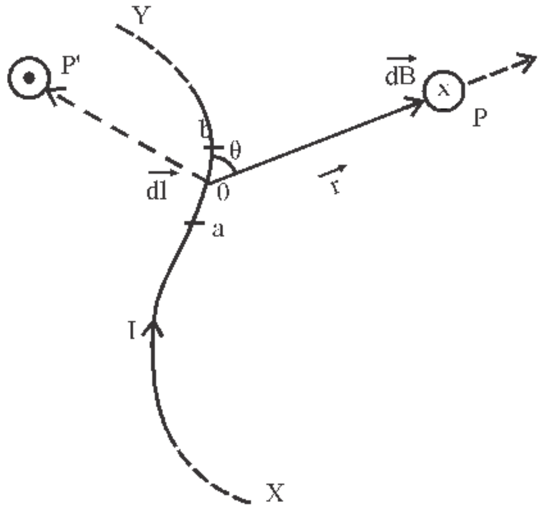


Fig 7.2 Biot-Savart law

The magnetic field $d\vec{B}$ due to a small length element $d\vec{\ell}$ of a conductor XY having a current I at a distance $\vec{r}$ from $d\vec{\ell}$, (shown in fig 7.2) in vacuum as -

(i) $d\vec{B}$ is directly proportional to I. $|d\vec{B}| \propto I$

(ii) Proportional to length elements $d\vec{\ell}$.

$$|d\vec{B}| \propto |d\vec{\ell}|$$

(iii) $d\vec{B}$ is proportional to sine of the angle between $d\vec{\ell}$ and $\vec{r}$.

$$|d\vec{B}| \propto \sin\theta$$

(iv) $d\vec{B}$ is inversely proportional to the square of the distance of the point P from $d\vec{\ell}$.

$$|d\vec{B}|\, \alpha \frac{1}{r^2}$$

So combining all above relations we get

$$|d\vec{B}|\, \alpha \frac{I\,|d\vec{l}\,|\sin\theta}{r^2} \quad \ldots (7.3)$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I\,|d\vec{l}\,|\sin\theta}{r^2} \quad \ldots (7.4)$$

Here $\frac{\mu_0}{4\pi}$ = is a proportionality constant. Its value for vacuum is 10^{-7} N/A^2 its unit is $\frac{N}{A^2}$ or

$\frac{Wb}{A \times m}$ or $\frac{T \times m}{A}$.

μ_0 is called magnetic permeability of free space (vacuum).

If the conductor is surrounded by another medium, then

$|d\vec{B}| = \frac{\mu}{4\pi} \frac{I\,|d\vec{l}|\,\sin\theta}{r^2}$ where $\mu = \mu_0 \mu_r$, is the magnetic permeability of that medium.

$\mu_r = \frac{\mu}{\mu_0}$ = relative permeability of that medium.

The Biot-Savart law in vector notation is

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2} \qquad \ldots (7.6)$$

or

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3} \qquad \ldots (7.7)$$

$$\left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$

From equation (7.6) it is clear that the direction of $d\vec{B}$ is always perpendicular to the plane of $d\vec{\ell}$ and $\vec{r}$ according to right hand screw rule. In the fig 7.2 the direction of $d\vec{B}$ at P, is perpendicular to the page and downwards shown by $\otimes$. At P' it is perpendicular to the page but upwards as shown by $\odot$.

Different Positions

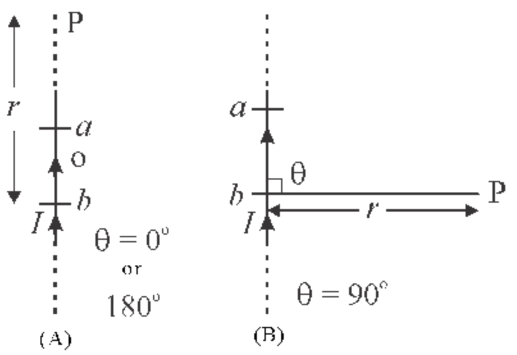


Fig 7.3 $d\vec{B}$ for (A) $\theta = 0°$, 180° (B) $\theta = 90°$

(i) If the P and P' are situated on the line of the current, then $\theta = 0°$ and 180° respectively.

$\sin\theta = \sin 0 = \sin 180° = 0$

hence $|d\vec{B}| = 0 \qquad \ldots (7.8)$

(ii) If the required point P is normal to $d\vec{\ell}$, as in [fig 7.3(B)], then $\theta = 90°$, and $\sin 90° = 1$.

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{|d\vec{\ell}|\sin 90°}{r^2}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{|d\vec{\ell}|}{r^2} \qquad \ldots (7.9)$$

This is the max. value.

(iii) The resultant magnetic field due to the whole of the conductor at a point P for is

$$\vec{B} = \frac{\mu_0}{4\pi} \sum \frac{I\, d\vec{l} \times \vec{r}}{r^3} \qquad \ldots (7.10)$$

Comparision of $d\vec{B}$ due to small current element $Id\vec{\ell}$, from Biot-Savart law equation (7.7), and the $d\vec{E}$ due to a small charge dq by coulomb's law $\left(d\vec{E} = \frac{kdq}{r^2}\hat{r}\right)$.

In both the cases there are two similarlities and two important differences. The current $Id\vec{\ell}$ produces magnetic field where as dq produces electric field. Both obey inverse square law. But there is difference in the direction of the field, due to dq the field $\vec{E}$ is radial, where as $d\vec{B}$ is normal to the plane of $\vec{r}$ and $d\vec{\ell}$. The second difference is that $d\vec{E}$ can be due to single charge or a charge distribution whereas the magnetic field is due to only current.

7.3.1 Direction of Magnetic Field

The direction of $\vec{B}$ can be given by following rules -

(i) Snow Rule - The direction of $d\vec{B}$ near a conductor can be given by the deflection of north pole of a magnetic needle placed near it. According to this law- "If the current in a conductor is form south to north and wire is situated over the compass needle, then the deflection of its north pole is towards west" (fig 7.1 B).

(ii) Right Hand Rule - According to this rule, if we hold a current carrying conductor by our right hand as shown in fig 7.4 and the direction of thumb indicates the direction of I, then the currled fingers will give the direction of magnetic field arround the conductor.

Fig 7.4 : Right hand thumb rule

(iii) Right hand palm rule for circular current- According to this rule, if the direction of currled fingers of right hand gives the direction of current, then the direction of the thumb gives the direction of magnettic field (fig 7.5).

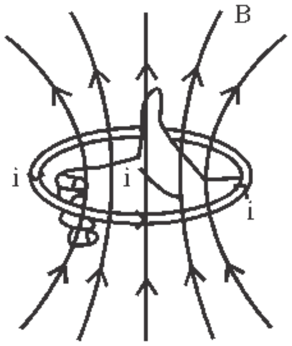


Fig 7.5 Right hand palm rule

(iv) Maxwell's Cork Screw Rule-Right handed screw rule- If the direction of linear motion of a right handed screw gives the direction of current in a conductor, then the direction of rotation of screw, gives the magnetic field produced by that current.

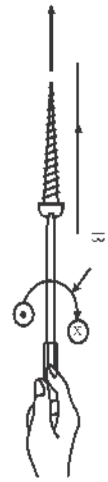


Fig 7.6 Maxwell's screw rule

7.4 Magnetic Field Due to a Long and Straight Current Carrying Conductor

7.4.1 Magnetic Field of a Straight Current Carrying Wire of Finite Length

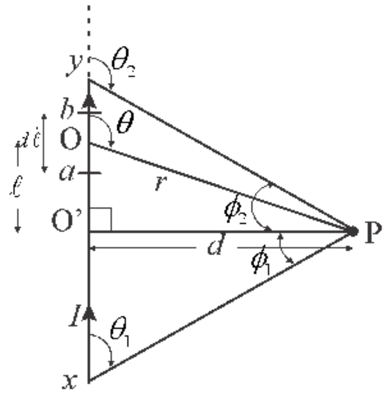


Fig. 7.7 Magnetic field due to long conductor

As per the figure 7.7, a straight, wire X Y lies in the plane of the paper. It carries current from x to y end, consider a point P, at a perpendicular distance d from it. The magnetic field due to an arbitrary length element $d\vec{\ell}$ (ab) whose mid point is O at a. Distance OP = r. From Biot-Savart's law

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2} \qquad \ldots (7.11)$$

here there are three variables, l, r and θ. We can change l and r in terms of θ, by the geometry in the figure.

From $\Delta OO'P$ &

$$\frac{OO'}{O'P} = \frac{\ell}{d} = \cot\angle(POO') = \cot(180-\theta) = -\cot\theta$$

$$\ell = -d\cot\theta \qquad \ldots (7.12)$$

$$\frac{d\ell}{d\theta} = -d(-\text{cosec}^2\theta)$$

$$d\ell = d\,\text{cosec}^2\theta\, d\theta \qquad \ldots (7.13)$$

Again from $\Delta OO'P$

$$\text{cosec}(180-\theta) = \frac{OP}{OO'} = \frac{r}{d}$$

$$\text{cosec}\,\theta = \frac{r}{d}$$

$$r = d\,\text{cosec}\,\theta \qquad \ldots (7.14)$$

Substituting in equation 7.13

$$dB = \frac{\mu_0}{4\pi}\frac{I(d\,\text{cosec}^2\theta\, d\theta)\sin\theta}{(d\,\text{cosec}\,\theta)^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I(d\,\text{cosec}^2\theta)\sin\theta\, d\theta}{d^2\,\text{cosec}^2\theta}$$

$$dB = \frac{\mu_0 I}{4\pi d}\sin\theta\, d\theta \qquad \ldots (7.15)$$

since the angle θ, changes from θ_1 to θ_2 for conductor XY- so to obtain the magnetic field due to wire at P_1 on integrating dB between limits θ_1 to θ_2

$$B = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi d}[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 I}{4\pi d}[\cos\theta_1 - \cos\theta_2] \qquad \ldots (7.16)$$

Again from geometry of the fig 7.7; $\theta_1 = 90° - \phi_1$

$(\because \theta_1 + \phi_1 = 90°)$

$\theta_2 = \phi_2 + 90°$

Substituting θ_1 and θ_2 in equation 7.16.

$$B = \frac{\mu_0 I}{4\pi d}[\cos(90° - \phi_1) - \cos(90° + \phi_2)]$$

$$B = \frac{\mu_0 I}{4\pi d}[\sin\phi_1 + \sin\phi_2] \qquad \ldots (7.17)$$

Here ϕ_1 and ϕ_2 angles subtended by ends *x* and *y* at P with O^1P.

7.4.2 Magnetic Field Due to Straight Current Carrying Conductor of Infinite Length

Since the length of the conductor is infinite, so the angles $\phi_1 = \phi_2 = \pi/2$. Using equation (7.17)

We get

$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$$

$$= \frac{\mu_0 I}{4\pi d}(1+1) \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

or $$B = \frac{\mu_0 I}{2\pi d} \qquad \ldots (7.18)$$

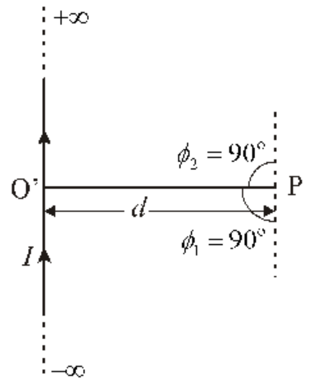


Fig 7.8 Magnetic field due to infinitely long conductor

Special Condition

(i)Magnetic field at a distance d from one end of the finite conductor.

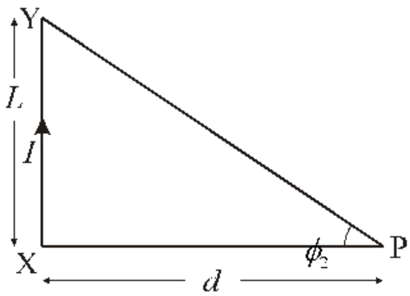


Fig 7.9 Half infinite wire

$$B = \frac{\mu_0 I}{4\pi d}(\sin O + \sin\phi_2)$$

$$B = \frac{\mu_0 I}{4\pi d}\sin\phi_2 \qquad \ldots (7.19)$$

From fig 7.9 $\sin\phi_2 = \frac{L}{YP} = \frac{L}{\sqrt{L^2 + d^2}}$

From fig 7.9 $B = \frac{\mu_0 I}{4\pi d} \frac{L}{\sqrt{L^2 + d^2}}$ . . . (7.20)

(ii) Magnetic field at a perpendicular distance d from one end of the infinite wire.

For this condition $\phi_2 = \frac{\pi}{2}$ and $\phi_1 = 0°$; hence from eqn. (7.17)

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin 0 + \sin\frac{\pi}{2}\right]$$

$$B = \frac{\mu_0 I}{4\pi d} \quad \text{. . . (7.21)}$$

(iii) Magnetic field due to a conductor of finite length when the point P is situated at a perpnedicular distance d from its mid point - here $\phi_1 = \phi_2 = \phi$

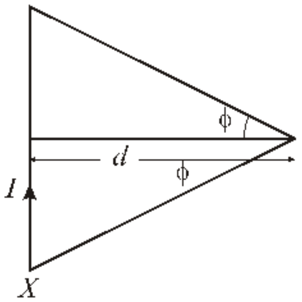


Fig 7.10 Wire of finite length

From eqn. (7.17)

$$B = \frac{\mu_0 I}{4\pi d}(\sin\phi + \sin\phi)$$

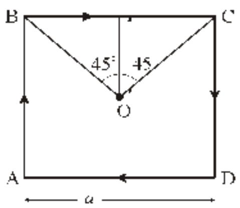
$$B = \frac{\mu_0 I}{4\pi d}(2\sin\phi)$$

$$B = \frac{\mu_0 I}{2\pi d}\sin\phi \quad \text{. . . (7.22)}$$

(iv) At the axial position of point P; $\phi = 0$ hence $B = 0$.

Example 7.1 : Find the magnetic field at the center O of square ABCD of a side a, which carries a current I A.

Solution :



The ends of each side makes an angle 45° at the center. Hence magnitude of magnetic field is same for all sides, from right hand palm rule the direction is also same, and downwards, hence $B_1 = B_2 = B_3 = B_4$ and. Total magnetic field at the center will be 4 times B_1

$$B = \frac{\mu_0 I}{4\pi d}(\sin\phi_1, + \sin\phi_2)$$

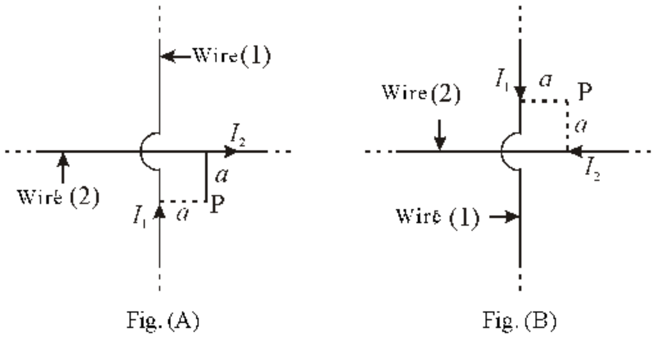
here, $B_1 = B_2 = B_3 = B_4 = \frac{\mu_0 I}{4\pi(a/2)}(\sin 45° + \sin 45°)$

$$= \frac{\mu_0 I}{2\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2\pi a}\left(\frac{2}{\sqrt{2}}\right)$$

$$|\vec{B}| = \frac{8\sqrt{2}\times 10^{-7} I}{a} Tesla$$

Example 7.2 : Find the net magnetic field at point P due to two perpendicular current carriers, in two situations given in fig (A) and fig (B).

Solution :



In fig (A), the magnetic field due to I_1 is $B_1 = \frac{\mu_0 I_1}{2\pi a}$

The direction of B_1 is perpendicular to page and downwards. The magnetic field due to I_2 at P is $B_2 = \frac{\mu_0 I_2}{2\pi a}$; again the direction of B_2 is same as that of B_1 according to right hand rule. Hence the net field at P is

$|\vec{B}| = B_1 + B_2 = \frac{\mu_0}{2\pi a}(I_1 + I_2)$ the direction is to page downwards.

Again for fig (B) the magnetic field at P, due to I_1 is

$B_1 = \frac{\mu_0 I_1}{2\pi a}$ the direction of B_1 is perpendicular to page upwards.

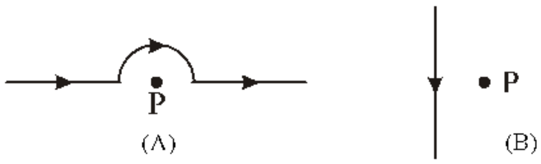
The magnetic field at P, due to I_2 is $B_2 = \frac{\mu_0 I_2}{2\pi a}$ the direction of B_2 is perpendicular to the page downwards.

hence the net magnetic field at P is

$$\vec{B} = \vec{B}_1 - \vec{B}_2$$

$$|\vec{B}| = B_1 - B_2 = \frac{\mu_0}{2\pi a}(I_1 - I_2)$$

Example 7.3 : Show the direction of magnetic fields at point P, as $\otimes$ and $\odot$.



Solution : In fig. (A) the direction of $\vec{B}$ will be downwards and given as $\otimes$. In fig (B) the direction of $\vec{B}$ at P is upwards, and given as $\odot$.

7.5 Magnetic Field Due to a Current Carrying Circular Coil

7.5.1 Magnetic Field at the Centre of Coil

To find the magnetic field at the center of a coil of radius R, having a current I, we consider the contribution of small lenght element $\delta\vec{\ell}$ at center O.

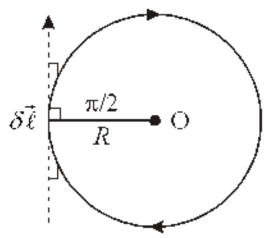


Fig 7.11 Magnetic field at center of a coil

(i) $\delta\vec{\ell}$ is at a distance R from the center.

(ii) $\delta\vec{\ell}$ is perpnedicular to R; i.e $\theta = \pi/2$.

hence from Biot and Savart's law

$$\delta B_{Centre} = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\delta\ell}{R^2}\sin\left(\frac{\pi}{2}\right) \qquad \because r = R \text{ and } \theta = \frac{\pi}{2}$$

$$\delta B_{Centre} = \frac{\mu_0}{4\pi}\frac{I\delta\ell}{R^2} \qquad \ldots (7.23)$$

Since all the length elements contribute in the same direction. The net magnetic field is sum of all contributions

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}\Sigma\delta\ell \qquad \ldots (7.24)$$

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}(2\pi R)$$

$$B_{Centre} = \frac{\mu_0 I}{2R} = \frac{\mu_0}{4}\frac{(2I)}{R} \qquad \ldots (7.25)$$

If the coils has N turns, the magnetic field at center is

$$B_{Centre} = \frac{\mu_0 NI}{2R} \qquad \ldots (7.26)$$

Dependance of B on radius of the coil.

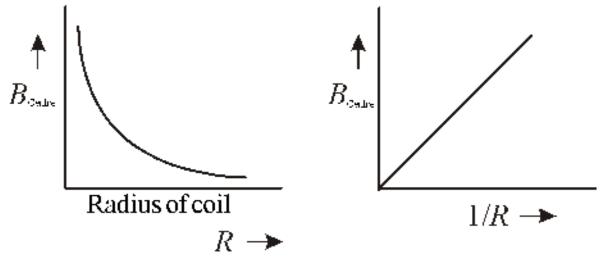


Fig 7.12 Dependance of magnetic field on radius of coil

As it is evident from equation (7.26), hence the graph between B and R is hyperbolic, and between B and 1/R it is straight line, as shown in fig 7.12.

Special : We can find B at the center of coil by another method. Let the angle subtended by $\delta\vec{\ell}$ at center be δα, then

$$\delta\alpha = \frac{arc\,\delta\ell}{radius\,R} = \frac{\delta\ell}{R} \quad \text{or} \quad \delta\ell = R\delta\alpha$$

using this relation in equation (7.24) we get

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}\times 2\pi R = \frac{\mu_0 I}{2R}$$

since $\sum\delta\ell = R\Sigma\delta\alpha = R(2\pi)$ for whole loop.

Magnetic field at center due to one fourth of coil will be and

$$\sum\delta\ell = \Sigma R\delta\alpha = R\times\frac{1}{4}(2\pi) = R\times\frac{\pi}{2}$$

$$B = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right) \qquad \ldots (7.27)$$

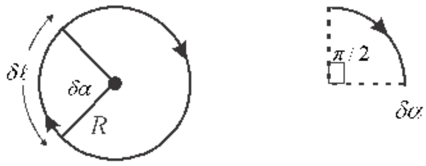


Fig 7.13 : Magnetic field due to a segment at centre

7.5.2 Magnetic Field Due to a Circular Current Carrying Coil at an Axial Point

A coil of radius R with a current I is considered in Y - Z plane with center at origin O. Consider a point P on the axis of the coil (as X-axis). Consider a small length element $LM - \delta\vec{\ell}$.

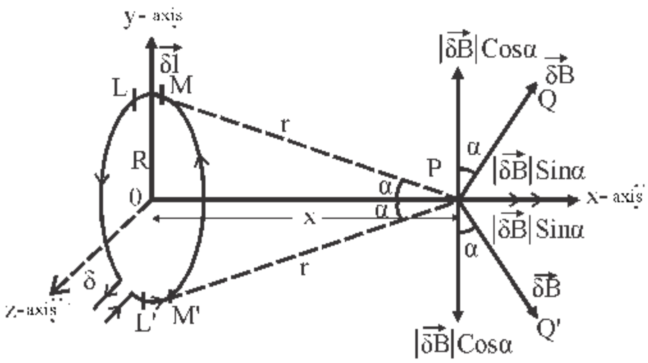


Fig 7.14 Magnetic field due to circular current at axial point

The distance of $\delta\vec{\ell}$ from P is r and angle between $\delta\vec{\ell}$ and $\vec{r}$ is 90^0 (i.e. 90^0) . From Biot and Savart's law the magnetic field at point P due to the above length element $\delta\vec{\ell}$ is

$$\delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\delta\vec{\ell}\times\hat{r}}{r^2}$$

$$|\,\delta\vec{B}\,| = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin 90^\circ}{r^2}$$

$$(\because |\,\delta\vec{\ell}\times\hat{r}\,| = \delta\ell\sin\theta \text{ and } \theta = 90^\circ)$$

$$|\,\delta\vec{B}\,| = \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2} \qquad \ldots (7.28)$$

The direction of $\delta\vec{B}$ will be always perpendicular to the plane of $\vec{r}$ and $\delta\vec{\ell}$, according to right hand rule, as shown by $\vec{PQ}$. Similarly the contribution due to diametrically opposite length element L'M' = $\delta\vec{\ell}$ will be -

$$|\,\delta\vec{B}\,'| = \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2}$$ and the direction will be the direction of $\vec{PQ}\,'$. As shown in the diagram. Now resolving, both $\delta\vec{B}\vec{PQ}$ into components, as parallel and perpendicular components, $\delta\vec{B}$ and $\delta\vec{B}\,'$ we see that cosine components being equal and opposite cancels each other, and the sine component, due to whole of coil contribute to the magnetic field at axial point P. Hence

$$B = \sum |\,\delta\vec{B}\,| \sin\alpha = \sum \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2}\sin\alpha$$

$$= \frac{\mu_0}{4\pi} I \sum \frac{\delta\ell}{r^2}\left(\frac{R}{r}\right)$$ (From the diagram for whole coil $\sin\alpha = \frac{R}{r}$)

$$= \frac{\mu_0}{4\pi}\frac{I\,R}{r^3}\sum \delta\ell = \frac{\mu_0}{4\pi}\frac{I\,R}{r^3}(2\pi R)$$

$$(\because \sum \delta\ell = \text{ circumfrence of the coil } = 2\pi R$$

$$= \frac{\mu_0}{4\pi}\frac{I(2\pi R^2)}{r^3}$$

$$B = \frac{\mu_0}{4\pi}\frac{I(2\pi R^2)}{r^3} \qquad \ldots (7.31)$$

from pythogorous theorem

$$r^2 = R^2 + x^2$$

$$r = (R^2 + x^2)^{\frac{1}{2}}$$

$$r^3 = (R^2 + x^2)^{3/2} \qquad \ldots (7.32)$$

By putting value or r in eqn. (7.31) and eqn. (7.32)

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(\pi R^2)}{(R^2 + x^2)^{3/2}} \qquad \ldots (7.33)$$

If the coil has N number of turns

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(N\pi R^2)}{(R^2 + x^2)^{3/2}} \qquad \ldots (7.34)$$

or $$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \qquad \ldots (7.35)$$

In vector notation

$$\vec{B}_{\text{Axis}} = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}\hat{x} \quad \ldots (7.36)$$

Due to the direction of current shown in the figure, the direction of $\vec{B}$ will be in positive x direction.

Special Conditions -

(i) Magnetic field at the center of the coil

In this case $x = 0$; hence $\vec{B}$ will be maximum

$$B_{\text{Centre}} = \frac{\mu_0 N I R^2}{2(R^2+0)^{3/2}}$$

$$B_{\text{Centre}} = \frac{\mu_0 N I}{2R} = B_{\text{maximum}} \quad \ldots (7.37)$$

(It is same as obtained earlier)

(ii) If the point P is at a large distance compared to R, i.e. $x >> R$,

hence $\frac{R^2}{x^2}$ is negligible

$$B = \frac{\mu_0 N I R^2}{2(o+x^2)^{3/2}} = \frac{\mu_0 N I R^2}{2x^3} \quad \ldots (7.38)$$

(iii) If the point P is at $x = R/2$

$$B_{x=R/2} = \frac{\mu_0 N I R^2}{2\left[R^2+\left(\frac{R}{2}\right)^2\right]^{3/2}}$$

$$= \frac{\mu_0 N I R^2}{2\left(R^2+\frac{R^2}{4}\right)^{3/2}} = \frac{\mu_0 N I R^2}{2\left(\frac{5R^2}{4}\right)^{3/2}}$$

$$B_{x=R/2} = \frac{4}{5\sqrt{5}}\frac{\mu_0 N I}{R} \quad \ldots (7.39)$$

Comparing with magnetic field at center we get

$$B_{x=R/2} = \frac{8}{5\sqrt{5}}B_{\text{Centre}} = 0.72 B_{\text{Centre}} \quad \ldots (7.40)$$

The Variation of B with Distance on Axis

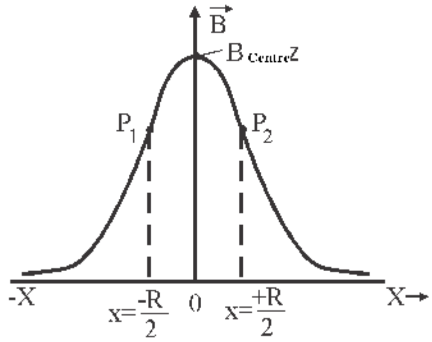


Fig 7.15 Variation B due to axial distance

The variation of B with distance is given by equation (7.36) and x is shown by figure (7.15). It is evident from the figure that B is maximum at center. B varies with distance on both sides, non-linearly and becomes zero at $x = \infty$. At a particular distance $x = \pm R/2$, we get two points on the curve P_1 and P_2, at these points $B \,\alpha\, 1/R$ and linear. At these two points, the sign of the slope of the curve changes from positive to negative. Hence the points are called the points of inflection.

At these points $\frac{dB}{dx}$ = constant; and $\frac{d^2B}{dx^2} = 0$;

The distance between these points is equal to R.

7.5.3 Comparision of Small Current Loop with a Magnetic Dipole

The magnetic field due to a circular coil at an axial point is given by

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(N\pi R^2)}{(R^2+x^2)^{3/2}}$$

If the loop is small $R^2 << x^2$; $\frac{R^2}{x^2}$ is negligible,

also $A = \pi R^2$ = is area of the current loop, we get

$$B = \frac{\mu_0}{4\pi}\frac{2NIA}{x^3}$$

or $$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3} \quad \ldots\ldots (7.41)$$

where M = NIA is the magnetic moment of current loop. This expression is exactly similar to the magnetic field produced by a small bar magnet at the distance on its axis from the center of the magnet.

Hence a small current loop is equivalent to a bar magnet (magnetic dipole).

7.5.4 Helmholtz Coils

Two identical coaxial coils held in vertical plane, such that their center are at a distance equal to the radius of the coil. These coils are called Helmholtz coils.

The plane of the coils is parallel to each other. The coils are connected in series, they produce exactly same magnetic field. Fig 7.16. The coils are used to produce uniform magnetic field, in the area between the coils.

Fig 7.16 Helmholtz coils

The Magnetic Field Between the Space of Coils

The magnetic field between the space of the coils is the vector sum of the fields produced by the two coils. The center of space is the area about the points of inflection.

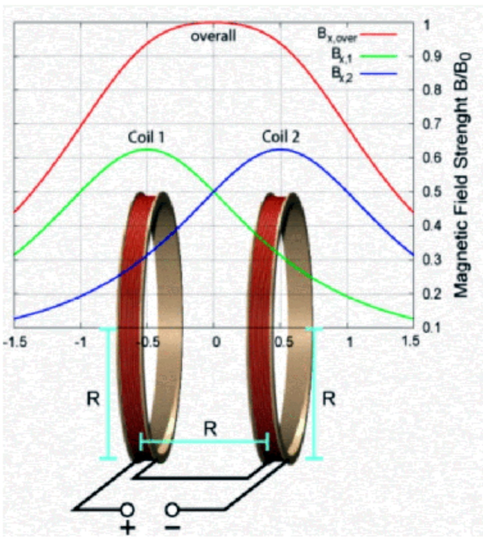


Fig 7.17 Uniform magnetic field between the coils

The magnetic field at the center of space i.e. point of inflection of both the coils, hence the magnetic field will be

$$B_1 = \frac{4\mu_0 N I}{5\sqrt{5} R} \qquad (B_x = R/2)$$

$$B_1 = B_2 = \frac{4\mu_0 N I}{5\sqrt{5} R}$$

$$B = B_1 + B_2 \ [\because \ \vec{B}_1 \text{ and } \vec{B}_2 \ ]$$

$$= 2B_1$$

$$= 2 \times \frac{4\mu_0 N I}{5\sqrt{5} R} = 0.716 \frac{\mu_0 NI}{R}$$

$$B = 1.432 \frac{\mu_0 N I}{2R} \qquad \text{. . . (7.42)}$$

$$B = 1.432 B_{Centre} \qquad \text{. . .(7.43)}$$

Which means the uniform magnetic field in the space between the coils is 1.432 times the maximum magnetic field produced at the center of each coil.

7.5.5 The Direction of Magnetic Field Due to Straight Current

(1) The form of magnetic field due to stright current can be understood by the following experiment -

Consider a straight current carryiing wire PQ passing through a cardboard ABCD, whose plane is perpendicular to the wire. Put some iron fillings on the cardborad, and establish a current in the wire. By taping the card board you will notice the iron filling to align like circular rings around the wire, as shown in the figure.

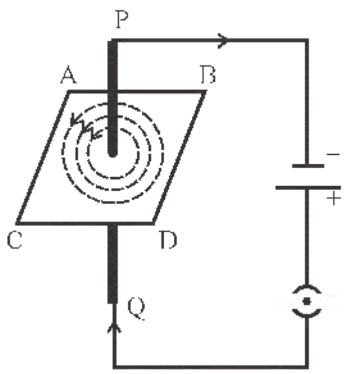


Fig 7.18 Magnetic field due to straight current

(2) Magnetic field in a current carrying coil. We look at the face of the coil. If the direction of current is N - wise (anti clock wise) then the face will behave like north pole of the magnet. And if the current in the face is clock wise i.e S-wise, the face (end) will behave like south pole.

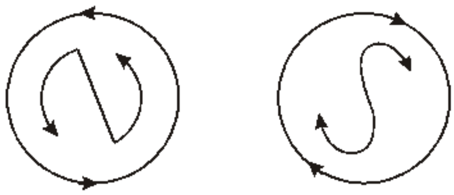


Fig 7.19 Deciding the pole of current coil

Example 7.3 : A circular coil of radius 10 cm, has 100 tightly wound turns. Find the magnetic field at the center of loop if the current in the coil is 1A.

Solution : given R = 10 cm = 0.1 m

N = 100, I = 1A,

$$B = \frac{\mu_0 N I}{2R} = \frac{4\pi\times10^{-7}\times100\times1}{2\times0.1}$$

$$= 2\pi\times10^{-4}\ \text{T}$$

$$= 6.28 \times 10^{-4}\ \text{T}$$

Example 7.4 : A helium nucleus revolves in a circular path of radius 0.8 m, in 2 s. Find the magnetic field at the center of the circle.

Solution : The charge on *He* nucleus is q = + 2e. The magnetic field at the center of circle r is

$$B = \frac{\mu_0 I}{2r}$$

$$I = \frac{2e}{t} \qquad \left|\because I = \frac{q}{t}\right|$$

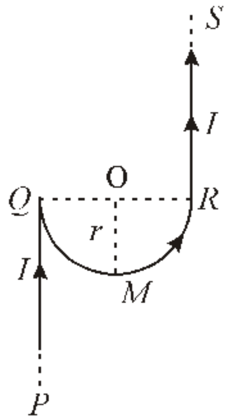
t = time taken in one revolution

$$B = \frac{\mu_0 (2e)}{2rt} = \frac{\mu_0 e}{rt}$$

$$= \frac{4\pi\times10^{7}\times1.6\times10^{19}}{0.8\times2}$$

$$= 12.56 \times 10^{26}\ \text{T}$$

Example 7.5 : Find the magnetic field at point O in the given figure.



$$\vec{B} = \vec{B}_{PQ} + \vec{B}_{QMR} + \vec{B}_{RS}$$

Solution : Magnetic field at O is due to the current in PQ, QMR and RS. The contribution of PQ is

$$|\vec{B}_{PQ}| = |\vec{B}_{RS}| = \frac{\mu_0 I}{4\pi R} \quad \text{downwards}$$

Contribution of RS is up-wards both cancel each other, being of same magnitude.

The only contribution is due to semi circle QMR, which is

$$|\vec{B}| = |\vec{B}_{QMR}| = \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) \text{which is upwards.}$$

Example 7.6 : At what distance from the center of the coil of radius R the magnetic field will be 1/27 of the field at center.

Solution : Magnetic field at the axis of a circular current carrying coil is :

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

Magnetic field at center

$$B_{center} = \frac{\mu_0 N I}{2R}$$

given-

$$B = \frac{1}{27} B_{center}$$

or $$\frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}} = \frac{1}{27}\frac{\mu_0 N I}{2R}$$

or $$\frac{R^2}{(R^2+x^2)^{3/2}} = \frac{1}{3^3}\frac{1}{R}$$

or $$(R^2+x^2)^{3/2} = 3^3 R^3 = (3R)^3$$

or $$(R^2+x^2)^{\frac{1}{2}} = 3R$$

or $$R^2 + x^2 = 9R^2$$

or $x^2 = 8\ R^2$

or $$x = 2\sqrt{2}R$$

Example 7.7 : In Helm holtz coils, each coil has 20 turns and radius 10 cm. If the current in the coil is 0.1 A, find the magnetic field in area between the coils.

Solution : Magnetic field in the required area is

$$B = \frac{8}{5\sqrt{5}}\frac{\mu_0 N I}{R}$$

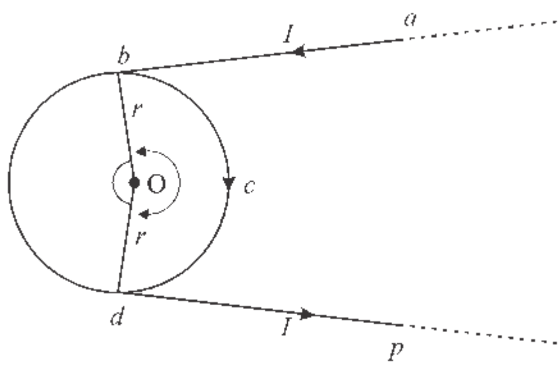
given - $N = 25, R = 10\ \text{cm} = 0.1\ \text{m}, I = 0.1\ \text{A}$

$$B = \frac{8}{5\sqrt{5}} \times \frac{4\pi \times 10^{-7} \times 25 \times 0.1}{0.1} T$$

$= 2.25 \times 10^{-5}$ T

Example 7.8 : A wire of infinite length is curved as shown in the figure. If the current is I, then find the angle, for which the magnetic field at center O is zero.

Solution :



The total magnetic field at center is

$$\vec{B}_0 = \overrightarrow{B_{ab}} + \overrightarrow{B_{bcd}} + \overrightarrow{B_{dp}}$$

$$B_{ab} = \frac{\mu_0 I}{4\pi r} \quad \text{(upwards)}$$

$$B_{dp} = \frac{\mu_0 I}{4\pi r} \quad \text{(upwards)}$$

If we take + *ve* sign for an up wards magnetic field and - *ve* sign for downwards magnetic field then -

$$B_{bcd} = \frac{\mu_0 I}{4\pi r^2} \quad \text{x (length of arc bcd)}$$

$$B_{bcd} = \frac{\mu_0 I}{4\pi r^2}(2\pi - \theta)r \quad \text{(downwards)}$$

Total magnetic field at O is

$$|\vec{B}_0| = \frac{\mu_0 I}{4\pi r} - \frac{\mu_0 I}{4\pi r}(2\pi - \theta) + \frac{\mu_0 I}{4\pi r}$$

$$\frac{\mu_0 I}{4\pi r} - \frac{\mu_0 I}{4\pi r}(2\pi - \theta) + \frac{\mu_0 I}{4\pi r} = 0$$

As given in question

$$\frac{\mu_0 I}{4\pi r}\left[1 - (2\pi - \theta) + 1\right] = 0$$

$$2 - (2\pi - \theta) = 0$$

$$\theta = 2\pi - 2 = 2(\pi - 1) rad.$$

7.6 Motion of a Charge in Magnetic Field

If a charge q is moving in both electric field $\vec{E}$ and magnetic field $\vec{B}$, then the net force on the praticle is

$[\vec{F}_e = q\vec{E}\ ;\ \vec{F}_m = q(\vec{v} \times \vec{B})]$

hence $\vec{F} = \vec{F}_e + \vec{F}_m = q[\vec{E} + \vec{v} \times \vec{B}]$ . . . (7.44)

This force was given by H.A. Lorentz, and hence the name, Lorentz force.

The magnetic force on a moving charge is given by

$\vec{F}_m = q(\vec{v} \times \vec{B}) = qvB \sin\theta\, \hat{n}$ . . . (7.45)

$\left|\vec{F}_m\right| = qvB \sin\theta$ . . . (7.46)

The direction of force is given by $\hat{n}$, which is a unit vector perpendicular to the plane of v & B according to right hand rule. If the charge is negative, the force is opposite to that on + ve charge.

Special Cases :

(7.5.1) If the charge is stationary; $|\vec{v}|=0$; $|\vec{F}|=0$, only a moving charge experiences magnetic force. 7.5.2. If $\theta=0°$ or $180°$ i.e. the charge is moving parallel or antiparallel to the field; $\sin\theta=0$ $|\vec{F}|=0$, hence the charge continue to move in staight line.

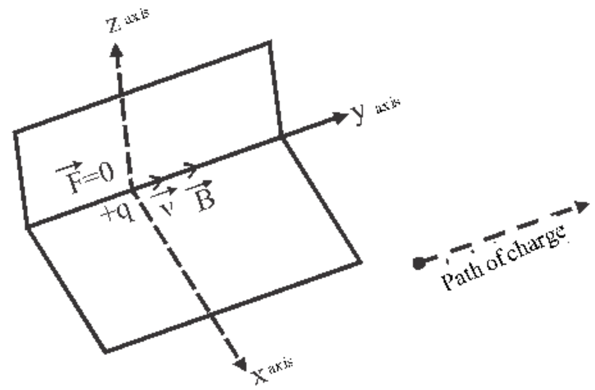


Fig 7.20 (A) Froce on charge in magnetic field
Fig 7.20 (B) Motion of charge parallel to $\vec{B}$

7.6.1 Motion of Charge in Perpendicular Magnetic Field

If $\vec{v}$ and $\vec{B}$ are mutualy perpendicular, then $\theta = 90^0$. the force on the charge will be maximum and equal to $F = qvB\sin 90°$; where is to the plane of $\vec{v}$ and $\vec{B}$

$$F = qvB = F_{max} \qquad \ldots (7.47)$$

The direction of this force is shown in Z direction in fig 7.21 (A) hence the charge will have a circular motion in X-Z plame. In fig 7.21 (B). The $\vec{B}$ is perpendicularly down-wards to the page.

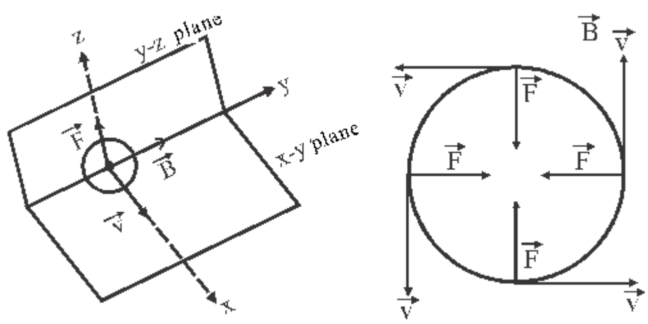


Fig 7.21 (A) Force on a charge in magnetic field
Fig 7.21 (B) Motion of charge in magnetic field

If the charged particle has a mass m, and moving on a circular path of radius r, then the magnetic force will act as centripetal force, hence

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad \ldots (7.48)$$

If kinetic energy of particle is E_k, than $p = \sqrt{2mE_K}$

The radius of circular path is

$$r = \sqrt{\frac{2m\,E_K}{qB}} \qquad \ldots (7.49)$$

It means that radius of circular path is proportional to the linear momentum of the particle. Since r is E_k will be constant. It menas the works done by this force, on particle is zero.

The circular motion of a charged particle behave like a current loop, and produces its magnetic field which affect the existing magnetic field.

The time period T of this circular motion is

$$T = \frac{2\pi\, m}{qB} \qquad \ldots (7.50)$$

and the frequency

$$v = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \ldots (7.51)$$

The angular frequency

$$\omega = 2\pi v$$

$$\omega = \frac{qB}{m} \qquad \ldots (7.52)$$

From equation (7.50) it is clear that time period T and frequency w or v is independent of speed and kinetic energy E_k. It is also independent of momentum.

This important concept is used in the design of cyclotron. T depends on B and specific charge of the particle q/m. $T \propto 1/B$ and $T \propto m/q$.

7.6.2 Motion of charged particle when $0° < \theta < 90°$

If the veloity of the particle makes an angle θ with

$\vec{B}$, then velocity v has, two components, v_{11} and $v_{\perp}$. The component $v_{11} = v\cos\theta$ is in the direction of B, and $F_m = 0$. The particle will move in a straight line with constant velocity. The other component $v_{\perp} = v\sin\theta$ make the particle to have a uniform circular motion.

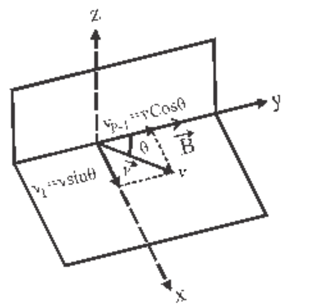


Fig 7.22 (A) Force on a charge when $0^0 < \theta < 90^0$

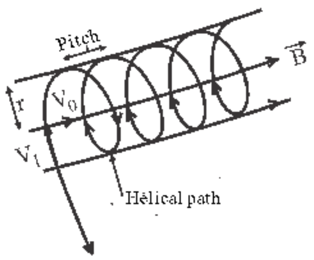


Fig 7.22 (B) Helical motion of the particle

The combination of these two motions is a helical motion, which is shown in fig 7.22(B). Radius of the helical path is

$$r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB} \quad ...(7.53)$$

and time period $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ ...(7.54)

The linear distance, between the consecutive revolutions is called pitch, which is

$$y = v_{11}T = (v\cos\theta)\frac{2\pi m}{qB}$$

$$y = \frac{2\pi mv\cos\theta}{qB} \quad ...(7.55A)$$

Or $$y = \frac{2\pi r}{\tan\theta} \quad ...(7.55B)$$

In the polar region, for example in Northern Canada and Alaska, sometimes a spectacular pattern of coloured light polar aura is seen, which is called AURORA BOREALIS scientists explained it a s a phenomem due to motion of charged particles from cosmic rays in the earth's magnetic field, which is strong at poles.

Example 7.8 : An electron of energy $10eV$ is moving on a circular path in perpendicular magnetic field $B = 10^{-5}T$. Find the velocity of electron and radius of circular path.

Solution : $E_K = \frac{1}{2}mv^2 = 10eV$

$$= 10\times1.6\times10^{-19}J$$

$$v = \sqrt{2E_k m}$$

$$v = \sqrt{2\times10\times1.6\times10^{-19}\times9.1\times10^{-31}}$$

$1\ eV = 1.6\times10^{-19}J$, $m = 9.1\times10^{-31}$ kg

$$v = 1.88\times10^{-6}\ m/s$$

radius of circular path

$$r = \frac{mv}{qB} = \frac{mv}{eB} \quad (\because q = e = 1.6\times10^{-19}c)$$

$$= \frac{9.1\times10^{-31}\times1.88\times10^{6}}{1.6\times10^{-19}\times10^{-5}}$$

r = 1.07 m

Example 7.9 : A beam of proton with velocity 4 x 10^5 m/s is moving in a uniform magnetic field 0.3 T at an angle 60^0 with B. Find (i) radius of the path and (ii) pitch.

Solution : The path is helical hence

$$v_{11} = v\cos\theta = 4\times10^5\cos60°$$

$$= 4\times10^5\times\frac{1}{2}$$

$$v_{11} = 2\times10^5\ m/s$$

$$v_{\perp} = v\sin\theta = 4\times10^5\times\frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3}\times10^5\ m/\sec$$

$$r = \frac{mv_{\perp}}{qB}$$

$$= \frac{(1.67\times10^{-27})\times(2\times\sqrt{3}\times10^{5})}{1.6\times10^{-19}\times0.3}$$

$$r = 12\times10^{-5}\,m$$

(ii) $$y = v_{11}T = v_{11}\times\frac{2\pi m}{qB}$$

$$y = \frac{2\times10^{5}\times2\times3.14\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.3}$$

The pitch $= 43.7\times10^{-3}\,m = 43.7\,mm$

Example 7.10 : An electron is moving with speed 3×10^7 m/s in a perpendicular uniform magneic field $B = 6 \times 10^{-4}$T. Find (i) radius of path (ii) frequency (iii) energy in KeV. ($m_e = 9\times10^{-31}$ kg ; $e = 1.6\times10^{-19}$ C) (1eV = 1.6×10^{-19} J)

Solution : $$r = \frac{mv}{qB} = \frac{9\times10^{-31}\times3\times10^{7}}{1.6\times10^{-19}\times6\times10^{-4}}$$

$= 2.81 \times 10^{-1}$m

$= 28 \times 10^{-2}$m = 28 cm

frequency $$v = \frac{qB}{2\pi m}$$

$$= \frac{1.6\times10^{-19}\times6\times10^{-4}}{2\times3.14\times9.0\times10^{-31}}$$

$= 17\times10^{6}$ Hz = 17 MHz

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}\times9\times10^{-31}\times(3\times10^{7})^2$$

$$= \frac{1}{2}\times9\times10^{-31}\times9\times10^{14}$$

$$= 40.5\times10^{-17}\,J$$

$$= \frac{40.5\times10^{-17}}{1.6\times10^{-19}}\,eV$$

$= 25.3 \times 10^{2}$eV = 2.53 KeV

7.7 Cyclotron

It is an electromagnetic device, used to accelarate, massive + *ve ly* charged particles like $\alpha -$ particles proton, deutron, at high velocities.

It was invented by E.O. Lawrance and M.S. Livingston, to investigate the structure of nucleus (1934).

7.7.1 Principle of Cyclotron

(i) The charged particles are compelled to move in a perpendicular magnetic field, with constant freqeucny/ time period.

(ii) The electric potential (AC potential of high frequency) provide energy twice in one cycle.

7.7.2 Construction

Two hollow, D shaped metalic containers called "Dees" are placed between poles of magnet such that B is perpendicular to "Dees". These "Dess" are placed in vaccum chamber to avoid collision of charged particles with air molecules.

An AC source of cyclotron frequency $v = \dfrac{qB}{2\pi m}$ is connected to "Dees".

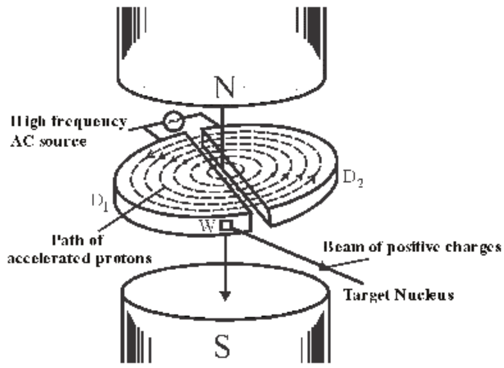


Fig 7.23 Cyclotron

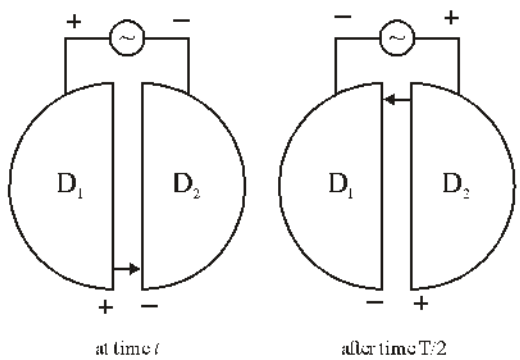


Fig 7.24 AC source at "Deed"

Working of Cyclotron-

The source of ions/particles (to be accelarated) is placed at the center of the circle made by the two "Dees". As soon as the particle is ejected from the source by its own velocty, it enters perpendicular magnetic field and starts circular motion inside Dee. After completing half circle in D_1, its enters the space between D_1 and D_2 where it is exposed to electric field of potential V. It experiences a kick and gains energy qV. The particle enters D_2 with increased velocity and moves in larger half circle in D_2. When the particle leaves D_2 and enters the space between Dees, the polarity of the applied voltage is reversed. The particle again gains energy qV and pre-enters D_1 with increased velocity. The particle gain an energy $2qV$ from electric field in one revolution.

If the particle had N revolutions before coming out of cyclotron its energy is increased by $2NqV$, which appears as kinetic energy of the particle.

Mathematical Analysis-

Let m, q an v be the mass, charge and the velocity of the charged particle. When particle enters a perpendicular magnetic field B,

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad \ldots (7.57)$$

$$(p = mv = \ )$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \text{(here v is velocity)} \ldots (7.58)$$

$$\frac{T}{2} = \frac{\pi m}{qB} \quad \text{and} \quad v = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \ldots (7.59)$$

which is called cyclotron frequency. It is independent of v and r. Note that the frequency of the applied AC voltage to "Dees" is equal to cyclotron frequency $v = \frac{1}{T} = \frac{qB}{2\pi m}$, then the cyclotron is said to be in condition of resonance. And the particle gains maximum energy from the system.

$$\omega = 2\pi v = \frac{qB}{m} \qquad \ldots (7.61)$$

Kinetic energy of the particle is $E_K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m\frac{q^2B^2r^2}{m^2} \qquad \left(7.57 \quad v = \frac{qBr}{m}\right)$$

$$E_k = \frac{1}{2}\frac{q^2B^2r^2}{m} \qquad \ldots (7.62)$$

When the particle is about to come out from cyclotron, $r = R$; The kinetic energy will be maximum

$$E_{max} = \frac{1}{2}\frac{q^2B^2R^2}{m} \qquad \ldots (7.63)$$

If the particle had completed N resovtions before coming out, the energy obtained from electric field is

$$E = (2qV)N \qquad \ldots (7.64)$$

Since energy is changed into kinetic energy, we get

$$(2qV)N = \frac{1}{2}\frac{q^2B^2R^2}{m}$$

$$N = \frac{1}{4}\frac{qB^2R^2}{mV} \qquad \ldots (7.65)$$

7.7.4 Limitations of Cyclotron

(i) It can't accelarate light particles like electrons, because to obtain required kinetic energy, we have to provide very high velocity to electrons. At such relativistic velocity, the mass of electron no more remain constant, and the cyclotron frequency changes, which disturbs the resonance of the cyclotron. To accelarate electrons, another device called Betatron is used.

(ii) Neutral particles like neutrons can not be accelarated by cyclotron.

Uses of Cyclotron

(i) The particles accelarated by cyclotron are used to study the structure of neucleus.

(ii) The accelarated ions are impregnated by bombarding into another materials to improve quality or synthesis of new materials.

(iii) To obtain new radio active materials, which has applications in several fields like research and medical sciences.

Example 7.11 : The cyclotron frequency is 10 MHz. To accelarate protons, what will be the value of magnetic field? Radius of Dees is 60 cm. also find the maximum kinetic energy of accelate protons in MeV.

(e = $1.6 \times 10^{\ 19}$c, $m_p = 1.57 \times 10^{\ 27}$kg, 1 MeV = 1.6×10^{-13} sJ)

Solution : The cyclotron freuqency is $v = \dfrac{qB}{2\pi m}$

hence $B = \dfrac{2\pi m v}{q}$

$$= \frac{2 \times 3.14 \times 1.67 \times 10^{\ 27} \times 10^{7}}{1.6 \times 10^{-19}}$$

$B = 0.66$ T

The maximum velocity of protons is

$$v = \frac{qBr}{m} = \frac{1.6 \times 10^{\ 19} \times 0.66 \times 0.60}{1.67 \times 10^{-27}}$$

= 3.78×10^{7} m/s

The maximum kinetic energy

$$E_k = \frac{1}{2} m v^2 = \frac{1.67 \times 10^{\ 27} \times (3.78 \times 10^{7})^2}{1.6 \times 10^{-13}}$$

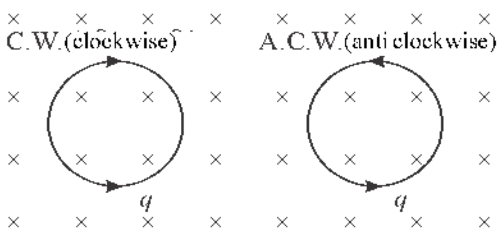
= 7 MeV

Example 7.12 : Discuss the path of a charge q (charged particle) entering a uniform magnetic field.

Solution : Case (i) If the particle enters the field, parallel or anti parallel to the field, then $F_m = o$, The path of the particle is a straight line.

Case (ii) When the particle enters the field perpendicularly $\theta = 0°$ then the force $F_m = qVB$ will be normal to V. The path will be a circle. It will move clock or anticlock wise according to direction of B.

Case (iii) When the particle enters magnetic field at an angle $\theta \neq 0°$, 180, 90. The path of the particle will be helical.



7.8 Force on Current Carrying Conductor in Magnetic Field

When a current carrying conductor is placed in a uniform magnetic field, the charge carriers, (the free electrons) moving with drift velocity v_d experience a force $\vec{F} = q(\vec{v}_d \times \vec{B})$. Hence, there will be a net force on the conductor.

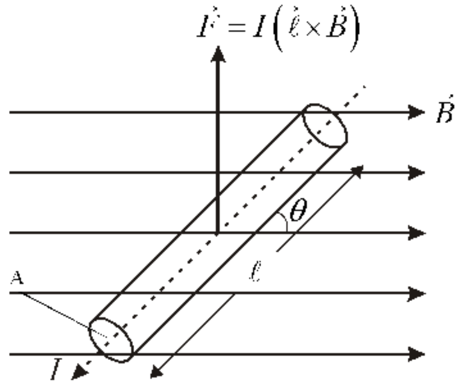


Fig 7.25 A current carrier in magnetic field

As in fig 7.25 conductor of cross section A, number of free electrons per unit volume 'n' carries a current I. Its $\vec{\ell}$ length l is placed in uniform magnetic field B at an angle θ. The total amount of charge of free dectrons will be $q = neAl$. Since the velocity of this charge is v_d , the net force on the conductor will be

$$|\vec{F}| = q\, v_d B \sin\theta$$

$$= neA\ell\, v_d\, B \sin\theta \qquad (\because q = neA\ell)$$

$$= (neA\, v_d)\ell\, B \sin\theta \qquad (\because I = neA\, v_d)$$

$$|\vec{F}| = I\ell\, B \sin\theta \qquad \ldots (7.66)$$

$$\vec{F} = I(\vec{\ell} \times \vec{B}) \qquad \ldots (7.67)$$

Here the direction of $\vec{\ell}$ is in the direction of current. Direction of the force will be perpendicular to the plane of $\vec{\ell}$ and $\vec{B}$, according to right hand rule.

7.8.1 Direction of Force on a Current Carrying Conductor in Magnetic Field

For this, two laws are in use -

7.8.1.1 Fleming's Left Hand Rule

Make thumb, index finger and middle finger of year left hand perpendicular to each other. If index finger indicates the direction of $\vec{B}$ (magnetic field) middle finger, the direction of I; then the thumbs will indicate the direction of force on the conductor.

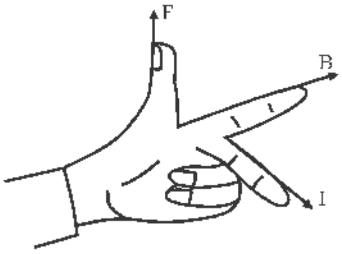


Fig 7.26 Fleming's left hand rule

7.8.1.2 Right Hand Palm Rule

If we spread our right hand in suh away that the finger are in the direction of magnetic field B, the thumb is in the direction of I, then the force on the conductor will be upward and perpendicular to the palm.

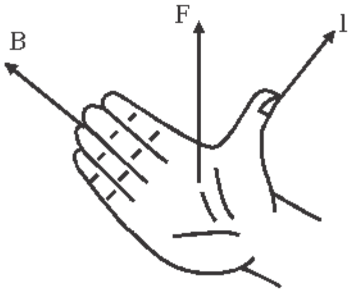


Fig 7.27 Right hand palm rule

7.9 Magnetic Force Between Two Parallel Current Carrying Conductors

Let two parallel conductors carrying currents I_1 and I_2 are in the plane of the paper at a distance d in air/vacuum. $\vec{B}_1$ and $\vec{B}_2$ are the magnetic fields produced by currents I_1 and I_2, at the location of II and I conductor. The force on the dl_1 length of the first conductor carrying currenr I in magnetic field $\vec{B}_2$ will be -

direction shown if fig (7.29) here $B_2 = \frac{\mu_0 I_2}{2\pi d}$

$$\delta \vec{F}_{21} = I_2\left(\delta \vec{\ell}_2 \times \vec{B}_1\right)$$

$$\left|\delta \vec{F}_{21}\right| = I_2 \left|\delta \vec{\ell}_2\right|\left|\vec{B}_1\right| \sin 90^\circ$$

$$\left|\delta \vec{F}_{21}\right| = I_2 \delta \ell_2 B_1$$

$$\left|\delta \vec{F}_{21}\right| = \frac{\mu_0 I_1 I_2 \delta \ell_2}{2\pi d} \qquad \ldots (7.69)$$

similarly the force on the length dl_2 due to B_2 and I_2

$$\delta \vec{F}_{12} = I_1\left(\delta \vec{\ell}_1 \times \vec{B}_2\right)$$

$$\left|\delta \vec{F}_{12}\right| = I_1 \left|\delta \vec{\ell}_1\right|\left|\vec{B}_2\right| \sin 90^\circ$$

$$= I_1 \delta \ell_1 B_2$$

hence $$\left|\delta \vec{F}_{12}\right| = \frac{\mu_0 I_1 I_2}{2\pi d} \delta \ell_1 \qquad \ldots (7.70)$$

(1) If the direction of current in the two conductors is same, they experience a force of attaction.

(2) If the direction of current in the two conductors is opposite to each other, they will experience a force of repulsion (Fig. 7.28).

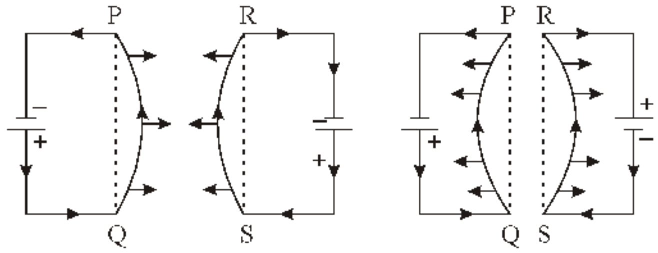


Fig 7.28 Force between two parallel currents

The direction of both forces is as per right hand palm rule, and shown in the figure 7.28. These forces are action reaction pairs, $\delta \vec{F}_{12}$ and $\delta \vec{F}_{21}$, and are opposite in direction.

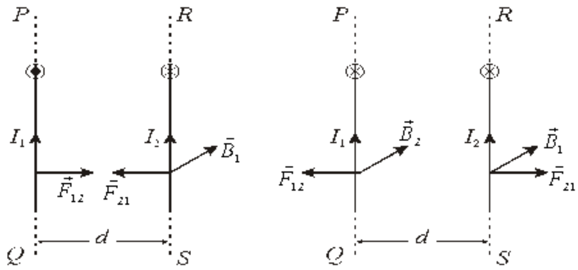


Fig 7.29 Force between two parallel currents

The force per unit length of the conductors is given

by $$\frac{\left|\delta F_1\right|}{\left|\delta \ell_1\right|} = \frac{\left|\delta F_2\right|}{\left|\delta \ell_2\right|} = \frac{\mu_0 I_1 I_2}{2\pi d} \text{N/m} \qquad \ldots (7.72)$$

7.9.1 Defination of Standard Ampere in S.I. Units

From equation 7.72, the force per unit length on two parallel currents in air/vacuum, is $\frac{\delta F}{\delta \ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$ N/m

If we put the condition that $I_1 = I_2 = 1\,\text{A}$ and

$$d = 1\,\text{m} \text{ in air } \frac{\delta F}{\delta \ell} = \frac{\mu_0 \times 1 \times 1}{2\pi \times 1} = \frac{4\pi \times 10^{-7}}{2\pi}$$

$$= 2 \times 10^{-7}\ \text{N/m}$$

From the above condition we can define 1 A. 1 A is that current maintained in two parallel conductors placed at a distance of 1 m in air, if it exerts a force per unit length equal to 2×10^{-7} N/m then the current in each conductor is 1 A. The latest definition of Ampere in SI units effective from 20-5-2019 can be searched at (http://physics.nist.gov).

7.10 Force and Torque on a Rectangular Current Loop in Uniform Magnetic Field

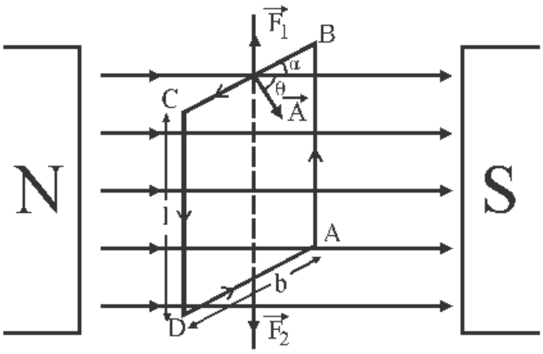


Fig 7.30 Torque on a current loop in magnetic field

Consider rectangular current loop ABCD of length l and breadth b and area A. The current in the loop is I. It is placed in a uniform magnetic field B. To find net force on the coil, we consider the force on each side of rectangle and just sum up. At any instant if the area vector $\vec{A}$ makes an angle θ with $\vec{B}$, the force on side BC is $\vec{F}_1 = I\left(\vec{b} \times \vec{B}\right)$, the direction is upward in the plane of the paper. Similarly the force $\vec{F}_2$ on DA is $\vec{F}_2 = I\left(\vec{b} \times \vec{B}\right)$, the direction is downward in the plane of the paper. They are equal, opposite and collinear hence get cancelled. The force on side CD and AB is $\left|\vec{F}_3\right| = I\ell B \sin 90^\circ = I\ell B$, again their sum is zero. But they are not colinear, hence they produce a torque on the coil. Net force on the loop is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

Calculation of Torque

As shown in the figure 7.31, the forces $\vec{F}_3$ and $\vec{F}_4$ are equal and opposite. They act on two different points, hence produce a torque $\vec{\tau}$.

$\tau =$ Force x perpendicular distance between forces

$$\tau = \left(I\ell B\right)\left(b \sin\theta\right)$$

$$= I\left(\ell b\right) B \sin\theta$$

$$\tau = IAB \sin\theta \qquad \ldots (7.73)$$

If the loop has number of N turns

$$\tau = NIAB \sin\theta \qquad \ldots (7.74)$$

$$\tau = MB \sin\theta \qquad \ldots (7.75)$$

Here $\vec{M} = NI\vec{A}$ ... (7.76)

is the magnetic moment of the current loop.

In vector form $\vec{\tau} = \vec{M} \times \vec{B}$ ... (7.77)

or $\vec{\tau} = NI\vec{A} \times \vec{B}$ ... (7.78)

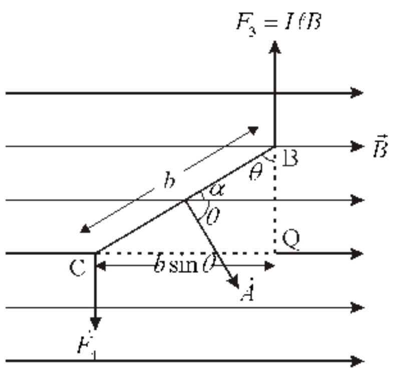


Fig 7.31 Torque on a current loop

Note - When we consider the angle between the plane of the coil and as the equation (7.75) will be

The direction of $\vec{\tau}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ as per right hand screw rule.

Comparing $\vec{\tau}$ on an electric dipole in uniform electric field, we see that it is $\left(\tau = PE \sin\theta\right)$

Special Conditions

(i) When the plane of the coil is perpendicular to $\vec{B}$, i.e. $\theta = 0^\circ$, 180° and $\alpha = 90^\circ$ from eq. 7.75, the

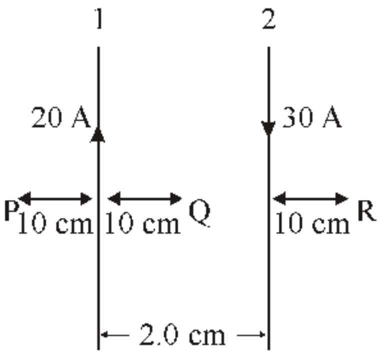
$\tau = \tau_{\min} = MB\sin\theta = 0 =$ (minimum).

(ii) When the plane of the coil is parallel to i.e $\theta = 90^\circ$; $\alpha = 0^\circ$ the torque will be maximum

$$\tau = \tau_{\max} = MB\sin 90^\circ = MB \qquad \ldots (7.79)$$

Electric motor and moving coil meters work on this principle.

Example 7.13 : Find the magnetic field $\vec{B}$ at points P, Q and R, due to two parallel currents as given in diagram.



Solution : Field $\vec{B}$ due to a straight current I

d at distance is given by $B = \dfrac{\mu_0 I}{2\pi d}$

A point P the net $B_P = B_{P_1} \sim B_{P_2}$

$$= \frac{\mu_0(20)}{2\pi(0.1)} \sim \frac{\mu_0(30)}{2\pi(0.3)}$$

$$= \frac{\mu_0}{2\pi}[200-100] = \frac{4\pi\times10^{-7}}{2\pi}\times100$$

$$= 2\times10^{-5}T$$

of B_P is given by as perpendicular to page upwards.

(ii) At point Q; net $\vec{B_Q} = \vec{B_{Q_1}} + \vec{B_{Q_2}}$ since both are in same direction.

$$B_Q = \frac{\mu_0(20)}{2\pi(0.1)} + \frac{\mu_0(30)}{2\pi(0.1)}$$

$$= \frac{\mu_0}{2\pi}[200+300]$$

$$= 2\times10^{-7}\times500$$

$$= 10\times10^{-5}T = 10^{-4}T$$

given by i.e. to page downwards.

(iii) Similarly on point R. The net $\overrightarrow{B_R} = \overrightarrow{B_{R_1}} + \overrightarrow{B_{R_2}}$

$$B_R = B_{R_1} \sim B_{R_2} = \frac{\mu_0}{2\pi}\left[\frac{I_1}{d_1} \sim \frac{I_2}{d_2}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{30}{0.1} - \frac{20}{0.3}\right]$$

$$= 2\times10^{7}\times2.33\times10^{2}$$

$$= 4.66\times10^{5}\ \mathrm{T}$$

The direction will to page upwards.

Example 7.14 : A 10m wire carries a current of 10A. It is placed at , with B. Find force per unit on the wire, if B = $5.5\times10^{-4}\,T$.

Solution : The force on whole wire is given by

$$\vec{F} = I\left(\vec{\ell}\times\vec{B}\right)$$

The for per unit length is;

$$\left|\vec{F}\right| = I\ell B\sin\theta$$

$$I = 10\,A,\ B = 5.0\times10^{-4}\,T$$

$$\ell = 10\,\text{m}\ \text{तथा}\ \theta = 30^\circ$$

$$\left|\vec{F}\right| = 10\times10\times5\times10^{4}\times\sin30^\circ$$

$$= 10\times10\times5\times10^{-4}\times\frac{1}{2}N$$

$$= 250\times10^{4}\,N$$

$$\frac{\left|\vec{F}\right|}{\ell} = \frac{250\times10^{4}}{10} = 25\times10^{-4} = 0.025\ \text{N/m}$$

7.11 Galvanometer

In previous chapters, we have studied about the physical quantities like electric current and potential. In this section we will study about the devices that measure these quantities.

Galvanometer is a device used to detect current in a circuit or potential difference between two points. It can be converted into a voltmeter and Ammeter. It is based on the principle of torque on a current in magnetic field. They are of two types - (i) Moving coil (ii) Moving magnet type.

In this section we will study only moving coil type galvanometer, which are again of two types -

(i) Suspended coil glavanometer (ii) Pivoted coil galvanometer. Both types are based on same principle, but differ in their construction and working.

7.11.1.1 Suspended Coil Galvanometer

As shown in the diagram 7.32, a rectangular or circular coil of insulated copper wire wounded over an alluminium frame, is suspended by a phospher bronze fiber between the poles of a horse shoe magnet. The alluminium frame of the coil is free to rotate about a fixed iron core. One end of the coil is connected to terminal T_1 via phospher bronze fiber and the other to terminal T_2 via an elastic spring.

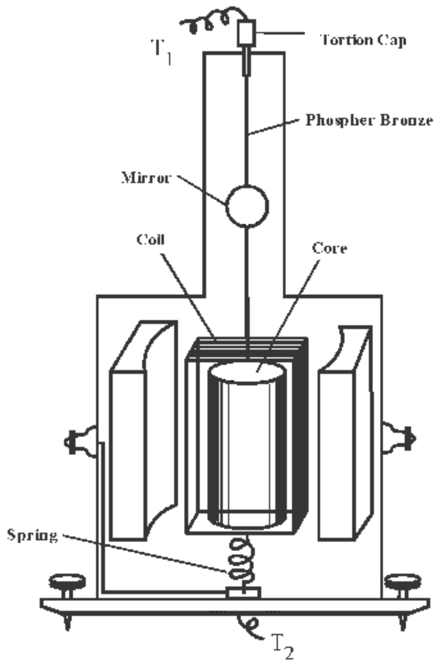


Fig 7.32 Suspended coil galvanometer

Principle - It is based on te principle of, torque on a current loop in magnetic field. This torque deflect (rotate) the coil, which is measured on a suitable scale.

A coil of N turns and area A, having current I experiences a torque $\tau_{def} = NAIB\sin\theta$ in magnetic field B. To aviod the monlinearity of, the poles of the magnet are made concave in shape, so that the field B is always radial and $\theta = 90^\circ$. So $\tau_{def} = NAIB$, which has a linear relation with I.

The role of phospher bronze fiber is to produce a counter torque (Restoring Torque) due to its tortion; so that the coil comes in equilibrium after a rotation of θ . θ is measured by lamp and scale arrangement, for that, a small mirror is attached (fixed) to phaspher bronze fiber.

If the restoring torque per degree due to phospher bronze fiber is C, then the restoring torque for deflection ϕ will be $\tau_R = C\phi$ . . . (7.80)

In equilibrium $\tau_{def} = \tau_R$

or $NAIB = C\phi$

$$I = \left(\frac{C}{NAB}\right)\phi \quad \ldots (7.81)$$

$$I = k\phi \quad \ldots (7.82)$$

here $k = \dfrac{C}{NAB}$ is a constant called reduction factor of the galvanometer. Hence $I \propto \phi$

7.11.1.2 Radial Field and Role of Iron Core

As shown in the fig 7.33, when the poles of the magnet are made concave in shape, then magnetic field is always radical and perpendicular to area vector $\vec{A}$ of the coil, the torque is maximum. $\tau_{def} = NAIB$ The soft iron core intensifies the effective magnetic field B, so that k is reduced and sensitivity of the galvanometer is increased.

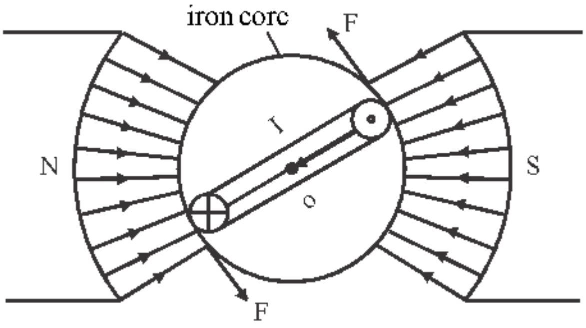


Fig 7.33 Radical magnetic field

7.11.1.3 Working

In this device the deflection is measured using lamp and scale arrangement. If the perpendicular distance of mirror from scale is D, and deflection of light spot on scale is d. $\tan(2\phi) = \frac{d}{D}$ and for small deflection (tan x ≃ x), $2\phi = \frac{d}{D}$; $\phi = \frac{d}{2D}$. (Here we have taken 2ϕ, because deflection of mirror by ϕ, deflects the reflected ray by angle 2ϕ) and $I \propto d \propto \phi$

7.11.1.4 Sensititvity of Galvanometer

If a small current causes large deflection in galvanometer it is said to be more sensitive. The sensitivity is inverse of figure of merit/reduction factor

From equ. 7.81 $I = \left(\frac{C}{NAB}\right)\phi = k\phi$

Hence, the current sensitivity

$$S_i = \frac{\phi}{I} = \frac{NAB}{C} = \frac{1}{k} \quad \ldots (7.84)$$

To increases sensitivity we can increase N, B and A to certain practical limit. C can be decreased by taking the fiber of phospher bronze as long and thin. Again to practical limit for ruggedness of galvanometer.

Voltage Sensitvity - It is defined as deflection per volt in galvanometer $V_s = \frac{\phi}{V}$

If the resistance of galvanometer coil is G, and current is I ; $V = IG$ and

$$V_s = \frac{\phi}{IG}$$

$$V_s = \frac{NAB}{CG} \quad \ldots (7.86)$$

From equation 7.84 and 7.86

$$V_s = \frac{S_i}{G} \quad \ldots (7.87)$$

7.11.2 Figure of Merit of Galvanometer

It is defined as the current required for unit deflection in galvanometer. Hence it is inverse of current sensitivity

$$X = \frac{1}{S_i} = \frac{I}{\phi}$$

$$X = \frac{C}{NAB} = k \quad \ldots (7.88)$$

7.11.3 Pivoted Coil Galvanometer

All the arrangements of coil, frame, iron core and concave shape of magnetic poles is same as that in suspended coil galvanometer, except that its coil is pivoted on two sharp point (instead of suspension) so that it is free to rotate. Coil is connected to terminals T_1 and T_2 via two hair springs, which provide restoring torque for equlibrium. A very light indicator needle of aluminimum is attached to the coil. The other end of the indicator needle gives deflection on a graduated dial. Such galvanolmeter is also called weston galvanometer. In spite of its less sensitivity compared to suspended coil galvanometer, it is most used because of convinience in use.

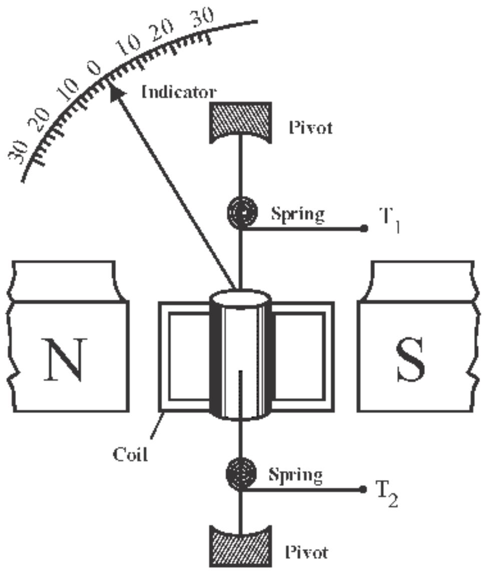


Fig 7.34 Pivoted coil galvanometer

Use of Shunt

Galvanometer gets damaged, due to excessive current. A copper wire (shunt) is connected between the terminals T_1 and T_2 which by passes the extra current and protect the galvanometer from damage. Fig 7.35 shows a shunted galvanometer.

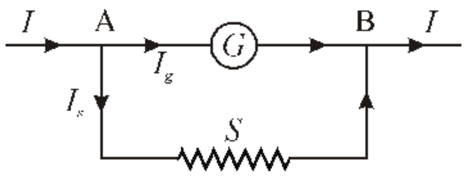


Fig 7.35 Shunted galvanometer

If G and R are resistances of galvanometer and shunt respectively and I, I_g and I_s are the main current, current in galvanometer and current in shunt. Then

$I_g G = I_s S$ (potential between A & B is same for both)

$$\frac{I_s}{I_g} = \frac{G}{S}$$

$$\frac{I_s}{I_g} + 1 = \frac{G}{S} + 1$$

$$\frac{I_g + I_s}{I_g} = \frac{G + S}{S}$$

$$\frac{I}{I_g} = \frac{G + S}{S} \qquad (\because\ I = I_g + I_S)$$

which gives $$I_g = \left(\frac{S}{G + S}\right) I \qquad \ldots (7.89)$$

Galvanometers of required range are fabricated using above relation.

7.11.4 Ammeter

An ammeter is a current measuring device, so it is always connected in series in the circuit. Resistance of an ideal ammeter should be zero, so that it does not effect the current in the circuit, (to be measured). But a practical ammeter has certain non-zero resistance (i.e. resistance of coil).

To make the effective resistance of the galvanometer as low as possible, a very low resistance is connected between the terminals T_1 and T_2 of the galvanometer it is called shunt. The value of shunt resistance is determined as per requirement of the range.

As shown in the fig 7.36, a shunt is connected between its terminals T_1 and T_2 whose value is determined by the relation $I_g G = (I - I_g) S$

$$S = \frac{I_g G}{(I - I_g)} \qquad \ldots (7.90)$$

Here I = range of a ammeter; Ig = current for full scale deflection.

$$I_g = \left(\frac{I - I_S}{G}\right) S \qquad \ldots (7.91)$$

The effective resistance of the ammeter is given by

$$R_A = \frac{GS}{G + S} \qquad \ldots (7.92)\ \text{(law of parallel combination)}$$

Since $S << G$

$$R_A = \frac{GS}{G} \approx S \qquad \ldots (7.93)$$

The converted ammeter is first calibrated. It is zero is marked to extreme left on the dial.

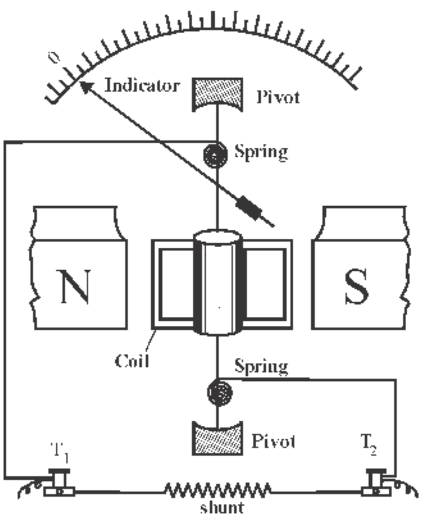


Fig 7.36 Conversion of galvanometer into ammeter

7.11.5 Voltmeter

It is a device to measure potential differnce between two points. A volt meter is always connected in parallel to the points. A voltmeter should not draw any current for it self (not to change the potential difference to be measure).

An ideal voltmeter should have infinite resistance. But an infinite resistance give I=0, and coil of voltmeter will not rotate. So for a particle moving coil galvanometer, it should as high resistance as possible (as per its range).

To convert a galvanometer to a voltmeter a very high resistance is connected in series with galvanometer.

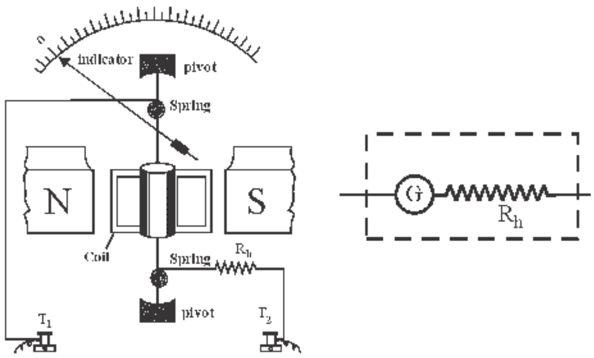


Fig 7.37 Conscrsion of a galvanometer to a voltmeter

As per diagram the potential across the series combinaion of G and R_H (applied potential) is -

$$V = I_g\left(R_H + G\right)$$

$$R_H + G = \frac{V}{I_g}$$

$$R_H = \frac{V}{I_g} - G \qquad \ldots(7.94)$$

$$R_V = G + R_H \qquad \ldots(7.95)$$

$$R_H >> G$$

Since $R_H >> G$. The effective resistance of the combination $R_V \approx R_H$.

Here V = range of voltmeter; G = resistance of galvanometer coil

and Ig = Current for full scale deflection of galvanometer

The practical voltmeter discussed above is unable to meaure potential difference very accuratly. For this we use potentiometer.

Example 7.15 : A deflection for certain current is 50. When it is short circuited by a reistance of $12\,\Omega$, the deflection reduces to 10. Find resistance of galvanometer.

Solution : For shunted galvanometer $I \propto \phi$

$$\frac{I_g}{I} = \frac{10}{50} = \frac{1}{5}$$

$$I_g = \frac{I}{5}$$

$$(I - I_g)S = I_g G$$

$$\left(I - I/5\right) \times 12 = \left(I/5\right)G$$

$$G = 4 \times 12 = 48\,\Omega$$

Example 7.16 : For a galvanometer the current for full scale deflection is 5mA, and resistance of its coil is $99\,\Omega$. Find the value of required resistance to convert it to (i) Ammeter of range 5 A. (ii) Volt meter of range 5V.

Solution : Given $I_g = 5\,\text{mA}$, $G = 99\,\Omega$, $I = 5\,A$

(i) convert to an ammeter the value of shunt

$$S = \frac{I_g G}{I - I_g} = \frac{5 \times 10^{-3}}{5 - 0.05} \times 99$$

$$= \frac{5 \times 10^{-3}}{4.95} \times 99 = 0.1\,\Omega$$

hence a resistance of $0.1\,\Omega$ is to be connected in parallel to galvanometer.

(ii) To convert to a volt meter

$$R_H = \frac{V}{I_g} - G$$

$$R_H = \frac{5}{5 \times 10^{-3}} - 99 = 1000 - 99$$

$$R_H = 901\,\Omega$$

be connected in series with galvanometer.

7.12 Ampere's Circuital Law

Just as Gauss's law in electrostatics help in tackling the problems regarding $\vec{E}$ due to a symetrical charge distributions, where Coulomb's law can't; there is a law called Ampere's law, which tackles problems regarding $\vec{B}$ due to symetric currents where Biot and Savarts law can't.

According to this law;

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma I \qquad ...(7.96)$$

It states that the line integral of magnetic field produced by electric currents in air/vacuum, over a closed path (loop) enclosing an area, is equal to the product of μ_0 and alzebric sum of the currents passing through that area. $\oint \vec{B}\cdot d\vec{\ell}$ is also called circulation of magnetic field.

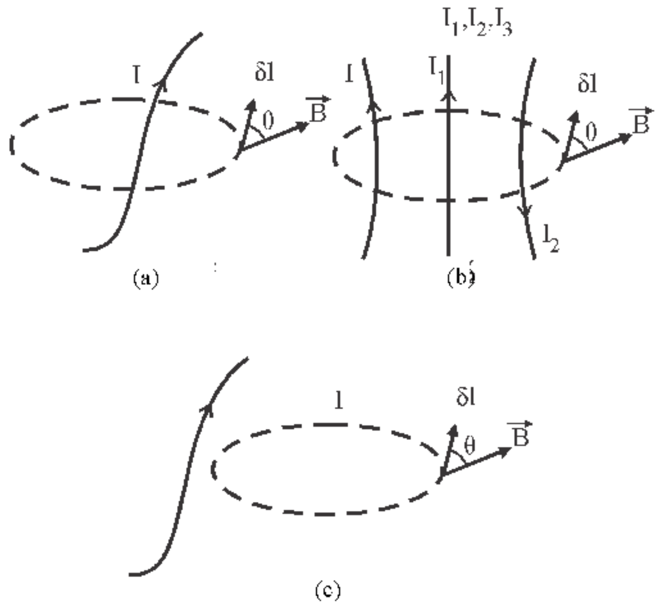


Fig 7.38 Ampere's Law

Here alzebric sum of currents $\Sigma i = I$, means I can be taken as +*ve* ore -*ve*; according to one convention. If integration is taken in anticlock-wise direction, the upward currents are taken as +*ve*; and vice-versa. For fig (a) $\Sigma i = I$; for fig (b) $\Sigma i = I_1 + I_2 + (-I_3)$; for fig (c) $\Sigma i = 0$; For $\oint \vec{B}\cdot d\vec{\ell} = 0$; it does not mean that there is no magnetic field it states that the loop choosen, contains no currents.

$B = \mu_0 H$ here H is magnetizing field or magnetic intensity.

The ampere's law in terms H is

$\oint \vec{H}\cdot d\vec{\ell} = \Sigma i$ and i.e. circulation of H is called mmf (magnetio motive force).

Ampere's law is same as Biot and Savarts law and both can be deduced from each other. While tackling problems, the Ampere's loop is selected in such a way that -

(i) B is same on each part of the loop.

(ii) $\vec{B}$ and $d\vec{\ell}$ should be parallel so that $\vec{B}\cdot d\vec{\ell} = Bd\ell$

(iii) $\vec{B}$ and $d\vec{\ell}$ should be so that $\vec{B}\cdot d\vec{\ell} = 0$

7.12.1 Magnetic Field Due to Infinity Long Current

The magnetic field produced by current I in conductor CD is in the form of concentric loops around the conductor, the conductor is the center of these loops. The loops are all along the length of the conductor.

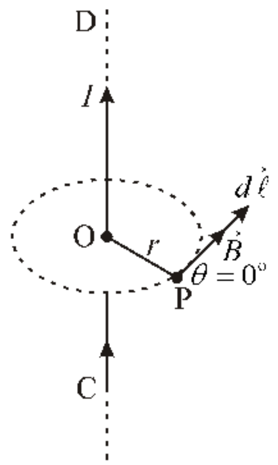


Fig 7.39 Magnetic field due to infinitly long wire

To find magnetic field at point P at a distance r from the conductor, we construct a circular Apmere loop taking wire as center. Now take a small element $d\vec{\ell}$ and find $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$ along the loop. From Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0\,\Sigma i$$

$$\oint B d\ell\cos\theta=\mu_0\,\Sigma i$$

$$\theta=0^\circ,\ \cos\theta=1$$

$$\Sigma i=I$$

$$\oint B d\ell=\mu_0 I$$

$$B\oint d\ell=\mu_0 I$$

$$\oint d\ell=\ell=2\pi r$$

$$B(2\pi r)=\mu_0 I$$

hence $$B=\frac{\mu_0 I}{2\pi r} \qquad \ldots(7.98)$$

The result is same as that obtained earlier by using Biot and Savart's law and long mathematical process. The magnetic field in this case is $B\propto I$ and $B\propto I\ /r$.

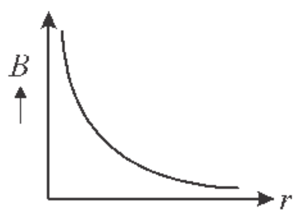


Fig 7.40 Variation of B with distancer

7.12.2 Magnetic field due to current carrying long cylendical conductor

The magnetic field due to the solid cylinder in which current I is uniformly distributed through whole cross section A, will be in the from of concentric circles, as the cylinder at the center. The loops of magnetic field lines will be there thoughout the whole length of cylinder -

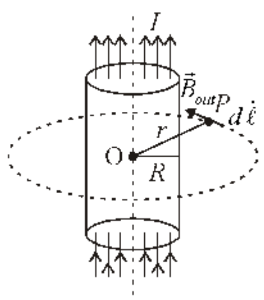


Fig 7.41 B due to cylindrical conductor

(i) To find magnetic field B at a point P, at a distance r, we construct a circular Ampere's loop of radius r, passing through point P. From Ampere's law

$$\oint \vec{B}_{out}\cdot d\vec{\ell}=\mu_0\,\Sigma i$$

$$\oint B_{out} d\ell\cos\theta=\mu_0\,\Sigma I$$

$$\theta=0^\circ,\cos\theta=1\ ;\ \Sigma i=I$$

$$\oint B d\ell=\mu_0 I$$

$$\oint d\ell=(2\pi r)$$

$$B_{out}\cdot 2\pi r=\mu_0 I$$

$$B_{out}=\frac{\mu_0 I}{2\pi r} \qquad \ldots(7.99)$$

Here again $B_{out}\propto\frac{1}{r}$ as expected.

(ii) To find $\vec{B}$ at the surface of the consuctor; $r=R$. by same consideration we obtain

$$B_S=\frac{\mu_0 I}{2\pi R} \qquad \ldots(7.100)$$

(iii) To find $\vec{B}$ at a point inside the cylinder; i.e. r < R; The same procdure is again adopted but right hand side of the Ampere's law should be taken care of. From Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0\,\Sigma I$$

$$\oint B_{in} d\ell\cos\theta=\mu_0\,\Sigma i$$

$$\theta=0^\circ,\quad \cos\theta=1$$

We consider an Ampers loop passing through that point, inside the cylinder Σi is the total currenr passing though the area of Ampers loop. From unitary method -

$$\Sigma i=\frac{I}{\pi R^2}\cdot\pi r^2$$

$$\Sigma i=\frac{Ir^2}{R^2}$$

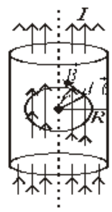


Fig. 7.42

$$\oint B_{in} d\ell = \frac{\mu_0 I r^2}{R^2}$$

$$B_{in}(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

$$B_{in} = \frac{\mu_0 I r}{2\pi R^2}$$

$$B_{in} = \left(\frac{\mu_0 I}{2\pi R}\right)\frac{r}{R}$$

hence $B_{in} = B_s \frac{r}{R}$ . . . (7.101)

$B_{in} \propto r$ and at center (undefined)

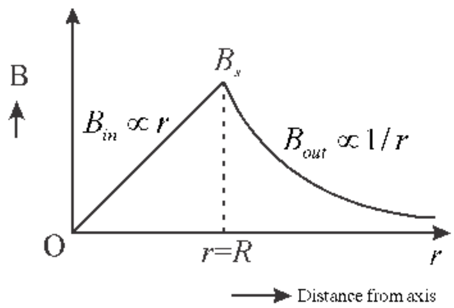


Fig 7.43 Variation of B for cylinder with r

(iv) If the conductor is a pipe (hollow cylinder) the field inside at all points is zero.

Example 7.17 : The magnetic field due to an infinitly long current carrying conductor at distance 10 cm is 10^{-5} T. Find the current in the conductor.

Solution : For infinitly long conductor

$r = 10\text{ cm} = 0.1\text{ m}$

$B = 10^{-5}$ T

$$B = \frac{\mu_0 I}{2\pi r}$$

$$10^{-5} = \frac{2\times10^{-7}\times I}{0.1}$$

$$I = \frac{10^{-5}\times0.1}{2\times10^{-7}} = 0.5\times10 = 5\text{ A}$$

7.13 Solenoid

A tightly wounded coil of insulated copper wire over a insulator pipe, where the turns are very close to each other is called solenoid.

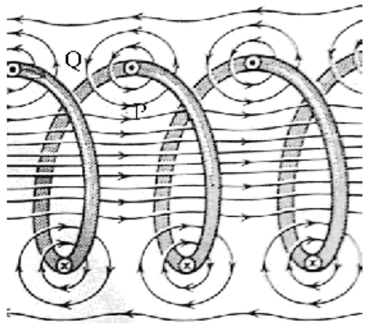


Fig 7.44 Magnetic field due to two loops

The plane of each circular loop of wire may be considered perpendicular to the axis of solenoid. To know about the magnetic field produced by solenoid we consider two current loops (1) and (2) and two points P and Q inside and outside solenoid. Fig 7.44. The constribution of both current loops at point P (inside) is in same direction, where as it is in opposite direction at Q (point outside). Super position of magnetic field due to all the loop gives B = 0 outside the solenoid; and B_{in} as strong and uniform, inside the solenoid for an ideal solenoid.

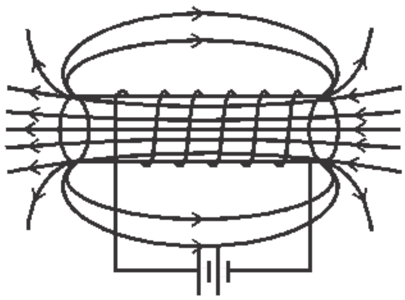


Fig 7.45 B due to loosely wounded solenoid

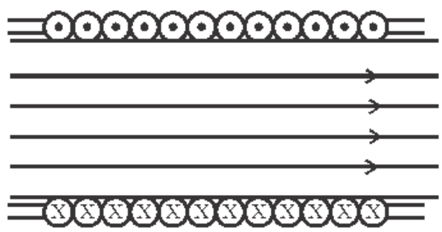


Fig 7.46 Field due to an ideal solenoid

7.13.1 Magnetic Field Inside an Infinitly Long Solenoid

To determine the field inside a soelnoid the field, we take a longitudial section (LS) of the solenoid, and show the direction of current in the loops as (x) (.) and the direction of magnetic field produced, as in fig 7.47. The current in the solenoid is I, and same in all loops. Asuming the solenoid as ideal one, we mark (show) field inside as B, nd out side as B = 0.

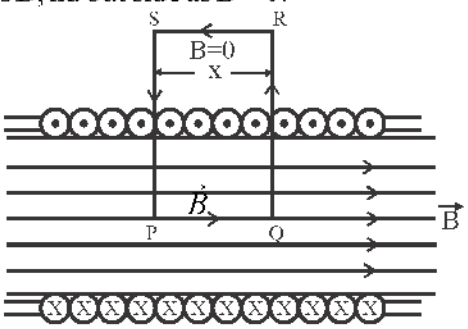


Fig 7.47 Magnetic field inside solenoid

To find B inside we construct an Amperian loop PQRS which enclose length x of th solenoid.

From Ampere's law $\oint_{PQRS} \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$

(= all currents enclosed by loop PQRS)

The defined quanity for solenoid is n, the turn density. (Not the total number of turns) hence the number of turns enclosed by Ampere loop PQRS is nx, hence

Again we take

$$\oint_{PQRS} \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$$

$$\int_P^Q \vec{B}\cdot d\vec{\ell} + \int_Q^R \vec{B}\cdot d\vec{\ell} + \int_R^S \vec{B}\cdot d\vec{\ell} + \int_S^P \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$$

$\int_P^Q \vec{B}\cdot d\vec{\ell} = \int_P^Q B d\ell$; (the contribution of remaining integrals it zero, due to either B=0 or $\vec{B}\cdot d\vec{\ell} = 0$)

we get, $B\int_P^Q d\ell + 0 + 0 + 0 = \mu_0 \Sigma i$ . . . (7.102)

since $\int_P^Q d\ell = x$

and $\Sigma i = nxI$

we get $Bx = \mu_0 nxI$

or $B = \mu_0 nI$ . . . (7.103)

which is uniform throught the space inside solenoid. It the solenoid is finite but its radius R << L, the result is valid at a point inside and far away from ends.

$$n = \frac{N}{L}$$

Also $B = \mu_0 \frac{N}{L} I = \mu_0 nI$ . . . (7.104)

$$H = \frac{B}{\mu_0}$$

$H = \frac{N}{L} I = nI$ . . . (7.105)

If a medium is placed inside the solenoid.

$$B_m = \mu \frac{N}{L} I$$

$B_m = \mu_r \mu_0 \frac{N}{L} I$ . . . (7.106)

where μ_r is relative magnetic permeability of the medium. For a short solenoid B at a point P is

$$B = \frac{1}{2}\mu_0 nI\left(\cos\phi_1 - \cos\phi_2\right)$$

where ϕ_1 and ϕ_2 are the angle between the axis and the line joining the point P to outer rims. Magnetic field at one end of the solenoid -

for this and

$$(\phi_1 = 90^\circ, \phi_2 = 180^\circ)$$

hence $B_{end} = \frac{1}{2}\mu_0 nI\left[\cos 90^\circ - \cos 180^\circ\right]$

$B_{end} = \frac{\mu_0 nI}{2}\left[0-(-1)\right]$ hence $B_{end} = \frac{1}{2}\mu_0 nI$

which is half the magnetic field inside the solenoid.

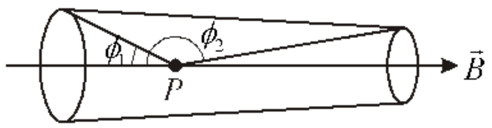


Fig 7.48 Magnetic field at the end of a solenoid

7.14 Behavioral Comparison of Bar Magnet and Current Solenoid

1. A freely suspended current carrying solenoid always stay in North-South direction. Just like a bar magnet.
2. If the ends of two current carrying solenoids are brought close to each other, they will attract or repel each other, just like magnets do.
3. Each end of solenoid behave like a North or South pole, depending upon the direction of current.
4. The ends of solenoid attracts feromagnetic materials, just like magnets do.
5. Fig 7.50 shows the relation of polarity with direction of current.

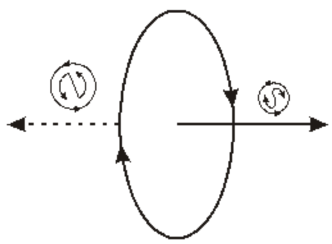


Fig 7.50 Magnetic behaviour of a current loop

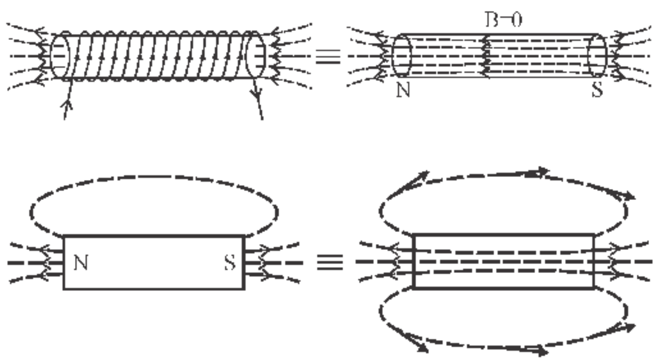


Fig 7.51 Comparison of solenoid with bar magnet

We conclude that a current carrying solenoid behave like a bar magnet. But there are some dissimilarties too -

(a) The field lines in solenoid are straight, but in a magnet some what curved.

(b) The magnetic field outside a solenoid is approximalty zero, but in case of a magnet we get magnetic field, but different at different points.

7.15 Magnetic Field on the Axis of a Toroid

If a solenoid is circularly curved and its both ends are joined it becomes Totoid. It behave like infinitly long solenoid.

It can also be fabricated by wounding insulated copper wire over a ring.

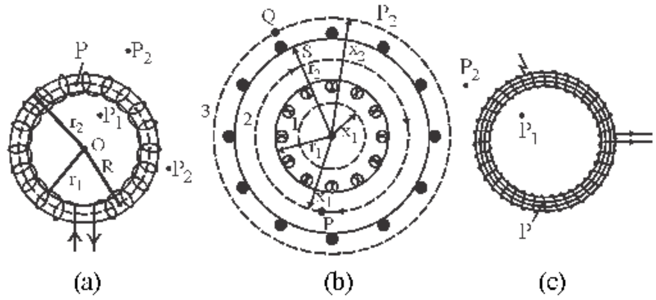


Fig 7.52 Toroid

N turns are uniformly wounded. Due to current I in each loop, they produce equal magnetic field at their centers; the contribution of all loops is in the form of concentric circles; whose center is the center of toroid. Hence the magnetic field inside (i.e. on axis) the solenoid is parallel to its axis. Tangent at a point on field line, gives the direction of magnetic field at that point.

To find magnetic field at the point on axis, we take the cross section of Toroid along/parallel to its plane. Show the direction of current as (x) and ( ) and also direction of $\vec{B}$ produced accordingly.

Now construct a circular Ampere's loop passing through the point of interest. We see that this Ampere's loop encloses all the turns and

From Ampere's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma i$

$$\oint B \cdot d\ell \cos\theta = \mu_0 \Sigma i$$

$$\theta = 0^\circ, \quad \cos\theta = 1$$

$$\Sigma i = NI$$

$$\oint B d\ell = \mu_0 NI$$

$$B(2\pi r) = \mu_0 NI$$

$$B = \mu_0 \frac{N}{2\pi r} I$$

$$B = \mu_0 nI \qquad \text{. . . (7.107)}$$

$$n = \frac{N}{2\pi r}$$, is the turn density

$$H = \frac{B}{\mu_0} \Rightarrow H = nI = \frac{N}{2\pi r} I \qquad \text{. . . (7.108)}$$

If a emdium is present

$$B_m = \mu \frac{N}{2\pi r} I$$

or $$B_m = \mu_0 \mu_r \frac{N}{2\pi r} I \qquad \text{. . . (7.109)}$$

Magnetic field out side Toroid is zero. Magnetic field depends on radius, current and medium inside the toroid.

Example 7.18 : A solenoid of length 0.5 m and radius 1 cm has 500 turns. It carries a current of 5A. Find magnetic field inside the toroid.

Soltuion : Given $L = 0.5m$, $r = 1cm = 0.01m$,

$N = 500$; $I = 5\,A$

$$A = \pi r^2 = 3.14 \times (10^{-2})^2 = 3.14 \times 10^{-4} m^2$$

$L = 0.5m$. Clearly $A << L$

Where, A=Area of cross section

$$B = \mu_0 \frac{N}{L} I$$

Clearly A<<L, hence the coil can be asumed to be very long (infinitly long)

$$B = 4\pi \times 10^{\ 7} \times \frac{500}{0.5} \times 5$$

$$B = 6.28 \times 10^{-3}\, T$$

Example 7.19 : Mean radius of a toroid is 10 cm and number of turns is 500. Find the magnetic field if the current in it is 0.1 A. $(\mu_0 = 4\pi \times 10^{-4} Wb / Am)$

Solution : Magnetic field inside toroid is

$r = 10cm = 0.1m$

$N = 500 \qquad B = \mu_0 \frac{N}{2\pi r} I$

$$B = 4\pi \times 10^{\ 7} \times \frac{500 \times 0.1}{2\pi \times 0.1}$$

$$B = 10^{-4}\, Wb / m^2$$

Important Points

1. Orested found in his experiment that a current in a conductor produces magnetic field around it. This phenomenon is called magnetic effect of electric current. If a moving charge experiences a force (neglecting electric and gravitational field), there exists a magnetic at that point.
2. SI unit of magnetic field is T or Wb / m^2 or N / A.m. The force on a moving charge in magnetic field is

   $$\vec{F} = q(\vec{v} \times \vec{B})$$

   $$|\vec{F}| = qvB \sin\theta$$

   whose direction is perpendicular to the plane of $\vec{v}$ and $\vec{B}$.
3. Stationary charge produces electric field only whereas a moving charge produces, both electric and magnetic field.
4. From Biot and Savarts law, the magnetic field due a current elements Idl -

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin\theta}{r^2}, \ \delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{\ell}\times\hat{r}\right)}{r^3}$$

5. The magnetic field due a conductor of finite length $B = \frac{\mu_0 I}{4\pi d}\left[\sin\phi_1 + \sin\phi_2\right]$

magnetic field due to straight infinite current

$$B = \frac{\mu_0 I}{2\pi d}$$

6. Magnetic field due to circular current loop at

(i) Its center is $B_C = \frac{\mu_0 NI}{2R}$

(ii) On a point at distance x from its center -

$$B_{axial} = \frac{\mu_0 NIR^2}{2\left(R^2 + x^2\right)^{3/2}}$$

(iii) If R << x; $B = \frac{\mu_0 NIR^2}{2x^3} = \frac{\mu_0}{4\pi}\left(\frac{2\pi NIR^2}{x^3}\right)$

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

Here $M = NI\left(\pi R^2\right)$ is the magnetic moment of the coil, it shows that coil behaves like a bae magnet.

7. Helmholtz coils, has two idendical coaxial coils whose centers are at a distance equal to their radius. Magnetic field in the region, between the coils is

$B_{(middle\ region)} = 1.432\ B_{(center)}$

8. Force on a charge moving perpendicaulr to magnetic field is $F = qvB$, and path of the particle will be circular and radius of path is $\left(r = \frac{mv}{qB}\right)$, period of revolution $T = \frac{2\pi m}{qB}$ and freqeuncy $\nu = \frac{qB}{2\pi m}$

This freqeuncy is called cyclotron frequncy. The frequecny does not depends on velocity and radius of path. This concpet is used in cyclotron.

9. Cyclotron is a device, used to accelarate heavy positive particles. The frequency of applied AC is equal to cyclotron frequency.

10. Force on a current carrying conductor in magnetic field $\vec{F} = I\left(\vec{\ell}\times\vec{B}\right)$

$$\left|\vec{F}\right| = I\ell B\sin\theta$$

11. The force between two parallel currents is attractive if the current in them is parallel, and repulsive if the current is antiparallel. The force per unit length on such conductors is

$$\frac{F}{\delta\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} \text{ N/m}$$

definition of 1 A is SI; 1 A is that current maintained in two parallel wires placed 1 meter apart in air/vacuum if a force of 2×10^{-7} N/m force act per unit length on each other, then current in both conductor is 1 Ampere.

12. A current loop in uniform magnetic field experiences no net force, but the torque on the loop is

$$\vec{\tau} = NI\vec{A}\times\vec{B}$$

$$|\vec{\tau}| = NIAB\sin\theta$$

Here N = number of turns, I = Current; A = Area, of current loop B = Magnetic field angle between $\vec{A}$ and $\vec{B}$ is θ

13. Galvanometer detects electric current. In moving coil galvanometer

(i) $I = k\phi = \frac{C}{NAB}\phi$

Here I = current, N = number of turns, A = area of coil, B = magnetic field, C = restoring torque per unit deflection, K = reduction factor of galvanometer.

(ii) Current sensitivity $S_i = \frac{\phi}{I} = \frac{NAB}{C} = \frac{1}{k}$

(iii) Figure of merit $X = \frac{1}{S_i} = \frac{I}{\phi} = \frac{C}{NAB} = k$

(iv) Voltage sensitivity $S_v = \frac{\phi}{V} = \frac{NAB}{CR} = \frac{S_i}{R}$

14. The value of shunt required to convert voltmeter into ammeter is $S = \left(\frac{I_g G}{I - I_g}\right)$

15. The value of reistance required for conversion of galvanometer to voltmeter is

$R_H = \frac{V}{I_g} - G$; Resistance of ideal voltmeter is infinite.

16. Ampere's circular law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i$

17. The magnetic field due to a cylendrical conductor -

(i) Outside cylender $B = \frac{\mu_0 I}{2\pi r}$; r = distance from axis.

(ii) On the surface $B = \frac{\mu_0 I}{2\pi R}$ R = radius of conductor

(iii) Inside the conductor $B_{in} = \frac{\mu_0 I}{2\pi R}\left(\frac{r}{R}\right)$; $B_{in} \propto r$

18. Infinitly long solenoid and Toroid are used to produce uniform magnetic field.

19. The magnetic field inside an ideal solenoid is $B = \mu_0 nI$ for a finite solenoid at one end $B = \frac{\mu_0 nI}{2}$.

20. Magnetic field due to current in a toroid.

(i) $B = \mu_0 nI = \frac{\mu_0 NI}{2\pi R}$; R = Radius; N = Number of turns

(ii) B = 0 magnetic field outside is zero.

Questions for Practice

Multiple Choice Questions -

1. A charge in uniform motion, produces -

(a) Only electric field

(b) Only magetic field

(c) Both electric and magnetic field

(d) EM waves along with electric & magnetic field

2. The magentic field due to a straight current at a distance r is B. If distance becomes $r/2$ keeping I constant, then magnetic field will be -

(a) 2 B (b) B / 2

(c) B (d) B / 4

3. The magnetic field at the center of a circular current carrying coil is B_O. The magnetic on the axis at a distance equal to radius of the same coil is B. The ratio B/B_0 will be -

(a) $1 : \sqrt{2}$ (b) $1 : 2\sqrt{2}$

(c) $2\sqrt{2} : 1$ (d) $\sqrt{2} : 1$

4. Helmholtz coils are used -

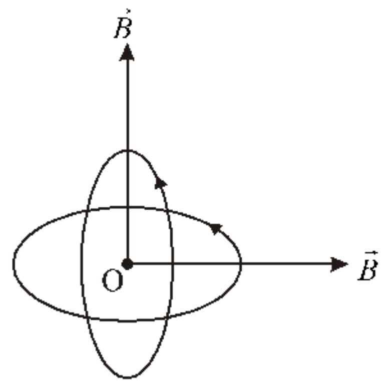
(a) To produce uniform magnetic field

(b) To measure electric current

(c) To measure magnetic field

(d) To find the direction of electric current

5. Two circular current coils are concentric and their planes are mutualy perpendicular and the magnetic field due to each coil is B, as hown in the figure: the net magnetic field at their common center will be -



(a) Zero (b) 2 B\

(c) $B/\sqrt{2}$ (d) $\sqrt{2}\,B$

6. Projected with same velocity in uniform and perpendicaulr magnetic field, which particle will experience maximum force?

(a) ${}_{-1}e^0$ (b) ${}_1H^1$

(c) ${}_2He^4$ (d) ${}_3Li^7$

7. Two wires of mains supply are at a them distance 12 cm. If the force per unit length between is equal to 4 mg weigth, the current in both the wires would be-

(a) Zero (b) 4.85 A

(c) 4.85 mA (d) 4.85×10^{-4} A

8. A proton of energy 100 eV is moving in circular path in perpendicular magnetic field of 10^{-4} T. The cyclotron frequency of the proton in Radian/s will be -

(a) 2.80×10^6 (b) 9.6×10^3

(c) 5.6×10^6 (d) 1.76×10^6

9. A galvanometer of resistance G, requires 2% of the main current as current for full scale deflection. The value of shunt will be -

(a) $\frac{G}{50}$ (b) $\frac{G}{49}$

(c) 49 G (d) 50 G

10. Magnetic field in a solenoid due to current I, is B. If the length and number of turns are doubled. To get the same magnetic field the current will be -

(a) 2 I (b) I

(c) $I/2$ (d) $I/4$

11. A toroid has a turn density n, the current is I. If the magnetic field inside (at axis) is B, the magnetic field out side will be -

(a) B (b) B / 2

(c) Zero (d) 2 B

12. A galvanometer is converted to voltmeter by connecting -

(a) High resistance in series

(b) Low resistance in series

(c) High resistance in parallel

(d) Low resistance in parallel

13. An ideal voltmeter and ideal ammeter should have -

(a) Zero and infinite resistance

(b) Infinite and zero resistance respectively

(c) Both should have zero resistance

(d) Both should have infinite resistance

Very Short Answer Questions

1. Write the name of sources, used to produce magnetic field.

2. Write the dimensions and unit of magnetic field.

3. Which field is produced by a moving charge.

4. A charge q enters a perpendicular to magnetic field B at velocity $\vec{v}$. What will be the force on the charge and path of the charge?

5. Define 1 Ampere in S.I. unit.

(Search latest definition at https://physics.nist.gov)

6. A proton is moving up word in vertical plane, it experiences a horizontal force in North direction. What will be the direction of magnetic field?

7. A charged particle is moving parallel to uniform magnetic field. What will be the path of particle?

8. A battery is connected to diametrically opposite points of circular coil. What will be the magnetic field at the center?

9. A coil of turns N and radius R is opened as a straight wire, how many times will be the magnetic field at a distance R, compared to the magnetic field at the center of the coil?

10. What will be the distance between, two points of inflations of a Helmholtz coil?

11. Write down the mathematical form of Ampere's circutal law.

12. Write down the value of magnetic field inside a copper pipe of radius R and current I.

13. Why the magnetic poles of moving coil galvanometer, made concave in shape?

14. How the current sensitivity of a galvanometer is increased?

15. At equilibrium what will be the position of coil and magnetic field in a galvanometer?

16. Why cyclotron is not used to accelarate light charged particles?

17. Which device you will use to produce uniform magnetic field?

18. How does the period of revolution of a charged particle depends on speed and radius of circular

path inside "Dee" of cyclotron.

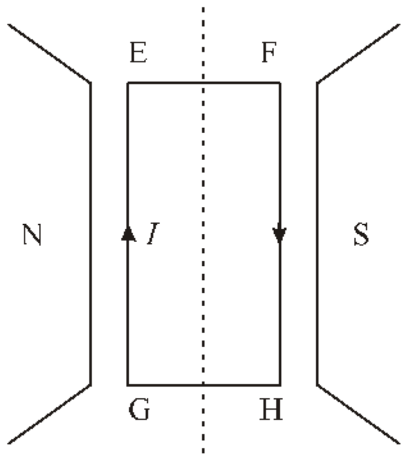
19. Write the expression for the high resistance used to convert a galvanometer in to voltmeter.

Short Answer type question-

1. Write down the conclusions from Orested's experiment.
2. Write down, Biot and Savart's law in vector form.
3. Explain the two laws to find the direction of magnetic field produced by electric current.
4. A charge enters a uniform magnetic field at an angle ($0 < \theta < 90^\circ$). What will be the path of the charge? Also find its pitch.
5. Find the relation between the magnetic field at R/2 on axis, and magnetic field at the center of coil. Here R is radius of the coil.
6. Show how a small current loop, behave like a bar magnet?
7. What is the circulation of magnetic field? Please explain.
8. What is the difference between a current carrying solenoid and a bar magnet?
9. Find the force per unit length on two parallel current carrying conductors.
10. Using Ampere's circutal law, find magnetic field inside a current carrying cylinder.
11. Show the period of half revolution of a positive charge in "Dee" of a cyclotron does not depend in the radius of circular path.
12. Explain the principle of cyclotron.
13. What is sensitivity and figure of merit of a galvanometer? How they are related to each other?
14. Find the expression for the resistance connected in parallel to convert a galvanometer to an ammeter.
15. A rectangular current loop EFGH is placed in uniform magnetic field, as shown in the figure.

(a) What will be the direction of torque on loop?

(b) When torque will be (i) maximum (ii) zero



Essay type Questions -

1. State Biot and Savant's law. Using this law find the expression for the magnetic field due to a finite straight current carrying conductor. Show that for infinitely long conductor, the field at a perpendicular distance d, is $B = \dfrac{\mu_0 I}{2\pi d}$
2. Using Biot and Savart's law, find the expression for magnetic field at an axial point of a circular current loop in vector form. Draw required diagram.
3. Describe the working of cyclotron. Draw the diagram showing path of particle in both "Dees". Derive expression for (i). Frequency of cyclotron (ii). Kinetic energy of ions in cyclotron.
4. Derive expression for force and torque on a current loop in uniform magnetic field. Draw required diagram. When the torque will be (i) maximum (ii) zero
5. Find the expression for force on a current carrying conductor in uniform magnetic field. Explain the right hand palm rule to explain the direction of force.
6. Write Ampere's circulate law. Find the expression for magnetic field in long current carrying solenoid.

Draw required diagram.

7. Describe construction of a toroid. Find expression for magnetic field at the axis of toroid of mean radius r number of turns N and current I. Show that the magnetic field out side, and in open area enclosed by toroid is zero.

8. What is a galvanometer? Describe the construction and working of a galvanometer using labbled diagram.

9. Describing the principle of a galvanometer, find the expression for its sensitivity and figure of merit. On what factors these depend.

Answer (Multiple Choice Questions)

(1) (C) 2 (A) 3 (B) 4 (A) 5 (D) 6 (D) 7 (B)
8 (B) 9 (B) 10 (B) 11 (C) 12 (A) 13 (B)

Very Short Answer Type -

1. Permanent magnet, current carrying conductor, moving charge, change in electric field.

2. $M^1L^0T^{-2}A^{-1}$ and Tesla.

3. Both, electric and magnetic field.

4. Force $\vec{F} = q(\vec{v}\times\vec{B})$; $|\vec{F}| = qvB\sin 90^\circ = qvB$ and the path will be circular.

5. If the force per unit length in air/vacuum, on two equal current carrying conductors palced at a distance 1m, is 2×10^{-7} N/m, then the current in conductor is 1 ampere.

   Search latest definition at (https//physics.nist.gov)

6. In horizontal plane towards East.

7. Rectilinear

8. Zero

9. A the center of coil $B_{centre} = \frac{\mu_0 NI}{2R}$ and due to straight wire, $B = \frac{\mu_0 I}{2\pi R}$ as required $\frac{B_{centre}}{B} = N\pi$

10. Equal to radius of coil R.

11. $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$

12. Zero

13. So that the field is radial and deflection on scale is linear.

14. By increasing number of turns, area of coil and taking soft iron core.

15. When the plane of the coil is perpendicular to B.

16. For a required energy more velocity, due to relativistic effect mass of the particle and resonance frequency changes.

17. Long solenoid.

18. Does not depend. It remains constant

$$\left(\because T = \frac{2\pi m}{qB}\right)$$

19. $R = \frac{V}{I_g} - G$; here I_g = current for full scale deflection.

    G = Resistance of galvanometer; V = Range of voltmeter.

Numerical Problems

1. Find the magnetic field at the center of a circular coil of radius 8.0 cm and 100 turns, having a current 0.40 A.

   ($3.1\times10^{-4}\ T$)

2. A circular coil made from 6.28cm length of wire has a radius of 0.10cm and current 1.0 A. Find magent field at its center.

   ($6.28\times10^{-5}\ T$)

3. A long straight wire has a current 35 A. Find magnetic field at a distance of 10cm.

   ($3.5\times10^{-5}\ T$)

4. A wire having current of 10 A is on the plane of table. Another wire in which current is 6A is just over the wire AB, at a distance of 2 mm. find the mass per unit length of CD, such that it held there by magnetic force. What will be the direction of

current in CD, with respect to the current in AB?

(m/l $= 6 \times 10^{4}$ kg / m; opposite to AB)

5. A wire on horizontal plane has a current of 50A in south to North direction. Find the magnitude and direction of magnetic field at point 2.5m towards east.

($4 \times 10^{-6}\ T$ downwards)

6. Two long parallel wires having current I and 3I in same direction are 4m a part. Find the point where magnetic due to both is Zero.

(1 cm from I, between them)

7. A proton enters a perpendicular magnetic field of 0.2T with velocity 6.0×10^{5} m/sec. Find acceleration of proton and radius of its path.

(1.15×10^{13} m/s^2 and 0.031 m)

8. A wire with current 8 A is placed in a magnetic field of 0.15T, at an angle 30^0 form B. Find the force per unit length, and direction of force.

(0.6 N/m)

9. Two identical coils of radius 8cm and number of turns 100, are fixed coaxially at distance 12cm. If the current in them is 1 A in the same direction, find magnetic field at mid point on the axis.

($8.04 \times 10^{-4}\ T$)

10. Two wire of length 2m each are parallel and are at a distance 0.2m from each other; if the current in both is 0.2A in the same direction. Find force per unit length on them.

(2×10^{7} N/m)

11. A Square coil of side 10 cm and 20turms have a current 12 A is suspended vertically. If its area vector makes an angle 30^0 with uniform magnetic field of 0.08T. Find the torque on the coil.

(0.96 N $\times$ m)

12. Find the ratio of the radii of circular paths traced by an α particle and beam of proton, entering with equal velocity v in perpendicular magnetic field.

($\frac{r_\alpha}{r_p} = \frac{2}{1}$)

13. Radius of "Dee" of cyclotron is 0.5m. A magnetic field of 1.7 T is perpendicular to it. Find the maximum energy gained by proton.

($5.53 \times 10^{12}\ J$)

14. Resistance of a galvanometer coil is 12 Ω, the current required for full scale deflection is 2mA. How you will convert it to a voltmeter of range 0-18 volt.

(By connecting 5988 resistor in series)

15. A galvanometer of resistance 99 Ω, requires 4mA current for its full scale deffection. what you will do to convert it into an ammeter of range 0-6 A?

(By connecting $6.6 \times 10^{2}\ \Omega$ resister in parallel)

16. A 1.0m long solenoid has 100 turns. Its radius is 1cm. Find magnetic filed at its axis, if current in it is 5A. Find the force on electron moving with velocity 10^4 m/s along the axis.

($B = 6.28 \times 10^{-3}\ T$ cy F = 0 N)

17. A solenoid has length 0.5m. Its winding is in double layer, each layer having 500 turns. Its radius is 1.4 cm. Find magnetic field at its center when current in it is 5A.

($12.56 \times 10^{3}\ T$)

Chapter - 8

Magnetism and Properties of Magnetic Substacnes

We have studied about magnets in earlier classes. The property of attracting iron, cobalt, nickel is called magntism and the material that shows this property is called magnet. In this chapter we will study about different physical quantities realted to magnetism. The earth also behave like a magnet, so we will study about elements of its magnetism. Every material show some magnetic property, so we will classify materials on this basis.

8.1 Natural Magnets

In anciant times in Greece, some deposits were found in island magnesia, which showed the property of attracting iron, cobatt and nickel. This material was named magnet after the place, and the property as magnetism. It was magnetite, an ore of iron (Fe_3O_4). Normally natural magnets are not used because of their irregular shape and weak strength.

8.2 Artifical Magnets

Artifical magnets are prepared either by rubbing a ferromagnetic material by a strong magnet, keeping in long contact with pole of a magnet or by keeping in magnetic field of a solenoid. They are of two types (i) permanent (ii) temporary.

(i) Permanent magnet - Their magnetism stay for a long time. They are prepared by hard steel, cobalt steel, tungustan or an alloy *ALNICO (Al + Ni + Co)* or other alloys. Their magnetism can't be controlled.

(ii) Temporary Magnet - These magnets shows the property as long as magnetizing field exist. Their shape an strength can be manipulated. They normally are of soft iron, and used in motor, generator, electric bell and electromagnetic relay.

8.2.1 Properties of Bar Magnet

Normally magnets are used in the form of a bar. They have following basic properties -

(i) Attraction property - These magnets attract iron, nickel and cobalt like feromagnetic materials. The strength is maximum at ends (called Pole) and zero at mid point.

(ii) Directional Property - When suspended freely from centre of gravity, the bar magnet, in equilibrium it always stay in N-S direction. The end which stay in north direction is marked as north pole, the end which stay in south direction as south pole.

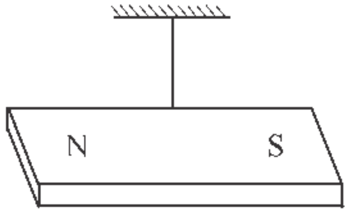


Fig 8.1 Directional property of a bar magnet

(iii) Existance as dispole - Magnets always exist as dipole. North pole and south pole are equal in magnetude (strength). Existance of monopole is not possible.

(iv) Attraction and repulsion - Same poles repell each other, while opposite poles attract.

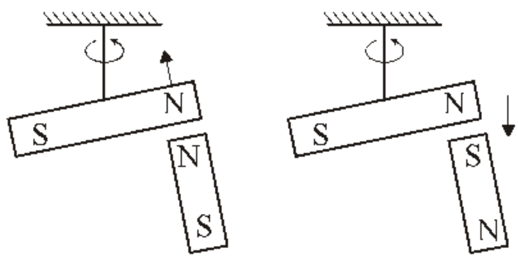


Fig 8.2 Attraction and repulsion light

(v) Equality of pole strength - The pole strength of both the poles are always same. As shown in the figure 8.3 (A) and (B). If devided transversaly, the two pieces behave like two separate magnets of same pole strength. And if devided longitudinally into two parts, their pole strength will be halved. Here m is taken as symbol of pole strength.

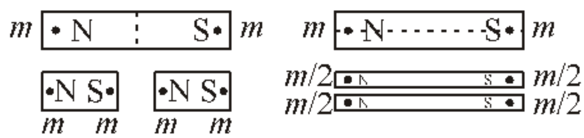


Fig 8.3 Division of pole strength of a magnet

(vi) Repulsion is a sure test to differentiate between a magnet and an iron bar- If you have two bars, one is certainly magnet, but the other you don't know. To test, bring one end of the second bar towards a magnet; if it gets attracted - other may be magnet or iron bar. If the second end is brought near the magnet -

(1) If it is attracted - the other bar is iron bar and

(2) If it gets repelled - the other bar is a magnet hence repulsion is sure test in this case.

(vii) Magnetic induction - When a magnetic material is kept near in contact with a bar magnet it acquires magnetic property. This phenomenon is called manetic induction. The other pole will be of opposite polarity.

(viii) Demagnetization - A magnet will loose its property by heating, beating by a hammer, kept under influence of AC current or by keeping dumped in earth for a long time.

(ix) Repulsion of some material - Some materials get repelled by a magnet these materials are called diamagnetic materials, water, gold and silver are some examples. This repulsion is extreamly weak and can only be observed by sensitive devices or arrangement.

8.2.2 Some Definations Realted to Magnet

(i) Magnetic Poles and Magnetic Axis - At the point, very near to the end of a bar magnet magnetism (field) is strongest. These points are called poles. All the field lines (lines of action of magnetic force) pass through these points.

The imaginary line passing through the poles and extending up to infinity is called axis of a bar magnet.

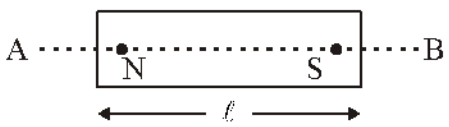


Fig 8.4 Magnetic axis and effective lenth

(ii) Effective length of a bar magnet - The distance between the two poles is called effecitve length of a bar magnet. It is aproximatly 5/6 of the physical length. (Since poles are situated slightly inside and not exactly at ends). It is a vector quantity $\vec{\ell}$ and its direction is from S pole to N pole.

(iii) Magnetic Meridian - The imaginary vertical plane passing through the axis of a bar magnet, when it is freely suspended and is in equilibrium is called magnetic meridian. Any other plane which is parallel to the aboce mentioned will also be magnetic meridian.

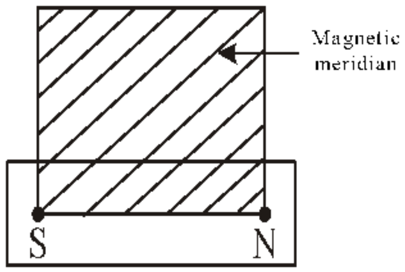


Fig 8.5 Magnetic meridian

(iv) Pole Strength - The attraction power of a magnet is expressed by a physical quantity called pole strength its symbol is m and unit $A\ m$.

(v) Coulomb's Law for the force between magnetic poles - Just as gravitational force between two masses and electrostic force between two electric charges, we have inverse square law for magnetism.

The Coulomb force is inversely proportional to the squared distance between the magnetic poles and directly proportional to the product of poles' strengths.

Let two poles of strength m_1 and m_2 are at a distance from each other in air/vacuum. (Situation is maginary since magnetic monopole does not exist). The attractive or repulsive force between them is

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = k\frac{m_1 m_2}{r^2}$$

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} \quad \text{. . . (8.1) is given by}$$

here $k = \frac{\mu_0}{4\pi} = 10^{-7}$ Web/Am is a constant

and μ_0 = magentic permiability of air/vaccum.

$$F = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{N}$$

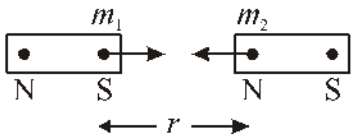


Fig 8.6 Force between magnetic poles

Definition of unit pole

If $m_1 = m_2 = 1\,\text{A m}$, $r = 1\,\text{m}$ and

$F = \frac{\mu_0}{4\pi} = 10^{-7}\,\text{N}$. Then using equation (8.1) we

get $m_1 = m_2 = 1\,\text{A m}$ (i.e unit pole)

If two equal poles separated by 1m in air/vacuum experience a force of 10^{-7} N, then both poles have unit pole strength.

8.3 Magnetic Field Lines (Magnetic Lines of Forces)

If a small magnetic compass needle is moved in a magnetic field, and path of one pole is traced using dots, joining these dots gives curves which are called magnic field lines. (Earliar it was defined as the locus of a free north pole in a magnetic field).

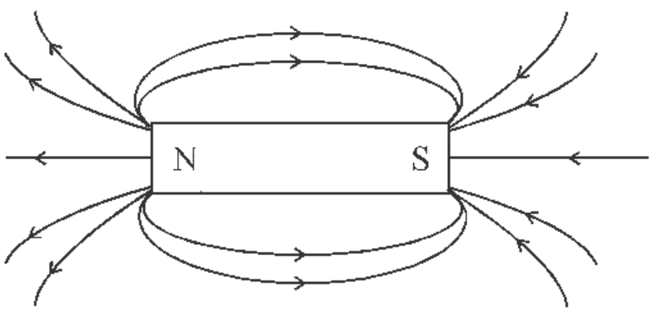


Fig 8.7 Magnetic field lines

Fig 8.7 show the magnetic field lines of a bar magnet. Place a paper or a glass sheet on a bar manget. Sprikle some iron fillings on the glass sheet. The pattern of the iron fillings will give you an idea of the field lines.

Properities of Field Lines

(i) They are imaginary and are in the form of closed loops.

(ii) Outside the magnet their direction is from N to S, while inside they are from S to N.

(iii) The tangent drawn at any point on the field line, gives the direction of magnetic field at that point.

(iv) They never intersect each other. If they do, two tangents can be drawn on the point of intersection which gives two directions of the field, which is not possible.

(v) The area where field lines are close, the field is strong and vice-versa. The field is strong near the poles.

(vi) The paralled field lines represent uniform magnetic field.

8.4 Neutral Point

The earth has its own magnetic field. So when we place a bar magnet on a paper placed on wooden table, there will be super position of two fields (i.e of earth and bar magnet). The pattern of field lines drawn, show some (one/two) points where the net magnetic field is zero. These points are called null points. At these points, the horizontal component of earth's magnetic field exactly cancells the magnet field of bar magnet i.e $B - B_{H} = 0$. The location of null points depends on the orientation of the bar magnet.

(i) When S pole of bar magnet is in Geographical North Direction - In this position/ orientation as shown in fig (8.8A). The field on equator (i.e $\perp$ line to axis) is strong near the magnet because both fields are in the same direction. We get null points on axis as N_1 and N_2.

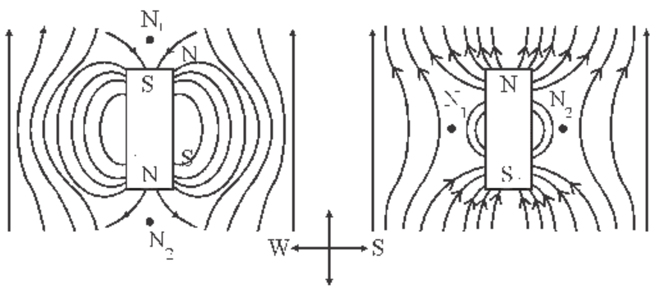


Fig 8.8 Neutral Point (A) When S pole of the magnet is towards geographical north. (B) When N pole of magnet is towards geographical north.

(ii) When North Pole of Bar Magnet is owards geographical North - In this case the null points N_1 and N_2 are obtained on equitorial line, as shown in fig 8.8 (B).

(iii) If the bar magnet is kept vertical and its north pole is downwards on the paper; the field lines will be as shown in fig 8.9. We get only one null point is in south direction. If south pole is kept downwards the position of null point is reversed, i.e. in north.

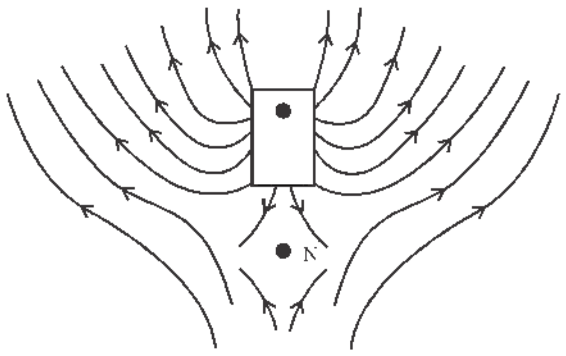


Fig 8.9 : Field lines when magnet is kept verticle

8.5 Magnetic Dipole and Magnetic Dipole Moment

8.5.1 Magnetic Dipole

In a bar magnet, both north pole and south pole exist. In a current carying solenoid, one end behaves like north pole and the other as south pole. In a current loop, one face behaves like north pole and other face as south pole. In nut shell magnetism exist as dipole and it is the elementary entity of magnetism. Mono pole is not possible. Even if we go on dividing a magnet into two, each piece has dipole.

8.5.2 Magentic Dipole Moment

It is the physical quantity which gives the strength of a magnet.

We have learnt that a coil having N turns, area A, current I, experiences a torque in external uniform magnetic field B;

$$\tau = NIAB\sin\theta \qquad \ldots (8.2)$$

Comparing it with the torque experienced by an electric dipole in external electrical field E,

$$\tau = pE\sin\theta \qquad \ldots (8.3)$$

We get an equivalent quantity to p as NIA, this quantity is similarly named as magnetic dipole moment M. Just as in electrostatics, we can write where m is pole strength and is a vector distance between two poles, which is from S to N.

In $\vec{M} = NI\vec{A}$ . . . (8.4)

the direction of $\vec{A}$ is $\perp$ to the plane of current loop. The unit of $\vec{M}$ is A m^2.

It can be defined using realtion

$\tau = MB\sin\theta$ . . . (8.5) for a dipole. If B = 1, $\theta = 90°$, then $\tau = M$. Hence the magnetic moment is equal to the torque experienced by a dipole in unit magentic field, when it is placed perpnedicular to the field.

8.5.3 Magnetic Moment of a Bar Magnet

It is given by $M = m \times \ell$ . . . (8.6)

where m = pole strength and ℓ is effective length of a bar magnet.

(A) If we bisect a bar magnet of dipole momnet M into two, perpendicular to its length. The dipole moment of each part is

$$M_1 = m \times \frac{\ell}{2} = \frac{m\ell}{2} = \frac{M}{2}$$

(B) If a bar magnet is dividend into two, by dividing along its length, again

$$M_2 = \frac{m}{2} \times \ell = \frac{m\ell}{2} = \frac{M}{2}$$

(C) If a bar magnet of length 2 ℓ and magnetic moment M is bent into a semi circle of radius r. Cirumferance of semi circle $\pi r = \ell$. Then $M_3 = m \times 2r$;

$$M_3 = m \times \frac{2\ell}{\pi} = \frac{2M}{\pi}$$

(D) If two bar imagnets are kept at an angle θ between their axis, the net magnetic moment will be vector sum of indivisul M.

$$M = \sqrt{M_1^2 + 2M_1M_2\cos\theta + M_2^2} \qquad \ldots (8.7)$$

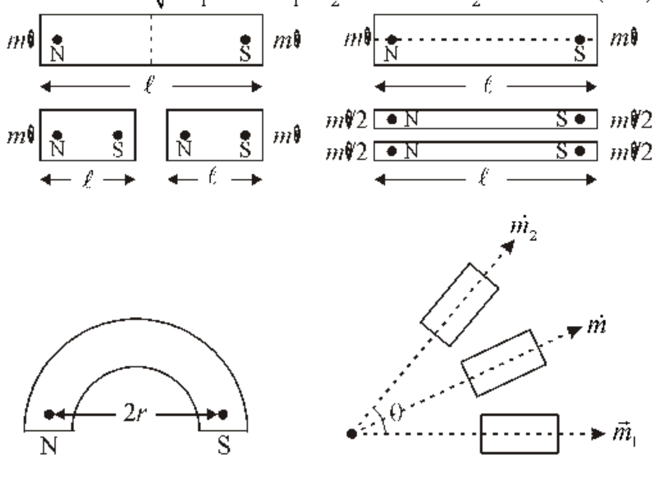


Fig 8.10 Magnetic moment of a bar magnet

8.5.2.2 Magentic Moment of a Revolving Electron

In an atom, electrons revolve round the nucleus, which is equivalent to current loop. Hence every orbit (having one electron) has a magnetic moment M = N I A. For an revolution (N=1), charge = e, and time t=T hence

$$I = \frac{e}{T} \text{ but } M = \frac{e}{T} \times \pi r^2 \text{ or } T = \frac{2\pi}{\omega}$$

where r = radius of orbit, v = linear velocity of electron and w = angular velocity.

$$\text{So } M = \frac{e}{2\pi/\omega} \times \pi r^2 = \frac{1}{2} e\omega r^2 \qquad \ldots (8.8)$$

$$\text{and } M = \frac{1}{2} evr \qquad \ldots (8.9)$$

since $v = r\omega$

Example 8.1 : Find the magnetic moment of a bar magnet with pole strength 40 Am and effective length 5 cm.

Solution : $M = m \times \ell$

$m = 40\text{ Am}$, $l = 5\text{ cm} = 0.05\text{ m}$

$M = 0.05 \times 40 = 2\text{ Am}^2$

Example 8.2 : A current carrying coil has magnetic moment 5 Am^2. If its radius is halved and the current is doubled, what will be new magnetic moment compared to previous one.

Solution : $M = NI\pi r^2$ The new magentic moment will be $M' = NI'\pi r'^2$

$$r' = \frac{r}{2} \quad I' = 2I$$

$$\frac{M'}{M} = \frac{N \times 2I \times (r/2)^2}{N \times I \times r^2} = \frac{1}{2};$$

deviding we get $M' = \dfrac{M}{2}$

Example 8.3 : Find the magentic moment of the elctron in first orbit of hydrogen atom.

$(r = 0.53\,\text{Å},\ v = 2.2 \times 10^6\text{ ms}^{-1})$.

Solution : $M = \dfrac{1}{2} evr$

$r = 0.53\,\text{Å}$

$r = 0.53 \times 10^{-10}\text{ m}$

$v = 2.2 \times 10^6\text{ m s}^{-1}$

$$M = \frac{1}{2} \times 1.6 \times 10^{-19} \times 2.2 \times 10^6 \times 0.53 \times 10^{-10}$$

$M = 0.93 \times 10^{-23}\text{ Am}^2$

Note : It is called Bohr Mahneton and is a fundamental constant in physics.

8.6 Intensity of Magnetic Field

Earlier it was defined as $\vec{B}$ (force on unit N pole). In analogy to (force per unit positive charge).

But now we define it using the realtion $\tau = \vec{M} \times \vec{B}$ (since M is the most basic quantity in magnetism and magnetic momopole does not exist.

B is equal to the torque experienced by a magnet of unit magnetic moment placed perpendicular to it.

Its SI unit is N / Am, or Tesla.

8.6.1 Magentic Field at an Axial Point of a Bar Magnet

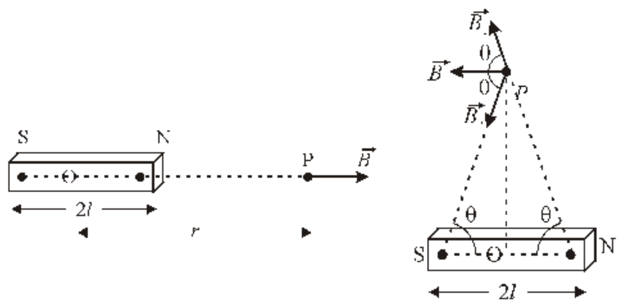


Fig 8.11 : Intensity at an equatorial point

As shown in fig 8.11, pole strength of a bar magnet is m, its effective length is $2l$. The magnet field at an axial point at a distance r from the center O, of the bar magnet is $\vec{B} = \vec{B}_1 + \vec{B}_2$, where $\vec{B}_1$ and $\vec{B}_2$ are the field due to north and south pole at p. Using Coulomb's law, and definition of B we get ; hence

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(r - l)^2}$$

and $B_2 = \frac{\mu_0}{4\pi}\frac{(m)}{(r+l)^2}$ . . . (8.9b)

By convention, taking pole strength of north pole is (+m) and for that of south pole as (-m).

We get $B = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2} + \frac{\mu_0(-m)}{4\pi(r+l)^2}$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$= \frac{\mu_0 m}{4\pi} \times \frac{m4rl}{(r^2-l^2)^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2} \quad [\because M = m \times 2l] \quad . . . (8.10)$$

For Special Condition of $l << r$ (for a very small magnet) we modify equ.8.10 by taking common from denominator-

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2}$$

here l^2/r^2 is a negligible quanitity

hence $B = \frac{\mu_0}{4\pi} \times \frac{2M}{r^3}$ . . . (8.11)

8.6.2 Magnetic Field due to Bar Magnet at its Equitorial Point

As clear from fig 8.11, the magentic field at P, due to north pole is $B_1 = \frac{\mu_0}{4\pi} \times \frac{m}{(r^2+l^2)}$ along line NP and away from N. Similarly field due to south pole is

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{m}{(r^2+l^2)}$$ (a long Ps, towards S).

Resolving $\vec{B}_1$ and $\vec{B}_2$ into components along axis and equator. We see that the sine components get cancelled being equal and opposite; and only cosine components contribute to $\vec{B}$ at equator. We get

$\vec{B} = \vec{B}_1 + \vec{B}_2$ (only cosine components of $\vec{B}_1$ & $\vec{B}_2$)

$\therefore \quad B = B_1\cos\theta + B_2\cos\theta$

$B = 2B_1\cos\theta \qquad [\because B_1 = B_2]$

$\therefore \quad B = \frac{\mu_0 \times 2m}{4\pi(r^2+l^2)} \times \cos\theta$

(but $\cos\theta = \frac{\ell}{(r^2+l^2)^{1/2}}$)

$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2ml}{(r^2+l^2)^{3/2}}$ . . . (8.12a)

Again for special condition for a small ba magnet $l << r$, l^2/r^2 being negligible, we get

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3} \quad [\because M = m \times 2l] \quad . . . (8.12b)$$

The direction of $\vec{B}$ here is opposite to that of $\vec{M}$.

We appreciate the similarity to that of the electric field due an electric dipole at axis and equator

$$\vec{E} = \frac{1}{4\pi \in_0}\frac{2\vec{p}}{r^3} \quad . . . (8.13a)$$

$$\vec{E} = \frac{1}{4\pi \in_0}\frac{\vec{p}}{r^3} \quad . . . (8.13b)$$

here M is replaced by P and μ_0 by $\frac{1}{\in_0}$.

8.7 The Torque on a Bar Magnet in Uniform External Magnet Field

As in fig 8.12 a bar magnet of pole strength m and effective length 2ℓ is placed in external uniform magnetic field $\vec{B}$, such that it is parallel to $\vec{B}$. The net force on the magnet is $F = F_N + F_S = mB + (-mB) = 0$. Also the torque is zero.

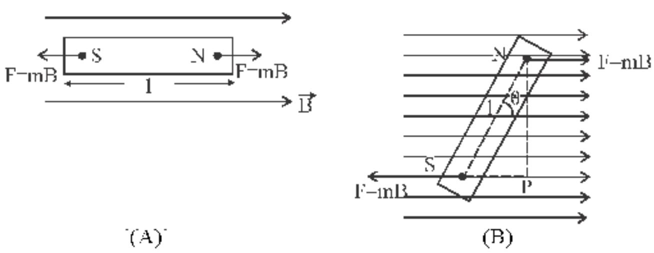


Fig 8.12 (A) and (B) Force on poles of a bar magnet in external magentic field

If the bar magnet is slightly rotated by an angle θ from equilibrium (θ is angle between $\vec{B}$ and 2ℓ), The net force on the magnet is again zero; but the magnet experiences a torque which is $\tau = mB \, x \, NP$(distance between forces).

$$\tau = mB \times NP$$

$$\tau = mB \times \ell \sin\theta$$

$$\tau = (m \times \ell)B \sin\theta$$

$$\tau = MB \sin\theta \qquad \text{. . . (8.14a)}$$

$$(M = m \times \ell)$$

$$\vec{\tau} = \vec{M} \times \vec{B} \qquad \text{. . . (8.14b)}$$

Special Conditions

(i) When $(\theta = 0)$ magnet being parallel to B

$$\tau = MB \sin 0 = 0$$

(ii) When magnet is perpendicular to $\vec{B}$, then $(\theta = 90^\circ)$ and $\tau_{max} = MB \sin 90 = MB$. The torque will be maximum.

The potential energy of a bar magnet (manetic dipole) in external magnetic field.

Um = work done in rotating the magnet by angle θ.

$U_m = \int \tau(\theta) d\theta$ here shows that τ is a function of θ.

$$U_m = \int \tau(\theta) d\theta$$

$$= \int MB \sin\theta \, d\theta = -MB \cos\theta$$

$$U_M = -\vec{M}.\vec{B} \qquad \text{. . . (8.15)}$$

for position $\theta = 0^\circ$; $(U_M)_{Min} = -MB$ (most stable condition for (ii) $\theta = \pi$, $(U_M)_{Max} = MB$)

(most unstable)

If we rotate the magnet by an angle to θ. then the definit integral will give you -

$$W = \int_0^\theta MB \sin\theta \, d\theta$$

$$W = MB\left[-\cos\theta\right]_0^\theta$$

$$= MB(\cos 0 - \cos\theta)$$

$$W = MB(1 - \cos\theta) \qquad \text{. . . (8.16)}$$

Example 8.4 : Find the torque on a bar magnet of pole strength 25 Am and effective length 10 cm, which is at an angle $\theta = 30^\circ$ from B_H(earth magentic field)

($B_H = 0.4 \times 10^{-4}$T).

Solution : $\tau = MB_H \sin\theta$

$$m = 25 \text{ Am}, \ \ell = 0.1\text{m}, \theta = 30^\circ$$

$$\tau = m\ell B_H \sin\theta$$

$$= 25 \times 0.1 \times 0.4 \times 10^{-4} \times 0.5$$

$$= 0.5 \times 10^{-4} \text{ N m}$$

Example 8.5 : A magnet of magnetic moment 5 Am^2 is placed in magnetic filled 0.2 T. Find the work done in rotating it, from parallel to antiparallel position. Also find the potential energy at the two positions.

Solution : The work done in rotating the magnet is

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$W = 5 \times 0.2(\cos 0 - \cos 180)$$

$$= 1.0\ (1 + 1) = 2 \text{ J}$$

Similarly the energy in position 1 i.e. θ=0

$$U_1 = -MB\cos\theta_1 = -MB\cos 0 = -MB$$

$$= -5 \times 0.2 = -1 \text{J}$$

$$U_2 = -MB\cos 180 = MB = 5 \times 0.2 = 1\text{J}$$

8.8 Earth's Magnetism

The earth behave like a bar magnet, it has its own magnetic field. It is called geomagnetism. The following facts confirm it -

(i) A bar magnet freely suspended from its C G always stay is NS direction in equilibrium.

(ii) An iron piece kept burried in earth for a long time aquires magnetism.

(iii) We get null points while ploting field lines of a bar magnet.

The magnetic field on the surfaces varies from place to place and is of the order 10^{-5}T.

8.8.1 Cause of Earth's Magnetism

The origion of geomagnetism is not well understood. There are certain explanations, the simplest one of existance of a giant magnet at the center of earth was rejected due to hot conditions inside earth where such magnet can't exist. The other one is "dynemo effect", given by Elsister is most accepted. According to it certain metals like iron and nickel exist in the outer core of earth in molten and ionic form. They rotate with earth and causes convention current which in turn produces magentic field.

Magnetic field lines of earth are maped, and their simplest version resembles to the field lines of an imaginary bar magnet placed inside the earth such that its axis makes an angle 11.30^0 to earth's axis and its north pole is towards south pole and south pole towards north pole of the earth. The location of these pole on earth surface are at lattitude $79.74°N$ and longitude $71.8°W$ a place in north canada. And $79.74°S$ and $108.22°E$ which is in Antartictica.

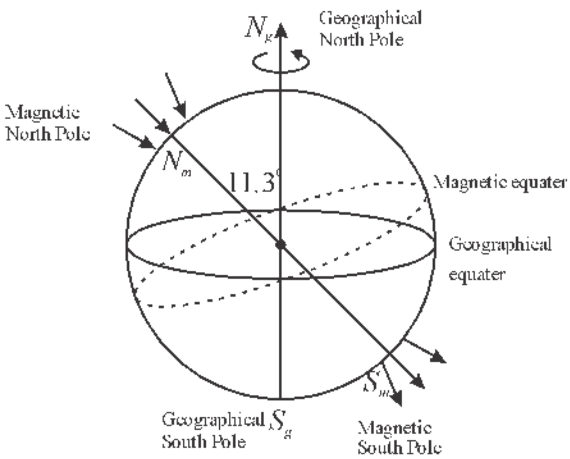


Fig 8.13 : Geomgnetism

8.8.2 Elements of Earth's Magnetism

To know about earth's magnetic field at certain place, need to know about three physical quantites about it. These are called elements of earth's magnetism. They are -

(1) Angle of declination (or simply declination).

(2) Dip angle.

(3) Intensity of earth's magnetism (or simpley horizontal component of it). First two gives the direction of earth's magnetic field in horizontal and vertical plane respectively, and third gives its magnitude (or magnitude in hortizontal direction).

(1) Angle of declination

Magentic meridian is a vertical plane passing through the axis of a freely suspended bar magnet from its CG and is in equilibrium.

Geographical meridian is a vertical plane passing through the axis of rotation of earth. It also contain longitude circle.

The angle of declination is the acute angle between magnetic meridian and geographical meridian. It is different at different places of earth. A Delhi it is $0^0.41$ E and at Mumbai it is $0^0 58'$ W. These small values shows that at these places, the direction shown by a compass needle is true north south.

(ii) Dip angle or angle of dip

If we take a compass needle, which is free to rotate about a horizontal axis, in vertical plane, then in equilibrium, the angle between its axis with horizontal is called angle of dip. It gives the direction of earth's magnetic field in verticle plane. Again the dip angle vaies from place to place on surface of the earth. It is O^0 at equator and 90^0 at poles.

(iii) Horizontal component of earth's magnetic field

At a place other then equator or pole the direction of magnetic field make certain angle with horizontal. We can resolve this magnetic field B, into two components, as B_V and B_H i.e. vertical component and horizontal component

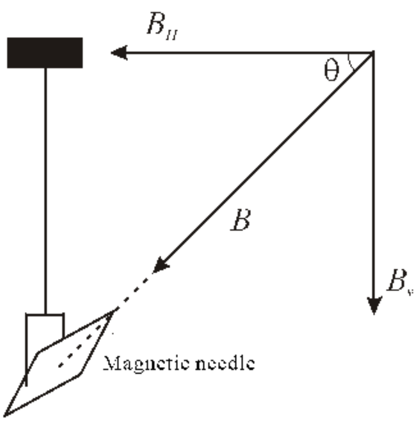


Fig 8.16 Components of earth's magnetic field

$$B_V = B\sin\theta \qquad \ldots (8.17)$$

$$B_H = B\cos\theta \qquad \ldots (8.18)$$

Such that $\vec{B} = \vec{B}_H + \vec{B}_v$

$$B = \sqrt{B_H^2 + B_v^2 + 2B_H B_v \cos 90^\circ}$$

$$B = \sqrt{B_H^2 + B_v^2}$$

$$\tan\theta = \frac{B_v}{B_H}$$

$$B_v = B_H \tan\theta \qquad \ldots (8.19)$$

Example 8.6 : At certian point on earth surface dip angle is 60^0 and horizontal component of earth's magnetic field is 0.25 G. Find vertical component of earth's magentic field at this place. Also find the resultant magnetic field at that point.

Solution : $B_v = B_H \tan\theta$

$$B_v = 0.25 \tan 60^\circ$$

$$= 0.25 \times \sqrt{3} = 0.25 \times 1.732 = 0.433\text{G}$$

$$B_H = B\cos\theta$$

$$B = \frac{B_H}{\cos\theta} = \frac{0.25}{\cos 60} = \frac{0.25}{0.5}$$

$$B = 0.50\,\text{G}$$

8.9 Magnetism and Gauss's Law

According Gauss's law in electrostatics i.e for a closed surface integral of electric filed is proportional to the algebric sum of charges enclosed by the surface.

If a surface encloses electric dipole . The incoming electric flux is equal to outgoing electric flux. Exaclty in the same wayfor magnetic field $\phi_B = \oint_S \vec{B}\cdot d\vec{S}$ because magnet always exist as dipole. The incoming magnetic flux through a closed surface is exactly equal to the outgoing flux, and net flux = 0. Hence Gausses law for magnetism is

$$\phi_B = \oint_S \vec{B}\cdot d\vec{S} = 0 \qquad \ldots (8.20)$$

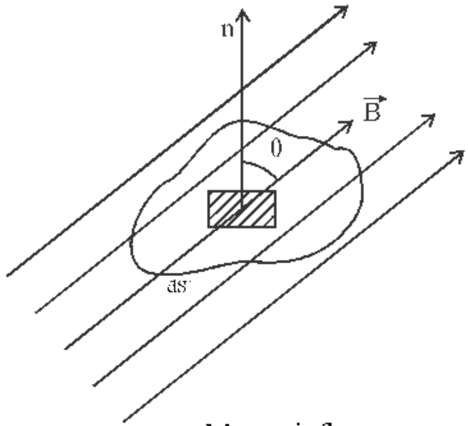


Fig 8.17 : Magnetic flux

8.10 Behaviour of Substances/Materials in External Magnetic Field

Farady found that all materials are effected by magnetic field or react to external magnetic field. However they interact with the field differently.

To invesitigate the behaviour of different materials following experimental set up is suggested -

A current carrying sole noid produces magnetic field, strong inside and weak near its ends. A test tube with sample to be tested is attached to a very sensitive spring balance near the end, as shown in fig 8.18.

When a current is set in the sole noid -

(i) Some materials like, iron, nickel and cobalt are strongly pulled inside the sole noid, these materials move from weak field to strong field.

(ii) Some other materials like aluminium, $CuCl_2$ etc are weakly attracted inside, i.e. with very weak force.

(iii) Majority of the materials like Z_n, B_i, gold etc are weakly repelled out in above experiment i.e they go from strong magnetic field to weak magnetic field.

There are many more types of materials, which you will know in chemistry or in higher classes. Here we restrict ourself to only three types mentioned above.

(i) The first type strongly attacted b y mangetic field are called Ferromagnetic materials.

(ii) The second type which are very weakly attracted by magnetic field are called paramagnetic materials.

(iii) The third type which is very weakly repelled by magnetic field is called dimagnetic, material.

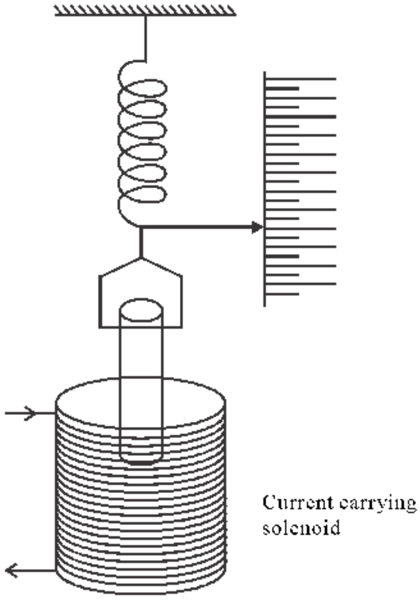


Fig 8.18 Behaviour of materials in a solenoid

8.11 Important Physical Quantities Related to Magnetism

8.11.1 Intensity of magnetisation I

We know that the circulating electron has a magnetic moment; when material is not magnetized, the magnetic dipole sum up to zero. When an external magnetic field is applied the magnetic moments are alligned in a particular direction, and the material gets a net non-zero dipole moment. The net dipole moment per unit volume is defined as magnetization or intensity of magnetisation. Its symbol is I, and its unit is Am^{-1}. It is a vector quantity. Its dimensional formula is $[M^0L^{-1}T^0A^1]$.

8.11.2 Magnetizing Field H

The magnetic field produced only by electric current (without any contribution of medium) in sole noid, it is nI or $\vec{H} = \dfrac{\vec{B}_0}{\mu_0}$. It is also called magnetic intensity. It is the external field that induce magentic property in material. Its unit is also Am^{-1}.

8.11.3 Magnetic Susceptibility

When a material is placed in external magnetizing field H, the material get magnetized. For small magnetizing field, the I aquired by the material is proportional to H. i.e $I \propto H$ or $I = \chi_m H$ here χ_m is a constant for a perticular material called the magnetic susceptibility of that material. It is defined as, if H = 1, then $\chi_m = I$ in words we say the magnetization I aquired by the material in unit magnetizing field is equal to its magnetic susceptibility.

8.11.4 Magnetic permeability μ

It is a measure of ability of the medium to allow magnetic field to set in the medium. μ_0 is magnetic permeability of vacuum/air. μ is magnetic permiability of medium. And $\mu_r = \dfrac{\mu}{\mu_0}$ is relative magnetic permiability of the medium.

8.12 Relation between different Magnetic Quantities

Imagine a core of a material is placed in-side the sole noid. The net magnetic field produced by the system is

$$B = B_0 + B_1 \qquad \text{. . . (8.25)}$$

B_0 is contribution of current B_m is contribution of material of core.

here $B_1 \propto I$

$$\Rightarrow \quad B_1 = \mu_0 I$$

again $B = \mu_0 H + \mu_0 I$

Substituting in equation 8.25

$$B = \mu_0 (H + I) \qquad \text{. . . (8.26)}$$

$$B = \mu_0 (H + \chi_m H) \qquad [\because I = \chi_m H]$$

$$\frac{B}{H} = \mu_0 (1 + \chi_m)$$

$$\mu = \mu_0 (1 + \chi_m) \qquad \because \frac{\mu}{\mu_0} = \mu_r$$

$$\mu_r = (1 + \chi_m) \qquad \text{. . . (8.27)}$$

Or $$\frac{B}{\mu_0} - I = H \qquad \text{. . . (8.28)}$$

Example 8.7 : The paramagentic material cromium has magnetic susceptibility as 2.7×10^{-4}. Find its magnetic permeability and relative magnetic permeability.

Solution : $\mu = \mu_0 (1 + \chi_m)$

$\chi_m = 2.7 \times 10^{-4}$

$\mu = 4\pi \times 10^{\ 7}(1 + 2.7 \times 10^{\ 4})$

$\mu = 12.56 \times 1.00027 \times 10^{\ 7}$

$\mu = 12.5634 \times 10^{\ 7}$ H / m

Relative permeability

$\mu_r = 1 + \chi_m$

$= 1 + 2.7 \times 10^{\ 4} = 1.00027$

Example 8.8 : Paramagnetic material Aluminium has magnetic susceptibility $2.3 \times 10^{\ 5}$. It is placed in a manetizing field 4×10^5 Am $^{\ 1}$. Find the magentization of the material.

Solution : $I = \chi_m H$

$\chi_m = 2.3 \times 10^{\ 5}$ and $H = 4 \times 10^5$ A / m

$I = 2.3 \times 10^{\ 5} \times 4 \times 10^5 = 9.2$ A / m

Example 8.9 : An iron wire of length $l = 1m$ and cross section 1mm² is placed inside a sole noid, which produced a magnetizing field 4×10^3 A / m. Find the magnetic moment of the wire.($16\pi \times 10^{-5}$ H/m)

Solution : $\chi_m = \frac{I}{H} = \frac{M}{H\ V}$

$M = \chi_m HV$

$\ell = 1\,\text{m}$,

$A = 1\,\text{mm}^2 = 10^{\ 6}$ m

$H = 4 \times 10^3$ A/m , $\mu = 16\pi \times 10^{-5}\ H/m$

$V = A\ell = 10^{-6} \times 1 = 10^{-6}$ m³

$$\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{16\pi \times 10^{-5}}{4\pi \times 10^{-7}} - 1 = 400 - 1 = 399$$

$M = \chi_m HV$

$= 399 \times 4 \times 10^3 \times 10^{-6} = 1.596$ A m²

Example 8.10 : A rod of cross section 0.40 cm² is placed in magnetizing field 4000 Am $^{\ 1}$. If the magnetic flux passing the rod is $5 \times 10^{\ 5}$ Wb, then find magnetic induction, magnetic susceptibility and magnetization of the material of rod.

Solution : $B = \frac{\phi_B}{A}$

given $\phi_B = 5 \times 10^{-5}$ Wb ,

$A = 4 \times 10^{-5}$ m², $H = 4000$ A/m

$$B = \frac{5 \times 10^{-5}}{4 \times 10^{-5}} = 1.25\ \text{Wb/m}^2$$

Magnetic permeability

$$\mu = \frac{B}{H} = \frac{1.25}{4000} = 0.3125 \times 10^{\ 3}$$

$= 3.125 \times 10^{-4}$ H / m

Magnetic susceptibility

$$\chi_m = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{3.125 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1$$

$\chi_m\ 1 = 248.8 - 1 = 247.8$

$I = \chi_m H = 247.8 \times 4000 = 9.90 \times 10^5$ A/m given

Example 8.11 : An iron rod of dimensions 5 cm × 1 cm × 0.5 cm is placed the in magnetizing field $10^{\ 4}$ A/m. If a magnetic momen of 10 A/m² is induced in it. Find magnetic induction.

Solution : $B = \mu_0 \left(\frac{M}{V} + H \right)$

$M = 10$ Am²,

$V = 5 \times 1 \times 0.5 \times 10^{\ 6} = 2.5 \times 10^{\ 6}$ m³

H = 10^4 A / m

$$B = 4\pi \times 10^{-7} \left(\frac{10}{2.5 \times 10^{\ 6}} + 10^4 \right)$$

$= 12.56 \times 10^{-7}(4 \times 10^6 + 10^4) = 5.036$ Wb/m²

8.13 Classification of magnetic materials

According to the behaviour of materials in external magnetic field, the materials are of mainly three types - (i) Diamagnetic (ii) Paramagnetic (iii) Ferromagnetic

8.13.1 Diamagnetic Substances

If placed in non uniform exrternal magnetic field, the materials moves from strong field to weak field, or they outstead by the field. Actually they aquire a small, net non-zero magnetization opposite to the applied field. These materials are called diamagnetic and the property, diamagnetism. examples- Cu, Zn, Sb, Bi, Hg, H_2, N_2, Au, Ag, air water diamond etc.

Explanation of Diamagnetism

Such materials have pairedelectron in their atoms, which revole in opposite direction. In the absence external magnetic field, the magnetic moment of those electron get cancalled being equal and opposite. In the presence of external magnetic field, the magnetic force on moving electron is opposite in both pairing electrons, on one electron, it is towards the nucleus, increasing its velocity, hence increasing magnetic moment; $m - 1/2evr$. One other it is away from nucleus, thus decreasing v and magnetic moment.

The magnetic moment in the direction of applied field get decreased, and that which is opposite, get increased. Hence the net magnetization induced in the material is opposite to the appilied field.

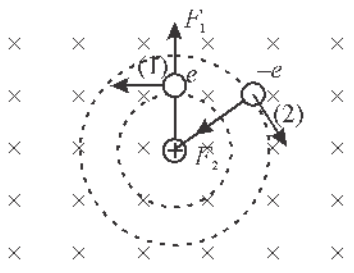


Fig 8.19 Explaination of diamagnetism

Diamagnetism is present in all substances in some it is not observed due to other dominet properties present. Super conductors are ideal diamagnetic substances. For them $\chi_m = -1$; $\mu_0 = 0$ this effect is called Meisner effect. Magnetic field lines of external field are completly expelled by them. Aliquid diamagnetic substance placed in watch glass over magentic poles, behaves as shown in the figure 8.20.

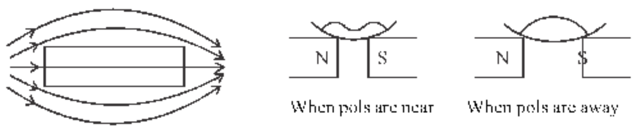


Fig 8.20 : Behaviour of diamagnetic materials

8.13.2 Paramagnetic Substances

These substances, when placed in external magnetic field move slightly from weak field to strong field or they are slightly, attracted by manetic field. We say that they aquire small net magnetic moment in the direction of applied field. These substances are called paramagnetic substances. Example - Na, Ca, Al, $CuCl_2$ etc.

Explanation of Paramagnetism-

This type of materials have unpaired electron in their atom. So every atom is a magnetic moment. The net magnetic moment, in the absence of external field is zero because of random orientations due to thermal agitation.

When external field is applied, the torque acting on them; align some of these atomic magnetic dipoles in the direction of applied field. All magnetic dipoles do not get aligned due to thermal effect. Hence the material gets some net non-zero magnetic moment in the direction of applied field.

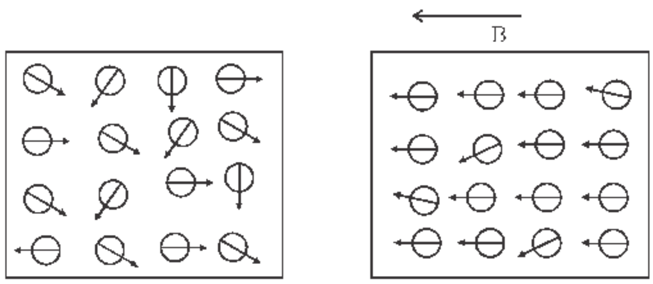


Fig 8.21 : Explanation of paramagnetism

Such materials allow. Some of the field lines of external field to pass through them. IF placed in watch glass over the poles of a magnet, shows the behaviour as shown in fig 8.22.

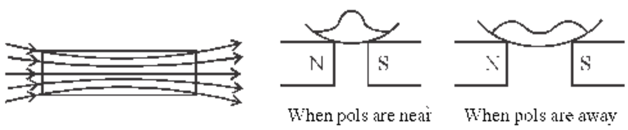


Fig 8.22 : Behaviour of paramagnetic materials

Temparature Dependance

Practically the magnetic intensity of a paramagnetic substance is proportional to the imposed magnetic field and inversly proportional to the absolute temparature

$$I \propto \frac{B_0}{T}$$

or $$I = C\frac{B_0}{T} \qquad \ldots (8.29)$$

here C = Curie Constant

But $B_0 = \mu_0 H$

$$I = C\frac{\mu_0 H}{T}$$

or $$\frac{I}{H} = C\frac{\mu_0}{T}$$

which gives $$\chi_m = \frac{C\mu_0}{T} \quad \ldots (8.30)$$

called curie law.

8.13.3 Ferromagnetic Substances

When placed in non uniform magnetic field, these materials rapildy move towards strong field ot they are strongly attracted by a magnetic field. Also they get magnetized in the direction of applied magnetic field.

Example - Fe, C_O, Ni, Fe_2O_3, gadolinium and magnetite (Fe_3O_4).

Explanation of Ferromagnetism

As in paramagnetism, the ferromagetic materials also have permanet dipole moment (of a group of atoms, oriented in same direction). The difference is of intensity. The orientation of these groups of atom is random; which make for whole sample.

Due to complex interaction between the atoms, one dipole, compels the other, to orient in the same direction. In this process, small colony of dipoles having same orientation is formed. This colony is called "domian". Whole sample is devided into domains having different orientations, making for whole sample. Size of one domian is of the order of few mm and it contain atom of the order of 10^{11}.

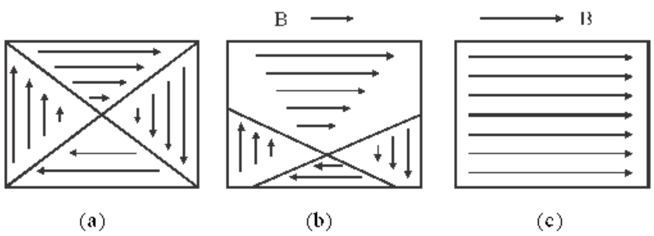


Fig 8.23 : Ferromagnetic materials

When a sample of ferromagnetic material is kept in an external magnetic field; if the applied field is weak, then the area or size of the domain having same orientation as that of applied field, increases and vice-versa. If the exrternal magnetic field is removed; the phenomenon is reversed due to thermal effect. Hence the process in reversible.

If the applied magnetic field is strong. The whole domain rotates in the direction of applied field. The first to rotate is that which makes minimum angle with applied field. If kept in the field for a long time, the whole sample becomes one domain.

If the external field is removed, the sample does not returns to original state, but some residual magnetism remains.

8.14 Curie Law and Curie Temparature

Pierre Curie studied the effect of temparature on magnetic materials and found that the magnetic susceptibility of diamagnetic materials does not depend on temparature, whereas the magnetic susceptibility of paramatic substance/material in inversly proportional to its absoulte temprature i.e

$$\chi_m \propto \frac{1}{T} \text{ Or } \chi_m = \frac{C}{T}$$

where C = Curie temparature

T = Absolute Temparature of the material

Temparature dependance of a ferromatic material obeys Curie - Weiss law. According to this law the magnetic susceptibility of a ferromagnetic material is given

by $$\chi_m = \frac{C}{T - Tc} \quad \ldots (8.31)$$

where C = Curie constant; T is the temparature of the material and T_C is curie temparature for that material. Below T_C the material behave like a ferromagnetic material and above T_C the material behave like a paramagnetic material. Above T_C all materials are paramagnetic. T_C is different for different materials. The curie temparature for some materials are given as -

Materials	Curie temprature
Iron	T_C = 1043 K
Cobalt	T_C = 1394 K
Nickle	T_C = 631 K
Gadolanium	T_C = 317 K

Example 8.12 : The curie temparature for some material is 300 K. If its magnetic susceptibility at 420 K is 0.4, then find curie constant.

Solution : manetic susceplibility $\chi = \dfrac{C}{T - T_C}$

$\chi = 0.4,\ T_C = 300\,K$ तथा $T = 420$ K

$C = \chi(T - T_C)$

$= 0.4\ (420 - 300) = 0.4 \times 120 = 48$ K

8.15 Magnetic Hysteresis Curve (B - H Curve)

When a ferromagnetic material is placed in magnetic field H (magnetizing field), magnetization of material takes place which produces magnetic field B. The curve showing of B and H is known as Hysteresis curve or B-H curve.

To find B-H curve of a ferromagnetic material we take its demagnetized from in the shape of a rod. We place this rod in a current carrying sole noid. The current in the sole noid produces $H\ (H - ni)$. This H magnetises the material which in turn produces B. We can change H by changing current, and we have a device to measure B.

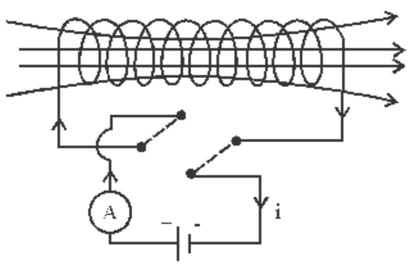


Fig 8.24 : Magnetic material inside a solenoid

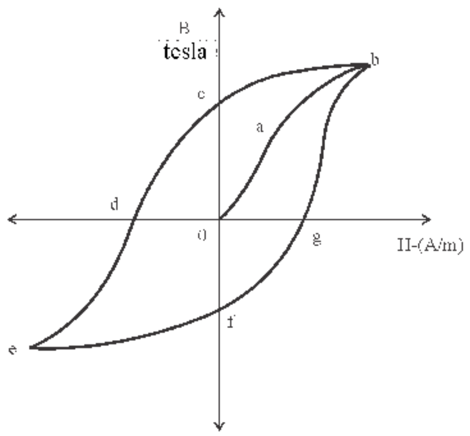


Fig 8.25 : Hystersiss loop

We start our experiment with $i - o, H - 0$, B will be zero i.e the origin O. Increasing H, B will increase but the relation is not lenear, it goes as per curve O ab in the diagram. After the point b, increasing H does increase B (the material is saturated and all the domains are aligned in one direction). The value of H after which B does not increases, but become constant, is called magnetic saturation.

Now if we reduce H to zero, the curve does not retrace the path b a o; but it goes from b to c. At point c, H = 0 but B ≠ 0 i.e some magnetization remains in the material. This remainent magnetism is called sesidual magnetism. The value of B at H = 0 is called retentivity or remanence, B_R. The domains are not completely randomised although external field H = 0.

Now if we reverse the direction of current in the sole noid, and increase it slowly, B decreases as curve cd and B becomes zero at certain value of H, which is called coercivity of the material. If we go on increasing H beyond d, the material is magnetized in opposite direction and get saturated at e (i.e all the domains are alligned in opposite direction). If we reduce H to zero again B will bot be zero but have a value of the remamant value in opposite direction. This is show as *of* in the curve. Again if we increase *i* in the original direction, the curve goes as *fgb*, and completes a cycle. Again the value of H, Og is coercivity of the material. We can know about the behaviour of material and its magnetic properties from its B-H loop *bede fgb*. Fig 8.26 shows B-H curves of soft-iron and steel.

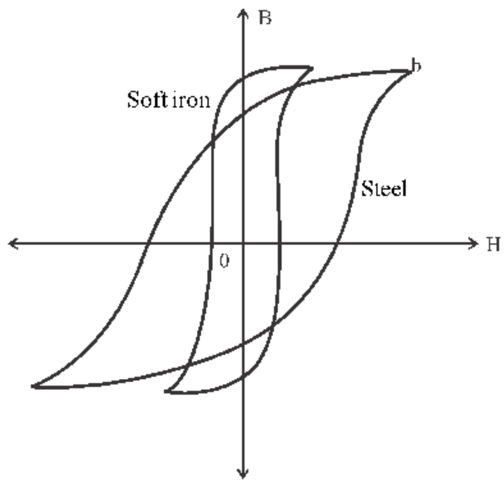


Fig 8.26 : Hystersiss loop for soft iron & steel

From this diagram it is evident that -

(1) For any value of H, the value of B for soft iron is more than that for steel, hence magnetic permeability μ, of soft iron is more than that of steel.

(2) Retentivity of soft iron is more than of steel.

(3) Steel has more coercivity than soft iron.

(4) For any value of H, the value of magnetization I is more for soft iron than steel; i.e. $\chi_m = I/H$ is more for iron which shows that the value of magnetic suceplibility of soft iron is more than that of steel.

(5) The area of B-H loop for soft iron is less than that for steel which shows that hysteresis loss is less in soft iron compared to steel. The area of B-H loop represents the enrgy loss per cycle per unit volume.

(6) B always lags behind H, this property is called hysteresis.

To select a material for core of electromagnet soft iron is suitable for its high permitivity μ, less coercivity and less hysteresis loss.

8.15.1 Hysteresis Loss

Energy is given to the material during magnezation. But during demagnezation the material does not release the whole energy it recieved. Some energy get lost (work done in rotating the domains). This loss of energy per cycle is called Hysteresis loss. This energy is converted to heat energy.

Area of B-H loop represents energy loss per cycle per unit volume. Hence the loss of energy per second is $Q = VAn;$ V = Volume of sample; A = Area of B-H loop and n is the frequency (number of cycles/s).

Electromagnet consists of insulated copper wire wound over a soft iron core. It is used in telephone electric bell, electric motor, dynamo, telegraphy and separation of magnetic materials from a mixture. It is also used in medical sciences.

For a permament magnet the material choosed, should have high retaintivity and high coercivity. It should also have high curie temparature and high saturation magnetism, so that it does not get demagnetized due to temparature, stray magnetic field and mechanical impulses. Hysteresis is meaningless for permanent magnet. For permanent magnet the suitable materials are steel and alnico (Al + Ni + Co). In these materials, the domains once get orienated remains as such and the external demagnetizing effects are minimum. Permanent magnets are used in galvanometers, ammeter, volmeter and loudspeaker.

The material for a transformer core, should also have the properties that are required for electromagnet i.e high μ, high χ_m, high retaintivity. An extra quality of low hysteresis is required. For transformer core the suitable materials are, superalloy, transformer steel (soft iron 4% silicon) and μ-metal (Cu+Fe+Ni+Mn).

Important Points

1. Magnets show directional and attractive properties. Both the poles can not be separated.
2. Magnetic field lines are imaginary close loops. The tangent drawn at any point gives the direction of magnetic field at that point.
3. Super position of magnetic fields of two magnets gives two points, where mahnetic field is zero, these points are called null points.
4. The magnetic moment of following dipoles is as given -

   (i) For bar magnet M = pole strenth x effective length

   (ii) For a revolving electron

   $M = 1/2evr = 1/2e\omega r^2$
5. When a magnet of manetic moment M is placed in niform magnetic field B (i) net force on it is zero.

   (ii) Torque $\vec{\tau} = \vec{M} \times \vec{B}$

   (iii) Potential energy $U = -\vec{M} \cdot \vec{B}$

(iv) work done in rotating the magnet is

$W = MB(1 - \cos\theta)$

6. Magnetic axis does not coincide with axis of rotation of earth, but it makes an able 11.3^0.
7. The quantities that give complete infromation about earth's magnetic field are called elements of earth's magnetism, they are (i) angle of declination (ii) dip angle (iii) Horizontal component of earth;s magnetic field.
8. The Gauss's law for magnetism is

$$\phi_B = \oint_S \vec{B}.d\vec{s} = 0$$

9. If a material is kept in magnetic field B_0 then magnetizing field or magnetic intensity $\vec{H} = \vec{B}_0 / \mu_0$.

Magnetic intensity $I = M / V$ i.e magnetic moment per unit volume.

10. Magnetic susceptibility $\chi_m = \dfrac{M}{H}$.
11. Relation between χ_m, and μ_0 is given as

$\mu = \mu_0(1 + \chi_m)$

$\mu = \mu_0\mu_r \quad \mu_r = 1 + \chi_m$

12. Magnetic materials are classified as diamagnetic paramagnetic and ferromagnetic according to their reaction to B.
13. Diamanetism is due to orbital motion of electrons, paramagnetism is due to orbital and spin motion of electron, ferromagnetism is due to domain property.
14. The hysteresis loop or B-H loop is used to study magnetic properties of of materials, and in selecting suitable materials for electric devices.
15. Diamagnetism does not depend on temparature for paramagnetic material $\chi_m \propto 1/T$ curie law. For

ferromagnetism $\chi_m \propto \dfrac{1}{T - T_C}$ (curie-wiess law).

where T_C = Curie temparature

Questions for Practice

Multiple Choice Questions -

1. If two magnetic poles of unit pole strength are at 1m distance in vacuum. The force between them will be -

(a) $4\pi \times 10^{\ 7}$ N (b) 4π N

(c) 10^{-7} N (d) $\dfrac{4\pi}{10^{-7}} N$

2. For super conductors, magnetic susceptibility is -

(a) + 1 (b) - 1

(c) Zero (d) Infinite

3. Magnetic susceptibility of free space is -

(a) +1 (b) - 1

(c) Zero (d) Infinite

4. The magnetic susceptibility is negative and very smallfor -

(a) Feromagnetic materials

(b) Paramagnetic materials

(c) Diamagnetic materials

(d) All of these

5. The relative permeability of a meterial is 1.0001 the material will be -

(a) Feromagnetic (b) Paramagnetic

(c) Diamagnetic (d) Non of above

6. The unit of magnetic moment is -

(a) Wb (b) Wb / m^2

(c) A / m (d) Am^2

7. Wb x A/m is equal to -

(a) J (b) N

(c) H (d) W

8. Magnetic field does not interact with -

(a) Another magnetic field

(b) Accelarated magnet

(c) A stationary chagre

(d) Moving electric charge

9. The cause of diamagnetism is -

(a) Orbital motion of electron

(b) Spin motion of electron

(c) Paired electron

(d) Non of the above

10. Magnetic moment of diamagnetic substances -

(a) Infinite (b) Zero

(c) 100 Am^2 (d) Non of the above

11. Relative permeability of ferromagnetisc substances is -

(a) > 1 (b) > > 1

(c) = 1 (d) = 0

12. The vertical component of earth's magnetic field is zero etc.

(a) Magnetic pole (b) Geographical pole

(c) Magnetic meridian (d) Non of the above

13. The area of hysteresis loop of a substance represents -

(a) Energy loss per cycle to magnetise the material

(b) Energy loss per unit volume per cycle in magnetizing the material

(c) Energy loss per unit volume in magnetizing

(d) Energy loss in magnetizing the material

14. Steel is used to prepare permanent magnet -

(a) Less energy loss

(b) Density of steel is more

(c) The residual magnetism is more

(d) Magnetism is not destroyed by ordinary external magnetic field

15. At curie temperature, a ferromagnetic substance becomes -

(a) Non-magnetic (b) Dia magnetic

(c) Paramagnetic (d) More ferromagnetic

Very Short Answer Type Questions -

1. A magnetic needle is free to rotate in vertical plane about horizontal axis. What direction it will indicate at magnetic poles?

2. Name the type of magnetic material which does not depend on temparature?

3. How the value of dip angle changes in going from equator to poles?

4. A magnetic material has magnetic susceptibility as -0.085. What type of materal it is?

5. What is retentivity or remanence?

6. Name two paramagnetic substances.

7. What is magnetic meridian?

8. Where on earth's surface the dip angle is zero and 90^0?

9. Write realtion between magnetic parmeability and magnetic susceptibility for a medium.

10. Write unit of pole strength.

11. What will be the value of dipangle, where the ratio of vertical and horizontal component of earth's magnetic field is $\frac{1}{\sqrt{3}}$.

12. What is magnetic Hysteresis?

13. What will be the ratio of magnetic fields at the points on axis and equator, equidistant from center of bar magnet?
14. What will be value of dip angle at a place where vertical and horizontal components of earth's magnetic field are equal?
15. What will be the change in magnetic moment of a bar magnetic if we bisected it along its length?

Short Answer Type Questions -

1. Find the expression for potential energy of a bar magnet placed in uniform magnetic field B, such that the angle between its dipole moment and B is .
2. How you will identify rods of paramagentic and diamagnetic substance?
3. Why we get two null points for a bar magnet? Can we get only one null point? How?
4. Why soft iron is used for electro magnets.
5. A bar magnet of magnetic moment $\vec{M}$ is placed parallel to uniform magnetic field $\vec{B}$. What will be the work done in rotating it by 90^0?
6. Difine angle of declination and dip angle.
7. Write down curie-wies law and write the value of curie temparature for iron.
8. Write four properties of magnetic field lines.
9. What is the behaviour of diamagnetic, paramagnetic and ferromagnetic substances in non-uniform magnetic field?
10. What is Gauss's law for magnetism? What does it indicates?
11. Why magnetic field lines are closed loops?
12. Compare magnetic fields of a bar magnet and a current carrying sole noid.
13. What is cause of earth's magnetism?
14. What are uses of hysteresis curve?
15. Find the expression for torque on a bar magnet placed at angle with uniform magnetic field. When it will be maximum?

Essay Type Questions -

1. What are elements of earth's magnetism? Define them and show with a labled diagram.
2. What is meant by hysteresis loop? Draw it and define its main physical quantities (specilities).
3. Explain diamagnetism, discuss its properties. Write five difference between paramagnetic and diamagnetic substances.
4. What is curie temparature? Explain how the magnetic susceptibility of paramagnetic, diamagnetic and ferromagnetic substances depend on temparature? Also write law regarding it.
5. Write specilities of the materials used for (i) Electri magnet (ii) Permanent magnet. Also write their uses.

Anwers (Mutliple Choice Questions)

1. (A) 2. (B) 3. (C) 4. (C) 5. (B) 6. (D) 7. (B) 8. (C) 9. (A) 10. (B) 11. (B) 12. (D) 13. (B) 14. (D) 15. (C)

Numerial Questions -

1. A bar magnet of magnetic moment 20 A m^2 is suspended in uniform magnetic field of 0.86T. Find the torque in rotating it by 60^0.

   $(86\sqrt{3}\ N\times m)$

2. The horizontal component of earth's magnetic field at certain place is $B_H = 0.5\times10^{-4}\ Wb/m^2$ and dip angle is 45^0. Find vertical component of earths magnetic field.

   $(5\times10^{-5}\ Wb/m^2)$

3. An iron rod of cross section 1 cm^2 is placed in magnetic field of 200 orested. It proudces a magnetic field of 3000 G. Find magnetic permeability and magnetic suceptibility of the material.

   (15 and 14)

4. For a sample of iron the following relation holds -

   $$\mu = \left[\frac{0.4}{H} + 12\times10^{-4}\right] \text{H/m}$$

   Find the value of H which produce a magnetic field of 1 T.

   (500 H/m)

5. A magnetic field of 2×10^3 A/m. produces a field

8π T in a sample of iron rod. Find relative magnetic permeability of the sample.

(10^4)

6. A sample of volume 30 cm^3 is placed in magnetic field of 5 orested. The magnetic moment induced is 6A/m^2. Find magnetic induction.

(0.2517 T)

7. A sample of ferromagnetic material of mass 0.6 Kg and density 7.8 x 10^3 kg/m^3 is placed in a alternating magnetic field of frequency 50Hz. If the area of hysteresis loop is 0.722m^2. Find hysteresis loss per second.

(2.777 x 10^{-4}J)

8. The curie temparature of ferromagnetic material is 300 K. If the magnetic suceptibility at 450 K is 0.6. Find curie constant for it.

(90 K)

9. Magnetic susceptibility for a paramagnetuic material is 0.60 at 120 K. Find its magnetic susceptibility at 27^0 C.

(0.24)

10. An iron rod of cross section 4 cm^2 is placed parallel to a magnetic field of 10^3 A/m. If the magnetic flux passing through it is 4×10^{-4}Web. Find magnetic permeability, relative magnetic permeability and magnetic susceptibility of iron.

(10^{-3} Web / A × m, 796, 795)

11. A circular coil of 100 turns and radius 0.05 m, has a current 0.1 A. Find the work done in rotating it by 180^0 in a field of 1.5 T.

(0.236 J)

12. A coil in the form of unilateral tringle of side l is suspended in magnetic $\vec{B}$ which is perpendicular to plane of coil. If the current in the coil is I and it experience a torque τ; find the expression for length of one side of the tringle.

$$\left[\left(2\frac{\tau}{\sqrt{3}BI}\right)^{1/2}\right]$$

Chapter - 9

Electromagnetic Induction

In previous chapter we have seen that a magnetic field is associated with moving charges and current. The above invention by orested raised questions to scientiests, is the converse effect possible i.e. can a magnetic field produce electric field which establishe electric current in a closed circuit? The answer is yes.

Experinments conducted independently by Michael Fraday and Joseph Henry conclusively showed that electric currents were induced in closed coils when subjected to varying magnetic fields. This phenomenon is called electro magnetic induction.

Experinments by Henry not only confirms the above inter-relationship but gave many practical utilities of phenomenon of electro magnetic induction. For example the electric generations which supply electric power to our homes and work places works on electro magnetic induction. The electric furnaces in which metals are melts in large amounts safely, works on electro magnetic induction. Now a days use of induction stoves is popular in kitchens, rapidly replacing the traditional stoves. In this chapter we will study the principles related to electro magnetic induction.

9.1 Magnetic Flux

Before studying the experinments performed by Faraday and Henry, we must get familiar with a physical quantity magnetic flux, which helps qualitative and quantitative explanation of these experinments. We will define magnetic flux as we have defined electric flux in chapter 2. Magnetic flux is measure of number of magnetic field lines passing through a surface. The magnetic flux $d\phi_B$ through an small area element dA placed in uniform magnetic field B can be given as

$$d\phi_B = \vec{B} \cdot d\vec{A} \qquad \ldots 9.1(a)$$

It is a scaler quantity. The area elements $d\vec{A}$ is a vector normal to its geomatrical area.

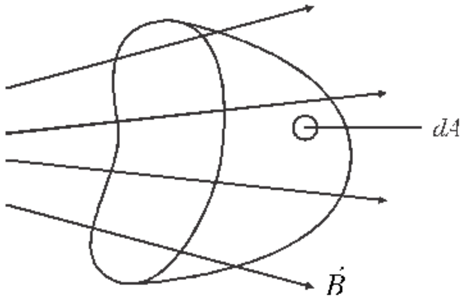


Fig. 9.1 (a) : An area placed in magnetic field are magnetic flux

The total magnetic flux passing through a surface placed in magnetic field $\vec{B}$ is summation of magnetic flux passing through all such small area elements.

Hence flux passing through a surface is given by

$$\phi_B = \int d\phi_B = \int \vec{B} \cdot d\vec{A} \qquad \ldots 9.1(b)$$

Here integral is considered over the surface area of surface under consideration.

As a special case magnetic flux passing through a plane surface placed in uniform magnetic $\vec{B}$ is :-

$$\phi_B = BA\cos\theta \qquad \ldots (6.2)$$

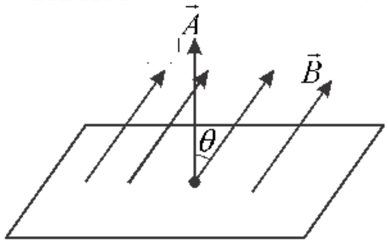


Fig. 9.1 (b) Plane surface of area $\vec{A}$ placed in uniform magnetic field, area vector $\vec{A}$ is normal to surface and outward

Here θ is angle between directions of normal to the surface and magnetic field $\vec{B}$. For a given area ϕ_B is maximum when $\cos\theta$ is maximum i.e. $\cos\theta = 1$ or $\theta = 0^\circ$. In this situation $\vec{B}$ and $\vec{A}$ are parallel and

$$\phi_{B\max} = BA\ 1$$

In the case of $\theta = 90^\circ$ (here $\vec{B}$ and $\vec{A}$ are normal to each other) the magnetic flux is minimum and zero. Hence

$$\phi_{B\min} = 0$$

Outgoing flux to the surface is taken as positive and flux entering the surface is taken as negative.

Because magnetic field lines are closed curve or loop, so the total magnetic flux associated to a closed surface is always zero.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

This is called Gauss law for magnetism. Magnetic flux is a scalar quantity, its dimensions are $\left[ML^2T^{-2}A^{-1}\right]$. Its S.I. unit is weber (Wb) or Tesla - meter^2. (Tm^2) as Joule/Ampere = $\frac{\text{Joule x sec}}{\text{Coulumb}}$ = volt × sec, these unit can also be used

Unit of magnetic flux in CGS is maxwel (Mx). maxwel and weber are related as follows :-
1 Wb = 1 V × s = 10^8 emu of potential = 10^8 Mx

Example 9.1 : A circular coil of area $(3\hat{i} + \hat{j} + 2\hat{k}) \times 10^{-2} m^2$ is placed in a magnetic field $(2\hat{i} - 2\hat{k}) \times 10^{-4}$ T. Find out magnetic flux passing through the coil.

Solution : Magnetic flux is given as :-

$$\phi_B = \vec{B} \cdot \vec{A}$$

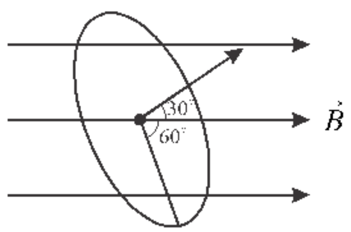
$$\therefore \quad \phi = (2\hat{i} - 2\hat{k}) \times 10^{-4} \cdot (3\hat{i} + \hat{j} + 2\hat{k}) \times 10^{-2}$$

$$= 8\ (6 - 4) \times 10^{-6}\ \text{Wb}$$

$$\phi_B = 2 \times 10^{-6}\ \text{Wb}$$

Example 9.2 : A circular coil is placed in a magentic field 5×10^{-3} T at 60^0 angle to the field. If the area of coil is $4m^2$ than find the amount of magnetic flux through the coil.

Solution :



$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

Here θ is angle between normal to the plane of coil and direction of magnetic field $\vec{B}$.

$$\theta = 30°$$

Putting values of B, A and θ we have

$$\phi_B = 5 \times 10^{-3}$$

$$= 5 \times 10^{-3} \times 4 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \times 10^{-3}\ \text{Wb}$$

9.2 Electroc Magnetic Induction

For the description of electro magnetic induction we will study first the three experiments carried out independently by Faraday and Henry. Understanding of different phenomenon related to electro magnetic induction are based on these experiments.

Experiment 1 : Let us consider a closed circuit of Galvano meter G and a coil C as shown in fig 9.2. Here coil is not connected with any source of e.m.f. (i.e. cell or battery). North pole of a bar magnet is kept towards the coil. As we move north pole of the magnet towards the coil, the pointer of the galvanometer shows deflection. But as the north pole of bar magnet moves away from the coil, the pointer of galvanometer deflects in opposite direction.

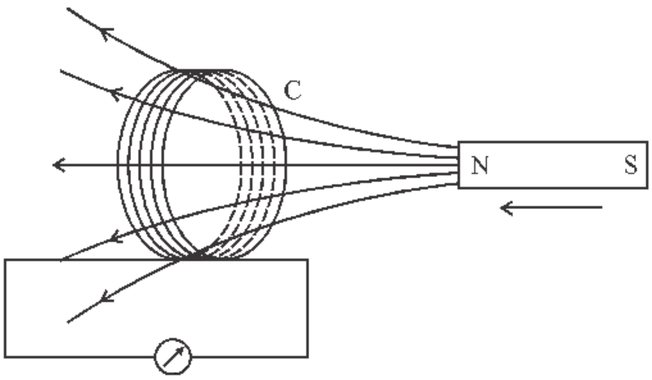


Fig. 9.2 : Electromagnetic induction north pole of bar magnet moving towards coil

If south pole of bar magnet moves toward or away from the coil, the direction of deflection in galvanometer is opposite to the direction of deflection for motion of north pole. If the magnet moves faster towards or away from the magnet, relatively large deflection is observed. If the bar magnet is held fixed and coil C is moved towards or away from the coil same effects are observed. However, when coil and bar magnet both remain stationary galvanometer shows no deflection.

Experiment - 2 : In this experiment bar magnet is replaced by another coil C_2 connected with a cell. When coil C_2 having current moves towrds or away from the coil C_1, the same effects are observed as in experiment one.

When coil C_2 is brought towrds C_1 galvanometer shows some deflection and when coil C_2 is brought away from C_1 galvanometer shows deflection in opposite direction. If coil C_2 is held fixed and coil C_1 moves deflection is observed in galvanometer. When both coil C_1 and C_2 are held fixed no deflection is observed in galvanometer.

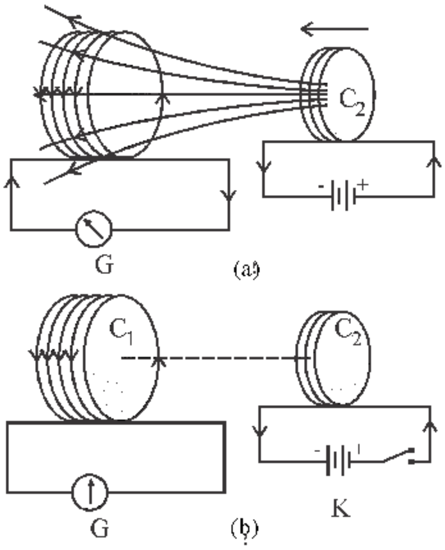


Fig. 9.3 (a) Induced current due to motion of C_2
(b) Induced current in C_1 due to change in current in C_2

Experiment - 3 : The above two experiment involve relative motion between magnet and coil and between two coils. But Faraday showed that for electro magnetic induction relative motion is not an absolute requirement. As shown in fig. 9.3 (b) if both coils C_1 and C_2 remains stationary and the key connected to coil C_2 is pressed, there is a momentary deflection in galvanometer. When the key is released there is again momentary deflection in galvanometer but now in opposite direction. When key is pressed continuously there is no deflection in galvanometer.

Deflection in galvanometer increases when number of turns of the coil increases and a soft iron rod is placed inside the coil.

9.2.1 Conclusions From Experiments

Faraday explained the results of experiments on the basis of magnetic flux. According to Faraday when magnetic flux associated with coil varies an emf is induced in coil due to which a current is also induced.

When coil and magnet remains fixed relative to each other or steady current flows through the secondary coil C_2 then magnetic flux through the first coil remains unchanged. When coil and magnet have relative motion or when varible current flows through the secondary coil,the magnetic flux varies through the first coil.

By moving the magnet near to the coil C_1 or by increasing current in secondary coil C_2, magnetic flux through coil C_1 increases. According to Faraday, variation of magnetic flux associated with coil C_1 develops induced emf in the coil.

Induced e.m.f is as larger as the rate of change of magnetic flux is faster.

The phenomenon of developing induced emf in coil due to variation in magnetic flux through the coil is called electromagnetic induction.

9.2.2 Faraday Laws of Electromagnetic Induction

From the experimental observations Faraday gave two laws for electro magnetic induction which are called Faraday's law of electro magnetic induction laws.

First Law :- When magnetic flux assoicated with a closed circuit varies, an emf is induced in the circuit. If the circuit is closed an induced current develops due to this induced e.m.f in the circuit, the induced current persists as long as the magnetic flux is varied.

Second Law :- According to this law "the magnitude of the induced emf is equal to rate of change of magnetic flux" associated with coil. If induced emf is denoted by ε , than mathematically it is given as

$$\varepsilon = \frac{d\phi_B}{dt} \qquad \text{. . . 9.3 (a)}$$

For a closely wound coil of N turns, variation of flux assoicated with each turn is same. The total induced emf is given by :-

$$\varepsilon = -N\frac{d\phi_B}{dt} \qquad \text{. . . 9.3 (b)}$$

(the negative sign in this equation is due to the Lenz's law discussed later)

From formula $\phi_B = BA\cos\theta$ flux can be changed by changing following processes-

(i) By changing magnetic field B.

(ii) By changing total area of the coil or part of area associated with in the magnetic field. For example by stretching or by shrinking the coil (by changing the shape of coil) or by pushing the coil inside the field or pushing it out of field.

(iii) By varing the angle between the direction of magnetic field B and normal to the plane of the coil (or plane of the coil itself).

For example rotating the coil in a magnetic field B in such a way that initially B remains normal to the plane of coil and afterwards it is in the plane of coil.

Example 9.3 : A coil is placed in magnetic field $\vec{B}$ so that its plane is normal to the field. If magnetic flux associated with coil is $\phi_B = (2t^2 - 6t + 9)$ mWb then find the induced emf at $t = 5$ sec.

Solution : Induced e.m.f $\varepsilon = -\frac{d\phi_B}{dt}$

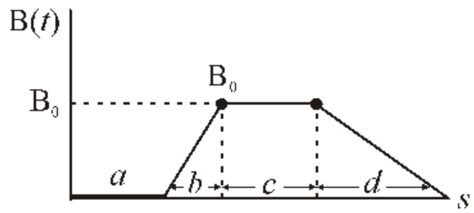
given $\phi_B = (2t^2 - 6t + 9)$ mWb

$$\varepsilon = -\frac{d}{dt}(2t^2 - 6t + 9) = -(4t - 6)\,\text{mV}$$

$$t = 5 \text{ s}$$

$$\varepsilon = -(4 \times 5 - 6) = -14\,\text{mV}$$

Example 9.4 : Graph given below shows a time dependent magnetic field B (t) that exists uniformly over a conducting loop. Direction of magnetic field is normal to the plane of loop. Arrange four parts a,b,c and d of figure in order of induced emf first greatest.



Solution : Magnitude of induced e.m.f is given by

$$|\varepsilon| = \left|\frac{d\phi_B}{dt}\right|$$

Here $\phi_B = B(t)A$

As B(t) is normal to the plane of loop and A is constant

$$\therefore \quad |\varepsilon| = A\left|\frac{dB}{dt}\right| \propto \left|\frac{dB}{dt}\right|$$

(i) In part (a) B(t) = o

$\varepsilon = 0$

(ii) In part (b) $\left|\frac{dB}{dt}\right| = \left|\frac{\Delta B}{dt}\right| = \frac{B_0 - 0}{2T - T} = \frac{B_0}{T}$

(iii) In part (c) $\frac{dB}{dt} = 0$ (here B = Bo = constant)

(iv) In part (d) $\left|\frac{dB}{dt}\right| = \left|\frac{b_0 - 0}{5T - 3T}\right| = \frac{B_0}{2T}$

Hence induced emf is in decreasing order as follows -

$$\varepsilon_b > \varepsilon_d > \varepsilon_a = \varepsilon_0 (= 0)$$

9.3 Lenz's Laws

By Faraday's law we can find the magnitude of induced emf but the direction of induced emf and induced current is given by Lenz's law.

"According to lenz's law the polarity of induced e.m.f and direction of induced current in the circuit is such that it opposes the change in magnetic flux that produced it."

From Faraday and Lenz's law :-

$$\varepsilon = -\frac{d\phi_B}{dt} \qquad \ldots (9.3)$$

For coil having N turns : -

$$\varepsilon = -N\frac{d\phi_B}{dt} \qquad \ldots (9.4)$$

When magnetic flux through the coil increases the direction of magnetic field lines due to induced current is opposite to the original magnetic field lines. When magnetic flux through the coil decreases the direction of magnetic field lines due to induced current is in the direction of original magnetic filed lines.

In fig. 9.4 north pole of a bar magnet moves towards on face of a coil, direction of induced current in the coil is such that this face of coil behaves like a north pole. There is a repulsion between bar magnet and coil so induced current in coil opposes the motion of bar magnet.

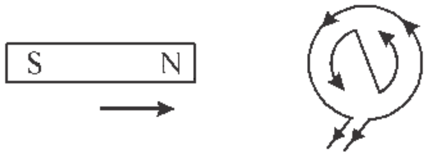


Fig. 9.4 : Moving magnet towards stationery coil

In fig. 9.5 when north pole of a bar magnet moves away form the coil, the direction of induced current in the coil is such that the face towards the magnet behaves as south pole. There is attraction between coil and magnet, so the induced induced current opposes the motion of magnet.

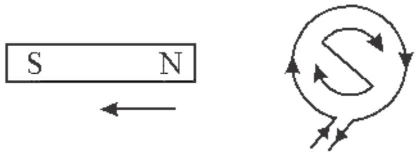


Fig. 9.5 Magnet moving away from stationery coil

In Lenz's law

$$\varepsilon = -d\phi_B / dt$$

The negative sign gives the direction of induced emf.

What will happen if an open circuit or open coil is used in place of a closed circuit. In this case emf is induced due to variation in magnetic flux but there is no induced current.

9.3.1 Lenz's Law and Conservation of Energy

Lenz's law is based on the law of conservation of energy. Let us imagine that the north pole of a magnet moves towards the coil and the direction of induced current is such that plane of coil towards magnet behaves as south pole and not as a north pole as discussed . The coil attracts the magnet and it is accelerated. Due to acceleration the induced current in the coil also increases which produces greater force on magnet and acceleration also increases. Hence kintetic energy of magnet and rate of heat i^2R, in the coil increases. Hence if initially we push magnet slightly towards the coil its velocity and kinetic energy increases continuously without spending any energy further which violets the law of consevation of energy. Hence our imgination is wrong.

In the experiments for electromagnetic induction we have seen that for keeping the magnet in motion external work is to be done against induced emf. Where does the energy spent in work goes? The mechnical energy spent in work converts in the form of electrical energy. Mechanical work done by external source is equal to energy dissipated by Joule heating due to induced current. Hence Lenz's law follow the law of conservation of energy.

9.3.2 Induced Current and Induced Charge

According to Faraday's law induced emf in coil having N turns is

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

If area vector $\vec{A}$ of coil is along magnetic field $\vec{B}$ than magnetic flux $\phi = BA$

$$\varepsilon = -N\frac{d}{dt}(BA)$$

If $\vec{A}$ is fixed and $\vec{B}$ is variable than

$$\varepsilon = -NA\frac{dB}{dt} \qquad \ldots (9.4)$$

If $\vec{B}$ is fixed and $\vec{A}$ is variable than

$$\varepsilon = -NB\frac{dA}{dt} \qquad \ldots (9.5)$$

If the total resistance of circuit is R than induced current is

$$I = \frac{\varepsilon}{R} = -\frac{N}{R}\frac{d\phi_B}{dt} \qquad \ldots (9.6)$$

Induced charge in time interval dt is

$$dq = I\,dt$$

$$dq = -\frac{N}{R}d\phi_B$$

If flux changes from ϕ_{B_1} to ϕ_{B_2} than induced charge is

$$\int dq = -\frac{N}{R}\int_{\phi_{B_1}}^{\phi_{B_2}} d\phi$$

$$q = -\frac{N}{R}(\phi_{B_2} - \phi_{B_1})$$

$$q = \frac{N}{R}(\phi_{B_1} - \phi_{B_2}) \qquad \ldots (9.7)$$

From above equation it is clear that magnitude of induced charge depend on change of flux but does not depends on rate of change of flux.

Example 9.5 : A coil of area 1.6 cm^2 having 50 turns placed in magntic field of 1.8 T in 0.3 sec. The plane of coil is normal to the direction of magnetic field lines. Find the amount of charge flown in coil if its resistance is $10\,\Omega$.

Solution : Flux passing through each turn of coil having area A placed normal to magnetic field is

$$\Phi_B = BA$$

$$\phi_B = 1.8 \times 1.6 \times 10^{-4} = 2.88 \times 10^{-4}\ \text{Wb}$$

From Faraday's law induced e.m.f

$$\varepsilon = -N\frac{d\phi_B}{dt} = -50 \times \frac{2.88\times 10^{\ 4}}{0.3} = -4.8\times 10^{-2}\ \text{V}$$

Magnitude of induced e.m.f

$|\varepsilon| = 4.8 \times 10^{-2}$ V

is Induced current in the coil

$$I = \frac{|\varepsilon|}{R} = \frac{4.8\times10^{-3}}{10} = 4.8\times10^{-3}\ \text{A}$$

Induced charge in coil

$$q = I\Delta t = 4.8\times10^{-3}\times 0.3 = 1.44\times10^{-3}\ \text{C}$$

9.4 Flemings Right Hand Rule

Fleming's right hand rule gives the the direction of induced current. According to this law if the fore finger, central finger and thumb of right hand are held perpendicular to each other as in fig (9.6). If fore finger shows direction of magnetic field and thumb shows direction of motion than central finger shows direction of induced current.

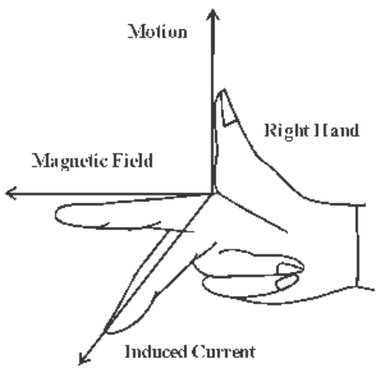
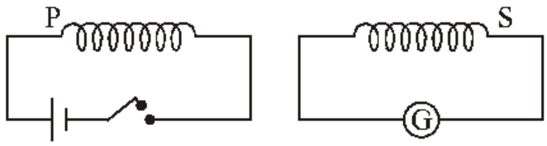
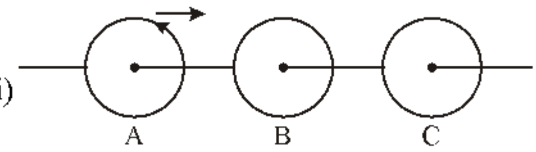


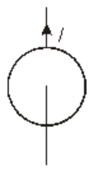
Fig. 9.6

Example 9.6 : Explain direction of induced current under following situations :-

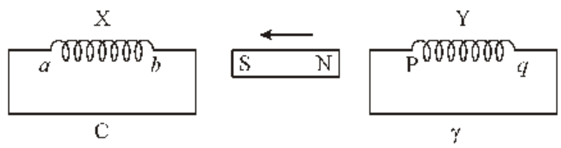
(i)


(ii)


(iii)


(iv)


direction of induced current in the loop when current decreases uniformly.

(v)


Solution :

(i) According to Lenz's law direction of induced current in loop is from B to A. Hence plate A will be positive and palte B negative.

(ii) Looking from the side of coil P direction of induced current in coil S is clock wise.

(iii) There is no induced current in coil B due to current in coil C because C coil is fixed. Looking from the side of coil A the direction of indcued current in B is anti clockwise.

(iv) The rate of change of magnetic flux associated with circular loop is constant hence current will not be induced in the loop.

(v) As south pole moves towards coil X the induced current is in the sense a c b and as north pole moves away from coil y the induced current in it is the sense q r p.

9.5 Induced emf in a Conductor Rod Moving in a Uniform Magnetic Field

In fig. 9.7 uniform magnetic field B is shown by dots. Its direction is normal to the plane of paper and outward. A conducting rod ab of length l is placed perpendicular to the field, and the rod is moved with a constant velocity $\vec{v}$ perpendicular to the direction of both ℓ and magnetic field $\vec{B}$.

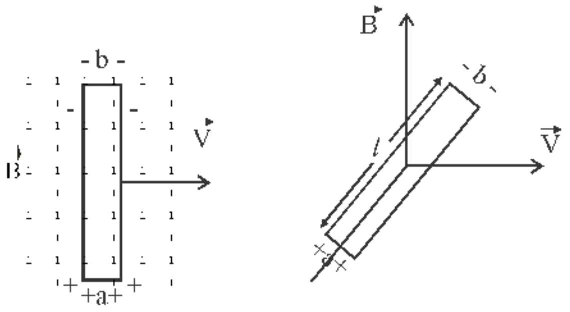


Fig. 9.7 : Motion of conducting rod in perpendicular magnetic field

The free electrons present in conducting rod also move with velocity $\vec{v}$ in magnetic field. Magnetic force on moving free electrons is

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \qquad \ldots (9.8)$$

here is q charge of an electron. According to figure for positive charge the direction of $\vec{v} \times \vec{B}$ is from b to a. Hence magnetic force on electrons because of negative charge is from a to b along the length of conductor. Due to drift of electrons there is excess of electrons's at and b and deficiency of electrons at and a. Hence there is excess of negative charges at end 'b' and excess of positive charge at end 'a'.

Due to accumulation of opposite charge at both ends of the rod a static electric field get developed between the edges. The drift process due to motion of conductor continuous till the force on electron due to electric field is balanced by the force of magnetic field. Force on electron of charge q due to electric field E is

$$\vec{F}_e = q\vec{E} \qquad \ldots (9.9)$$

In the state of equilibirium

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

i.e. the direction of $\vec{E}$ is opposite to direction of $\vec{v} \times \vec{B}$ or inside conductor it is from end a to end b ends magnitude of electric field $E = vB$.

Due to this electric field E, an emf ε is induced between two ends of conductor.

Hence

ε = work done in displacing a unit positive charge against electric field from one end to another end.

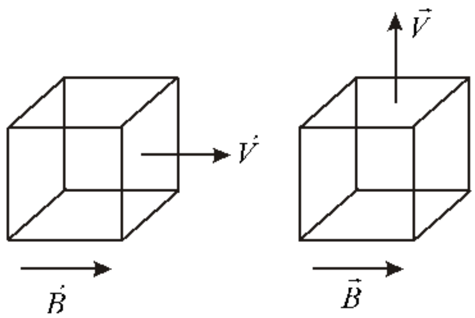
= Force on unit positive charge × displacement.

$$\varepsilon = E\ell$$

or $$\varepsilon = vB\ell \qquad \ldots (9.10)$$

If the direction of ℓ is taken from negatively charge end to positively charged end and the conducting rod is moving at an angle θ with the direction of magnetic field lines than component of v perpendicular to the direction of B is $v\sin\theta$. In such a case induced emf between the ends of conductor is given by $Bv\ell\sin\theta$. The induced emf is zero when conductor moves along the direction of magnetic field.

Example 9.7 : A cube is made by joining twelve straight conducting wires. Length of each arm is 5 cm. Cube is moving in a magnetic field of 0.5 T with velocity 5m/sec.



(i) If cube moves along the direction of magnetic field than what is induced emf in each arm.

(ii) If cube moves perpendicular to the direction of magnetic field than what is induced emf on each arm.

Soltuion : (i) In first case the velocity of conductor is parallel to magentic field hence induced emf along each arm is zero.

(ii) Induced emf will not develop along arms AB,CD,EF and GH because they are parallel to magnetic field. Arms AE, DH, BF and CG ara parallel to direction of velocity, hence induced emf along these arms is zero. Arms AD, BC, EH amd FG are perpendicular to both velocity v and magnetic field B, induced emf along each arm is given by -

$$\varepsilon = Bv\ell$$

$$= 0.05 \times 5 \times 5 \times 10^{-2}$$

$$= 1.25 \times 10^{-2} \text{ V}$$

Example 9.8 : A conducting rod of length 40 cm is placed perpendicular to a magnetic field of 0.5 T. The rod is moving with velocity 15m/s at an angle of 30^0 with magnetic field. Find induced emf across the rod.

Solution : $\varepsilon = BVl \sin\theta$

$$= 0.5 \times 15 \times 0.4 \sin 30^\circ$$

$$= 0.5 \times 15 \times 0.4 \times \frac{1}{2} = 1.5 \text{ V}$$

Example 9.9 : Two lines of a railway track are separated from each other and also from earth. They are connected by a milivoltmeter. When a train moves on this track with speed 180 km/hr then what is the reading in milivoltmeter. Distance between the railway lines is 1 m and vertical component of earths magnetic field is 0.2×10^{-4}T.

Solution : Induced potential difference between railway lines - $\varepsilon = Bvl$

given $v = 180 \text{ km/h} = \frac{180 \times 5}{8} = 50 \text{ m/s}$

$B = 0.2 \times 10^{-4}$ T and $l = 1$ m

$$\varepsilon = 0.2 \times 10^{-4} \times 50 \times 1$$

$$= 1 \times 10^{-3} \text{ V}$$

$= 1$ mV

9.6 Induced emf and Current in a Rectangular Loop Moving in a Non Uniform Magnetic Field

In fig 9.8 a conducing rectangular loop or coil is placed perpendicular to a non uniform magnetic field. Suppose magnetic is B_1 of arm ab and B_2 on arm cd.

Coil is moved with *e* velocity *v* perpendicular to magnetic field in such a way that direction of velocity is perpendicular to arm ab and cd.

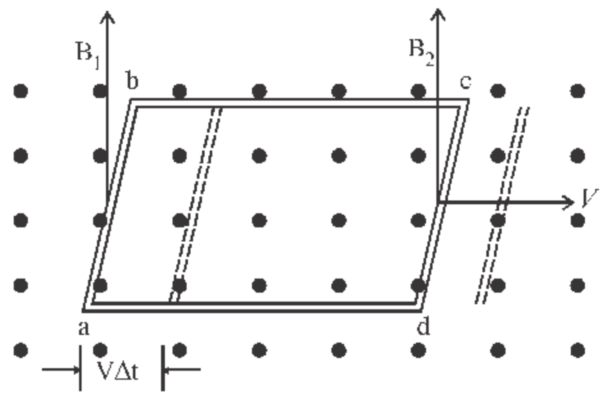


Fig. 9.8 Motion of rectangular loop in non uniform magnetic field

Suppose length of arm ab and cd is *l*. Distance traversed by coil in small time interval Δt is $v\Delta t$. Area crossed by arm ab and cd is $\Delta A = \ell v \Delta t$. Magnetic fields in these small areas can be taken as B_1 and B_2. From fig 9.8 it is clear that the area which comes out of magnetic field B_1 on left side and same amount of area enters in magnetic field B_2 on right side. Due to motion of coil decrease in magnetic flux crossing through coil on left side is

$$\phi_{B_1} = B_1 \times \Delta A = B_1 \ell v \Delta t \qquad \ldots(9.12)$$

Increase in magnetic flux crossing through coil on right side is

$$\phi_{B_1} = B_1 \times \Delta A = B_1 \ell v \Delta t \qquad \ldots(9.13)$$

Change in magnetic flux passing through coil

$$\Delta\phi_B = \phi_{B_2} - \phi_{B_1} = (B_2 - B_1)\ell v \Delta t \quad \ldots(9.14)$$

Hence $\frac{\Delta\phi_B}{\Delta t} = (B_2 - B_1)\ell v$

According to Faraday's law, the induced emf

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = \frac{-d\phi_B}{dt}$$

or $\varepsilon = -(B_2 - B_1)\ell v$

$$\varepsilon = (B_1 - B_2)\ell v \qquad \ldots(9.15a)$$

If resistance of coil is R than induced current in the coil

$$I = \frac{E}{R} = \frac{(B_1 - B_2)\ell V}{R} \qquad \ldots (9.15b)$$

9.7 Energy Conversation

In fig 9.9 (a) conducting loop abcd is placed perpemdicular to non uniform magnetic field. Suppose loop moves with velocity v. Due to motion of wire 'ab' in magnetic field positive charges accumulate at end a and negative at end b. In the same way on wire cd positive charges accumulate at end b and negative at c.

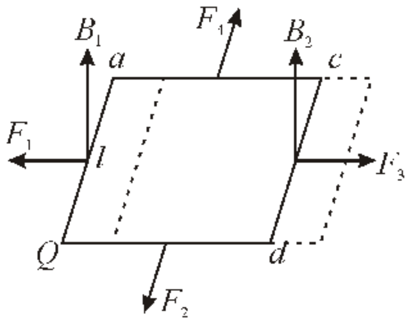


Fig. 9.9 Force on current carrying conducting in magnetic field

If $B_1 > B_2$ than amount of positive charge at end a is greater than at end d.

Induced current in loop flows in direction adcba. Let this induced current be I.

By force on current carrying conductor in magnetic field, the force on length l of arm ab is

$F_1 = IlB_1$ (towards left side)

In the same way froce on arm cd

$F_3 = IlB_2$ (towards right side)

Resultant of these two forces

$F = F_1 - F_3$

$F = IlB_1 - IlB_2$ (toward left side)

Forces F_2 and F_4 on arms bc and cd are equal in magnitude and opposite in direction. Hence they cancel each other.

When loop travels a distance $v\Delta\tau$ in time Δt towards right side work done against force F is

$$W = F\Delta x = I\ell(B_1 - B_2)v\Delta t$$

$$I = \frac{E}{R} = \frac{(B_1 - B_2)\ell V}{R}$$

$$W = (B_1 - B_2)^2 \frac{\ell^2 v^2}{R}\Delta t \qquad \ldots (9.16)$$

Energy spent in this work done is converted into electrical energy in the circuit and ultimately dissipated in the form of heat.

$$H = I^2 R\Delta t$$

On putting value of I

$$H = \frac{(B_1 - B_2)^2 \ell^2 v^2}{R^2} \times R\Delta t$$

$$H = \frac{(B_1 - B_2)^2 l^2 v^2}{R}\Delta t \qquad \ldots (9.17)$$

From equations 9.16 and 9.17

W = H

We see that power delivered by the external force is equal to the thermal power developed in the loop. This is consistent with the low of energy conservation.

Example 9.10 : Length of arm of a square loop is 1.5 m. Half of the loop is in magnetic field 2.5 T and remaining half part is in 1 T. It is moved with a velocity 7.2 km/hr perpendiculatr to magnetic field. Find induced emf.

Solution : Induced emf in a loop moving in a non uniform magnetic field

$$\varepsilon = (B_1 - B_2)v\ell$$

Here $B_1 = 2.5\,\text{T},\ B_2 = 1\,\text{T},\ \ell = 1.5\,\text{m}$

$$V = \frac{7.2 \times 5}{18} = 2\,\text{m/s}$$

$$\varepsilon = (2.5 - 1) \times 2 \times 1.5 = 4.5V$$

Example 9.11 : 2m long conducting rod is placed perpendicular to a magnetic field of 1 T. Rod is moved with a velocity 0.6 m/s perpendicular to its length and magnetic field. If conducting rod is connected across a resistance wire of 12 Ω. What is the required force and power for motion of rod. What is rate of heat production in the circuit.

Solution : Force on current carrying conducitng palced in magnetic filed

$$F = I(\ell \times B)$$

$$F = I\ell B \sin 90^\circ = I\ell B$$

or $$F = 0.1 \times 2 \times 1 = 0.2 \text{ N}$$

For uniform motion of rod an opposite force of same magnitude as of above force is required.

Required power for motion of rod

$$P = Fv = 0.2 \times 0.6 = 0.12 \text{ W}$$

Rate of heat produced in circuit

$$H = I^2R = (0.1)^2 \times 12 = 0.12 \text{ W}$$

9.8 Induced emf in a metal rod rotating in a Uniform Magnetic Field

In fig. 9.10 uniform magnetic field B is shown by cross (x) its direction is inward perpendicular to the plane of paper. A conducting rod OA of length L is rotating with uniform angular velocity ω anti clock wise in this magnetic field. Let us consider a small element dl of rod moving perpendicular to magnetic field with velocity v. Magnitude of induced emf in this small element.

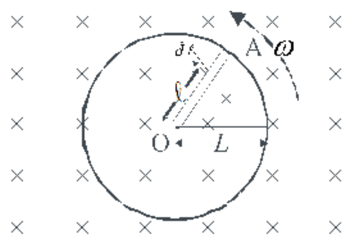


Fig. 9.10 Conducting rod rotating perpendicular to a uniform magnetic field

$$d\varepsilon = Bvd\ell$$

If small element is at distance l from centre than

$$v = \omega\ell$$

Hence $$d\varepsilon = B\omega\ell.d\ell$$

Induced emf across the rod is equal to the integration of above equation from 0 to L.

$$\int d\varepsilon = \int_0^L B\omega\ell\, d\ell$$

$$\varepsilon = \frac{1}{2}B\omega L^2 \qquad \text{. . . (9.18)}$$

By Fleming's right hand rule and in view of direction of magetic field and direction of rotation the induced current is directed from A to O and the end O of rod if positively charged and A is negatively charged for this case.

If frequency of rotation of rod is f than

$$\omega = 2\pi f$$

$$\varepsilon = \frac{1}{2}B \times 2\pi f \times L^2$$

$$= B \times \pi L^2 \times f$$

Suppose area of circle traversed by rod in magnetic field is A

$$\pi L^2 = A$$

and $$\varepsilon = BAf \qquad \text{. . . (9.19)}$$

9.9 Induced emf in a metal Disc Rotating in a Uniform Magnetic Field

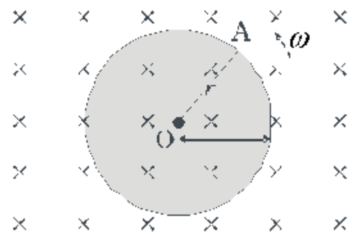


Fig. 9.11 Conducting disc rotating perpendicular to magnetic field

A uniform magnetic field B perpendicular to plane of paper and inward is shown in fig 9.11 by cross (x). A metallic disc of radius r is rotating in this magnetic field with angular velocity ω in the plane of paper anticlock wise. Disc can be considered as made up of many rods having one common end at the centre O of the disc and other end at the circumference. Length of each such rod L is equal to radius r. Due to rotation in magnetic field emf will induced across each rod. The end at the centre is positive and at circumference is negative.

emf across each rod is given by -

$$\varepsilon = \frac{1}{2}B\omega r^2 = BAf \qquad \text{. . . (9.20)}$$

Because emf across each rod is same and these are connected in parallel. Hence resultant emf across centre and circumference of disc is $\varepsilon = BAf$

$$|\varepsilon| = B \times \pi r^2 \times \frac{\omega}{2\pi}$$

or $\varepsilon = \frac{1}{2} B\omega r^2$ . . . (9.21)

Example 9.12 : A 0.5 m long conductor rod is placed in uniform magnetic field of 0.04 T. The rod id rotating about its one end perpendicular to the plane of magnetic field with angular velocity 40 revolutions per second. Find emf induced across the rod.

Soltuion : Induced emf $E = BAf$

$$E = B\pi\ell^2 f$$

given $B = 0.04T$, $\ell = 0.5m$, $f = 40$ revolutions/sec

$$E = 0.040 \times 3.14 \times = 3.14 \times 0.4 = 1.256 \text{ V}$$

Example 9.13 : Diameter of a metallic gramo phone disc is 0.20 m, the disc is rotating at the rate 40 revolutions/minute in horizontal plane. Vertical component of earth magnetic field is 0.01 T. Find emf induced between centre and circumference of the disc.

Soltuion : Given in the question

$$B = 0.01T, \text{radius } (r =) = \frac{0.20}{2} = 0.01 \text{m}$$

$$f = \frac{40}{60} \text{ rev./sec}$$

Induced emf $E = B\pi r^2 f$

$$= 0.01 \times 3.14 \times (0.1)^2 \times \frac{40}{60} = 2.09 \times 10^{-4} \text{ V}$$

9.10 Induced emf Due to Rotation of a rectangular Coil in Uniform Magnetic Field

In fig 9.12 (a) rectangular coil abcd is placed in uniform magnetic field such that its axis of rotation is perpendicualr to magnetic field. When coil rotates at angular velocity ω, the angle between plane of coil and direction of magnetic field changes continuously. Hence magnetic flux associated with coil also varies with time which produces induced emf in coil.

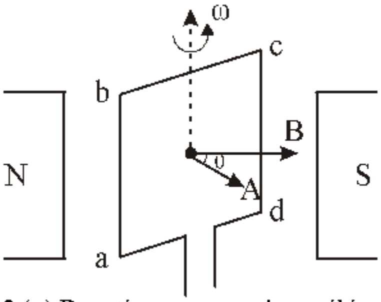


Fig. 9.12 (a) Rotating rectangular coil in uniform magnetic field

Suppose at any instant t the area vector $\vec{A}$ is at an angle θ with magnetic field $\vec{B}$. Number of turns in coil is N then flux passing through coil

$$\phi_B = N(\vec{B} \cdot \vec{A}) = NBA\cos\theta$$

$$\phi_B = NBA\cos\omega t \quad \text{. . . (9.22)}$$

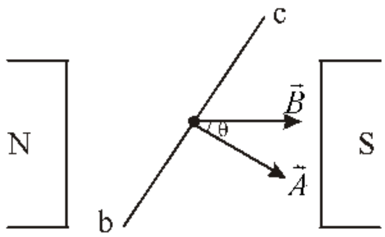


Fig. 9.12 (b) Position of coil at time t (top view)

Since magnituded of flux changes with time t the induced emf given by Faraday's law is-

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\varepsilon = -N\frac{d}{dt}(BA\cos\omega t)$$

$$\varepsilon = NBA\omega\sin\omega t \quad \text{. . . (9.23)}$$

$$\varepsilon = \varepsilon_0 \sin\omega t \quad \text{. . . (9.24)}$$

Here ε_0 is maximum (Peak value of) induced emf.

$$\varepsilon_0 = NBA\omega \quad \text{. . . (9.25)}$$

A graph drawn between induced emf ε and time t is as shown in fig 9.13

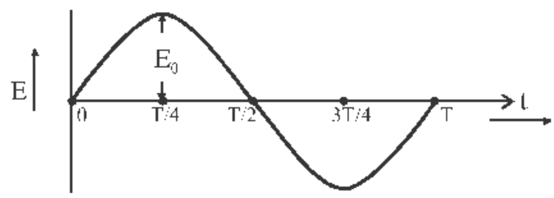


Fig. 9.13 Graph between induced emf and time

From fig 9.22 and 9.24 we see that induced emf is zero (minimum) when flux passing through coil is maximum and when flux passing through coil is minimum induced emf is maximum. If a resistor R is connected with coil then current in the circuit

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\sin\omega t$$

$$I = I_0 \sin\omega t \quad \text{. . . (9.26)}$$

Here I_0 is maximum (peak value of) induced emf.

The emf and current given by equations (9.23) and (9.26) are called alternating emf and alternating current respectively. This is also the principle of alternating current generator.

Example 9.14 : A coil of radius 0.15 m having 3000 turns is rotated at 250 rev/sec in the horizontal component of earth's magnetic field $B_H = 4\times10^{-5}$ T . Find out maximum induced emf.

Solution : Induced emf in a rotating coil

$$E = NBA\omega \sin \omega t$$

Maximum induced emf

$$E_0 = NBA\omega$$

We have given $N = 300, B = 4\times10^{-5}$ T, $r = 0.15$ m

$f = 250$ rev/sec

$$E_0 = 3000\times4\times10^{-5}\times3.14\times(0.15)^2\times2\times3.14\times250$$

$$= 13.31 \text{ V}$$

Example 9.15 : If a conducting coil after rotating once on a frictionless axle continue to do so with angular frequency ω without any external torque. If coil is in magnetic field and not in a closed circuit than explain (i) Whether emf will be induced in the coil (ii) current will be induced in the coil (iii) is there a need of external torque of for continuous rotation. (iv) If the coil is in closed circuit than how its motion is effected.

Solution :

(i) Due to rotation of coil in magnetic field the angle between area of coil and manetic field changes, hence flux through coil also changes which produces induced emf.

(ii) In open circuit there is no induced current

(iii) When current is not flowing, energy is not spent and there is no need of torque for continuous rotation.

(iv) If circuit is closed induced current flows in the circuit hence according to Lenz's law angular velocity of coil decreases and external torque is required for continuous rotation.

9.11 Eddy Currents

When magnetic flux associated with a closed electric circuit varies with time current is induced in circuit. In the same way when bulk pieces of metallic conductor's are subjected to changing magnetic flux, induced currents are produced in them, their flow pattern resembles with swirling eddies in water. These plane of flow is normal to the direction magnetic field lines. These are called eddy currents. These currents opposes the motion of metallic pieces and also change in magnetic flux. These currents were discovered in 1895 by Focault so also called as Focault current.

9.11.1 Experimental Demonstration of Eddy Currents

Experiment - 1 In fig. 9.14 a metallic plane plate PQRS is placed perpendicular to uniform magnetic field B, extended in a limited region, when plate is pulled out of the field, the area of plate inside the field reduces hence flux associated with plate also reduces which produces eddy current, in the plate. Direction of eddy current is such that it oppose the motion of plate. This type of damping is called electromagnetic damping. In fig. 9.14 direction of eddy currents are according to fleming's right hand rule.

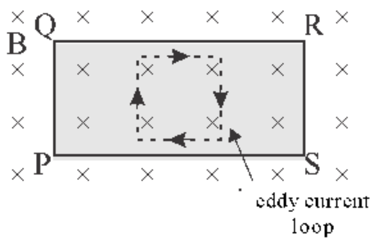


Fig. 9.14 Experimental demonstration of eddy currents

Experiment - 2 A copper or aluminium plate is allowed to swing like a pendulum between poles of a magnet. The area of plate associated with magnetic field changes with time. When plate enters and comes out of field, the flux through plate is minimum and when it is completely inside flux is maximum. Due to change in flux, eddy currents are induced in the plate, which damps its motion. When the plate swings into the region between the poles and when it swing out of the region some part of mechanical energy is converted into heat and plate stops after few oscillations. Here the oscillatory motion is damped. If rectangular slots are made in copper plate it swings comparatively more freely because this reduces the possible paths of the eddy currents considerably.

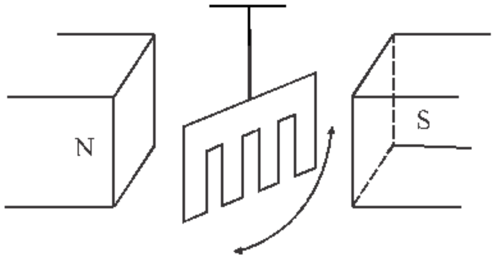


Fig. 9.15 Sloted copper plate to minimise eddy current

In electric devices, due to eddy current electrical energy is dissipated in the form of heat, hence eddy currents are reduced by laminations of core of electric devices. Core of transformer is made by thin plates placed one over other and separated by an insulating material like lacquer. Plane of thin plates of the core is kept parallel to magnetic field lines which increases resistance and reduces intensity of eddy currents.

9.11.2 Application of Eddy Currents

(i) Dead Beat Galvanometer : When current passes through a galvanometer its coil deflects and oscilates, when current is removed coil takes some time in coming to equilibirium position which is not required. So coil is made by winding its turns on a copper frame. Eddy currents are induced in the frame when coil rotates in magnetic field which damps the motion of coil and it comes to equilibirium position with in no time.

(ii) Brake in Electric Train :- The wheels of electric operated trains are joined with metallic drums. These drum rotates with wheels. Strong electro magnets are situated above the rails. When electro magnets are activiated, strong eddy current are induced in the drums which opposes the motion of drum which apply brake on trains. As there is no mechanical linkage the braking effect is smooth, free from wear-tear due to friction . Also the breaking action is efficient at high sbeeds as magnetic force increases with speed.

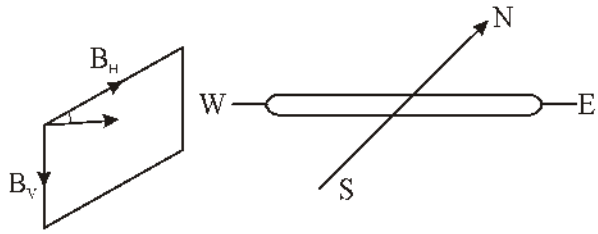
(iii) Diathermy :- Diathermy is production of heat in body tissuses for therapeutic purposes. A coil is wound on the part of the body when current flows through the coil which induced eddy current in tissues which produced heating of soft tissues.

(iv) Induction Furnace :- In induction furnace we get metals from ore. The metals to be melted are placed in high frequency variable magnetic field, which induces strong eddy currents in metals. These eddy currents produces large amount of heat which melts the metals.

9.11.3 Motion of Conducting Rod in Earth's Magnetic Field

When a conducting rod of length l moves with velocity v in earths magnetic field following cases are worth considering :-

Case - I Motion of conducting rod when placed in East-West direction and moves on perpendicular direction :-



(i) For the motion along East or West direction the conductor moves along its length hence area generated A = 0

$$\therefore \ \varepsilon = 0$$

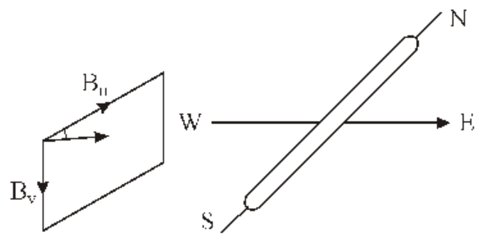
(ii) For the motion along north or south direction the conductor cut's earths vertical component B_v perpendicularly

$$\therefore \ \varepsilon = B_v Vl$$

(iii) The conductor crosses horizontal component of earth magnetic field B_H when it moves vertically upward or downward.

$$\varepsilon = B_H vl$$

Case II : When conductor is placed horizontal and moves in north-south direction and moves -



(i) While moving along east or west directions the conductor crosses vertical component of earth's magnetic field B_v

$$\varepsilon = B_v Vl$$

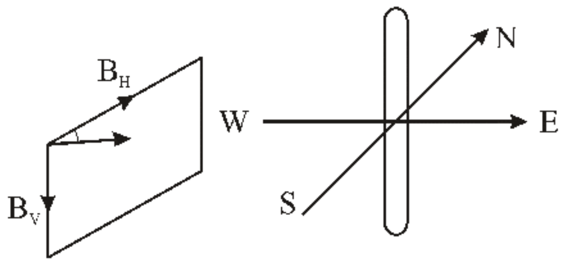
(ii) Along its length in north or south directions, then

$\therefore \ \varepsilon = 0$

(iii) Vertically upward or downward in magnetic meridian no component of earth's magnetic field is cut.

$\therefore \ \varepsilon = 0$

Case III : When conducting rod placed vertically and moves



(i) Along east or west direction the conductor cuts horizontal compnent of earths magnetic field B_H.

$\therefore \ \varepsilon = 0$

(ii) Along north and south direction, conductor moves in magnetic meridian

$$\varepsilon = B_H V \ell$$

(iii) The conductor moves along its length in vertically upward or downward directions, then

$\therefore \ \varepsilon = 0$

9.12 Self Induction

When current in a circuit or coil changes with time the magnetic field produced and flux asociated with the coil also changes. Due to this an emf is induced in the coil. This phenomenon is called self induction. According to Lenz's law the direction of induced emf is such that it opposes the change in linked magnetic flux.

Experimental Demonstration

In fig 9.16 a conducting coil is connected with a battery and key in series. As the key is made on a current flows throught the coil and the magnetic flux is associated with coil. Initially as key is made on current rises with time and induced magnetic flux also rises and emf is induced in the coil and induced current opposes the battery current in coil. when key is made off current in coil reduces to zero, the flux associated with coil also reduces and induced current flows in the direction of battery current. Direction of induced emf is shown in fig 9.16 when key is made on or off.

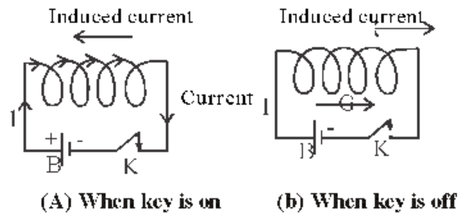


Fig 9.16 Phenomenon of self induction

9.12.1 Coefficient of self induction

Suppose current I flows through a coil. The magnetic field due to current is proportional to current and there by the magnetic flux associated with coil is directly proportional to current I, i.e.

$$\phi_B \propto I$$

$$\phi_B = LI$$

Here L is e constant of proportionality called coefficient of self induction or self inductance. Self inductance (L) depends upon shape and size of coil, marterial of core, medium and number of turns.

If I = 1 amp then $\phi_B = L$

Hence self inductance of a coil is equal to the magnetic flux linked with the coil when unit current flows through the coil. Due to variation of current, magnetic flux linked with coil changes hence by Faraday's law of electro magnetic induction -

Induced emf $\varepsilon = -\dfrac{d\phi_B}{dt}$

$$\varepsilon = -\frac{LdI}{dt} \qquad \text{. . . (9.28)}$$

negative sign shows that induced emf opposes the change in current

If $-\dfrac{dI}{dt} = 1$ than $|\varepsilon| = L$

Numberically, the self induction of a coil is equal to magnitude of induced emf due to unit rate of change of current.

When key is made on current flows in the coil, induced emf opposes the variation in current hence for the continuation of current work is required to be done against induced emf. This work is stored in coil in the form of magnetic potential energy.

Work done in time dt for maintaining current I :-

$$dW = |\varepsilon| I\,dt = \left(L\frac{dI}{dt}\right) I\,dt = LIdI$$

Hence work done to raise current from 0 to I :-

$$W = \int LIdI$$

$$W = \frac{1}{2}LI^2$$

If $I = 1$ than $L = 2W$

Hence self inductance of a circuit is equal to twice the work done against induced emf to maintaine unit current. Inductance is a scalar quantity its SI unit is Henery (H) or Vs/A or Wb/A and its dimensions are $[M^1L^2T^{-2}A^{-2}]$

9.12.2 Self Inductance of a Plane Circular Coil

Suppose a current I flows in a plane circular coil of radius r having N turns. Magnetic field at the centre of coil

$$B = \frac{\mu_0 NI}{2r}$$

magnetic flux linked with each turn of coil due to its own current

$$\phi'_B = BA$$

Total magnetic flux linked with coil is then

$$\phi_B = N\phi'_B = NBA = \frac{\mu_0 NI}{2r} \times N(\pi r^2)$$

$$\phi_B = \frac{\mu_0 N^2 I \pi r}{2}$$

From definition of self inductance

$$\phi_B = LI$$

$$L = \frac{\mu_0 \pi N^2 r}{2}$$

If some material of magnetic permeability μ is filled in the coil then

$$L = \frac{\mu \pi N^2 r}{2} \qquad \ldots (9.30)$$

9.12.3 Self Inductance of a Current Carrying Solenoid

Suppose I current flow through a solenoid of area of cross section A, length l and having number of turns N, than magnetic field inside the solenoid at its axis is

$$B = \mu_0 nI$$

Where $n = \frac{N}{\ell}$ is number of turns per unit length.

Total magnetic flux linked with solenoid

$$\phi_B = N(BA)$$

$$\phi_B = \frac{\mu_0 NI}{\ell} \times NA$$

$$\phi_B = \frac{\mu_0 N^2 A}{\ell} I$$

If self inductance of solenoid is L than

$$\phi_B = LI$$

$$L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell \qquad \ldots (9.31)$$

If solenoid is filled with material of magnetic permeability μ than

$$L = \mu n^2 A\ell \qquad \ldots (9.32)$$

In resistance box the resistance coil are doutly twisted to remove the effect of self induction.

In wheat stone bridge experiment, first we press the battery key and then galvanometer key to remove the effect of induced current.

9.13 Mutual Inductance

If a variable current flows in a circuit or coil, the flux linked with an other coil in its vicinity also changes and emf is incuded in the second coil or circuit. This phenomenon is called mutual induction.

Coil in which current varies is called primary coil and the coil in which emf is induced due to mutual induction is called secondary coil.

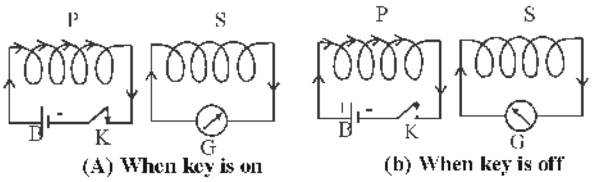


Fig. 9.17 Experiment for mutual induction

In fig. 9.17 coil P in primary circuit is connected to battery B through key K and in secondary circuit coil S is connected to galvanometer G. Both coils are placed near be each other. As key K in primary circuit is pressed there is momentary deflection in the galvanometer G in secondary circuit. When key K is released there is again momentary deflection in galvanometer but in opposite direction. when key K is kept pressed continuously constant current flows in primary coil P, there is no deflection in galvanometer.

Reason for the above results is that when intially key is on made current flows in primary coil magnetic flux around the coil increases and flux linked with secondary coil also increases, which induces emf and current in secondary coil. In the same way when key is made off the magnetic flux through secondary coil decays hence emf and current are induced in secondary coil. In both cases the direction of induced emf and current is such that they opposes the change in magnetic flux. The current is induced in secondary coil only when the magnetic field or magnetic flux of primary coil changes due to its current. This phenomenon is used in transformers and induction coils.

9.13.1 Coefficient of Mutual Inductance

If position, shape and orientation of primary coil C_1 and secondary coil C_2 remains unchanged and current in Coil C_1 is I_1 than magnetic flux ϕ_2 associated with coil C_2 is directly proportional to I_1.

$$\phi_2 \propto I_1$$

$$\phi_2 = MI_1 \qquad \text{. . . (9.33)}$$

Here M is a constant of proportionality called coefficient of mutual induction or mutual inductance between the coils. It depends on number of turns in both the coils, area of secondary coil and nature of medium.

If current in coil C_1 changes with time than flux ϕ_2 linked with coil C_2 also changes, therefore induced emf in coil C_2 is

$$\varepsilon_2 = -\frac{d\phi_2}{dt}$$

or
$$\varepsilon_2 = -M\frac{dI_1}{dt} \qquad \text{. . . (9.34)}$$

Negative sign shows that the direction of induced emf in secondary coil is such that it opposes variation of current in primary coil -

From eq. 9.33 If $I_1 = 1$ than $M = \phi_2$, numerical value of mutual inductance between two coils is equal to the flux linked with secondary coil when unit current passes through primary coil.

From eq. 9.34

If $\frac{dI_1}{dt} = 1$ than $|\varepsilon| = M$

Hence numerical value of mutual inductance is equal to induced emf in secondary coil when there is unit rate of change of current in primary coil. SI unit of M is Wb/A or Vs/A or Henery (H) and its dimension are $[M^1L^2T^{-2}A^{-2}]$.

9.13.2 Mutual Inductance between Two Caoxial Solenoids1

Suppose there are two air cored coaxial solenoids S_1 and S_2. Number of turns in S_1 and S_2 are N_1 and N_2 and length and area of cross section of both the coils are and A respectively. Both the coils are wound in such a way that when current flows in coil S_1 it produces magnetic flux which is completely linked with coil S_2. Magnetic field at the axis of coil S_1 when I_1 current flows through it -

$$B_1 = \frac{\mu_0 N_1}{\ell} I_1 = \mu_0 n_1 I_1$$

Magnetic flux associated with S_2 due to field B_1 -

$$N_2\phi_2 = N_2 B_1 A = (\mu_0 n_1 I) N_2 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell}$$

According to defination of mutual induction

$$N_2\phi_2 = MI_1$$

Hence mutual inductance

$$M = \frac{\mu_0 N_1 N_2}{\ell} A$$

Example 9.16 : Self inductance of a coil is 20 H. For obtaining 100 V induced emf to what value the current is to be reduced in it in 1 second from an initial value of 10 A.

Solution : Induced emf $\varepsilon = L - \frac{dI}{dt} = L\frac{\Delta I}{\Delta t}$

Here given $L = 20$ H

$I_1 = 10$ A, $I_2 = I$

E=100 V

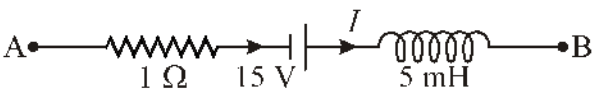
$dt = 1$ s

$$100 = 20\left[\frac{10 - I_2}{1}\right]$$

$10 - I_2 = 5$

$I_2 = 10 - 5 = 5$ A

Example 9.17 : In the fig shown current at some instant in circuit is $I - 5$ A and is decaying at a rate $10^3 A/S$. Then find $V_B - V_A$.



Solution : Rate of change of current in coil

$$\frac{dI}{dt} = -10^3 \text{ A/s}$$

Voltage accross resistance R

$V = IR = 5 \times 1 = 5$ V

Voltage accross termials of cell = 15 V

Voltage across indutance coil

$$= -L\frac{dI}{dt} = -\left(5\times10^{-3}\right)\times\left(-10^3\right) = 5 \text{ V}$$

terminal B is at higher potantial

$$V_B - V_A = 5\text{ V} + 15\text{ V} + (-5)V = 15V$$

Example 9.18 : An air cored solenoid of radius 1 cm has 100 number of turns. Its length is 60 cm. Find self inductance of solenoid.

Solution : Self inductance $L = \frac{\mu_0 N^2 A}{\ell}$

Given $N = 100,\ \ell = 0.60$ m,

$A = \pi r^2 = 3.14 \times (.01)^2 m^2$

$$L = \frac{4\pi \times 10^{-7} \times (100)^2 \times \pi(0.01)^2}{0.60}$$

$= 65.73 \times 10^{-7}$

$= 6.573 \times 10^{-8}$ H

Important Points

1. When vector area $\vec{A}$ is placed in magnetic field $\vec{B}$, $\vec{A}$ is at an angle θ with $\vec{B}$, magnetic flux passing through $\vec{A}$ is given by

   $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

2. Faraday Law :- According to Faraday's law of electro magnetic induction induced emf in a coil of N turns is equal to rate of change of magnetic flux passing through the coil

   Induced emf $\varepsilon = -N\frac{d\phi_B}{dt}$

3. When magnetic flux linked with a circuit changes, emf is induced in the circuit. If circuit is closed current is also induced. The phenomenon is called electro magnetic induction.

4. Lenz's law :- In electro magnetic induction, direction of induced emf and current is such that they oppose the cause due to which these are produced.

5. In electro magnetic induction

Induced current $I = \frac{E}{R} = -\frac{N}{R}\frac{d\phi_B}{dt}$

Induced charges $q = I\,dt = \frac{-N}{R} d\phi_B$

6. Right hand rule :- According to this rule the index finger, central finger and thumb of right hand are held out perpendicular to each other. If index finger showns direction of magnetic field and thumb shows direction of motion then central finger shows direction of induced current.

7. If conducting rod of length l moves with velocity v in uniform magnetic field B perpendicular to direction of field and its own length than induced emf accross the rod

$\varepsilon = \mathrm{B\,v\,l}$

Induced emf in a rod moving at an angle θ with the direction of magnetic field is given by -

$\varepsilon = \mathrm{B\,v\,l} \sin\theta$

8. Induced emf due to motion of rectangular loop with velocity v in non uniform magnetic field

$\varepsilon = (B_1 - B_2)\,vl$

here B_1 and B_2 are magnetic field on the two arms respectively.

9. Work done for moving a rectangular loop in magnetic field appears as electrical energy in the circuit and finally spent in the form of heat energy

$$W = H = \frac{(B_1 - B_2)^2 \ell^2 v^2 \Delta t}{R}$$

10. Due to rotation of a conducting rod of length L with with angular velocity ω in uniform magnetic field B, induced emf between the ends is given by -

$$E = \frac{1}{2} B\omega L^2 = BAf$$

11. Induced emf between centre and circumference of a metallic disc of radius r rotating with angular velocity ω in uniform magnetic field B

$\varepsilon = 1/2\, \mathrm{B}\, \omega\, \mathrm{L}^2 = \mathrm{BAf}$

12. If a rectangular conducting coil of N turns and area of cross section A rotates with angular velocity ω in uniform magnetic field B then induced emf

$\varepsilon = \mathrm{NBA}\omega \sin \omega t$

13. Circulating currents are induced in bulk metallic pieces placed in changing magnetic field. In these loops, electrical energy is spent in form of heat, these currents are called eddy currents. They are used in brakes of electric train, induction furnace, galvanometer etc.

14. Inductance is equal to ratio of linked magnetic flux and current. It is of two types (i) self inductance (ii) Mutual inductance.

15. Self inductance of a coil is equal to the magnetic flux associated with coil when unit current flows in it.

16. When current through a coil changes it produces an opposing emf which is given by

$$E = -L\frac{dI}{dt}$$

17. Work done against induced emf to maintain current I in the coil

$$W = \frac{1}{2}LI^2$$

18. Self inductance of a solenoid of length l and number of tunrs per unit length n is

$$L = \mu_0 n^2 Al$$

here A is area of cross section of solenoid.

19. When current in a coil or circuit changes then associated magnetic flux in another coil in its vicinity also changes due to this change, emf or current is induced in the second coil. This phenomenon is called mutual induction.

20. Induced emf due to phenomenon of mutual induction

$$E_2 = -M_{21}\frac{dI_1}{dt}$$

here M_{21} is mutual inductance of second coil relative to first coil.

21. Mutual inductance between two coaxil solenoid.

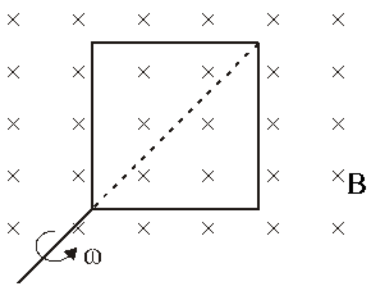
$$M_{21} = M_{12} = \frac{\mu_0 n_1 n_2 A}{l} \text{ H}$$

Questions for Practice

Multiple Choice Questions -

1. A conducting rod is moving with velocity V in a magnetic field B. An emf is induced across its ends when -
   (a) v and B are parallel
   (b) v and B perpendicular to each other
   (c) v and B are in opposite direction
   (d) All of the above

2. A square loop of length X. Loop is rotating with angular velocity ω about its diagonal in a perpendicular magnetic field as shown in fig. Find magnetic flux associated with loop at any moment number of turns in the loop is 20.



   (a) 20 B x (b) 10 Bx^2
   (c) 20 $Bx^2 \cos \omega t$ (d) 40 Bx^2

3. The unit of ratio of magnetic flux and resistance is same as which of the following quantity -
   (a) Charge (b) Potential difference
   (c) Current (d) Magnetic field

4. In electromagnetic induction the magnitude of induced emf depends only on -
   (a) Resistance of the conductor
   (b) Magnitude of magnetic field
   (c) Orientation of conductor realtive to direction of magnetic field
   (d) Rate of change of linked flux

5. When a bar magnet enters inside a coil, the induced emf in coil does not depends on
   (a) Velocity of magnet
   (b) Number of turns in coil

(c) Magnetic moment of magnet

(d) Specific resistance of wire of coil

6. A copper wire coil is moving in a uniform magnetic field parallel to the field then what is the value of induced current -

(a) Infinite (b) Zero

(c) Equal to magnetic field

(d) Equal to area of cross section of coil

7. Lenz's law gives -

(a) Magnitude of induced current

(b) Magnitude of induced emf

(c) Direction of induced current

(d) Magnitude and direction of induced current both

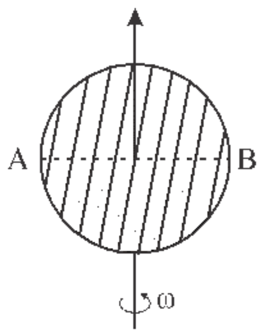
8. A copper wire coil C and a wire are place in the plane of paper as shown in fig. If current in wire increases from 1 A to 2 A along the direction shown in fig, than what is the direction of induced current in coil -

(a) Clockwise (b) Anticlock wise

(c) Current will not be induced

(d) None of above

9. If a disc is rotated about its axis and if magnetic field is uniform and along the axis of rotation than what the potential difference between the edges of diameter AB -



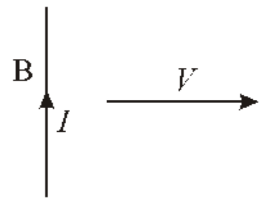
(a) Zero

(b) Half of potential difference between centre and circumference

(c) Double of potential difference between centre and circumference

(d) None of above

10. A conducting wire is moving towards right in magnetic field B. If direction of induced current is as shown in fig then the direction of magnetic field is-



(a) In the plane of paper towards left

(b) In the plane of paper towards right

(c) Perpendicular to the plane of paper, downward

(d) Perpendicular to the plane of paper, upward

11. In a electric transmission line current is flowing along north direction. On considering earth magnetic field negligible be find the direction of magnetic field above the electric line -

(a) Along east (b) Along west

(c) Along north (d) Along south

12. A coil is rotating in a uniform magnetic field. What is the phase differnce between induced emf and linked magnetic flux-

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) π

13. Current in a coil of self inductance 2×10^{-3} H rises uniformly in 0.1 sec 1 A than what is the magnitude of induced emf.

(a) 2 V (b) 0.2 V

(c) 0.02 V (d) Zero

14. If a coil having 100 turns produces a magnetic flux 5×10^{3} Maxwell, by 5 A current. What is its self inductance

(a) 0.5×10^{-3} H (b) 2×10^{-3} H

(c) Zero (d) 10^{-3} H

15. The magnetic flux passing perpendicularly through a coil changes with time as $\phi = 10t^2 + 5t + 1$ here t is in seconds and ϕ mWb then induced emf at t = 5s is -

(a) 1 V (b) 0.105 V

(c) 2 V (d) 0 V

Very Short Answer Type Questions -

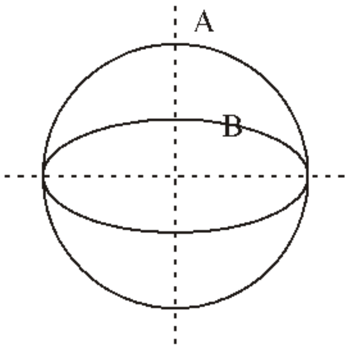
1. If current in inductance is doubled than how many times the stored energy increases?
2. When an electric circuit is suddenly breaked than why there is sparking?
3. How the mutual inductance between two coils can be increased?
4. On doubling the area of cross section while keeping the some number of turns in a coil, what will be the value of self inductance?
5. How we can reduce the effect of eddy currents in the core of transformer?
6. A metallic and a non metallic coins are droped from same height towards earth surface. Which coin reaches earlier on earth and why?
7. Why self induction is called electrical inertia?
8. On what factors and in what way the the self inductance of a solenoid depends?
9. In a wire current flows at high voltage. As current is swithed on in the wire why the bird sitting on wire fly?
10. Write down dimensions of L/R here L is self inductance and R is resistance.
11. When a rectangular loop moves in a uniform magnetic field with constant speed than what is the magnitude of induced emf?
12. In what way two coils are wound so that induced emf is maixmum?
13. If a coil or rectangular loop rotates in magnetic field what factors effects theinduced emf in it?
14. A straight and long wire is dropped freely in gravitional field keeping in north-south direction, why emf is induced in the wire?
15. How eddy currents are used to make the galvanometer dead beat?

Short Answer Type Questions -

1. What do you mean by electro magnetic induction? Write down Faraday's laws for electro magnetic induction and magnitude of induced emf?
2. If a coil is removed from magnetic field (i) with high rapidly (ii) slowly, then in which situation induced emf and work done is more.
3. Write down Lenz's law for electro magnetic induction and explain that Lenz's law follows the law of energy conservation.
4. When a metallic plate is pulled out of a uniform magnetic field or enters in a uniform magnetic field why we experience opposing force?
5. What is the reason

   (i) Resistance wire coils are doubly twisted in resistance boxes

   (ii) In wheatstone bridge why cell key is pressed first and then galvanometer key?
6. Write down Fleming's right hand rule for the direction of induced current?
7. Define mutual inductance and write its unit and dimensions.
8. A conducting rod of length L is rotating with uniform angular velocity ω in magnetic field B in such a way that the plane of rotation is perpendicular to magnetic field then find out induced emf between its ends?
9. Coils A and B are placed perpendicular to each other as shown in fig. If current in any one coil varies than will current be induced in other coil and why?



10. What factors effects the mutual inductance between two coil S_2 explain?
11. If the self inductance of a coil is 1 H, what do you understand by it.
12. Prove that induced charge $q = \frac{N}{R}(\phi_1 - \phi_2)$

   when flux associated with a coil changes from

ϕ_1 to ϕ_2 . Here N is number of turns in coil, R is its resistance.

13. Prove that law of conservation of energy holds good when a rectangular coil moves perpendicular to a non uniform magnetic field with constant velocity?

Essay Type Questions -

1. Find out induced emf due to motion of conducting rod in uniform magnetic field with a constant velocity. How, we can find the direction of induced emf.
2. A rectangular loop is moving perpendicular to a non uniform magnetic field with constant velocity. Find out expression for induced emf and current and also prove that the law of conversation of energy holds good here.
3. If a rectangular coil of area A and number of turns N is rotating in a uniform magnetic field with a constant angular velocity ω . Prove that induced emf in the coil is $NBA\omega \sin \omega t$.
4. What is meant by self induction? Explain the phenomenon self induction through an experiment and find out self inductance of solenoid?
5. What are eddy currents? Write down thier two uses. How unwanted eddy currents are reduced in transformers?

Answer (Multi Choice Questions)

1. (b) 2. (c) 3. (a) 4. (d) 5. (d) 6. (b)
7. (c) 8. (a) 9. (c) 10. (c) 11. (a) 12. (b)
13. (c) 14. (d) 15. (b)

Numerical Questions

1. A window of metallic frame (120 x 50 cm) is on a wall which is parallel to magnetic meridian. Total resistance of frame is 0.01 Ω. When window is opened at 90^0 then find the amount of charge flown in the frame.

(If H = 0.36 G)

2. The magnetic flux passing through a coil of 50 turns is given by $\phi_B = 0.02 \cos 100\pi t$ Wb

Find out -

(a) Maximum induced emf

(b) Induced emf at time t = 0.01 S

(c) Induced current at t= 0.005 S (if external resistance is 100 Ω)

(3.14 V, zero 13.14 A)

3. A coil of 50 turns and area 0.2 m^2 in placed perpendicular to a 0.6 T magnetic field the resistance of circuit of coil is 10 Ω then find out induced charges - (a) When coil rotates by 180^0 (b) Coil is pulled out of magnetic field

(1.20 C, 0.60 C)

4. A conductor of length $-3\hat{k}$ m is moving with velocity $i + 2\hat{j} + 3\hat{k}$ m/s in $\hat{i} + 3\hat{j} + \hat{k}$ T magnetic field. Find out potential difference across the ends of conductor.
5. A rectangular coil of 1000 turns and 0.02 x 0.1 m^2 size is rotating with 4200 revolutions per mintues in 0.2 T magnetic field. Find the maximum induced emf in coil.

(1758.4 V)

6. One meter long conducting rod rotating with angular velocity 50 rotations/sec in a place perpendicular to a magnetic field of 0.001 T about its one of end. Find the induced emf across it ends.

(0.157 V)

7. Length and diameter of a solenoid are 1 m and 0.05 m respectively there are 500 turns/cm in the solenoid. Find the magnetic flux when 3 A current flows through it.
8. Length of a solenoid of radius 2cm and 100 number of turns is 50cm. Find the self indicates of solenoid in vacuume.
9. Two coils are wound on iron core. Their mutual inductance is 0.05 H. If current through one of the coil changes from 2 A to 3 A in 10^{-2} sec than find out induced emf in the other coil.

(-50 V)

10. Wires are wound on a soft iron rod of length 0.1 m and radius 0.01 m, to form a coil. If relative permeability of soft iron is 1200 then find out number of turns in coil.

(Self inductance of coil 0.25 H)

11. A metallic disc of diameter 15 cm is rotating in

horizontal plane with $\frac{100}{3}$ rotations per minutes. It vertical component of magnetic field is $0.01 Wb/m^2$ than find out induced emf between centre and circumference of the coil.

(9.75×10^{-3} V)

12. A 20 cm conducting long wire is placed perpendicular to 5×10^{-4} Wb / m^2 magnetic field. Wire is moving perpendicular to its length and magnetic field. If wire moves 1m is 4 s than find induced emf between its ends.

(2.5×10^{-5} V)

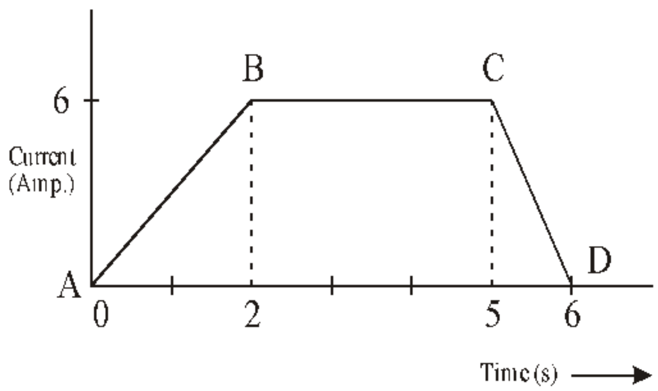
13. A 12 m long metallic rod is moved from west to east with speed 15 km/h by keeping it (i) vertical (ii) horizontal. If horizontal component of earths magnetic field is 0.5×10^{-5} Wb / m^2 there find out induced emf across rod in each situation.

(4.16×10^{-5} V, 0)

14. If current through primary coil is reduced from 5 A to zero is 2 ms than induced emf in the secondary coil is 25 kV. Find out mutual inductance of the coils.

(10 H)

15. Self inductance of a coils is 2 H, variation of current with time in the coil is shown in following graph. Draw graph for the variation of induced emf with time.



Chapter - 10

Alternating Current

Whenever a voltage source is connected to a circuit, then free electrons in conductors have a motion in a particular direction, along with random motion. The rate of flow of charge at any point of the circuit is called current. In direct current circuits, the current source is a cell or a battery and a resistor R is used to control the current. Generally, the electrical energy is generated as alternating current due low cost of production and conveniance in transmission to long distances. It can be converted to direct current easily when required. Generally alternating current and voltage varies sinusoidal with time. To control it capacitor C and inductor L are also used, together with R. In a R-L-C circuit it is not necessary that current and voltage are in phase, i.e. it is not necessary that the current to be maximum when voltage is maximum. A transformer is used to step-up or step down the AC voltage, so that its transmission over a long distance is possible economically and at low energy loss.

In this chapter we will study phase relationship between AC voltage and current in different circuits, power, watt less current, transformer etc.

10.1 Direct Current

The current/voltage whose direction of flow does not change with time is called Direct Current. This current is produced by the such voltage sources whose terminals have constant polarity with time. If this current is ploted with time we get a straight line parallel to the time axis. This current (or voltage) is called unidirectional or direct current (or voltage) . Its frequency f is zero. (fig. 10.1)

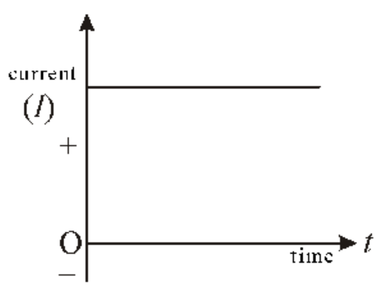


Fig 10.1 : Direct current

If the current from some special devices like rectifier is studied than it is found that it has a defenite direction but its value has a small peroidic/ pulsatingchange such currents are called direct current (or voltage) of unequal fluctuations or pulsating DC.

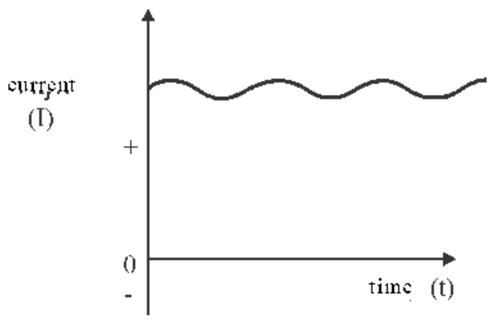


Fig 10.2 : Nonuniform DC

10.2 Alternating Current

The current (voltage) which changes its direction peoriodically with time and alternatively becomes positive and negative in each half cycle is called alternating current. It is obtained from the sources whose terminals change their polarity periodally with time.

Alternating current may be of many types according to their wave forms-few of them are following-

10.2.1 Square Wave Current

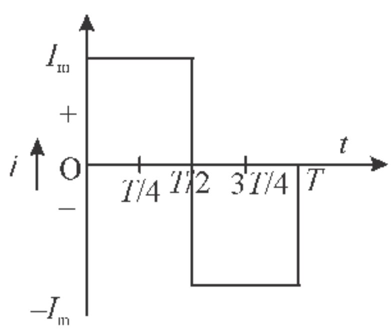


Fig 10.3 : Square ac wave

In this type of current, current will remain I_m (maximum) from t = 0 to t = T/2 and at T/2 it suddenly becomes - I_m, (minimum) which remains same up to t = T, again becomes zero at time T.

Thus for $0 \le t \le T/2$, $I = I_m$

for $T/2 \le t \le T$, $I = -I_m$

10.2.2 Triangular ac Wave Current

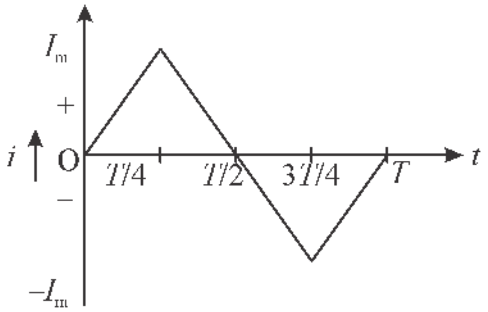


Fig 10.4 : Triangular ac wave current

In this type of current, the current linearly increases from 0 to I_m, from t = 0 to t = T/4: then linarly decreases to 0 at T/2 and becomes - I_m at t = 3T/4. Ultimately at t = T it becomes zero. (Fig 10.4)

at	$t = 0$	$I = 0$
at	$t = T/4$	$I = I_m$
at	$t = T/2$	$I = 0$
at	$t = 3T/4$	$I = -I_m$
at	$t = T$	$I = 0$

10.2.3 Sinusoidal Wave ac Current

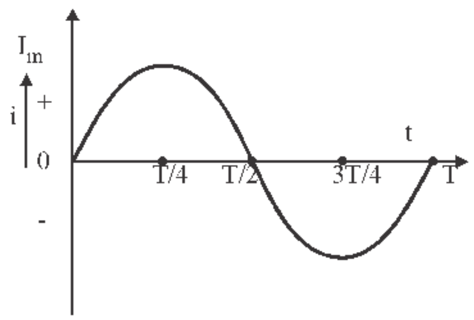


Fig 10.5 : Sinusoidal wave ac current

This is the simplest and basic form of ac It varies as sine or cosine functions. (Fig 10.5), hence are called sinusoidal current.

In this chapter, we will study sinusoidal ac.It is to be mentioned that all forms of ac i.e. square or triangle all are mathematically produced by super position of sinusoidal waves of different amplitude and frequencies.

The frequecny of ac used for domestic use is 50 Hz in India. In U.S.A it is 60 Hz. At any instant t the ac current and voltage are given by the equation :-

$$I = I_m \sin(\omega t + \phi) \qquad \ldots (10.1)$$

$$V = V_m \sin(\omega t) \qquad \ldots (10.2)$$

here I_m and V_m are the maximum value of ac current and voltage and are also called peak values of current and voltage.

The symbol of a.c. voltage is ~

This type of ac can be produced by a rotating coil in uniform magnetic field or by electronic oscillatory circuits.

10.3 Instantaneous, Peak, Average and Root Mean Square of Alternating Voltage and Current

10.3.1 Instantaneous Value

The value of current or voltage at any instant in an ac circuit is called instantaneous value.It can be zero, positive or negative. Equation (10.1) and (10.2) gives instantaneous values in a simple periodic form. Here ϕ is the phase difference in voltage and current at any instant t.

10.3.2 Peak Value

The maximum value of ac voltage or current in a complete cycle is called its peak value. It also represents the amplitude of alternating change. In equations (10.1) and (10.2) I_m and V_m are the peak values of alternating current and voltage respectively.

10.3.3 Average Value

In an ac circuit the magnitude and direction of voltage/current changes periodically with time. The average of all these values for a complete cycle is called average value of AC. For a complete cycle the average is -

$$I_{av} \text{(For a complete cycle)} = \frac{\int_0^T I dt}{\int_0^T dt} = \frac{I_m}{T}\left[\int_0^T \sin \omega t \, dt\right]$$

$$= \frac{I_m}{T}\left(\frac{-\cos \omega t}{\omega}\right)_0^T = \frac{-I_m}{\omega T}(\cos \omega T - \cos 0)$$

I_{av} (for a complete cycle) $= \frac{-I_m}{\omega T}(0)$

$(\because \omega T = 2\pi \text{ and } \cos 2\pi = 1)$

I_{av} (for complete cycle) = 0

Hence the average of AC for a complete cycle is always zero.

Average value for first positive half cycle-

$$I_{av} = \frac{\int_0^{T/2} I_m \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{I_m}{T/2}\left(-\frac{\cos \omega t}{\omega}\right)_0^{T/2}$$

$$= -\frac{2I_m}{\omega T}\left(\cos\frac{\omega T}{2} - \cos 0\right) = \frac{2I_m}{\pi} = 0.636\, I_m$$

similarly, for second half cycle the value will be

$$I_{av} = -0.636\, I_m$$

10.3.4 Root Mean Square Value

In an ac circuit for a complete cycle the square root of average of squares of current and voltage is called root mean square value of current and voltage current I_{rms} and voltage V_{rms}.

$$I_{rms} = \sqrt{I^2_{av}} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt} = \sqrt{\frac{I_m^2}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T}$$

$$= \sqrt{\frac{I_m^2}{2T}\left(T - \frac{\sin 2\omega T}{2\omega}\right)} = \sqrt{\frac{I_m^2}{2T}(T-0)} = \sqrt{\frac{I_m^2}{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m \qquad \ldots (10.3)$$

Similarly $V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m \qquad \ldots (10.4)$

The values shown by all ac meters in a circuit are the RMS values of these quantities.

The rms value is also called virtual or effective value which means that rate of heating effect in a resistor is same as that of equivalent value of dc and I_{rms}. The rate of heat generated by ac in a complete cycle is -

$$H_{av} = \frac{\int_0^T I^2 R\, dt}{\int_0^T dt} = I^2_{rms} \times R$$

from a dc the rate current the raate of heat produced is $= I^2_{dc} R$

Hence both rates are equal - $I_{dc} = I_{rms}$

Thus $I \quad = I \quad = I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \ldots (10.5)$

If we want to measure the ac voltage and current using dc moving coil meters, they will give zero reading, since the torque on the coil changes so rapidly, that it will not respond due to inertia of the coil.

To measure ac we use hot wire meters, which are based on heating effect. The heat produced depends on V^2_{rms} or I^2_{rms}, hence the scale of meters is nor linearly marked. The relative distances between the making will be in the ratio of 1 : 4 : 9 : . . . etc. for the currents I, 2I, 3I, ... etc.

The domestic supply of ac in India has $V_{rms} = 220\text{ V}$. Hence its peak value will be

$$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} \times 220 = 311\text{ V}$$

10.3.5 Properties of ac

Merits : -

(i) Alternating voltage can be steped up or steped down, so the transmission of electri power is possible at high voltage and low current with very small power loss for long distance transmission.

(ii) It can be easily converted to dc by a rectifier.

(iii) Alternating current generators and motors are more rugged, and convenient in operation, also their cost is less than dc generators and motors.

Demerits : -

(i) Alternating current voltage of any value is more dangerous than its equivalent dc voltage, because its peak value is $\sqrt{2}$ times the rms value.

(ii) Skin effect :- High frequency ac current does not uniformly pass through the whole cross-section of the conductor. It prefers the surface layer (skin) of the conductor. So a thick wire in replaced by a bunch of thin wires to reduce this effect.

(iii) It can't be used directly for elecrolysis, electro plating and making electro magnet.

Example 10.1 : Find RMS value of the AC current given by $I = I_1 \cos\omega t + I_2 \sin\omega t$.

Solution : $I = I_1 \cos\omega t + I_2 \sin\omega t$

hence

$$I^2 = I_1^2 \cos^2\omega t + I_2^2 \sin^2\omega t + 2I_1I_2 \sin\omega t \cos\omega t$$

$$\overline{I^2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^T (I_1^2 \cos^2\omega t + I_2^2 \sin^2\omega t) + 2I_1I_2 \sin\omega t \cos\omega t}{\int_0^T dt} dt$$

$$\overline{I^2} = I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 0$$

since the average value of $\sin^2\omega t$ & $\cos^2\omega t$ for a complete cycle is 1/2 and that of $\sin 2\omega t = 0$.

$$I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2}} = \frac{1}{\sqrt{2}}\left(I_1^2 + I_2^2\right)^{\frac{1}{2}}$$

Example 10.2 : The rms value of a sinusoidal ac of frequecny 50 Hz is $200\sqrt{2}$ V . Write down the equation for its instantaneous value at time t.

Solution : Given

$V_{rms} = 200\sqrt{2}$ V and $f = 50 Hz$

So $V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 200\sqrt{2} = 400$ V

$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/s

hence $V = V_m \sin\omega t$

$V = 400 \sin 314t$ V

Example 10.3 : Find the frequency of ac voltage given by $V = 400 \sin 100\pi t$

Solution : General equation of AC voltage is

$V = V_m \sin\omega t = V_m \sin 2\pi ft$

and the given equation of voltage

$V = 400 \sin 100\pi t$

comparing the two equations we get

$2f = 100$

Thus

$f = 50$ Hz

Example 10.4 : The peak value of ac current in a circuit is 5 A. What will be the value of current given by (i) ac ammeter (ii) dc ammeter.

Soution : (i) Since ac ammeter always meausres rms value

then $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 2.5 \times \sqrt{2} = 3.535$ A

(ii) dc ammeter meausres averages value for one cycle hence $I = 0$.

Example 10.5 : The rms voltage in a circuit is 220 V then find peak value of voltage.

Solution : $V_m = \sqrt{2} \times V_{rms}$

Given $V_{rms} = 220$ V

$V_m = 220 \times \sqrt{2} = 311.08$ V

Example 10.6 : The ac current is given by $I = 3 \sin 2\pi t$ A , then find (i) rms value of current (ii) instantaneous value of current at $t = \frac{1}{2} s$.

Solution : (i) $I_{rms} = \frac{I_m}{\sqrt{2}}$, then

Given $I_m = 3$ A, $t = \frac{1}{2} s$

$$I_{rms} = \frac{3}{\sqrt{2}} = 2.12 \text{ A}$$

(ii) $I = 3 \sin 2\pi \times \frac{1}{2} = 3 \sin\pi = 0$

Example 10.7 : Find the time to reach from zero to its maximum value of ac current of frequency 50 Hz .

Solution : Time taken by current to reach from zero to its maximum value is

$$t = \frac{T}{4}$$

thus $t = \frac{1}{4f}$

given $f = 50$ Hz

thus $t = \frac{1}{4 \times 50} = 0.005\,s$

10.4 Phase Relation between alternating voltage and alternating current in different types of ac circuits and phasor diagram

10.4.1 Pure resistive ac Circuit

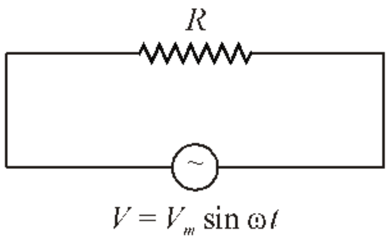


Fig 10.6 : Pure resistor in ac circuit

In fig 10.6 a pure resistor R is connected to an ac source voltage $V = V_m \sin \omega t$. If the current in the circuit be I, then using Kirchhoff's loop law, the voltage developed across R is equal to voltage applied.

$$V_m \sin \omega t = IR$$

$$I = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \ldots (10.6)$$

here $I_m = \frac{V_m}{R}$ is the peak value of ac current. It is clear from equation (10.6) that on applying sinusoidal voltage, we get sinusoidal current, and both are in same phase. It means that value of V and I will be zero simultaniously and maximum simultaniously.

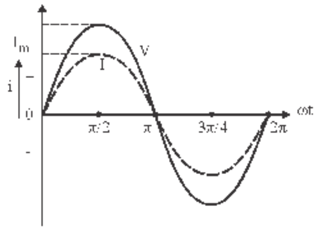


Fig 10.7 : Sinusoidal nature of ac voltage and current and their phase relation in pure resistive ac circuit

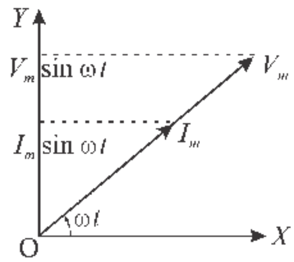


Fig 10.8 : Phasor diagram in pure resistive ac circuit

Fig 10.7 represent the sinusoidal nature of ac voltage and current. As mentioned earlier it is not always necessary that V and I are in same phase in ac circuits. For such ac circuits the concept of phasor makes the analysis simple. To represent ac quantities (V, I, R, X, Z etc), a rotating vector is used whose magnitude equals to the peak value and rotational frequency is equal to the frequency of ac voltage or current. Such a rotating vector is called phasor, and the related diagram is called phasor diagram. If the tail of rotating vector is at origin, and vector coincide with X axis at t = 0, then at instant t it makes an angle $\theta = \omega t$ with X-axis, and the y component gives the instantaneous value at instant t. Fig 10.8 shows the values of ac voltage and ac current at an instant t, represented by their respective phasors. These phasor are shown as making an angle ωt with x-axis. Both phasor rotates in anticlock-wise direction with frequency ω. The vertical components of the phasor represents instantaneous values of AC voltage or current that's why they are taken in Y-axis. Fig 10.8 shows the phasor diagram for a pure resistive circuit, V and I are in same direction and phase difference between them is zero. But as you will see later, they will not be in same phase for inductive or capacitive circuits.

A pure resistance obstructs the flows of current which is independent of the frequency of applied ac.

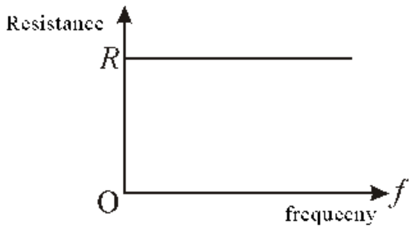


Fig 10.9 : Dependence of R on frequecny

10.4.2 A Pure Inductive ac Circuit

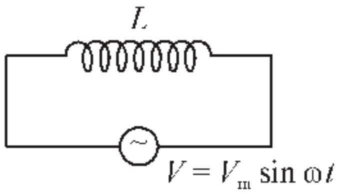


Fig 10.10 : Pure inductive ac circuit

Fig 10.10 represents pure inductor (coil of thick copper wire of resistance, R=O) is joined to an ac voltage $V = V_m \sin \omega t$. Inductance of the coil is L. Since the voltage changes with time, the current will also change. The voltage developed

across the inductor $\left(-L\dfrac{dI}{dt}\right)$. From Kirchhoff's

law $V_m \sin \omega t = \dfrac{LdI}{dt}$,

$$dI = \frac{V_m}{L}\sin \omega t\, dt \quad \ldots (10.7)$$

The current $\int dI = \int \dfrac{V_m}{L}\sin \omega t\, dt$

$$I = \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right)$$

$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots (10.8)$$

here $I_m = \dfrac{V_m}{L\omega}$. The Lω has the same dimensions as resistance and called Inductive reactance, represented by X_L.

$$X_L = L\omega \quad \ldots (10.10)$$

and $$I_m = \frac{V_m}{X_L} \quad \ldots (10.11)$$

X_L controlls current in ac circuit in the same way as a resistor, (but with a different mechanism given later). Unit of X_L is ohm.

From equation (10.8) it is evident that the current in pure inductive circuit is also sinusoidal with same frequency that of applied voltage, but lags behind the applied voltage by a phase of $\pi/2$. Which means that the current gets its maximum value compared to voltage after a time interval of T/4 ; (Fig 10.11).

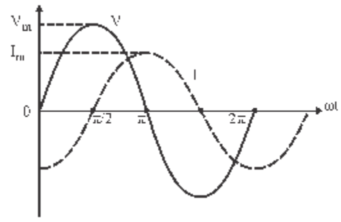


Fig 10.11 : Graph of V and I with ωt

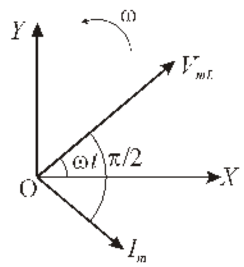


Fig 10.12 : Phasor diagram for pure inductive circuit

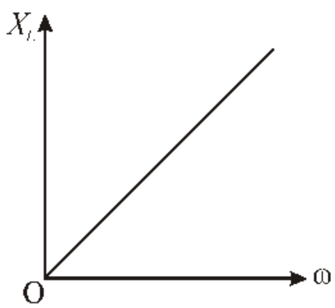


Fig 10.13 : Variation of X_L with frequency

From equation 10.10 $X_L = L\omega$

$$X_L = L \times 2\pi f \quad \ldots (10.12)$$

Fig 10.13 represents graph between X_L and ω. Slope of this graph $\tan\phi$, represents cofficient of self inductance of the coil.

For direct current, f = 0, hence X_L = 0, so a pure inductor short circuits the dc circuit, but opposes the flow of ac current.

In a pure inductive circuit alternative voltage is $V = V_m \sin \omega t$ and the current is given by

$I = I_m \sin(\omega t - \frac{\pi}{2})$ so voltage leads the current by $\pi/2$ or 90^0.

The magnetic flux $\phi = LI$; $\phi \propto I$ and the power P = VI.

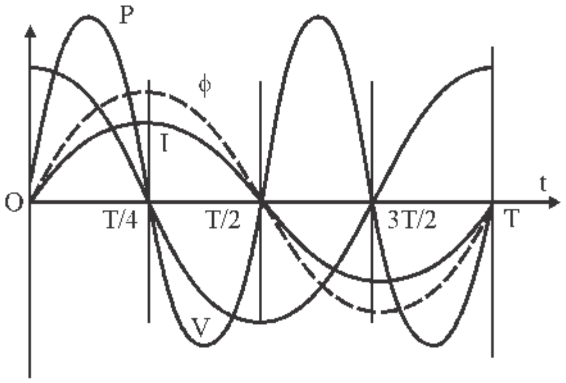


Fig 10.14 : Graph of Power in pure inductive ac circuit

Fig 10.14 shows all the four quantities in one complete cycle of ac, for an inductor.

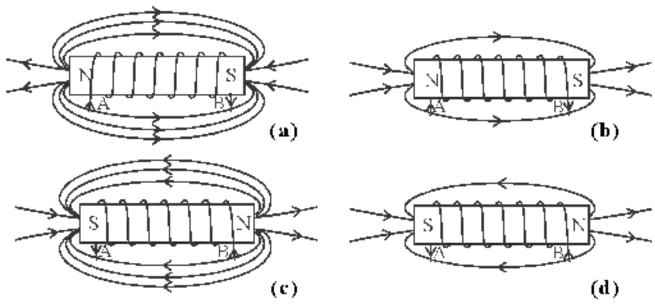


Fig 10.15 : Change in and for an inductor for one complete cycle. (a) 0 to T/4 (b) T/4 to T/2 (c) T/2 to 3T/4 (d) 3T4/4 to T

To understand the power and flux change in an inductive circuit. Let us consider the fig 10.15 in which change in current is considered with its respective change in flux.

In fig (A) the current enters at A and reach the maximum value, so the flux. The core is magnetized, voltage and current are both positive hence their product, power is also positive. It means the circuit absorbs energy from source.

Fig (B) current decreases from T/4 to T/2. At T/2 the core is demagnetized and the total flux becomes zero. Voltage is negative and current is positive, which means that power is negative, it implies that circuit returns rhe absorbed energy to the source.

Fig (C) the current is increasing in opposite direction during T/2 to 3T/4 and flux also, core is magnetized in opposite direction. Both voltage and current are negative, their product power is positive. The circuit absorbs energy from source.

Fig (D) from 3T/4 to T current decreases to zero, V is positive while I is negative, Power is negative, core is demagnetized and the circuit returns the absorbed energy to the source.

For a complete cycle, average power in an inductor is zero. We will prove this in section 10.7.

10.4.3 Pure Capacitive ac Circuit

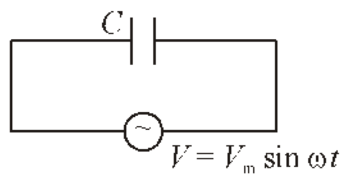


Fig 10.16 : A capacitor connected to ac source

Fig 10.16 shows a capacitor of capacity C is connected to ac voltage $V = V_m \sin \omega t$. If the current in the circuit is I, and voltage developed across the capacitor is V_C, then from Kirchhoff's law $V - V_C = 0$; If change on capacitor in q at time t, then instantaneous voltage accross it is

$$V_C = \frac{q}{C}$$

so $\frac{q}{C} = V_m \sin \omega t$ or $q = V_m C \sin \omega t$

Thus, current in the circuit is

$$I = \frac{dq}{dt} = \frac{d}{dt}(V_m C \sin \omega t)$$

$$I = V_m C\omega \cos \omega t$$

since $\quad \cos \omega t = \sin (\omega t + \frac{\pi}{2})$

hence $\quad I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots (10.13)$

where $\quad I_m = V_m C\omega$

or $\quad I_m = \frac{V_m}{1/C\omega} \qquad \ldots (10.14)$

The dimension of $1/C\omega$ is that of a resistor and its unit is ohm, and it is the measure of obstacle produced by a capacitor in the ac circuit. It is called capacitive reactance, and expressed as X_C.

$$X_C = \frac{1}{C\omega}$$

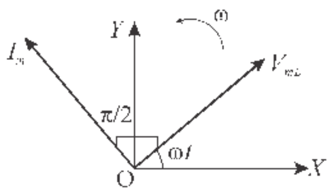


Fig 10.17 (A) Phasor diagram for pure capacitive ac circuit

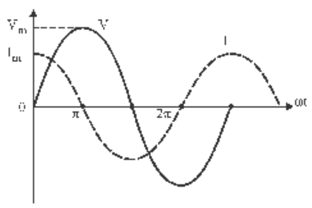


Fig 10.17 (B) V and I plotted against ωt

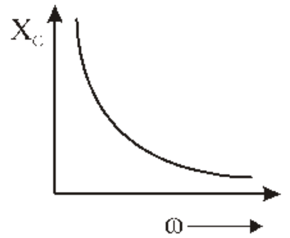


Fig 10.17 (C) Variation of X_C with ω

As it is evident from the equation 10.13, that if a sinusoidal voltage is applied to a pure capacitor the current will also be sinusoidal, but leads the voltage by $\pi/2$. It means that current reach the maximum value earlier by time T/4 compared to the voltage.

From equation (10.15) $X_C = \frac{1}{C\omega}$ and graph (10.17) (C) show the variation of X_C with ω. For dc, $f = 0$ ($\omega = 2\pi f = 0$) $X_C = \infty$ so dc is not allowed by a capacitor. But for ac, X_C has some finite value, so a capacitor allows it to pass through.

For a pure capacitive circuit

$$V = V_m \sin \omega t; \qquad \text{and} \qquad I = I_m \sin(\omega t + \frac{\pi}{2}) \;,$$

hence the voltage lags the current by $\pi/2$ radian.

q = CV and $P - VI$. All the four quantities are plotted against time in fig (10.18) for a complete cycle.

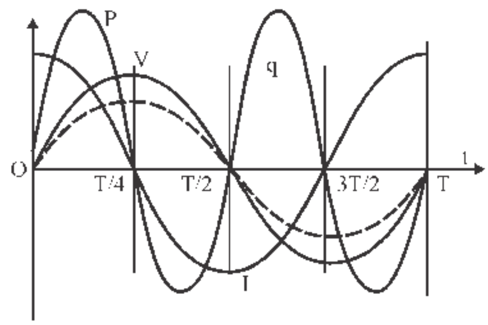


Fig 10.18 : Graph for power in a capacitor

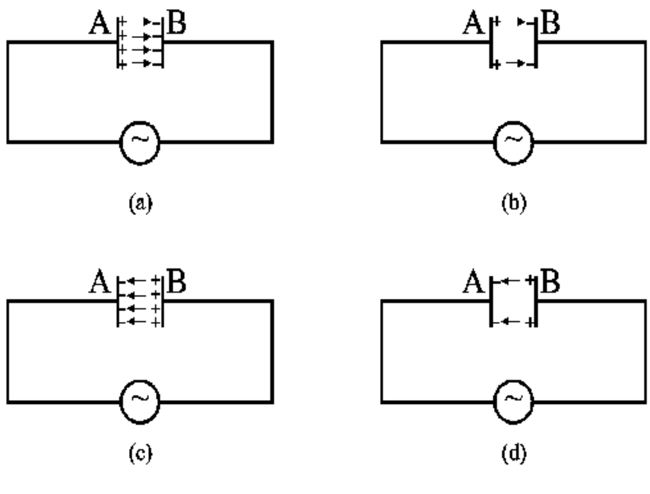


Fig 10.19

Fig 10.19 : (a) shows that from t = 0 to T/4 current increases from zero to maximum, hence the charge on the plates and the potential. Plate a becomes positive while B is negative. Circuit absorbs energy from the source.

Fig 10.19 (b) the current is in opposite direction, the charge on the plate decreases to zero hence the potential. Circuit returned energy to the source from $T/4$ to $T/2$. Fig 10.19 (c) the charge on the plates rises to its maximum value, hence the potential and field between the plates. The circuit absorbs energy from the source from $T/2$ to $3T/4$. Plate A becomes negative while the plate B is positive. Voltage and current both are negative. So power is positive.

Fig 10.19 (d) again shows that during $3T/4$ to T, the stored charges decreases, as the voltage. At T,the capacitor is completely discharged. Current is positive while potential is negative, hence power is negative. It means that during this interval. Circuit returns the abrobed energy to the source.

In this way the net energy absorbed during one complete cycle by a capacitor is always zero.

Example 10.8: A resistanceless coil of inductance $L = \frac{5}{\pi}$ m H is connected to an ac source of frequency 50 Hz. Find the inductive reactance of the coil. If the current in the circuit is 0.5 A. Find the voltage across the coil.

Solution : Inductive reactance $X_L = L\omega$

or $\quad X_L = L \times 2\pi f$

given $\quad f = 50\,\text{Hz},\ L = 5/\pi\ \text{mH},\ I = 0.5\ \text{A}$

so $\quad X_L = \frac{5}{\pi} \times 2\pi \times 50 \times 10^{-3} = 0.5\,\Omega$

Voltage developed across inductor is

$$V_L = I \times L = 0.5 \times 0.5 = 0.25\ \text{V}$$

Example10.9 : Capacity of a capacitor is 50 pF. Find its capacitive reactance at 5 kHz.

Solution : Capacitive reactance

$$X_C = \frac{1}{C\omega} = \frac{1}{C \times 2\pi f}$$

given $\quad C = 50\ \text{pF} = 50 \times 10^{-12}\ \text{F},\ f = 5 \times 10^3\ \text{Hz}$

so $\quad X_C = \frac{1}{50 \times 10^{-12} \times 2 \times 3.14 \times 5 \times 10^3}$

$$= 6.37 \times 10^4\,\Omega$$

Example 10.10 : A capacitor of capacity 1 μF is connected to a source $V = 200\sqrt{2}\sin 100t$ V. Find the currrent in the circuit.

Solution : $V_{rms} = \frac{V_m}{\sqrt{2}}$

given $\quad V_m = 200\sqrt{2}$ V, $\omega = 100$ rad/s

$\quad C = 10^{-6}$ F

Now $\quad V_{rms} = \frac{200\sqrt{2}}{\sqrt{2}} = 200\ \text{V}$

and $\quad X_C = \frac{1}{C\omega} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

hence $\quad I_{rms} = \frac{V_{rms}}{X_C} = \frac{200}{10^4} = 0.02\ \text{A}$

Example 10.11: A coil is used with a 50 Hz ac source. What will be value of inductance to obtain a reactance of 100 Ω?

Solution : Inductive reactance $X_L = L \times 2\pi f$

given $\quad X_L = 100\Omega,\ f = 50\ Hz$

$$L = \frac{X_L}{2\pi f} = \frac{100}{2 \times 3.14 \times 50} = 0.318\ \text{H}$$

10.4.4 L-R Series ac Circuit

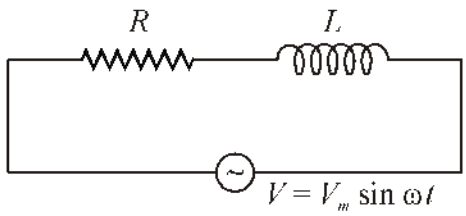


Fig 10.20 : R-L series ac circuit

Alternating voltage $V = V_m \sin \omega t$ is applied to the R-L series circuit. At any instant t the current in the circuit is I, V_L and V_R are the potential at L and R. The net potential developed across R and L is V_{RL}. Then from Kirchhoff's law :-

$$V - V_{RL} = 0$$

From phasor diagram 10.21 (A) we find that V_{mR} and I_m are in phase. But V_{mR} and V_{mL} have a phase difference of $\pi/2$. They are normal to each other. The resultant voltage, $V_{mRL} = \sqrt{V_{mR}^2 + V_{mL}^2}$

But $\quad V_{mR} = I_{mR}R$ and $V_{mL} = I_m X_L$

hence $\quad V_{mRL} = \sqrt{I_m^2 R^2 + I_m^2 X_L^2}$

and $\quad I_m = \dfrac{V_{mRL}}{\sqrt{R^2 + X_L^2}}$ ... (10.16)

here $\sqrt{R^2 + X_L^2}$, is the effective obstacle of the L-R combintion for ac current; which is given by Z and called impedance of the circuit.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (L\omega)^2} \quad ...(10.17)$$

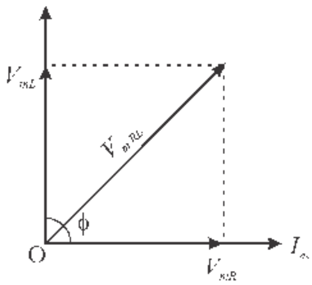


Fig 10.21 (A) Phasor diagram

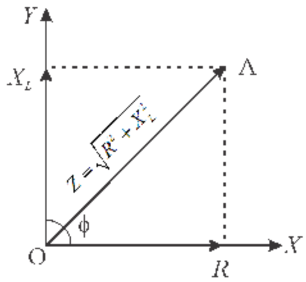


Fig 10.21 (B) Inpedance diagram

From Fig 10.21 (A) it is clear that the current in L-R series circuit is leading the applied voltage by ϕ.

Hence $I = I_m \sin(\omega t - \phi)$ ...(10.18)

Similarly, from fig 10.21 (B) we find the impedance Z of the circuit. R and X_L are in X and Y direction. The resultant Z makes an angle ϕ with X-axis.

So $\tan\phi = \dfrac{V_{mL}}{V_{mR}} = \dfrac{I_m X_L}{I_m R} = \dfrac{X_L}{R}$

hence $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right)$ ...(10.19)

In R-L series ac circuit, the graph between ac voltage and current with ωt is given by fig10.22 (A) where the current lags behind the applied voltage by an angle ϕ. Fig 10.22 (B) represents phasor diagram of ac voltage and current.

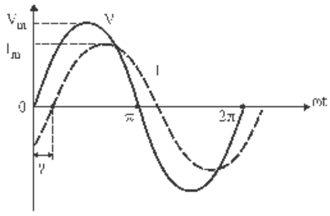


Fig 10.22 (A) Alternating voltage and current in a series R-L circuit

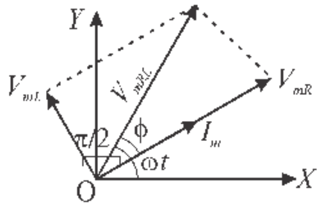


Fig 10.22 (B) Series R-L circuit phasor diagram

Example 10.12 : A 0.5 H inductor gives 0.5 A in a 100 V dc source. If the current in the circuit.

Solution : For dc source $X_L = 0$,

thus $R = \dfrac{V}{I}$

given is $L = 0.5$ H, $V = 100$ V

$f = 50$ Hz,

$$R = \frac{100}{0.5} = 200\,\Omega$$

In ac circuit, $Z = \sqrt{R^2 + L^2\omega^2}$

where $\quad \omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/s

so $\quad Z = \sqrt{(200)^2 + (0.5 \times 314)^2} = 254.26\,\Omega$

$$I = \frac{V}{Z} = \frac{100}{254.26} = 0.39\text{ A}$$

Example 10.13 : An electric bulb has rating 100V, 10A. If it is used at 200 V; 50 Hz ac circuit, then find the inductance of the choke coil used in series.

Solution : Impedance $Z = \sqrt{R^2 + (L\omega)^2}$ for bulb

$V = 100$ V, $I = 10$ A

$f = 50\,\text{Hz}$, $V = 200$ V

Resistance of the bulb R = V/I

$$R = \frac{100}{10} = 10\Omega$$

$$Z = \frac{\text{ac voltage}}{\text{current}} = \frac{200}{10} = 20\text{W}$$

$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314\,\text{rad/s}$$

using $\quad Z = \sqrt{R^2 + (2\pi f L)^2}$

$$20 = \sqrt{(10)^2 + (314 \times L)^2}$$

$$L^2 = \frac{300}{314 \times 314}$$

$$L = \frac{\sqrt{300}}{314} = 0.055\,\text{H}$$

Example 10.14 : A coil of self inductance $1/\pi$ H is in series with a 300 Ω resistor. A 200 V, 200 Hz ac is applied to the combination. Find phase difference between voltage and current.

Solution : $\tan\phi = \frac{L\omega}{R} = \frac{2\pi f L}{R}$

$$L = \frac{1}{\pi}\text{H},\ f = 200\ \text{Hz},\ R = 300\,\Omega$$

hence $\quad \tan\phi = \frac{2\pi \times 200 \times 1}{300 \times \pi} = \frac{4}{3}$

so $\quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$

Example 10.15 : A coil is connected to a 220 V and 50 H_z alternating source develops 200 watt power it draws a current of 2 A. Find the resistance and inductance of the coil.

Solution : Power $P = I_{rms}^2 R$

given $\quad V_{rms} = 220\,\text{V},\quad f = 50\,\text{Hz},\ ,\ I_{rms} = 2\,\text{A}$

$$P = 200\ \text{W}$$

$$R = \frac{P}{I_{rms}^2} \frac{200}{(2)^2} = 50\Omega$$

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{220}{2} = 110\Omega$$

From $\quad Z^2 = R^2 + X_L^2$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(110)^2 - (50)^2} = 98\Omega$$

$$X_L = L \times 2\pi f$$

$$L = \frac{X_L}{2\pi f} = \frac{98}{2 \times 3.14 \times 50}$$

$$L = 0.312\ \text{H}$$

Example 10.16 : A coil of inductance 0.4 H and negligible resistance is in series with 120 Ω resistor. If it is connected to $200/\pi$ Hz; 100 V ac source, then find total impedance, phase angle and current in the circuit.

Solution : Impedance $Z = \sqrt{R^2 + (L\omega)^2}$

given is $L = 0.4\,\text{H}, R = 120\ \Omega, f = \frac{200}{\pi}\text{Hz}$

$$V_{rms} = 100\,\text{V}$$

$$Z = \sqrt{(120)^2 + (400 \times 0.4)^2}$$

$$Q = 200\,\Omega$$

$$\omega = 2 \times 3.14 \times \frac{200}{\pi} = 400\ \text{rad/s}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{400 \times 0.4}{120}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{200} = 0.5\,\text{A}$$

10.4.5 R-C Series ac Circuit

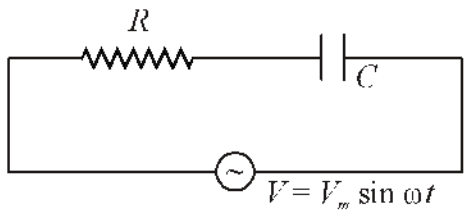


Fig 10.23 A series RC ac circuit

In fig 10.23 an ac voltage $V = V_m \sin \omega t$ is applied to a series combination capacitance C and resistance R. At any instant t, if the voltage across R and C are V_R and V_C and the current is I the resultant voltage developed across the R-C combination is V_{RC}. From Kirchhoff's law we get $V - V_{RC} = 0$.

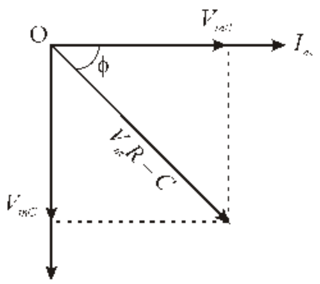


Fig 10.24 Phasor diagram of RC series ac circuit

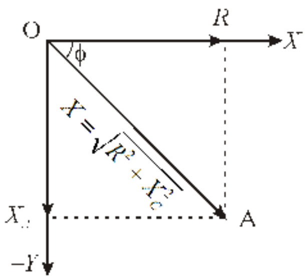


Fig 10.25 Impedance diagram

From phasor diagram 10.24, it is evident that V_{mR} is in phase with I_m, But V_{mC} lags behind the current by $\pi/2$. The resultant potential across the combination is

$V_{mRC} = \sqrt{V_{mR}^2 + V_{mC}^2}$. But $V_{mR} = I_m R$ and $V_{mc} = I_m X_C$

$$V_{mRC} = \sqrt{(I_m R)^2 + (I_m X_C)^2}$$

$$V_{mRC} = I_m \sqrt{R^2 + X_C^2}$$

$$= \frac{V_{mRC}}{I_m} \qquad \ldots (10.20)$$

or $$\sqrt{R^2 + X_C^2}, = \frac{V_{mRC}}{I_m} = Z$$

here $\sqrt{R^2 + X_C^2}$, is the effective obstacle in the circuit and is called impedance Z of the circuit. Hence

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} \qquad \ldots (10.21)$$

From fig 10.24, it is clear that on applying sinusoidal voltage to the RC series combination sinusoidal current is obtained. But the current leads the voltage by an angle ϕ.

$$I = I_m \sin(\omega t + \phi) \qquad \ldots (10.22)$$

From fig 10.25 we get the imdepance of the circuit. Resistance is shown on X-axis, while X_C on Y-axis. The resultant Z is represented by OA, which makes an angle ϕ with X-axis.

$$\tan \phi = \frac{V_{mC}}{V_{mR}} = \frac{I_m X_c}{I_m R} = \frac{X_c}{R}$$

and $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) \qquad \ldots (10.23)$$

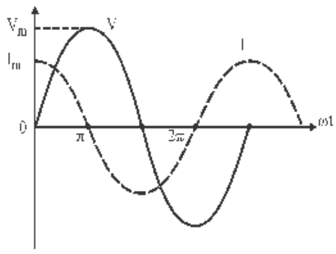


Fig 10.26 : V-I graph for series ac circuit

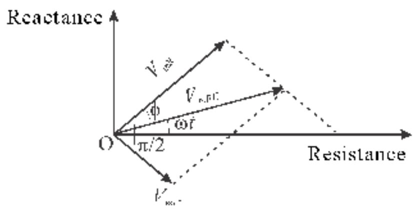


Fig 10.27 Phasor diagram for the circuit

The variation of ac voltage and current in series R-C circuit with ωt is as shown in Fig 10.26. It is clear that both the voltage and current have same freqency, but current leads the potential by an angle ϕ. Fig 10.27 shows the phasor diagram for the combination.

Example 10.17: A series combination of 100 μF capacitor and 40 Ω resistor are joined to 110 V, 60 Hz ac source. Find maximum current in the circuit.

Solution : $$I_m = \frac{V}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$$

given $V = 110$ V, $C = 100 \times 10^{-6}$ F

$R = 40\ \Omega$, $f = 60$ Hz

$$I_m = \frac{110}{\sqrt{(40)^2 + \frac{1}{(100\times10^{-6}\times2\times3.14\times60)^2}}}$$

$$= \frac{110}{\sqrt{(40)^2 + \frac{1}{(376.8\times10^{-4})^2}}} = \frac{110}{\sqrt{(40)^2 + (26.54)^2}}$$

$= 2.29$ A

10.4.6 L-C-R Series ac Circuit

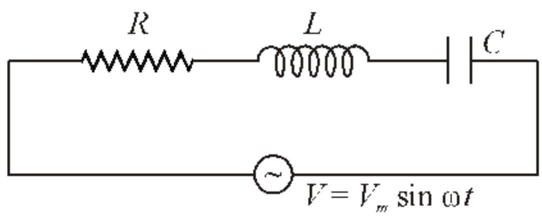


Fig 10.28 : L-C-R series ac circuit

The AC voltage $V = V_m \sin \omega t$ is applied to the series L-C-R circuit. At any instant t, the cutrrent in the cuircuit is I. The voltage developed across the elements is V_R, V_L and V_C. $V_R - IR$, $V_L - IXL$ and $V_C - IX_C$. The net voltage developed across the series combination is V_{LCR}. From Kirchhoff's loop law we get $V - V_{LCR} = 0$

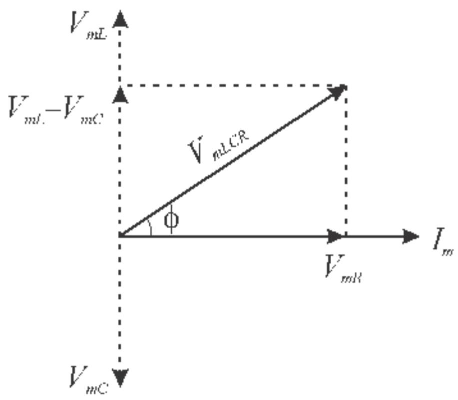


Fig 10.29 : Phasor diagram for $X_L > X_C$

From the phasor diagram (10.29) we see that V_{mR} is in phase with I_m while V_{mL} leads the current by $+\pi/2$ and V_{mC} lags the current by $-\pi/2$. Voltage across L and C are opposite to each other, hence V_{LC}= (V_{mL} - V_{mC}); which is perpendicular to V_{mR}.

(For $X_L > X_C$)

So $V_{mLCR} = \sqrt{(V_{mR})^2 + (V_{mL} - V_{mC})^2}$ and

$V_{mR} = I_m R$, $V_{mL} = I_m X_L$ and $V_{mC} = I_m X_C$

$$V_{mLCR} = \sqrt{(I_m R)^2 + (I_m X_L - I_m X_C)^2}$$

$$= I_m \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_m = \frac{V_{mLCR}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \ldots (10.24)$$

$\sqrt{R^2 + (X_L - X_C)^2}$ = Z is the effective resistance of the series combination, and is called impedance of the circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \ldots (10.25)$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2} \quad \ldots (10.26)$$

The current I_m lags the voltages V_{CR} by an angle ϕ.

If $X_C > X_L$; $V_{mC} > V_{mL}$ the current will lead the voltage.

$$\tan\phi = \frac{V_{mL} - V_{mC}}{V_{mR}} = \frac{I_m X_L - I_m X_C}{I_m R}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad \ldots (10.27)$$

From eq. (10.26) and (10.27) it is clear that both Z and ϕ depends on the three element R, X_L and X_C.

Special Conditions -

(i) If $V_{mL} > V_{mC}$;k $X_L > X_C$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ From eq. (10.25)

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ From eq. (10.27)

The phasor diagram is fig. 10.29. The value of ϕ

will be positive and have the value between 0 to $\pi/2$; the circuit will behave like L-R circuit and current lags the potential. In this condition Z is given by fig. (10.30); ac voltage and currents are given by phasor diagram fig.(10.31). The current in the circuit is given by

$$I = I_m \sin(\omega t - \phi) \qquad \ldots (10.28)$$

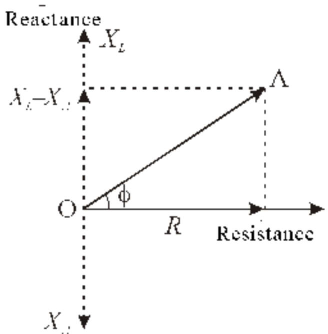


Fig 10.30 ($X_L > X_C$) impedance diagram

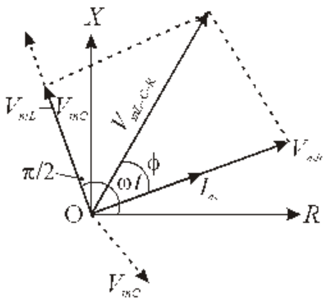


Fig 10.31 Phasor diagram ($X_L > X_C$)

(ii) If $X_C > X_L$ and $V_{mc} > V_{ML}$ then from equation (10.25) $Z = \sqrt{R^2 + (X_C - X_L)^2}$

and From (10.27) $\phi = \tan^{-1}\left(\dfrac{X_C - X_L}{R}\right)$

In this current leads the applied voltage and value of ϕ will be between 0 to $\pi/2$. And the circuit will behave like R-C circuit. And the current in the circuit is given by

$$I = I_m \sin(\omega t + \phi)$$

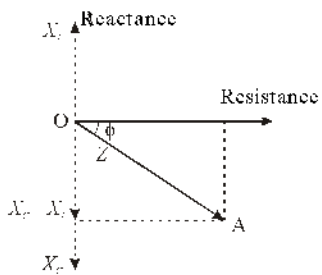


Fig 10.32 ($X_C > X_L$) impedance diagram

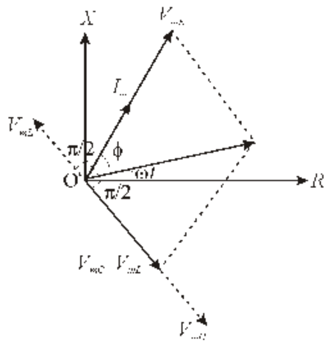


Fig 10.33 Phasor diagram for ($X_C > X_L$)

(iii) If $X_L = X_C$ or $V_{mL} = V_{mC}$ then $\phi = 0$, and the resultant voltage and current are in phase i.e they have the some phase. This condition is called resonance.

10.5 L-C-R Series Resonance Circuit

If an ac circuit contains L,C and R in series, then normally a phase diffence exists between voltage and current due to imdepance of these elements.

If the frequency of the applied ac voltage is increased ωL get increased and $1/\omega C$ decreased whereas R is unaffected. By decreasing frequency, ωL is decreases $1/\omega C$ increases. A condition will reach when $X_L = X_C$; so that the resultant reactance $X_L - X_C = 0$, $\phi = 0$. The current in the circuit is maximum. This condition is called resonance. And circuit is called series resonant circuit.

For resonance $X_L = X_C$ $\qquad \ldots (10.29)$

and $X = X_L - X_C = 0$ $\qquad$ From equ. (10.25)

$Z = Z_{min} = R$ $\qquad \ldots (10.30)$

which means the impedance of circuit will be minimum and equal to resistance.

From eq. (10.27) $\phi = \tan^{-1}(0) = 0$ $\quad \ldots (10.31)$

which indicates that the resultant voltage and current are in same phase. Hence from equ. (10.28)

$$I = I_m \sin \omega t \qquad \ldots (10.32)$$

At resonance angular frequency ω_r; $X_L = X_C$

$$L\omega_r = \frac{1}{C\omega_r} \text{ or } \omega_r = \frac{1}{\sqrt{LC}} \quad (\omega_r = 2\pi f_r) \ldots (10.33)$$

and the resonant frequency f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \ldots (10.34)$$

The peak value of current is $(I_m)_{\max} = \dfrac{V_{mLCR}}{R}$

or $\qquad (I_{rms})_{\max} = \dfrac{V_{rms}}{R}$

For the condition of resonance, impedance and phasor diagrams are given by Fig. (10.34) and Fig. (10.35).

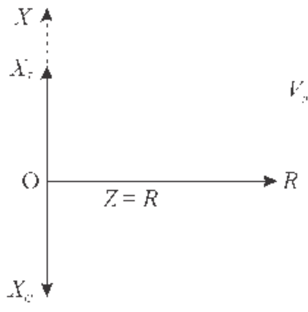


Fig 10.34 Impedance at resonance

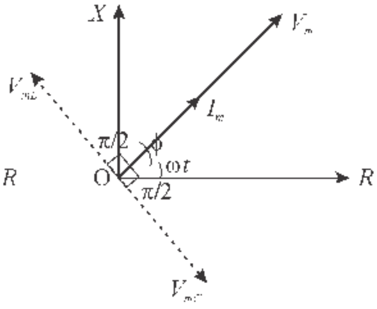


Fig 10.35 Phasor diagram for resonant LCR

The variation of I_m and impedance Z, of he series L-C-R circuit is given by fig. 10.37 and fig. 10.36. At resonant frequency (I_m) will be maximum and Z will be minimum.

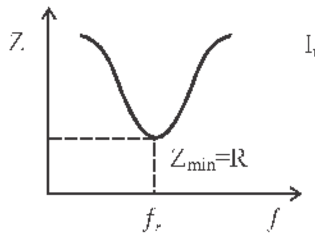


Fig 10.36 Graph between f and Z

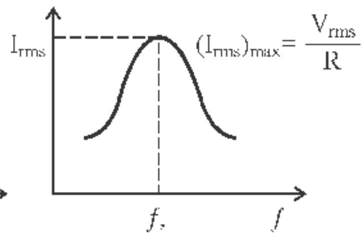


Fig 10.37 Graph between f and I_m

Analytical Solution of L-C-R Series Circuit

For L-C-R series circuit the voltage equation is

so $L\dfrac{dI}{dt}+IR+\dfrac{q}{c}=V_m \sin\omega t$ but $I=\dfrac{dq}{dt}$;

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=V_m\sin\omega t \qquad \text{. . . (i)}$$

This equation is similar to the equatition of forced oscillations, hence its solution is

$$q=q_m\sin(\omega t+\theta) \qquad \text{. . . (ii)}$$

and $$\frac{dq}{dt}=q_m\omega\cos(\omega t+\theta) \qquad \text{. . . (iii)}$$

$$\frac{d^2q}{dt^2}=-q_m\omega^2\sin(\omega t+\theta) \qquad \text{. . . (iv)}$$

Substituting these values in equ. (i)

$$q_m\omega[R\cos(\omega t+\theta)+(X_C-X_L)\sin(\omega t+\theta)]=V_m\sin\omega t$$

$$q_m\omega z\left[\frac{R}{Z}\cos(\omega t+\theta)+\left(\frac{X_C-X_L}{Z}\right)\sin(\omega t+\theta)\right]=V_m\sin\omega t$$

Let us $\dfrac{R}{Z}=\cos\phi,\ \dfrac{X_C-X_L}{Z}=\sin\phi$

$$\tan\phi=\frac{X_C-X_L}{R}$$

$$\phi=\tan^{-1}\left(\frac{X_C-X_L}{R}\right) \quad \text{Or}$$

$$q_m\omega z[\cos(\omega t+\theta)\cos\phi+\sin(\omega t+\theta)\sin\phi]=V_m\sin\omega t$$

$$q_m\omega z\ \cos(\omega t+\theta-\phi)=V_m\sin\omega t$$

Comparing both sides of the above equation

$$V_m=q_m\omega z=I_m z$$

So $\quad I_m=q_m\omega$

and $\quad \theta-\phi=-\dfrac{\pi}{2}$

$$\theta=-\frac{\pi}{2}+\phi$$

From equatiom (iii)

$$\frac{dq}{dt}=I=I_m\cos(\omega t+\phi-\frac{\pi}{2})$$

Or $\quad I=I_m\sin(\omega t+\phi)$

$$\frac{R}{z}=\cos\phi \quad \text{and} \quad \frac{X_C-X_L}{Z}=\sin\phi$$

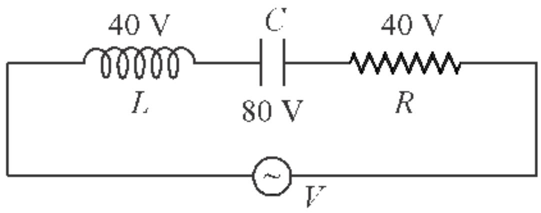
Squaring and adding

$$\frac{R^2}{Z^2}+\frac{(X_C-X_L)^2}{z^2}=1$$

$$Z^2=R^2+(X_C-X_L)^2$$

$$Z=\sqrt{(R^2)+(X_C-X_L)^2}$$

Example 10.18: Find the voltage of the given ac circuit.



Solution : From Phasor

$$V_{rms} = \sqrt{V_R^2 + (V_C - V_L)^2}$$

here $V_R = 40$ V,

$V_L = 40$ V, $V_C = 80$ V

$$= \sqrt{(40)^2 + (80-40)^2}$$

$$= 40\sqrt{2} = 56.56\,\text{V}$$

Example 10.19 : A series L-C-R circuit contains R= 12 Ω , X_L= 18 Ω and X_C = 23 Ω . Find the impedance of the circuit and phase difference.

Solution : Impedance of the circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$R = 12\Omega, X_C = 23\Omega, X_L = 18\Omega$$

$$Z = \sqrt{(12)^2 + (23-18)^2}$$

$$= \sqrt{144+25} = \sqrt{169} = 13\,\Omega$$

and phase difference is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{5}{12}$$

$$\phi = \tan^{1}\left(\frac{5}{12}\right)$$

Example 10.20 : A voltage source of 110 V; 50 Hz is connected to series combination of R =10 Ω; L = 2 / π H ; and C = 1 / π μF. Find phase difference between V and I.

Solution : Phase difference is given by

$$\tan\phi = \frac{X_C - X_L}{R}$$

Given $R = 10\Omega, f = 50\,Hz, L = \frac{2}{\pi}H, C = \frac{1}{\pi}\times 10^{6}F$

$$\tan\phi = \frac{\frac{1}{2\ \ f \text{x} C} - L \text{x} 2\ \ f}{R}$$

$$\tan\phi = \frac{\frac{1}{2\pi\times 50\times\frac{1}{\pi}\times 10^{6}} - \frac{2}{\pi}\times 2\pi\times 50}{10}$$

$$\frac{10^4 - 200}{10} = \frac{980}{10} = 980$$

$$\phi = \tan^{1}(980)$$

Example 10.21 : For a L-C-R series circuit, voltage and current are given by $V = 300\sin 100t$ and $I = 6\sin(100t - \phi)$. If the resistance in the circuit is of 40 Ω, find (i) impedance (ii) reactance (iii) phase difference between voltage and current.

Solution : Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

given $V_m = 300\,\text{V}, I_m = 6\text{A}, \omega = 100$ rad/s and R = 40 Ω

(i) $$Z = \frac{V_m}{I_m} = \frac{300}{6} = 50\Omega$$

(ii) $$Z^2 = R^2 + (X_L - X_C)^2$$

$$\therefore X_{LC} = \quad (X_L - X_C) = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(50)^2 - (40)^2} = 30\,\Omega$$

Phase difference is given by

(iii) $$\phi = \tan^{1}\left(\frac{X_L - X_C}{R}\right)$$

$$\phi = \tan^{1}\left(\frac{30}{40}\right)$$

$$\phi = \tan^{-1}\left(\frac{3}{4}\right)$$

Example 10. 22: Find angular frerquency and frequency for maximum current in an ac circuit containing an inductor of L = 0.5H and a capacitor of C = 8 μF .

Solution : Resosant angular frequency is given by

$$\omega_r = \frac{1}{\sqrt{LC}}$$

because current is maximum at resonant frequency,

so, $\omega_r = 2\pi f_r$

$L = 0.5$ H, $C = 8\times10^{-6}$F

$$\omega_r = \frac{1}{\sqrt{0.5\times8\times10^{-6}}} = \frac{10^3}{2} = 500\,\text{rad/s}$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{500}{2\pi} = \frac{250}{\pi}\text{Hz}$$

Example 10. 23: At resonance the values of R = 20 Ω, L = 0.1 H and C = 200 μF. If the inductor is replaced by L = 100 H, find value of C for same resonance frequency.

Solution : Given for first condition L =0.1 H, C = 200 μF for second condition L = 100 H, C = ?

Since resonance frequency is same

$$f_r = \frac{1}{2p\sqrt{LC}} = \frac{1}{2p\sqrt{L'C'}}$$

$LC = L'C'$

$0.1 \times 200 \times 10^{-6} = 100 \times C'$

$$C' = \frac{0.1\times200\times10^{-6}}{100} = 0.2\mu\text{F}$$

Example 10.24 : A wave of wavelength 300 m is being transmitted form a center. We have a capacitor of C = 2.4 μF. Find the value of inductor to tune (resonance circuit) the station.

Solution : $f = \frac{1}{2\pi\sqrt{LC}}$

given $\lambda = 300\,\text{m}$, $C = 2.4\ \mu\text{F}$

$$f = v = \frac{c}{\lambda} = \frac{3\times10^8}{300} = 10^6\,Hz$$

$$L = \frac{1}{4\pi^2 f^2 C}$$

hence $L = \frac{1}{4\times(3.14)^2\times(10^6)^2\times(2.4\times10^{-6})} = 10^{-8}\,\text{H}$

Example 10.25 : An ac circuit of 220 V, 50 Hz has a resistor of 11 Ω , inductor of $2/\pi^2$ H. For what value of C the circuit will be at resonance? At so find the current in the circuit.

Solution : $f = \frac{1}{2\pi\sqrt{LC}}$

given $V_{rms} = 220\,\text{V}, f = 50\,\text{Hz}, R = 11\Omega, L = \frac{2}{\pi^2}\text{H}$

$$C = \frac{1}{4\pi^2 L f^2} = \frac{1}{4\pi^2\times\frac{2}{\pi^2}\times50\times50} = 50\ \mu\text{F}$$

The current in resonant circuit

$$I_{rms} = \frac{V_{rms}}{Z_{\min}} = \frac{V_{rms}}{R}$$

$$I_{rms} = \frac{220}{11} 20\text{A}$$

10.6 Half Power Point Frequencies, Bandwidth and Quality Factor of a Series Resonance Circuit

10.6.1 Half Power Points of Frequency

Fig 10.38 shows variation of current I_{rms} with freqency in L-C-R series circuit. At resonance frequency the current in the circuit is maximum i.e. $(I_{rms})_{\max}$. The power dissipation will be $(I^2_{rms})_{\max}R$, and will be maximum.

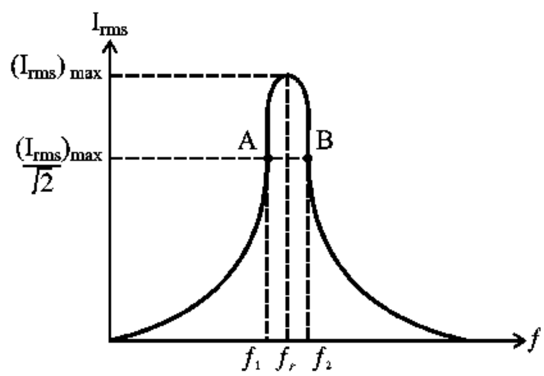


Fig 10.38 : Current I_{rms} and frequency f graph for L-C-R circuit

There exist two frequencies $f_1 < f_r$ and $f_2 > f_r$ at which the current in the circuit will be 1/2 of the maximum value. And the circuit comsumes half the power of its maximum value. These frequencies are thus called half power frequencies. At these freqencies the effective current is given as I_{rms}.

$$I_{rms}^2 R = \frac{1}{2}\left(I_{rms}\right)_{\max}^2 R$$

$$I_{rms} = \frac{\left(I_{rms}\right)_{\max}}{\sqrt{2}}$$

$$= 0.707\left(I_{rms}\right)_{rms}$$

Hence the current at half power frequencies will be of the maximum value.

10.6.2 Band Width

L-C-R resonant circuit is able to absorb more energy from the source in the frequency interval $(f_2 - f_1)$. This gap between the half power frequencies $f_2 - f_1$ is called band width.

the current at half power frequencies f_1 and f_2 is

$$I_{rms} = \frac{(I_{rms})_{\max}}{\sqrt{2}}$$

$$\frac{V_{rms}}{\sqrt{R^2 + (L\omega - \frac{1}{c\omega})^2}} = \frac{V_{rms}}{\sqrt{2}R}$$

Or $$R^2 + (L\omega - \frac{1}{c\omega})^2 = 2R^2$$

Or $$(L\omega - \frac{1}{c\omega})^2 = R^2$$

Or $$\left(L\omega - \frac{1}{c\omega}\right) = \pm R$$

R.

for f_1 $$L\omega_1 - \frac{1}{c\omega_1} = -R \qquad \text{. . . (10.35)}$$ R_2

for f_2 $$L\omega_2 - \frac{1}{c\omega_2} = R \qquad \text{. . . (10.36)}$$

Adding the equation 10.35 and equation 10.36 we get

$$L(\omega_1 + \omega_2) - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

Or $$L\left(\omega_1 + \omega_2\right) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right)$$

Or $$\omega_1\omega_2 = \frac{1}{LC} \qquad \text{. . . (10.37)}$$

Similarly subtracting equ. 10.35 from equ. 10.36

$$L(\omega_2 - \omega_1) + \frac{1}{c}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

Or $$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right) = 2R$$

We get

$$2L(\omega_2 - \omega_1) = 2R$$

$$(\omega_2 - \omega_1) = \frac{R}{L} \qquad \text{. . . (10.38)}$$

hence band width $= \omega_2 - \omega_1 = \dfrac{R}{L}$

Or $$f_2 - f_1 = \frac{R}{2\pi L} \qquad \text{. . . (10.39)}$$

The above equation gives the expression for band width.

10.6.3 Quality Factor

Behaviour of L-C-R circuit depends on the value of R. At different values of R, we see that as the value of R decreases, sharpness of resonance curve increases. Resonance current will be maximum and band width $f_2 - f_1 = \dfrac{R}{2\pi L}$ will be minimum.

Hence at low values of R, the resonances curve will be more sharp, as given by fig 10.39.

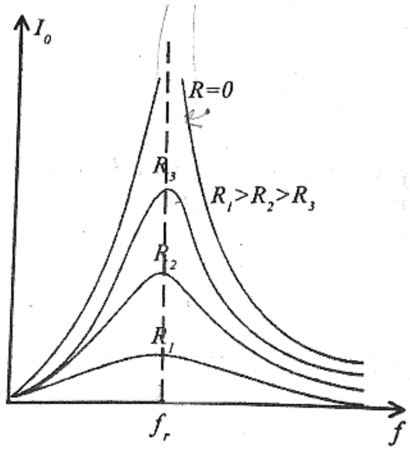


Fig 10.39 Comparision of Band with for $R_1 < R_2 < R_3$

For large value of R, the value of $(I_{rms})_{max}$ decreases, and band width increases.

By changing the frequency below or above f_r, the change in $(I_{rms})_{max}$, sharpens the curve. If the change in I_{rms} is slower, flater the curve. The sharpness of the resonance curve is given by a characterstic factor called qualiy factor Q.

$$Q = \frac{\text{resonant frequency}}{\text{band width}} = \frac{f_r}{f_2 - f_1} = \frac{Lw_r}{R} \quad \ldots (10.40)$$

$$\because \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{L\omega_r}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \ldots (10.41)$$

Or $$Q = \frac{\text{Capaitive or inductive reactance at resonant frequency}}{\text{resistance}}$$

Or $$Q = \frac{\text{Potential on L or C}}{\text{Applied potential}} = \frac{V_L}{V} = \frac{V_C}{V}$$

Normally $Q > 1$; hence V_L or $V_C > V$ so there is a voltage amplification and Q is also a measure of voltage amplification. Series resonant circuit is used to tune a radio. It is also called selective circuit. The signal recieved by the antenna work as a source. To select a specific station we tune first by changing inductor L and then change the capacity of the capacitor such that frequency of the circuit is equals to the transmitting frequency of that station. So the signal of this frequency gives maximum current, and the other near by frequencies are supressed by dominance of f_r. The side frequencies are more supressed if Q is large.

Example 10.26 : An L-C-R circuit contains R = 100 Ω, L = 1 mH and C = 1000 µF. Find the resonant frequency and band width.

Solution : $f_r = \frac{1}{2\pi\sqrt{LC}}$

Given $R = 100\,\Omega, L = 1\,\mu\text{H}, C = 1000\,\mu F$

$$f_r = \frac{1}{2\pi\sqrt{10^{-3} \times 1000 \times 10^{-6}}} = \frac{1000}{2\pi}\text{Hz}$$

Band width $f_2 - f_1 = \frac{R}{2\pi L} = \frac{100}{2\pi \times 10^{-3}} = \frac{5000}{\pi}\text{Hz}$

Example 10.27 : A series L-C-R circuit contains R = 14 Ω and inductor of L = 7 mH. The frequency of the source is equal to resonance frequency. If the quality factor is 1/2 then find (i) Band width (ii) capacitive reactance.

Solution : Band width $\omega_2 - \omega_1 = \frac{R}{L}$

given $R = 14\Omega, L = 7 \times 10^{-3} H, Q = \frac{1}{2}$

$$\omega_2 - \omega_1 = \frac{14}{7 \times 10^{-3}} = 2 \times 10^3 \text{ m}$$

$$Q = \frac{1}{c\omega \times R}$$

$$\frac{1}{C\omega} = Q \times R = \frac{1}{2} \times 14 = 7\,\Omega$$

Example 10.28 : Resonant frequency of an L-C-R circuit is 600 Hz. At frequencies 570 Hz and 620 Hz the current in the circuit is $1/\sqrt{2}$ of its maximum value at resonance. Find quality factor, X_L, X_C, L and C at resonance. $(R = 3\,\Omega)$.

Solution : $Q = \frac{f_r}{f_2 - f_1}$

given $f_r = 600\,\text{Hz}, f_1 = 570\,\text{Hz}, f_2 = 620\,\text{Hz}$

$$Q = \frac{600}{620 - 570} = 12$$

$$Q=\frac{L\omega_r}{R}$$

$$L\omega_r=Q\times R$$

$$L\omega_r=\frac{1}{C\omega_r}$$

$$\frac{1}{C\omega_r}=X_C=36\ \Omega$$

Also at resonance $X_C=X_L$ hence XC = 36.

$$L\omega_r=36$$

$$L=\frac{36}{\omega r}=\frac{36}{2\pi fr}=\frac{36}{2\times3.14\times600}=9.56\,\text{mH}$$

$$\frac{1}{C\omega_r}=36$$

$$C=\frac{1}{36\times2\pi fr}=\frac{1}{36\times2\times3.14\times600}=7.37\,\mu F$$

10.7 Average Power in AC Circuit

The rate of absorption of energy is called power of the circuit. It is the product of voltage and current in the circuit. The voltage and current has a phase difference, depending on its elements, hence the power also depends on phase. Let the voltage and current at any instant be

$$V=V_m\sin\omega t$$

$I=I_m\sin(\omega t-\phi)$ here current lags the potential by ϕ

Hence instantaneous power is

$$P=VI=V_m\sin\omega t\times I_m\sin(\omega t-\phi)$$

$$=V_mI_m\sin\omega t\sin(\omega t-\phi) \quad \text{. . . (10.42)}$$

using $\left[\sin C\sin D=\frac{1}{2}\{\cos(C-D)-\cos(C+D)\}\right]$

$$P=\frac{1}{2}V_mI_m\left[\cos\phi-\cos(2\omega t-\phi)\right]$$

$$P=\frac{1}{2}V_mI_m\cos\phi-\frac{1}{2}V_mI_m\cos(2\omega t-\phi) \quad \text{. . . (10.43)}$$

It shows that instantaneous power has two components. One is constant and other varies with time, periofically. The average power over one complete cycle is -

$$\overline{P_{av}}=\frac{1}{2}V_mI_m\cos\phi-\frac{1}{2}V_mI_m\overline{\cos(2\omega t-\phi)}$$ the

since average of cosine function over one complete cycle is zero.

$$=\frac{1}{2}V_mI_m\cos\phi-\frac{1}{2}V_mI_m\times0$$

$$\left[\because\overline{\cos(2\omega t-\phi)}=0\right]$$

$$P_{av}=\frac{1}{2}V_mI_m\cos\phi$$

$$P_{av}=\frac{V_m}{\sqrt2}\frac{I_m}{\sqrt2}\cos\phi=V_{rms}I_{rms}\cos\phi \quad \text{. . . (10.44)}$$

$$P_{av}=P_{vir}\cos\phi \quad \text{. . . (10.45)}$$

here P_{av} is called virtual power and $\cos\phi$ is power factor of the circuit.

10.8 Power Factor

The cosine of the phase angle between AC voltage and AC current in the circuit is called power factor of the circuit. It depends on the elements of the circuit. From equation (10.45)

$$\cos\phi=\frac{P_{av}}{P_{vir}}=\frac{\text{average power}}{\text{virtual power}}$$

The ratio of P_{av} and P_{rms} is equal to power factor $(\cos\phi)$. It is a dimensionless quantity and its value is between 0 and 1. If power factor of a circuit is zero, energy loss in the circuit is zero.

For L-C-R series Ac circuit

$$\tan\phi=\frac{X_L\sim X_C}{R}$$

$$\cos\phi=\frac{R}{\sqrt{R^2+(X_L\sim X_C)^2}}$$

$$\cos\phi = \frac{R}{Z} \qquad \ldots (10.46)$$

The ratio of resistance and impedance in L-C-R circuit is called power factor.

Special Conditions

(i) For pure resistive circuit- In this circuit voltage and current are always in phase, i.e. $\phi = 0$ $\cos\phi$ = 1, power factor is 1. $P_{av} = V_{rms} I_{rms} = P_{vir}$ which means that in a pure resistive circuit power factor is unity, and the circuit consumes maximum power. And $P_{av} = P_{vir}$. The fig. (10.40) gives instantaneous values of V and I and it shows that power consumed by the circuit is maximum.

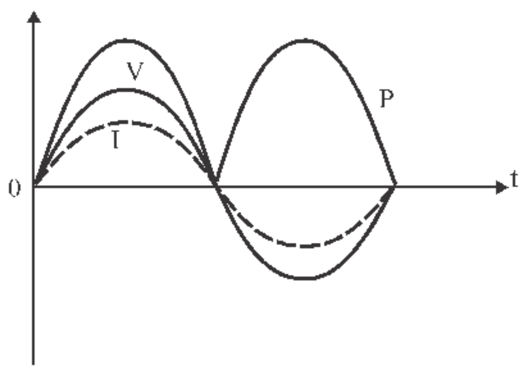


Fig. 10.40 Power in resistive circuit

(ii) For pure inductive circuit - In this circuit the applied voltage leads the current in the circuit by $\phi = \pi/2$; hence the power factor $\cos\phi = 0$ and P_{av} = 0. In pure inductive circuit the average power consumed over a complete cycle is zero. Fig (10.14) shows that the area of positive loop (energy absorbed) is exactly equal o the area of negative loop (energy returned).

(iii) For pure capacitive circuit - In this type of circuit, the current leads the applied potential by $\pi/2$ i.e. $\phi = -\pi/2$ hence the power factor $\cos\phi = 0$

From equ. (10.44) $\quad P_{av} = 0$

In a pure capacitive circuit $\cos\phi = R/Z$ also the average power over one cycle is zero. As evident from fig 10.18. The area of positive loop of power curve is equal to the area of the negative loop of the same curve.

(iv) For L-C-R series circuit - In this circuit, at resonance, V and I are in same phase, i.e $\phi = 0$ and $\cos\phi = 1$, the circuit has maximum power factor.

In electric fan and induction motor, the value of L is very large due to many turns of its winding, hence its ϕ is increased and power factor is decreased. To reduce ϕ, a condensor (capacitor) is used so that ϕ approaches zero and power factor approaches 1. The motor gets maximum power and fan moves faster. That is the reason of changing the condensor of a fan, when such problem arises.

10.9 Wattless Current

The AC circuit which has an inductor and a capacitor but no resistance, both voltage and current are present, but average power is zero.

$$P_{av} = V_{rms} I_{rms} \cos\left(\pm\frac{\pi}{2}\right) = 0$$

which means that although the currnet exists, its contribution to power (watt) is zero. Hence called wattless current.

Even when the current is having a phase $\phi = \pi/2$ 0, with voltage, there is a component of I, which has a phase difference of $\pi/2$ with voltage. this component does not contribute to the power, and called wattless current.

From section 10.7 we have

instantaneous power $P = V_m I_m \sin\omega t \sin(\omega t - \phi)$

$$P = V_m I_m \sin\omega t(\sin\omega t\cos\phi - \cos\omega t\sin\phi)$$

$$P = V_m I_m \sin^2\omega t\cos\phi - V_m I_m \sin\phi\sin\omega t\cos\omega t$$

$$P = V_m I_m \cos\phi\sin^2\omega t - \frac{V_m I_m}{2}\sin\phi\sin 2\omega t$$

$$P = P_1 - P_2$$

The average value of component P_1 for one cycle is

$$\overline{P_1} = V_m (I_m \cos\phi)\overline{\sin^2\omega t} \qquad \ldots (10.47)$$

$$= \frac{V_m (I_m \cos\phi)}{2} \qquad (\because \overline{\sin^2\omega t} = \frac{1}{2})$$

Similarly average of P_2-

$$\overline{P_2} = \frac{V_m (I_m \sin\phi)}{2}\overline{\sin 2\omega t} \qquad \ldots (10.48)$$

The contribution of component

$P_2 = 0$ $(\because \overline{\sin 2\omega t} = 0)$ for a complete cycle average value of $(\because \overline{\sin 2\omega t} = 0)$ hence $P_2 = 0$. This component of current, is called the wattless current in this case.

From the above calculation it is clear that if a resistance is used with a reactive element X, the current can be resolved in two components, one component $I_{rms}\cos\phi\left(\frac{I_m}{\sqrt{2}}\cos\phi\right)$ is in phase with the applied voltage and called working current. Similarly the other component of current $I_{rms}\sin\phi$ has a phase difference of $\pi/2$ with voltage and contribution to power is zero, and is called wattless current.

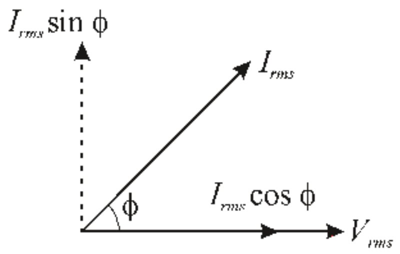


Fig 10.41 V-I graph for RC ac circuit

Example 10.29 : An ac circuit of source 200 V; 50 Hz has resistance of 10 Ω, and inpedance 14.14Ω. Find (i) Power factor (ii) Virtual power (iii) Average power (iv) the value of wattless current.

Solution : Given

$$V_{rms} = 200\,\text{V},\ f = 50\ \text{Hz}, R = 10\ \Omega$$

and $Z = 14.14\ \Omega$

(i) $\cos\phi = \frac{R}{Z} = \frac{10}{14.14} = \frac{1}{\sqrt{2}}$

(ii) Virtual Power $P_{vir} = V_{rms} I_{rms}$

$$= V_{rms} \cdot \frac{V_{rms}}{Z} = \frac{200200}{14.14} = 2820\text{W}$$

(iii) Average Power $P_{av} = V_{rms} I_{rms}\cos\phi$

$$= 2820 \times \frac{1}{\sqrt{2}} = 2000\ \text{W}$$

(iv) Wattless Current $= I_{rms}\sin\phi = \frac{V_{rms}}{Z} \times \sin\phi$

$$\left(\cos\phi = \frac{1}{\sqrt{2}}\right)$$

$$\frac{200}{14.14} \times \sin\frac{\pi}{4} = \frac{200}{14.14} \times \frac{1}{\sqrt{2}} = 10\ \text{A}$$

Example 10.30 : For an AC circuit, voltage and current are given by $V = 100\sin\omega t$ V

$$I = \sin(\omega t + \frac{\pi}{3})\ \text{A}$$

Find : (i) Power factor (ii) Average Power (iii) Wattless current

Solution : Given $V_m = 100\,\text{V}, I_m = 1\,\text{A},\ \phi = \frac{\pi}{3}$

(i) Power factor $= \cos\phi \quad \cos\frac{\pi}{3} = \frac{1}{2}$

(ii) Average Power $= V_{rms} I_{rms}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\cos\phi$

$$= \frac{100}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} = 25\ \text{W}$$

(iii) Wattless current $= I_{rms}\sin\phi = \frac{1}{\sqrt{2}} \times \frac{\sin\pi}{3} = \frac{\sqrt{3}}{2\sqrt{2}}$

$$= 0.61\ \text{A}$$

10.10 Choke Coil

A coil with very high self inductance and very low reisitance is called a choke coil. It is made by winding thick, insulated copper wire over a laminated iron core.

Due to high self inductance and very low resistance, $\phi \simeq 90^\circ$, and $\cos\phi \simeq 0$. There is a negligible power dissipation in the coil. Also due to high L, its inductive reactance X_L, is large, and it is highly effective in controlling AC current without significant losses. It is used in flourosent lamp and mercury/sodium lamp. In choke the most of the current is wattless current with high value of ϕ. If a resistance is used to controll the current, instead of choke, there is very high joule loss.

Metal Detector

It works on condition of resosance in AC circuit. When some metal/A person with metal comes in contact with coil of the circuit, its impedance changes which bring about a change in current, which is heared as a beep.

10.11 Transformer

A device by which we can change ac voltage. It is based on the principle of mutual inductance. It is called transformer.

10.11.1 Construction

As in fig. 10.42, it is made up of a rectangular or any other shaped laminated soft ion core. The soft iron laminea are placed one over the other and an insulating liquid (Lekar) is poured between them. Two coils of insulated wire of Cu/Al are wound over the iron core. The coil on which input AC is applied is called primary coil, while other, from which AC out put is drawn is called secondary coil.

10.11.2 Principle and Working

When AC is applied to the primary coil, the changing magnetic flux produced in iron core is associated with secondary coil. This changing magnetic flux produces *emf* in secondary coil according to Faraday's law. The magnetic flux is confined to iron core and there is very little leakage due to packed winding. The frequency of AC in secondary is same as that of AC in primary.

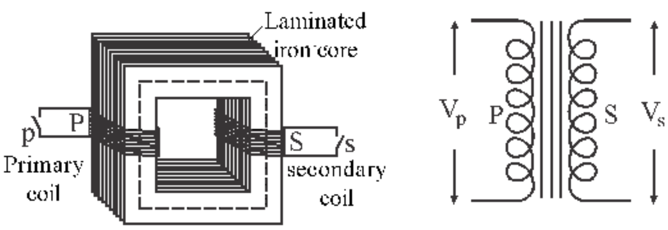


Fig 10.42 Transformer

Let the number of turns in primary and secondary be N_P and N_S. At any instant t, the voltage applied on primary coil V_P produces a flux ϕ, due to which emf induced between the ends of secondary coil of N_S turns

$$E_S = -N_S \frac{d\phi}{dt} \qquad \ldots (10.49)$$

The changing flux ϕ also produces *emf* in primary which is called back *emf*.

$$E_P = -N_P \frac{d\phi}{dt} \qquad \ldots (10.50)$$

When the secondary coil is in open circuit or very small current is drawn, then $E_P = V_P$ and $E_S \simeq V_S$.

$$E_P = V_P = -N_P \frac{d\phi}{dt} \qquad \ldots (10.51)$$

$$E_S = V_S = -N_S \frac{d\phi}{dt} \qquad \ldots (10.52)$$

deviding we get $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ $\qquad \ldots (10.53)$

If there is no energy loss in transformer, the input power is equal to out put power. (In ideal transformer)

$$I_P V_P = I_S V_S \qquad \ldots (10.54)$$

From equ. 10.53 and 10.54 we get

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} \qquad \ldots (10.55)$$

The ratio of voltage in primary and secondary is equal to the ratio of number of turns in primary and secondary coils. The ratio of currents is primary and secondary coils is equal to the inverse of the ratio of number of turns in them.

10.11.3 Type of Transformers

(i) Step-up Transformer - When $N_S > N_P$, then $V_S > V_P$ and current $I_S < I_P$. Such type of transformer is called step up transformer.

(ii) Step-down Transformer - When the number of turns in secondary N_S is less than the number of turns in primary N_P, the out put voltage V_S is less than input voltage V_P and the current in secondary I_S is greater than current in primary I_P. Such transformer is called step-down transformer.

Hence when $N_S < N_P$ then $V_S < V_P$ and $I_S > I_P$. If there is no loss of energy in transfering power from primary coil to secondary coil, $P_P = P_S$. The efficiency of such transformer is 100%.

Practically no transformer has 100 % efficiency. There is always an energy loss in one way or the other.

The following are the energy losses in transformer.

(i) Copper losses : The primary and secondary coils have very small but non zero resistance, due to which heat is produced due to joule effect (power loss – I^2R). This loss is called copper loss. To reduce this loss, thick, copper wire is used for windings.

(ii) Losses due to leakage in magnetic flux - The whole flux produced by primary does not passes through secondary, due to faulty winding or air between laminea of iron core, there is a leakage of flux. To reduce this loss the two coils are wound over each other.

(iii) Eddy current losses - The change in magnetic flux through the iron core, induces small current loops in it, called eddy currents, which dissipates energy in the form of heat. To reduce this loss, the transformer core is laminated.

(iv) Hysteresis Losses - The magnetization of iron core is periodically reversed as per the AC frequency. Hysteresis loss ocures in each cycle, due to reversal of magnetization. The area of hysteresis loop, represents the energy loss per second per unit volume. The soft-iron core is used to reduce this loss, which has minimum area of its hysteresis loop.

Transformers are used in power transmission and impedance matching. Audio frequency transformer are used in telephony and radiotelephony and radio frequency transformers are used in radio communication. The transmission of electrical power to distant areas is done at very high voltage and low current, to reduce the transmission loss. For this step up transformers are used. It is clear from example 10.32.

Example 10.31 : The current in primary coil of a transformer is 1 A at input power of *4kW*. The voltage at secondary coil is 400 *V*. If number of turns in primary coil is 100, find number of turns in secondary coil.

Solution :

$$P_{in} = V_P I_P$$

given $I_P = 1$ A, $P_{in} = 4000$ W, $V_S = 400$ W

$N_P = 100$

$$V_P = \frac{P_{in}}{I_P} = \frac{4000}{1} = 4000\,\text{W}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$N_S = \frac{400}{4000} \times 100 = 10$$

Example 10.32 : A transmission line has a resistance of 20 Ω. The power to be transmitted is 6.6 kW. If the power is transmitted at (i) 22000 V (ii) 220 V. Find power loss and voltage drop in both the cases. What conclusion is drawn from this?

Solution : In first case, $P = VI$

$$I = \frac{P}{V}$$

given $P = 6.6\,\text{KW}, R = 20\,\Omega,$

$V_1 = 22000$ V, $V_2 = 220$ V

$$I = \frac{6600}{22000} = 0.3\,\text{A}$$

Power loss due to heat is $I^2R = (0.3)^2 \times 20 = 1.8$ W

Voltage drop on line is IR = 0.3 x 20 = 6 V

For case (ii) $I = \frac{6600}{220} = 30$ A

Power loss due to heat is $I^2R = (30)^2 \times 20 = 1800$ W

Voltage drop on the line is IR = 30 x 20 = 600 V

It is concluded that all type of losses are less when power transmission is done at very high votage, i.e 220 kV or 400 kV.

Important Points

1. Sinosuidal Ac voltage or current is expressed by -

$$V = V_m \sin \omega t$$

or $$I = I_m \sin \omega t$$

2. Average value of AC voltage/current for whole cycle is zero for positive and negative half cycles the average value is

$$I_{av} = \pm \frac{2I_m}{\pi} = \pm 0.637 I_m$$

3. Values given for AC voltage and currents are their I_{rms} values, it is also called effective values.

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

4. The obstacle produced by an inductor in AC circuit is called inductive reactance X_L and by capacitor its capacitive reactance X_C.

$$X_L = L\omega = L \times 2\pi f$$

$$X_C = \frac{1}{c\omega} = \frac{1}{c \times 2\pi f}$$

5. Obstacle produced in R-L, R-C and L-C-R circuit is called impedance Z.

for R-L circuit $Z = \sqrt{R^2 + X_L^2}$

R-C circuit $Z = \sqrt{R^2 + X_C^2}$

L-C-R circuit $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

6. Phase angle for different AC circuits the phase difference between voltage and current is as given -

for pure resitaive circuit $\tan\phi = 0,\ \phi = 0$

pure inductive circuit $\tan\phi = \infty,\ \phi = \pi/2$

Pure capacitor circuit $\tan\phi = -\infty\ \ \phi = -\pi/2$

R-L circuit $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

R-C circuit $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

L-C-R circuit $\phi = \tan^{-1}\left(\frac{X_L \sim X_C}{R}\right)$

7. For resonant L-C-R circuit

$X_L = X_C$ $\omega_r = \frac{1}{\sqrt{LC}}$

$Z_{min} = R$ $f_r = \frac{1}{2\pi\sqrt{LC}}$

$\phi = 0$

$(\cos\phi)_{max} = 1$

8. The frequencies lower and higher than resosnant frequency, where the current in the circuit becomes $\frac{1}{\sqrt{2}} I_{max}$, are called half power frequencies.

9. The difference between half power frequencies is called band width.

$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$ or $\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$

$f_2 - f_1 = \frac{R}{2\pi L}$

10. In resosnant circuit, the ratio of resosnant frequency f_r and band width is called quality factor Q.

$Q = \frac{fr}{f_2 - f_1} = \frac{L\omega_r}{R}$

11. In ac circuit the average power $P_{av} = V_{rms} I_{rms} \cos\phi$ for resistive circuit $P_{av} = V_{rms} I_{rms}$ for inductive or capacitive circuit $P_{av} = 0$

12. Power factor $\cos\phi = \frac{P_{av}}{P\text{आभासी}} = \frac{R}{Z}$ for pure resistance $\cos\phi = 1$; for inductive or capacitive circuit $\cos\phi = 0$

13. The component of the current $I_m \sin\phi$, which is out of phase by $\pi/2$ with voltage is called wattless current since it don't contribute to power.

14. The transformer works on the principle of mutual induction, with its help the AC voltage can be steped up or steped down.

Transformer formula $\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$

15. Transformers are of two types, step-up and step-dn transformer for step up $N_S > N_P$ for step *dn* $N_S < N_P$

16. The energy loss in a tansformer is due to (i) joule effect (ii) eddy current (iii) hysterissis (iv) flux leakage due to faulty winding.

Questions for Practice

Multiple Choice Question -

1. RMS value of alternating current is -
(a) Double the peak value
(b) Half of the peak value
(c) Equal to peak value
(d) $\frac{1}{\sqrt{2}}$ of the peak value

2. Due to which component of the current in AC circuit leads the voltage in phase -
(a) Pure resistor (b) Pure Inductor
(c) Pure capacitor (d) None of these

3. The AC current lags the voltage by $\frac{\pi}{2}$ in phase, when the circuit has -
(a) Only resistor (b) Only inductor
(c) Only capacitor (d) Capacitor and resistor

4. The unit of ωC is -
(a) Ohm (b) mho
(c) Volt (d) Amp

5. Role of capacitor a circuit -
(a) Allows ac current to pass through
(b) Stops ac current
(c) Allows dc current
(d) Stops ac current and allows dc current

6. Which of these does not have same units -
(a) $\frac{1}{\sqrt{LC}}$ (b) $\sqrt{LC}$
(c) RC (d) $\frac{L}{R}$

7. An ac circuit is resonant at 10 k Hz. If frequency is raised to 12 k Hz. The impedance of circuit will -
(a) Remain uneffected
(b) Increased by 1.2 times
(c) Will increase and becomes capacitive
(d) increases and becomes inductive

8. In a circuit current lags the voltage by $\pi/3$, the elements in the circuit are -
(a) R and C (b) R and L
(c) L and C (d) Only L

9. Power factor of a pure inductor or pure capacitor is -
(a) One (b) Zero
(c) π (d) Greater than zero

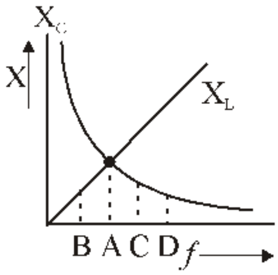
10. The current can be reduced in AC circuit witout power loss by -
(a) Using a pure inductor
(b) Using pure resistor
(c) Using a resistor and inductor
(d) Using resistor and capacitor

11. In an ac circuit the voltage and current are $V = V_m \sin \omega t$ and $I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$; the power dissipated in the circuit will be -
(a) $\frac{V_m I_m}{R}$ (b) $\frac{V_m I_m}{\sqrt{2}}$
(c) $\frac{VI}{2}$ (d) Zero

12. In LCR series circuit the value of C = 1F, L = 1 H at resosnace, the freqency will be -
(a) 10^6 (b) ½ $2\pi \times 10^6$
(c) $\frac{10^6}{2\pi}$ (d) $2\pi \times 10^{-6}$

13. The transformer core is laminated, so that -
(a) Magnetic field is increased
(b) Residual magnetization is reduced in the core
(c) Magnetic saturation of core is increased
(d) Energy loss due to eddy currents is reduced

14. In the diagram, the point which indicated, resosnance is -



(a) A

(b) B

(c) C

(d) D

15. The ratio of currents in primary and secondary coils of a transformer with 100% efficiency is 1 : 4. the ratio of the voltage across the coils will be -

(a) 1 : 4 (b) 4 : 1

(c) 1 : 2 (d) 2 :1

Very Short Answer type questions -

1. The AC voltage is given by equation $V = 200\sqrt{2}\sin 100\pi t$ write its RMS value and frequency.
2. Write down the realtion between Peak and RMS value of AC current.
3. The current in an inductive circuit is given by $I = I_m \sin \omega t$. Give the equation for voltage.
4. The voltage in an AC circuit is given by $V = 200\sin 314t$. Give the frequency of AC voltage.
5. How the inductive and capacitive reactances are effected by increasing AC frequency?
6. A coil has inductance of 0.1 H. Find its reactance at 50 Hz.
7. What will be the phase difference between voltage and current in series L-C-R circuit?
8. What will be phase difference between voltage across inductor anf voltage across capacitor in series L-C-R resosnant AC circuit?
9. What will be impedance in series L-C-R resonant circuit at resonance?
10. What will be value of power factor in Ac circuit for inductor, capacitor and resistor?
11. What is the unit of $\sqrt{LC}$?
12. In series L-C-R resosnant circuit, capacity is changed to 4 times. What will be new inductance for the same resonanace frequency?
13. What will be RMS value of wattless current?
14. The ratio of number of turns in primary and secondary of a transformer is 1:4. What type of transformer it is?
15. Write down the expression for wattless current in ac circuit.

Short Answer Type Question -

1. Why ac is prefered to dc? Please explain.
2. 220 V ac is more danserous then 220 V dc. Why?
3. Draw graph between frequency and X_L and X_C.
4. A capacitor stops DC current, while it allows ac current. Why?
5. A coil has ohmic resistance of 6Ω, if its impedance is 10Ω, find inductive reactance X_L.
6. If f_r is the resonant frequency of ac circuit. Give phase realtionship between voltage and current for (i) $f = f_r$ (ii) $f < f_r$ (iii) $f > f_r$.
7. What is band width? Write its expression for L-C-R series circuit.
8. What are half-power frequencies? What will be the current at these frequencies?
9. If the value of resistance and reactance are same for a coil, what will be its power factor?
10. In the transmission of electrical energy, less power factor of circuit means, more power loss. Please explain.
11. Write down the expression for impedance, frequency and power factor for a series L-C-R ac circuit?
12. Write down the principle of a transformer and its uses.
13. Find average value of ac current in its first positive half cycle.
14. On what factors, the power losses in a transformer depends? How its can be reduced?
15. Find the expression for impedance and phase difference between voltage and current in series R-L ac circuit.

Essay Types Question -

1. Find current, phase difference, reatance and average power used in a pure inductive AC circuit. Also draw phasor diagram.
2. Derive expression for impedance and current in a R-L series AC circuit. Draw phasor diagram.
3. What do you mean by resosnant circuit. Give the required condition for series L-C-R resonant circuit derive the expression for its resonance frequency. Where this circuit is used?
4. Draw a graph between frequency and current for a series L-C-R ac circuit. Derive expression for band width, showing half power frequencies on the graph.
5. Derive expression for power in AC circuit. How this formula will change if the circuit does not have reactance and resistance? Also define power factor.

Answer (Mutliple Choice Questions)

1. (D) 2. (C) 3. (B) 4. (B) 5. (A) 6. (A) 7. (D) 8. (B) 9. (B) 10. (A) 11. (D) 12. (C) 13. (D) 14. (A) 15. (B)

Very Short Answer Type Question -

(1) 200 V, 50 Hz (2) $I_{rms} = \frac{I_m}{\sqrt{2}}$

(3) $V = V_m \sin\left(\omega t + \frac{\pi}{2}\right)$

(4) $2\pi f = 314$ so $f = 50 Hz$

(5) Inductive reactance increases and capacitive reactance decreases.

(6) 31.4 Ω

(7) Between 0 and $\pm\frac{\pi}{2}$

(8) 180^0

(9) Equal to resistance

(10) Zero, zero and one

(11) Second

(12) $\frac{L}{4}$ (13) $\frac{I_m}{\sqrt{2}}\sin\theta$ (14) Step-up

(15) $I_{rms}\sin\phi$

Numerical Questions -

1. If $V = 50\sin(157t + \phi)$ V for ac circuit, find (A) RMS value of ac voltage (B) Frequency of ac voltage.

(35.35 V, 25 Hz)

2. At what instant the value of ac current will be equal (i) half its peak value (ii) $\frac{\sqrt{3}}{2}$ of its peak value for a sinosuidal AC current.

(T/12 s; T/6 s)

3. $V = 100\cos\omega t$ is applied to a circuit containing 10Ω resistor and 100 mH inductor in series. find the current in the circuit and phase difference between voltage and current. (ω =100)

$(\pi/4)$

4. Find inductive reactance for 100 mH inductor at 1 kHz frequency. Find current in the inductor if voltage applied is 6.28 V.

(X_L = 628 Ω; I = 0.1 A)

5. An inductor has inductance 1 H (i) At what frequency its reactance will be 3140 Ω ? What will be capacity of a capacitor to have same reactance at same frequency?

(500 Hz; 0.11 F)

6. A capacitor of capacity 120 μF is joined to an AC source of freqency 50 Hz. Find its capacitive reactance. If the frequency is changed to 5 MHz, what will be the change in its reactance?

(26.54Ω, reactance will reduced to $2.654\times10^{-4}\Omega$)

7. An inductor of $R = 10\Omega$ and 0.4 H is connected to 6.5 V, $\frac{30}{\pi} Hz$ Ac source. Find average power dissipated in the circuit. (5/8 W)

8. A 60 V, 10 Ω bulb is connected to 10 V, 60 Hz, AC source. A coil is connected in series. Find the value of inductance of the coil for full illumination of the bulb. (1.28 H)

9. A series L-C-R circuit containing $R = 20\Omega$, L = 200 m H and C = 40 F is joined to a 120 V, 60 Hz AC source.

Find (i) Total reactance (ii) impedance (iii) power factor (iv) average power.

($X = 9\Omega, Z = 21.94\Omega,\ \cos\theta = 0.912$ P = 598.58W)

10. Find the resosance frequency of series L-C-R series circuit containing $L - 0.1\ H, C - 20\ \mu F;$ $R = 10\Omega$. (112.6 Hz)

11. A source of $V - 15\cos\omega t$ V is connected to a series L-C-R circuit having $L - 10\ mH, R - 3\ \Omega$ and C = 1μ F. Find the peak value of current at frequency 10% less than the resonant frequency.

(0.704 A)

12. A series L-C-R circuit of $L - 200\ mH, C - 500\ \mu F;$ $R = 100\Omega$ is connected to 100 V ac source. Find -

(i) Frequency at which power factor of circuit is 1.

(ii) Peak value of current at this frequency.

(iii) Quality factor

(15.9 Hz, 1.414 A, 0.2 A)

13. A coil has a power factor 0.707 at 60 Hz. What will be power factor at 120 Hz?

(0.44)

14. A series L-C-R circuit of $L - 5\ H, C - 80\ \mu F,$ $R = 40\Omega$ is joined to a source of 230 V. Find (i) Resonant frequency (ii) impedance of circuit and peak alue of current at resonance (iii) RMS value of voltage at all the three elements.

(50 Hz, 40 Ω, 8.1 A, 230 V, 1437.5 V, 1437.5 V)

15. A transformer having 5000 turns in its primary steps down 2200 V to 220 V. If the efficiency of the transfer is 80% and out put power is 8 kW, find (i) N_S (ii) I_P (iii) I_S (iv) Input power where the symbols have their usual meaning.

(500, 4.54 A, 36.36 A, 10KW)

Chapter - 11
Ray Optics

Introduction : Light is a form of energy. We can see objects when light fall on them. It was always a matter of curiosity to know about the production and nature of light and related phenomenon. From our common experience. We know that it travels in straight line and its speed is extreamly high.

The straight path traversed by light is called a ray and denoted by a straight line with an arrow. Group of rays is called a beam. Some of the optical phenomenon like reflection, refraction, dispersion etc. can easily be understood, using ray concept of light. In this chapter we will study reflection, refraction, dispersion and scattering using ray concept. In the later section we will study the working of optical instruments like microscope, telescope and human eye. The study involves the study of optical phenomenon related to daily life. The study is governed by the laws of geometry hence it is also called geometrical optics.

11.1 Reflection of Light

The light travelling in a medium, returns back at the boundary as shown in fig. 11.1. This phenomenon is called reflection. It obeys following laws .

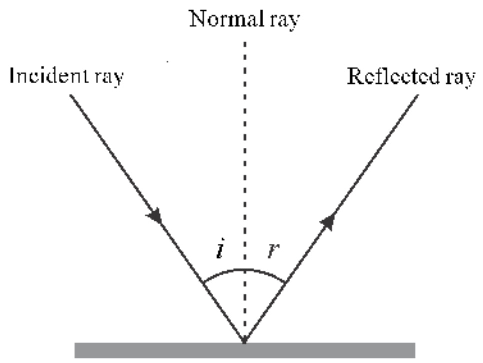


Fig. 11.1 : Reflection at a plane surface

(i) The angle of incidence i is equal to angle of reflection r.

$$\angle i = \angle r$$

(ii) The incident ray, reflected ray and the normal on reflecting surface lies in same plane called plane of incidence.

In reflection, there is no chnage in frequency and wavelength of light. If surface absorb light then intensity of reflected light decreases.

11.1.1 Formation of Image by a plane mirror

As shown in fig. 11.2 consider a point object at P, which is in front of a plane mirror AB. to construct the image we need two rays. The two rays PQ and PQ^1 get reflected by mirror and seems to be coming from P^1. Here P^1 is virtual image of the object. The image is virtual, erect and laterally inverted.

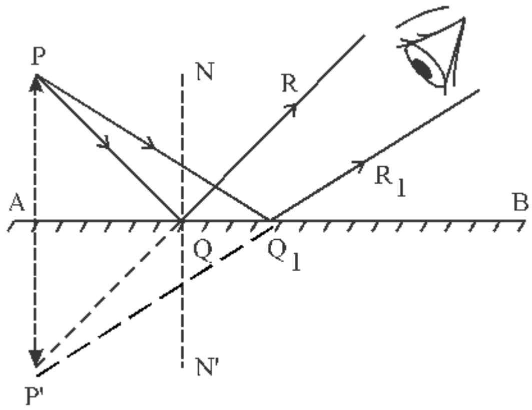


Fig. 11.2 : Image fromed by a plane mirror

11.2 Spherical Mirror

It is a part of a hollow sphere, whose one side is polished, normally these mirrors are of glass whose one side is silvered. If the reflecting surface is convex, the mirror is called convex mirror, and if the surface is concave, the mirror is called concave mirror. The figure 11.3 shows reflection of parallel rays by a concave mirror. Since the parallel rays get converged at a point these mirrors are also called convergent mirror. Similarly fig. 11.4 shows the reflection from a convex or divergent mirror.

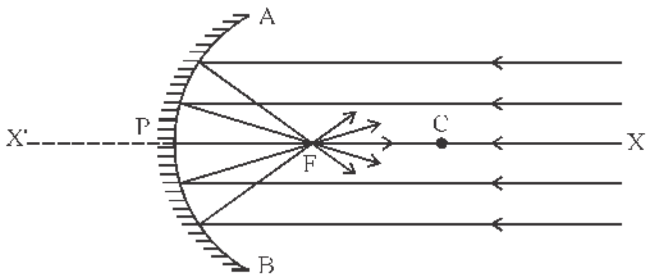


Fig. 11.3 : Concave mirror

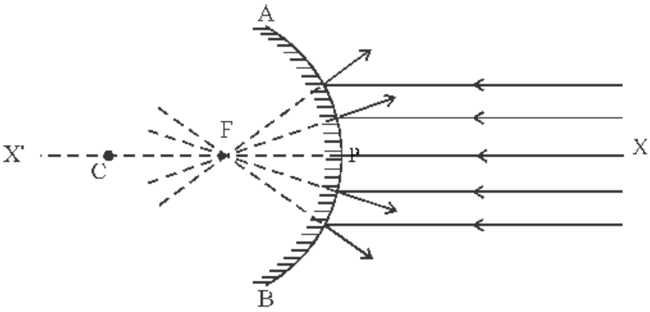


Fig. 11.4 : Convex mirror

11.2.1 The Terms and their Defenitions Related to Spherical Mirror

1. Aperture : The whole reflecting surface of a mirror is called its aperture.

2. Pole : The central point of the mirror is called pole. In fig. 11.3 and 11.4, the point P indicate pole.

3. Center of Curvature : The centre of the sphere, whose the mirror is a part, is called centre of curvature of the mirror. It is indicated as point C in fig 11.3 and 11.4.

4. Radius of Curvature : It is the distance between pole and centre of curvature of the mirror, indicated as CP in the figures.

5. Principle axis : The straight line passing through pole and centre of curvature is called principle axis. Shown as XX^1 in fig 11.3 and 11.4.

6. Principle focus : The incident rays parallel to principle axis, meet at a point on the principle axis after reflection from a concave mirror. This point is called principle focus of concave mirror. It is indicated as F in fig 11.3.

In case of a convex mirror, the incident rays parallel to principle axis are diversed after reflection from the mirror and seems to be coming from a point on principle axis. This point is called principle focus of a convex mirror and indicated as F in fig. 11.4.

7. Focal length: It is the distance between pole and the focus point. Its symbol is f.

11.2.2 Sign Convention

To obtain relation between different quantities like object distance, image distance, focal length, magnification etc. we should adopt a sign convention. Here we will follow cartesian sign convention. According to it -

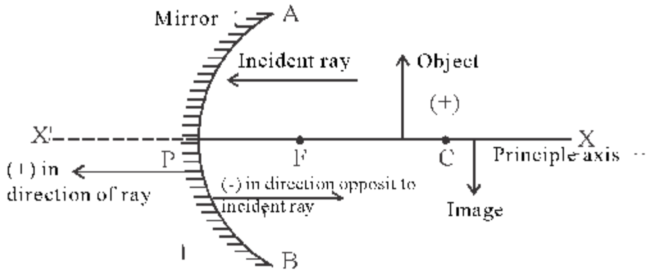


Fig. 11.5 Sign convention

1. All the distances are measured from pole.
2. The distance measured in the direction of incident rays is taken as positive and the distance measured in opposite direction of incident ray is taken as negative.
3. The height measured upwards with repect to X-axis and normal to principle axis is taken as positive. The height measured downwards is taken as negative.

According to above sign convention the object distance is taken as negative, but the image distance v can be positive or negative as per situation. Focal length of concave mirror is taken as negative while that of a convex mirror is positive.

11.2.3 Formation of images by Spherical Mirrors

To find the position of image we can take any two rays originating from object. After reflection from mirror these rays determines the position of image -

1. The incident ray which is parallel to principle axis pass through the focus (in concave mirror) or seems to be coming from focus (in convex mirror), after reflection from spherical mirror.
2. The incident ray passing through focus, becomes parallel to principle axis after reflection from spherical mirror.
3. The incident ray passing through center of curvature returns back on the same path after reflection from the spherical mirror.

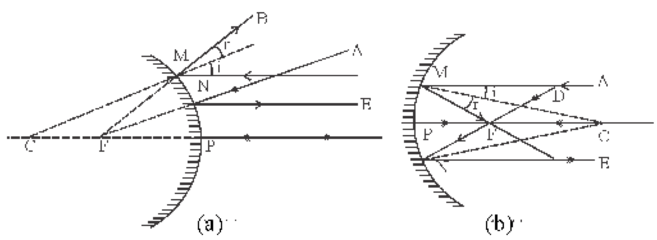
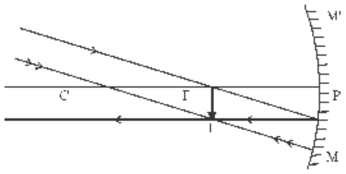
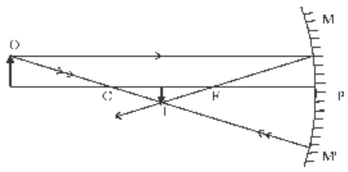
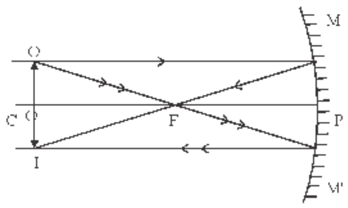
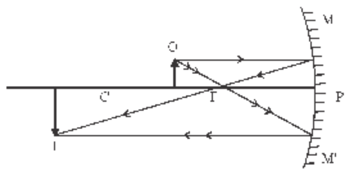
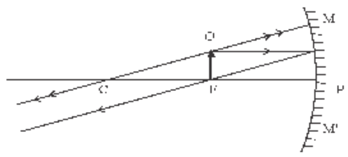
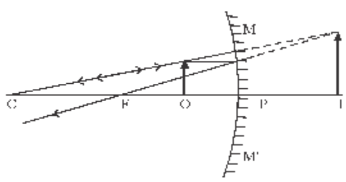


Fig. 11.6 : Rays used for the formation of images

If the two rays meet at a point after reflection a real image is formed. And if the rays do not meet but seems to be coming from that point, then a virtual image is formed. The real image is inverted while the virtual image is erect.

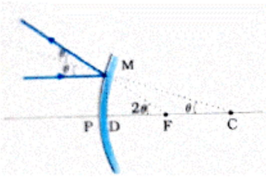
The different positions of images formed by concave and convex mirror are shown in table 11.1.

Table 11.1 (a) For concave mirror

Position of object	Ray diagram	Form of image
1. At infinity		Real, inverted, very small at focus
2. Between ∞ and C		Real, inverted, very small between F and C
3. At C		Real, inverted, and of same size, at C
4. Between F and C		Real, inverted, magnified between C and ∞
5. At focus		Real, inverted and very large at ∞
6. Between F and P		Virtual, erect, very large, behind the mirror

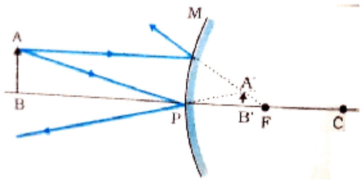
(b) For convex mirror

1. At ∞



virtual, erect, very small at focus

2. In front of mirror



Virtual, erect, small between pole and focus

11.3 Mirror Formula

11.3.1 Relation between R and F for a mirror

Consider a mirror of very small aparture (Fig. 11.7). AM is incident ray, MB is reflected ray and MC is perpendicular at M.

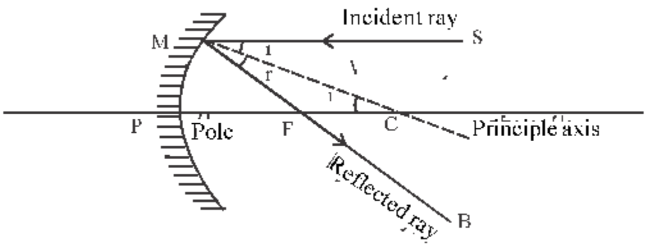


Fig. 11.7 : Reletion between R and f

From law of reflection $\angle$AMC = $\angle$FMC or $\angle i = \angle r$ But $\angle$AMC = $\angle$MCF (alternative angle).

Hence from $\angle$MCF, $\angle$FMC = $\angle$MCF and MF= FC

If the point M is very near to P; MF = PF = f (focal length) but fron fig. 11.7.

$PF - FC - R$

$f \quad f - R$

$$f = \frac{R}{2} \qquad \ldots (11.4)$$

Hence the focal length is half of the radius of curvature for a spherical miror.

11.3.2 Mirror Equation

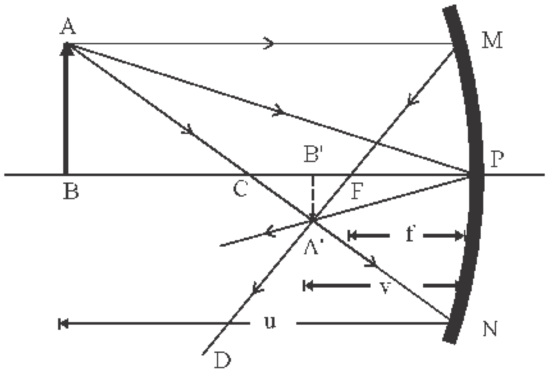


Fig. 11.8 : Ray diagram for image in concave mirror

In fig. 11.8 A^1B^1 is real image of the object . u, v and f are object distance, image distance and focal length respectively. To find relation between u, v and f, consider the similar tringle A^1B^1F and MPF (assuming MP as a straight line).

$$\frac{B'A'}{PM} = \frac{B'F}{FP} \text{ OR}$$

$$\frac{B'A'}{BA} = \frac{B'F}{FP} \quad (\because PM = BA) \qquad \ldots (11.5)$$

Again from Δ PAB and Δ PA^1B^1 (being similar) we get

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \qquad \ldots (11.6)$$

Comparing equation 11.5 and 11.6 we get

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP} \qquad \ldots(11.7)$$

using sign convention B[1]P = - v; FP = -f and BP = - u

$$\frac{-v+f}{-f} = \frac{-v}{-u} \text{ OR } \frac{v-f}{f} = \frac{v}{u}$$

or $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \ldots(11.8)$$

This relation is called mirror equation, and is valid for both concave and convex mirrors.

Magnification :- Linear magnification m is defined as the ratio of image height h[1] and object height h.i.e

$m = \frac{h'}{h}$ As per sign convention, the distance above the principle axis is taken as position and vice versa.

From fig. 11.8 $\frac{B'A'}{BA} = \frac{-v}{-u}$

from similar triangle A[1]B[1]P and AB P we get

$$\frac{-h'}{h} > \frac{-v}{u}$$

(using sign convention) $$m = \frac{h'}{h} = -\frac{v}{u} \qquad \ldots(11.10)$$

Here - ve sign indicate that the image is inverted.

Using mirror equation (11.8) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ and equation (11.10) we can obtain magnification in terms of f and v;

and f and u. $i.e$ $$m = -\frac{v-f}{f} \qquad \ldots(11.10(a))$$

and $$m = -\frac{f}{u-f} \qquad \ldots(11.11a)$$

Example 11.1: Radius of curvature of a concave mirror is 50 cm.Find its focal length.

Solution : Given R= 50 cm

We know that $f = \frac{R}{2}$ So $f = \frac{R}{2} = \frac{50}{2} = 25\,\text{cm}$

$f = 0.25\,\text{m}$

Example 11.2 : An object is placed at a distance of 10 cm from a convex mirror of radius of curvature 15 cm. Find the nature, position and magnification of image.

Solution : Given $f = \frac{-15}{2}\,\text{cm}$ and $u = -10\,\text{cm}$

From mirror equation $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

hence $$\frac{1}{v} = -\frac{2}{15} - \left(\frac{1}{-10}\right) = \frac{-2}{15} + \frac{1}{10} = \frac{-4+3}{30}$$

$$\frac{1}{v} = -\frac{1}{30} \text{ hence } v = -30\,\text{cm}$$

$$m = -\frac{v}{u} = -\left(\frac{-30}{-10}\right) = -\frac{30}{10} = -3$$

so the image will be at 30 cm in front of mirror, it is real and inverted and three times the size of object.

Example 11.3 : The image of an object placed in front of a concave mirror is obtained at 100 cm on the same side. If focal length of the mirror is 98 cm, find the object distance.

Soltuion : Given f = - 98 cm; v = - 100 cm

using mirror equation we get $\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$

$$\frac{1}{u} = \frac{1}{-98} - \frac{1}{-100} = -\frac{1}{98} + \frac{1}{100}$$

$$\frac{1}{u} = \frac{-100+98}{9800} = \frac{-2}{9800}$$

$$u = \frac{9800}{2} = -4900\,\text{cm}$$

Example 11.4 : Radius of curvature of a concave mirror in an amusement park is 4 m. A girl standing in front of this mirror sees herself 2.5 times taller. If the image is erect, find her distance from the mirror.

Solution : Given m = - 2.5 and R = - 4 cm

$$f = \frac{R}{2} = \frac{-4.0}{2} = -2.0\,\text{m}$$

$$m = \frac{h_2}{h_1} = -\frac{f}{u - f}$$

Substituting we get

$$2.5 = -\frac{(-2.0)}{u - (-2.0)} \Rightarrow u = -1.2\,\text{m}$$

Example 11.5 : An object is placed at a distance f in front of a convex mirror of focal length f. Find the image distance.

Solution : Taking $u = -f$ and the mirror equation we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}\ ;\ -\frac{1}{f} + \frac{1}{v} = \frac{1}{f}\ ;\ \frac{1}{v} = \frac{2}{f}\ \ i.e.\ v = \frac{f}{2}$$

Note : The image of a real object by a convex mirror is between pole and focus. The image is aint, virtual and erect.

Example 11.6 : The image of an object placed at a distance 20 cm from a concave mirror is obtained at 40 cm. Find the focal length of the mirror.

Solution : Given u = - 20 cm; v = - 40 cm

from mirror equation we get

$$\frac{1}{f} = \frac{1}{(-20)} + \frac{1}{(-40)} = -\frac{1}{20} - \frac{1}{40} = \frac{-2-1}{40}$$

$$\therefore \quad f = -\frac{40}{3} = -13.333\,\text{cm}$$

11.4 Refraction of light

Light travels in a straight line in a homogeneous medium. When light ray is incident at the interface of two media, a part of it is reflected and a part enters the other medium by bending from original direction. This phenomenon of bending of ray at the interface of two media is called refraction.

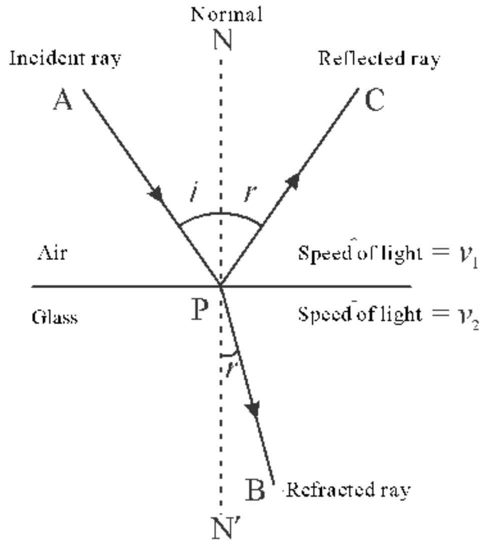


Fig 11.9 : Refraction of light

Following laws are obtained by simple experiments.

(i) Incident ray, the refracted ray and the normal on the interface at that point lie in the same plane.

(ii) The ratio of since of incident angle and rise of angle of refraction is a constant. This law is called Snell's law.

$$i.e.\ \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21} \qquad \ldots(11.11)$$

Here i and r are angle of incident and angle of refraction respectively. And n_1 and n_2 are refraction indexes of the first and second medium. Where as n_{21} is the refractive index of second medium with respect to the first medium.

If the light enters a medium from air or vaccum, then the refractive index is called the absolte refractive index of the medium. The Snell's law can be conveniently written as

$$n_1 \sin i = n_2 \sin r \qquad \ldots(11.12)$$

When $n_{21} > 1$; $r < i$, the refracted ray bends towards the normal, the second medium is denser than first medium.

If the ray enters a rarer medium from a denser medium then $r > i$ and ray bends away from the normal.

Speed of light C, is maximum in air/vaccum. The speed of light v is lesser in all other media.

Absolute refractive index of a medium is given by

$$n = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in that medium}} = \frac{c}{v} \quad \ldots (11.13)$$

The relative refractive index of a medium is given by

$$n_{21} = \frac{\text{Speed of light in first medium}}{\text{Speed of light in second medium}} = \frac{v_1}{v_2}$$

Refractive index is a scalar and dimensionless quantity and it depends on the nature of the medium and the wavelength/frequency of the light. During refraction only wavelength of the light changes due to change in speed, but frequency remains unchanged.

11.4.1 Phenomena related to refraction of light

1. Twinkling of stars

Image of stars on retina are point images. Light from the stars travels a long distance in a turbulant atmosphere. Due to this the apparent position of the image on the ratina fluctuates and the amount of light entering the eye thickers. This is called twinkling of star.

Why don't planets twinkle? The planets are comparatively nearer and behave like an extended source. The light from a planets seems to be coming from large number of points. The change in positions of all the points, nullify the effect of twinking.

2. Sunrise and sunset

The sun is vissible to us a few minutes before the actual sunrise. It is also vissible for a few miniutes after the sunset. This is due to refraction of light by different layers of atmosphere having different refective indices.

When the Sun is still at horizon, the ray enters from denser layer to a rarer layer thus bending away from the normal. Due to this gradual bending by different layers. We see the Sun at at S[1] instead of S.

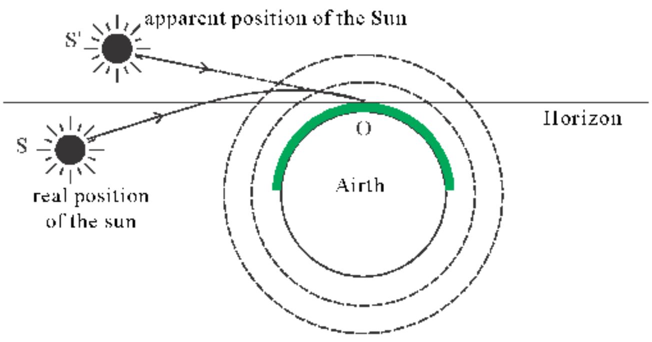


Fig. 11.10 The apparent position of the sun

3. Appearance of bottom of liquid as raised

The bottom of pot, filled with a liquid seems to be raised when viewed from above. This is due to refraction. As from fig. 11.11, the ray of light coming from the point P on the bottom, bends away from the normal at the liquid-air interface and seems to be coming from point P[1]. Hence the bottom seems to be raised.

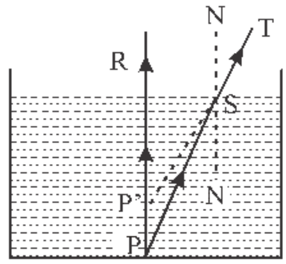


Fig. 11.11 : Virtual position of bottom of a liquid

4. Bending of a rod/spoon in a liquid

When a rod is partially dipped in a liquid, the rod appears to be bent. This is because the apparent positions of its different parts in liquid seems to be raised according to their depths. The rod ABE appears us as ABE[1] (Fig. 11.12).

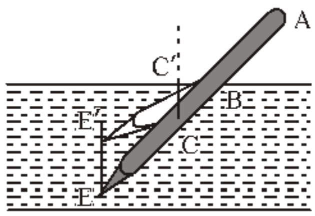


Fig. 11.12 : Bending of a rod in liquid

5. Refraction through a glass slab

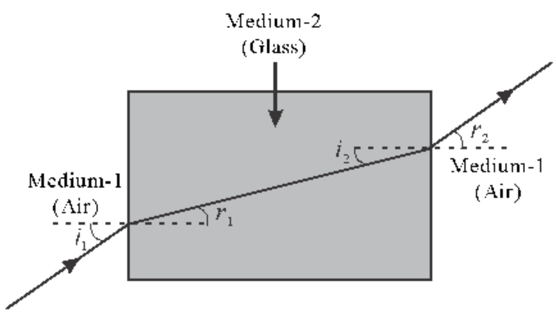
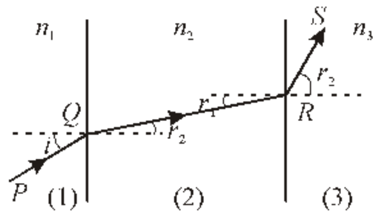


Fig. 11.13 : Refraction through a glass slab

As shown in the figure 11.13. The refraction of light occurs twice. First when the ray enters the slab and when it comes out of the slab. Here angle of incidence i_1 and angle of emergence r_2 are equal. The incident and emergent rays are parallel to each other, but there is a lateral shifting of emergent ray.

Example 11.7 : The given figure shows the path of a ray passing through three different media. The figure is based on scale. What do you conclude about the refractive indices of the three media.



Solution : As seen in the given figure, the refrated ray QR bends towards the normal, hence $n_2 > n_1$ and medium (2) is denser than medium (1).

Similarly the refracted ray RS is bending away from the normal hence $n_3 < n_2$ and the medium (3) is rarer than medium (2).

Also since $\angle r_2 > \angle$ i hence $n_2 > n_3$

Example 11.8 : Wavelength of a colour of light in air is 6000 Å, which changes to 4500 Å in water. Find the velocity of light in water.

Solution : We know that $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

hence $\frac{v_\omega}{v_a} = \frac{\lambda_\omega}{\lambda_a}$

$$v_\omega = \left(\frac{4500}{6500}\right) 3\times10^8 = 2.25\times10^8 \text{ m/s}$$

Example 11.9 : The absolute refractive indexes of water and glass are 4/3 and 3/2 repsectively. Find refractive index of water when the ray enters from glass to water.

Solution : Given $n_\omega = \frac{4}{3}$ and $n_g = \frac{3}{2}$

hence the refractive nindex of water with respect to glass

$$n_{\omega g} = \frac{n_\omega}{n_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

11.5 Total Internal Reflection

We have learned in previous section, that when a ray enters a rarer medium from a denser medium it bends away from normal. Let us deal the phenomenon in detail.

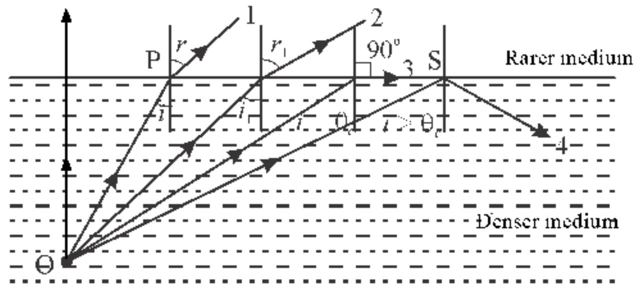


Fig. 11.14 Total internal reflection

A ray of light is incident on interface of denser and rarer medium. If the ray enters rarer medium from denser medium and $\angle i = 0$; the ray go undeviated. If we increase the incident angle of the refraction will increase. At a certain angle of incident, the angle of refraction r becomes 90^0, up to this stage both reflection and reflection simultaneously takes place. The angle of incident for which the corresponding angle of refraction becomes 90^0 is called critical angle and represented by θ_C.

If $i > \theta_C$ then $r > 90^0$. and the ray is totally reflected back (refraction ceases). This phenomon is called total internal reflection. for TIR two basic requirements are there - (1) The ray should enter a rarer medium from a denser medium.

(2) $\angle i > \theta_C$ *i.e.* angle of incidence should be greater than critical angle.

If the rarer medium is air, then from Snell's law

$$n_m \sin\theta_C = n_a \sin 90^\circ$$

$$n_m = \frac{1}{\sin\theta_C} \quad [\because\ n_a \approx 1] \qquad \ldots(11.14)$$

When n_m = refractive index of denser medium

The critical angle for some media are given in table 11.2 for air as rarer medium.

Substance	Refractive Index	Critical Angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Fliut glass	1.62	37.31°
Diamond	2.42	24.41°

11.5.1 Some Applications of Total Internal Reflection

1. Mirages

On a hot summer day, during driving on a coaltar raod, we experience an optical illusion in which we see that the road ahead of us is wet. Such type of experience is also there in desert during summer. This optical illusion is called mirage.

This phenomenon can be explained by assuming that the atmosphere near the earth surface consist of so many layer having different refractive indices, as we go up the refractive index go on increasing. The sun rays entering from uper layer have a refraction from denser to rarer layer hence bends away from the normal, a stage will come when the ray is totally reflected, and we feel the inverted image of tree or other object as if coming after reflection from a water surface.

2. Brilliance of diamond

Brilliance in diamond is due to total internal reflection. Reflective index of diamond is very large total approximately 2.42 hence the critical angle is small *i.e.* 24.41°. It is cut and polished in such a way that the light entering its different faces get totally internal reflected. Hence the brilliance.

3. Optical Fibre

Optical fibres are used to carry optical signals to a very long distance. It consists of central part core and outer coating, cladding. Refractive index of core is slightly greater than the refractive of cladding. An optical signal entering one end of the optical fibre at an angle $i > \theta c$ get multiple total internal reflection and ultimately comes out of the other end without much loss of energy.

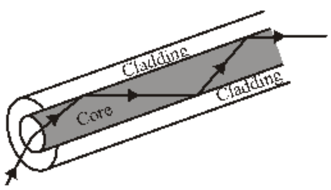


Fig.11.15 Optical Fibre

Example 11.10 : The refractive index of diamond is 2.42. Find critical angle for it.

Solution : Given n = 2.42

We know that $\sin C = \frac{1}{n}$

$$\sin C = \frac{1}{2.42} = 0.4132$$

$$\therefore \quad C = \sin^{-1}(0.4132)$$

From sine table we get θ_c =24.4°

Example11.11 : The refractive indices of core and cladding of an optical fibre are 1.47 and 1.31 respectively. find the angle of incidence for which total internal reflection takes place in optical fibre.

Solution : For total internal reflection $\theta > \theta_c$

$$\theta > \theta_c = \sin^{-1}(n_2 / n_1)$$

$$\theta > \sin^{-1}(1.31/1.47) = \sin^{-1}(0.88)$$

$$\theta > 63^{0} \text{ (From sine table)}$$

11.6 Refraction at Spherical Surface

In this section we will study refraction at the spherical interface of two transparent homogeneons media. The law of refraction is applicable at every point of a spherical surface.

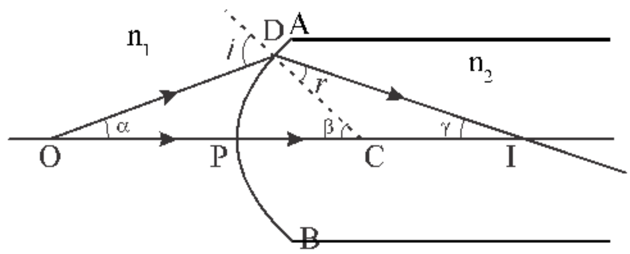


Fig. 11.16 Refraction at sperical surface

As in fig. 11.16 we consider a spherical surface AB of radius of curvature R, separating two media of refractive index n_1 and n_2. A point object is placed on the pricniple axis at O. The rays OD and OP meet at I after refraction and form an image. the point D is assumed very near to P. So that PD is straight line.

From fig. 11.16 $\tan \angle DOP = \tan \alpha = \frac{DP}{OP}$

$$\tan \angle DCP = \tan \beta = \frac{DP}{PC}$$

and $$\tan \angle DIP = \tan \gamma = \frac{DP}{PI}$$

In Δ DOC $\angle i$ is external angle hence

$$i = \angle DOP + \angle DCP = \alpha + \beta \quad ...(11.15)$$

For small angle (in radian) $\theta \approx \tan \theta$

$$i = \frac{DP}{OP} + \frac{DP}{PC} \quad ...(11.16)$$

Similarly in Δ DCI, is external angle hence $\beta = \gamma + r$

hence $$r = \angle DCP - \angle DIP \quad ...(11.17)$$

Or $$r = \frac{DP}{PC} - \frac{DP}{PI} \quad ...(11.18)$$

From Snell's law $n_i \sin i = n_2 \sin r$

$$n_i i = n_2 r \quad \text{(for small angle)} \quad ...(11.19)$$

$$n_1\left(\frac{DP}{OP} + \frac{DP}{PC}\right) = n_2\left(\frac{DP}{PC} - \frac{DP}{PI}\right)$$

$$\frac{n_1}{OP} + \frac{n_2}{PI} = \frac{(n_2 - n_1)}{PC} \quad ...(11.20)$$

Using cartesian coordinate sign convention and taking OP = - u; PI = v; and PC = + R

We get $$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_i}{R} \quad ...(11.21)$$

The above relation is valid for both convex and concave surfaces.

11.7 Lens

A transparent homogeneous medium enclosed by two curved surfaces is called lens. Out of the two surfaces, one can be a plane surface. The curved surface may be spherical, cylinderical or parabolic. We will discuss only spherical surface. The face in which light enters is called first face and the other is second face.

According to curved surfaces, lenses are of two types -

(1) Convex lens/ convergent lens. These lenses are thin on the outer rim and thick at the center.

(2) Concave lens/divergent lens. These lenses are thin at the center and thicker on the outer perifery.

11.7.1 Refraction Through Thin Lens

Consider a thin lens of refractive index n_2 placed in rarer medium of refractive index n_1. P_1 and P_2 are poles and R_1 and R_2 are radius of curvature of two surfaces of the lens.

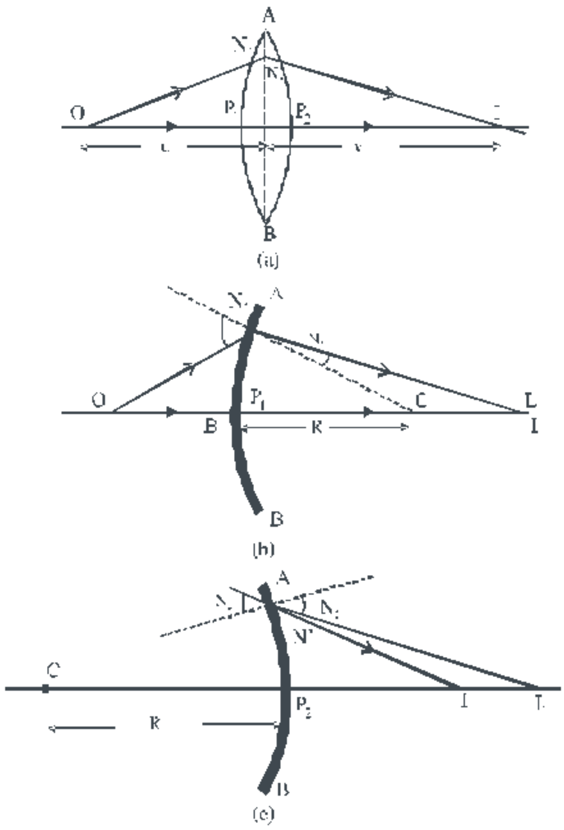


Fig. 11.17 : (a)Position of object and image
(b) Refraction through first surface and
(c) Refraction through second surface

In fig. 11.17 (a) the object is at O and image is at I. The formation of image in considered in two parts (i) action of the first surface and (ii) action of second surface.

As in fig. 11.17 (b) the object is at O and image is formed at I[1]. Applying equation 11.21 for this surface we get

$$-\frac{n_1}{OP_1} + \frac{n_2}{P_1 I_1} = \frac{n_2 - n_1}{R_1} \quad ...(11.22)$$

For the second surface the image at I[1] works as an object and final limage is formed at I.

Applying equation 11.21 for second surface we get

$$-\frac{n_2}{P_2 I_1} + \frac{n_1}{P_2 I} = \frac{n_1 - n_2}{R_2} \quad ...(11.23)$$

Adding equation 11.22 and equation 11.23, we get

$$-\frac{n_1}{OP_1}+\frac{n_1}{P_2I}=(n_2-n_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.24)$$

since $OP_1 = u$ and $P_2I = v$ we get

$$-\frac{1}{u}+\frac{1}{v}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.25)$$

If the object is at ($u=-\infty,\ v=f$) the image will be formed at focus ($v=f$)

$$\text{We get }\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.26)$$

The above relation is known as lens makers formula. Because for a particular value of refractive index, the required focal length is obtained by grinding the two surfaces. The above relation is also valid for concave lens.

A lens has two focii F and F[1] equidistance from the optical center on both sides. The focus on object side is called first focus and that of other side is called second focus.

As in case of a mirror, the magnification by lens is given by

$$m=\frac{\text{height of image}}{\text{height of object}}=\frac{h'}{h}=\frac{v}{u}\quad\ldots(11.28)$$

From sign convention we see that for virtual image m is positive, while for a real image m is negative.

11.7.2 Power of lens

The power of a lens is known by its ability to bend the incident ray by refraction. It is measured by the angle of deviation produced by lens.

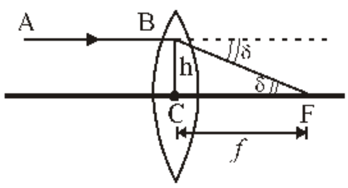


Fig. 11.18 : Power of lens

As in fig. 11.18 a ray parallel to principle axis is incident on lens at a point B which is at a distance h from the optical center C. The ray will pass through F after refraction from lens. Hence for a small angle of deviation δ,

$$\delta=\tan\delta=\frac{h}{f}$$

$$\text{So}\quad P=\delta=\frac{h}{f}$$

The power is defined taking $h = 1\,m$. hence

$$h=1\ P=\frac{1}{f(m)}\quad\ldots(11.29)$$

The focal length is taken in metre. From eq. (11.29) it is evident that the power of a lens is inversly proportional to its focal length. If the focal length of a lens is 1m, its power is one diaptre *i.e.* 1D. Power is a dimentionless scalar quantity. The sign of power is according to sign of focal length. *i.e.* power of a convex lens is positive while the power of concave lens is negative.

11.7.3 Combination of thin lenses

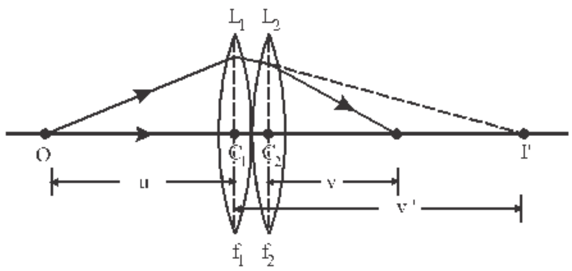


Fig. 11.19 : Combination of two lenses

The two lens kept in contact, behave like a single lens.

As in fig. 11.19 an object O is placed at a distance u from the combination of two lenses L_1 and L_2 of focal length f_1 and f_2. In the absence of lens L_2 the image formed by L_1 is at a distance V' from optical center C_1. Hence for the first lens

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1}\quad\ldots(11.30)$$

Now the image I[1] behave as an object for the second lens since the lenses are assumed to be thin, C_1 and C_2 are very near to each other. So we, can take the object distance for second lens as V'. Hence for the second lens we get

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}\quad\ldots(11.31)$$

Adding eq. 11.30 and eq. 11.31 we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots (11.32)$$

The combination behave as a single lens for which the object distance is u and image distance is v. Hence f is focal length of the combination.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \ldots (11.33)$$

So we get $$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots (11.34)$$

for combination of many lenses of focal lengths $f_1, f_2, f_3 ...$, etc. The resultant focal length and power will be -

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots \quad \ldots (11.35)$$

and $P = P_1 + P_2 + P_3 +$ ...(11.36)

Here P is the net power of the combination. and the total magnification m is given by

$$m = m_1 \times m_2 \times m_3 \times \ldots \quad \ldots (11.37)$$

where m_1, m_2, m_3 are magnification by individual lenses.

Such combinations are used in camera and optical microscopes (in eye pieces and objectives).

11.7.4 Image formation by lens

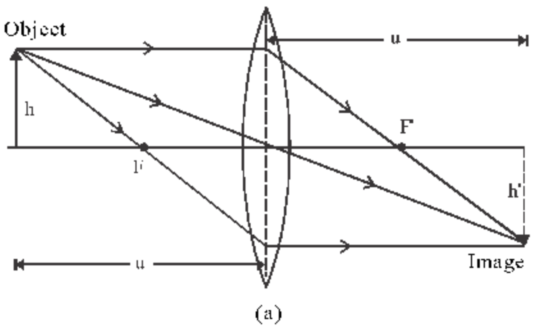


Fig. 11.20 (a) For convex lens

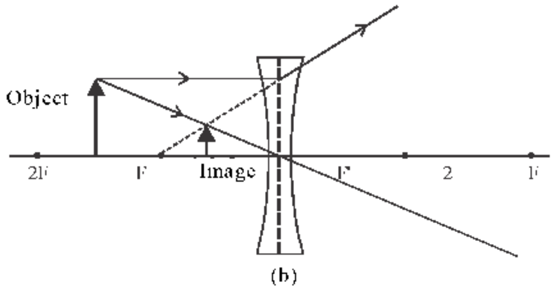


Fig. 11.20 (b) For concave lens

Formation of image by lens can be explained by tracing the path of at least two rays orriginating from the object. In case of a convex lens these two rays meet at a point after refraction and form a real image.

In case of a concave lens,these two rays from the object diverge after refraction, but seems to be coming from a point, where virtual image is formed. The path of the rays will be as follows-

(i) the incident ray parallel to principle axis will pass through the focus after refraction from convex lens.

But seems to be coming from focus in case of concave lens.

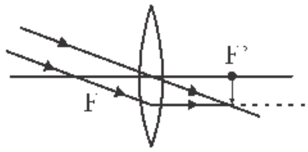
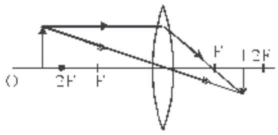
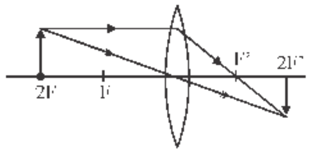
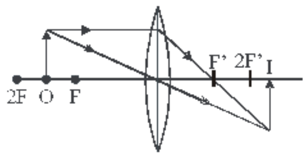
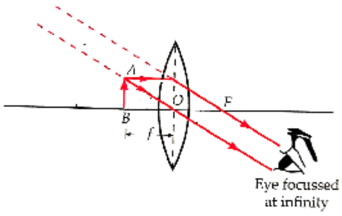
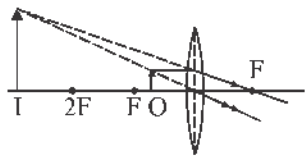
(ii) The incident ray passing through optical center will go undeviated in both types of lenses.

(iii) The incident ray passing through focus will go parallel to principle axis after refraction from convex lens.

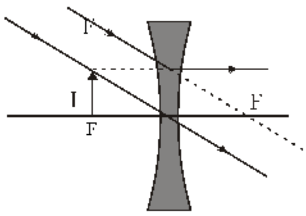
In case of concave lens,the incident ray targeted towards focus, will go parallel to principle axis after refraction.

Table 11.2 For the positions of object and image

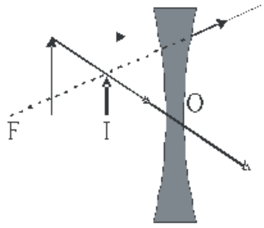
(a) For a converging or convex lens

S.No.	Position of object	Ray diagram	Position of image
1.	At ∞		Real, inverted, point image, at F (m<<-1)
2.	Between ∞ and 2 F		Real, inverted, small (m <-1) between F and 2 F
3.	At 2 F		At 2 F, inverted, Real (m = - 1)
4.	Between 2 F and F		Real,inverted, between 2 F and ∞
5.	At F		Real inverted,very large m >> - 1 at ∞
6.	Between F and O		Virtual,erect,large m > + 1 on the side of object between ∞ and object

(b) For divergent or concave lens

Position of object	Ray diagram	about image
1. At ∞		virtual, erect, small m << 1 at F (in the same side)

2. In front of lens



virtual, erect, small m < + 1 between F and O

Example 11. 12 : An object of height 6.0 cm is placed at 30.0 cm from a lens. the height of the image is 2.0 cm and is inverted. Find focal length of the lens.

Solution : $m = \frac{h_2}{h_1} = \frac{f}{u+f}$

Substituting $u = 30.0$ cm, $h_2 = 2.0$ cm and $h_1 = 6$ cm

$$\frac{(-2.0)}{(6.0)} = \frac{f}{(-30.0)+f}$$

$$f = \frac{60}{8.0} = 7.5\,\text{cm}$$

Example 11.13 : Radius of curvature of a convex lens is 20 cm and 30 cm. Refractive index of the material of lens is 1.5. If this lens is put in water (n = 1.33). Find its focal length.

Solution : Given $n_2 = 1.5$; $n_1 = 1.33$; $R_1 = 20$ cm and $R_2 = -30$ cm

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{20} + \frac{1}{30}\right) = \frac{1}{8}\left(\frac{5}{60}\right)$$

$$f = 96\,\text{cm}$$

Example 11.14 : What will be the object distance, when the magnification by a lens of focal length 10 cm is 2.

Solution : $f = +10$ cm ; $|m| = 2$

$$m = \frac{v}{m} = \frac{f}{u+f}$$

$$+2 = \frac{10}{u+10} \Rightarrow 2u + 20 = 10\ ;\ u = -5\,\text{cm}$$

$$\text{In } m - \text{-}2 \text{ then } -2 = \frac{10}{u+10} \Rightarrow -2u - 20 = 10$$

$$u = -15\,\text{cm}$$

Example 11.15 : A convex lens of focal length 5.0 cm is kept in contact with a concave lens of focal length 10.0 cm. What will be the focal length of this combination?

Solution : For combination of lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

given $f_1 = +5\,\text{cm},\ f_2 = -10\,\text{cm}$

$$\frac{1}{f} = \frac{1}{5} - \frac{1}{10} = +\frac{1}{10} \text{ hence } f = +10\,\text{cm}$$

Example 11.16 : Where should a candle of length 3 cm be placed from a lens of focal length 10 cm. So that its 6 cm long image is obtained on a screen placed at appropriate position?

Solution : h = 3 cm; h[1] = -6 cm; f= + 10;

$$m = I = \frac{h_2}{h_1} = \frac{v}{u};\ \frac{-6}{3} = \frac{v}{u};\ v = -2u$$

using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{-2u} - \frac{1}{u} = \frac{1}{10}$$

$$\frac{-1-2}{2u} = \frac{1}{10}$$

$$-\frac{3}{2u} = \frac{1}{10}$$

$$u = \frac{-3\times 10}{2} = -15\,\text{cm}$$

$$v = -2u = -2\times(-15) = 30\,\text{cm}$$

hence $u = -15$ cm

Example 11.17 : Find the focal length of a double convex lens made up of glass whose radius of curvature and refractive index are 20 cm; 30 cm and 1.5 respectively.

Solution : Given $R_1 = +20\,\text{cm}$, $R_2 = -30\,\text{cm}$

$n = 1.5$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-30}\right) = 0.5 \times \left(\frac{30+20}{600}\right)$$

$$\frac{1}{f} = 0.5 \times \frac{50}{600} = \frac{25}{600} = \frac{1}{24}$$

hence $f = +24\,\text{cm}$

Example 11.18 : A lens of refractive index 1.5 has focal length of 0.3 m. What will be its focal length in water?

Solution : Focal length of lens in air is

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{30} = 0.50 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (1)$$

When the lens is in water

$$\frac{1}{f'} = (n_{g\omega} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$n_{g\omega} = \frac{n_g}{n_\omega} = \frac{1.50}{1.33} = 1.1278$$

$$\frac{1}{f'} = (1.278 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f'} = 0.1278 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (2)$$

deviding equation (1) by eq. (2) we get

$$\frac{f'}{30} = \frac{0.50}{0.1278} = 3.912$$

$$f' = 3.912 \times 30 = 117.36\,\text{cm}$$

11.8 Prism

A homogeneous and transparent medium enclosed by two inclined plane surfaces is called prism. The angle betwee these two inclined planes is called prism angle,and the planes are called refracting planes. The section which is perpendicular to these planes is principle section of the prism.

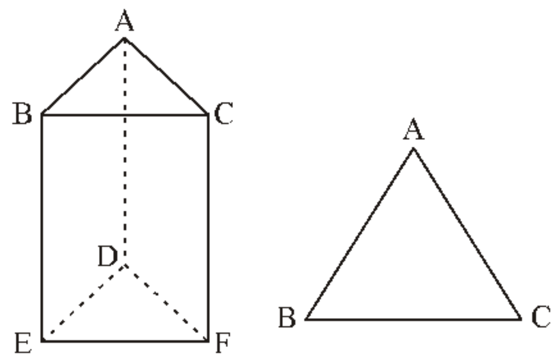


Fig. 11.21 Prism

ABC is the principle section and $\angle$BAC; $\angle$ABC OR $\angle$ACBis prism angle depending upon the refraction planes used. The prism can have any shape including tringular.

11.8.1 Refraction in prism

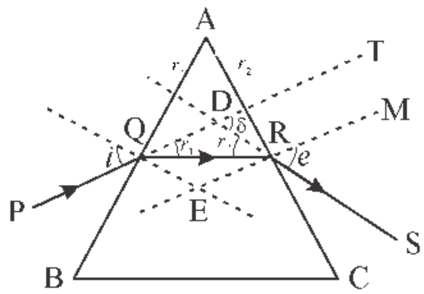


Fig. 11.22 Refraction by prism

In the above figure the planes AB and AC are used as refracting surfaces, hence $\angle$A is taken as prism angle. The rays PQ, QR and RS are incident ray, refracted ray and emergent ray respectively. Similarly angle of incident e, angle of refraction r_1 and r_2 and angle of emergence e are marked in the figure. The deviation of the incident ray caused by prism is given as δ. The refractive index of the material of prism is n.

To find the relation between the deviation angle δ, prism angle A and refractive index n, we make use of few equations from the above diagram.

From quadriangle

AQERA; $\angle A + QER = 180^0$ . . . (11.38 a)

But in Δ QER; $r_1 + r_2 + \angle QER = 180$ . . . (11.38 b)

comparing eq (11.38a) and eq. (11.38 b) we get-

$$A = r_1 + r_2 \quad \ldots (11.39)$$

similarly in Δ QDR; $\delta = \angle DQR + \angle DRQ$

or $\delta = (i - r_1) + (e - r_2)$

$= (i + e) - (r_1 + r_2)$

but from eq. 11.39; $r_1 + r_2 = A$

hence $\delta = i + e - A$ . . . (11.40)

The above relation shows that the angle of deviation depends on angle of incidence i. This variation is shown by the following graph.

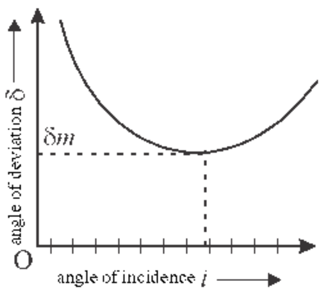


Fig. 11.23 Variation of δ with i for a triangular prism

From the above graph it is clear that the angle of deviation is minimum for a particular angle of incidence. In this condition of minimum deviation, $\delta = \delta_m$, the refracted ray QR is parallel to base of prism BC. Hence from geometry $i = e$ and $r_1 = r_2 = \frac{A}{2}$

Hence when the prism is in the position of minimum deviation, it obey

$$\delta_m = 2i - A \;; \quad i = \frac{A + \delta_m}{2} \text{ and } r = A/2$$

From Snell's law we get

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} \quad \ldots (11.41)$$

Practically we use this relation to find the refractive index of the material of the prism.

For small prism angle (expressed in radian) $(\sin\theta \approx \theta)$; the relation betwen A, n and δ_m can be expressed as

$$\delta_m = (\mu - 1)A \quad \ldots (11.42)$$

It shows that for a thin prism, deviation is independent of angle of incident i.

Example 11.19 : A thin prism of refractive index $n = \sqrt{3}$ has prism angle $A = \delta_m$. Find prism angle.

Solution : Given $n = \sqrt{3}$ and $A = \delta_m$

$$n = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin\frac{A}{2}}$$

$$= \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\sqrt{3} = 2\cos\frac{A}{2} \Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{A}{2} = 30^\circ \;;\; A = 60^\circ$$

Example11.20 : A ray is incident on a prism of small prism angle A, and emerges perpendicular to other surface. Refractive index of the material of prism is n. Find angle of incidence.

Solution : Given $r_2 = 0$; but $r_1 + r_2 = A$, hence $r_1 = A$

From Snell's law $n = \frac{\sin i}{\sin r_1} = \frac{i}{r_1}$;

$i = n\, r_1$ (for small angle in radian) hence $i = nA$ radian

11.8.2 Dispersion of Light

Splitting of light into its constituant colours is called dispersion. We have learnt that the refractive index also depends on the wave length of light passing through it. Refractive index of a material is maximum for violet light and minimum for red light, *i.e.* $n_v > n_r$.

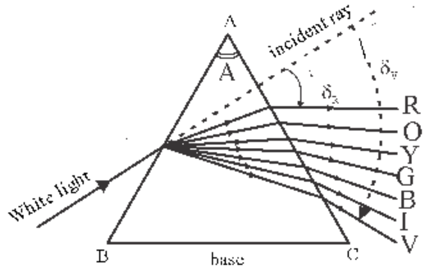


Fig. 11.24 : Dispersion by prism

White light consists of seven broad categories of colours known as VIBGYOR (for violet,indigo,blue, green,yellow, orange and red). The wavelengths of these categories is in increasing order and the refractive index of the material for these categories is in decreasing order.

From $\delta_m = A(n-1)$ it is clear that different colours deviate differently according to n for that colour. $\delta_v = A(n_v - 1)$ is maximum while the deviation of red $\delta_R = A(n_r - 1)$ is minimum. The angle between these extreme ends is known as angle of dispersion. *i.e.*

$$\theta = \delta_v - \delta_R \qquad \ldots (11.43)$$

The pattern consisting of constituant colours in the given order is called spectrum.

The ratio of dispersion angle and the angle of deviation for yellow light is called dispessive power ω of the material of the prism.

$$w = \frac{q}{d_Y} = \frac{d_V - d_R}{d_Y} = \frac{n_v - n_r}{n_Y - 1} \qquad \ldots (11.44)$$

It is evident from above relation that dispersive power is independent of prism angle.

Example11.21: Refractive index of the material of prism for red and blue colours are 1.58 and 1.60 respectively. If the prism angle is 2^0. Find the deviation of two colours and dispersion angle.

Solution : Given $n_R = 1.58$, $n_B = 1.60$ and $A = 2^o$

Deviation of red is

$$\delta_R = (n_R - 1)A = (1.58 - 1) \times 2 = 1.16^o$$

Deviation of blue is

$$\delta_B = (n_B - 1)A = (1.60 - 1) \times 2^o = 1.20^o$$

Angle of disperson is

$$\theta = (\delta_B - \delta_R) = (1.20 - 1.16) = 0.04^o$$

Example 11.22 : The refractive index of crown glass for red and violet colours are 1.514 and 1.523 repectively. Find dispersion angle by the prism of prism angle 6^0.

Solution : Given

$$n_R = 1.514,\ n_v = 1.523 \text{ and } A = 6^o$$

$$\theta = (n_v - n_R)A$$

$$= (1.523 - 1.514) \times 6^o = 0.009 \times 6^o = 0.054^o$$

11.9 Scattering of Light

When a beam of light fall on the particles of atmosphere (gases and other suspended particles) it spreads in all directions. This phenomenon is called scattering of light. It is not simple reflection, basically the particles first absorbs the incident light and then re-emit in all directions.

Intensity of scattering depends on the incident wavelength and size of the particles. If the particle size is smaller than wavelength, the scattering is proportional to $1/\lambda^4$. This law is called Rayleigh's law. For the particles of size a >> λ *i.e.* rain drops, large dust or ice particles; this law is not true and nearly all colours are equally scattered.

11.9.1 Phenomenon Related to Scattering

(i) Blue appearance of sky : When sunlight fall on atmospheric particles,the smaller particles scatters the short wavelength of vissible light and out of these blue colour is most dominent. Hence sky appears blue to the observer on earth. Had there been no atmosphere the sky would have appeared black, and we could see stars during day time.

(ii) Red appearance of sun at sunrise and sunset : During sunset and sunrise the sun is at horizon. The sun rays have to travel a large distance to reach the observer, during this, most part of the shorter wavelength is scattered away and only the longer wavelength remains which reach the observer, hence the red appearance of the sun. This does not happen during day time because the sun is relatively nearer.

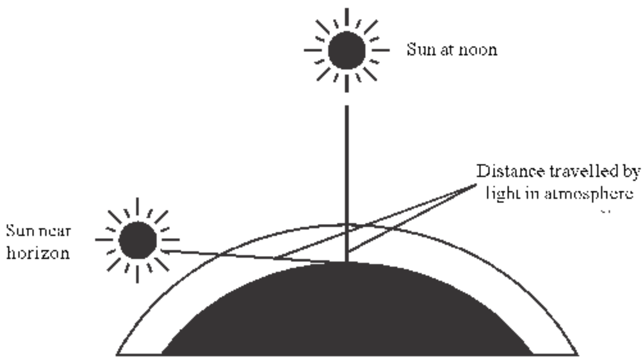


Fig. 11.25Travelling large distance by sun rays at sunrise and sunset

(iii) Red colour (light) : Red light is used for danger signal,because this colour is least scattered by atmospheric particles and the signal is vissible in the most adverse atmospheric conditions.

(iv) White appearance of clouds : Clouds consist of water droplets,water vapours which has the particle size a >> λ; so all the wavelengths of sunlight are equally scattered. Hence the cloud appears white.

11.10 Rainbow

After stoping of rain, sometimes we see an arch shaped band of seven colours in the sky in the direction opposite to the sun. It is called rainbow. The center of the rainbow lies between the sun and the observer. When the order of the colours is from red to violet, it is called primary rainbow. Sometimes we see another rainbow just above the primary rainbow in which colours of the strips are in revers order. It is called secondary rainbow.

When the cloud is exhausted of water vapours. The small water doplets do not grow further to fall down. They remain suspended because the mg force is balanced by bouyant force. When sunlight falls on these tiny droplets, the phenomenon of refraction, dispersion and total internal reflection takes place and rainbow is formed in the sky.

All droplets are not able to form rainbow. Only those droplets contribute to rainbow which lies between angle 40^0 to 42^0 from horizon (because of the conditions of TIR). Only one colour from one droplet reaches our eyes; So billions of droplets contribute to the whole rainbow. For secondary rainbow TIR occurs twice in a single droplet, that's why the order of colours is reversed and it appears faint.

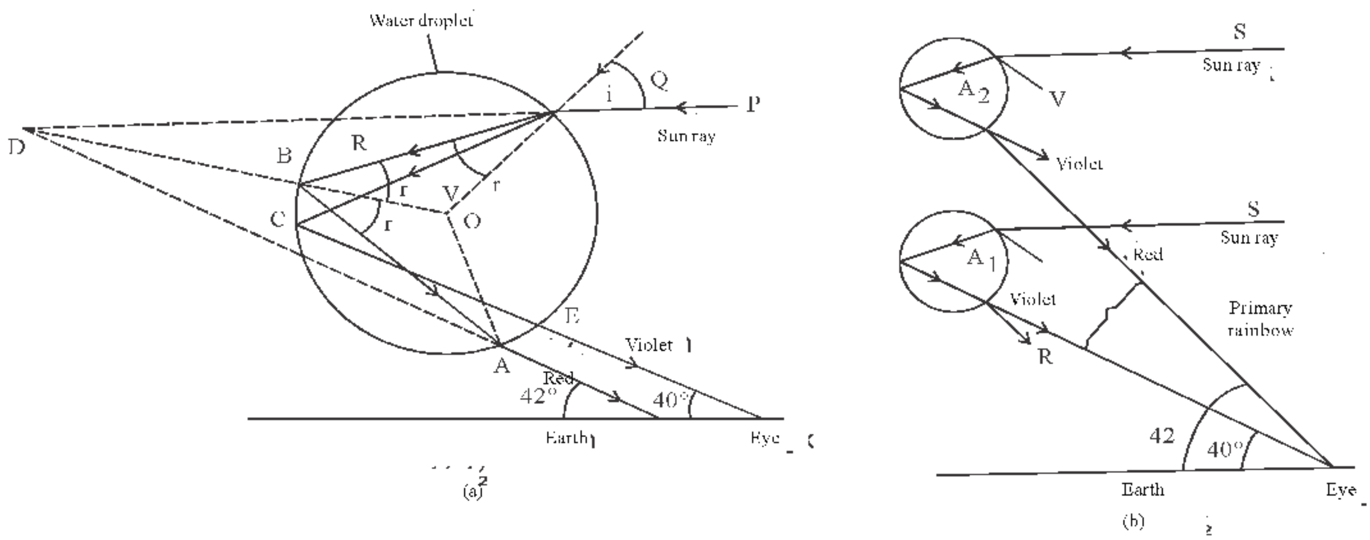


Fig. 11.26 : Formation of rainbow

11.11 Optical Instruments

11.11.1 Human Eye

The shape of our eyes is approximately spherical. As shown in the figure 11.27 (a),the front part of human eye is more sharply curved,which is covered by a transparent layer called cornea. Then an aparture called pupil controlls the amount of light entering our eyes as required. A transparent and fexible convex natural lens is held by cilliary muscles. A flued called aqueous humor is there between lens and cornea. After lens there is another fluid called vitreous humour. The refractive index of both these fluids is aproximately n = 1.336. At the end of the eye ball there is a screen called retina, which consist of photoreceptor cells called rods and cones. These photorecpter cells are connected to optical nerves, which caries the electrical singals to brain.

Cilliary muscles,which hold the lens can change the radius of curvature and focal length of the lens. They controlls the accomodation power of the lens. Lens obey the eq. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$; since the distance between lens and retina v is fixed, we have to adjust the focal length f of the lens, according to the object distance u. This is what the cilliary muscles are doing.

A healthy human eye can see the object placed between 25 cm and infinity from eye. This distance 25 cm is called near point and infinity is far point of heahlty, human eye. For children the near point may be 6-7 cm.

When we see at infinity (clouds and far distant object) cilliary muscles are relaxed,focal length is maximum. The muscles are heighly stressed when we see at near point.

Due to various reasons,such as ageing, weaking of cilliary muscles and hardening of lens,various defects of vission (refractive errors) arises in eyes; the few are :

(i) Presbyopia : The effectivity of cilliary muscles and flexibility of lens reduces with age. In some people the near point which is 25 cm; reaches 200 cm at the age of 60 yrs. This error (defect) is called presbyopia, and can be corrected by using convex lens of appropriate focal length.

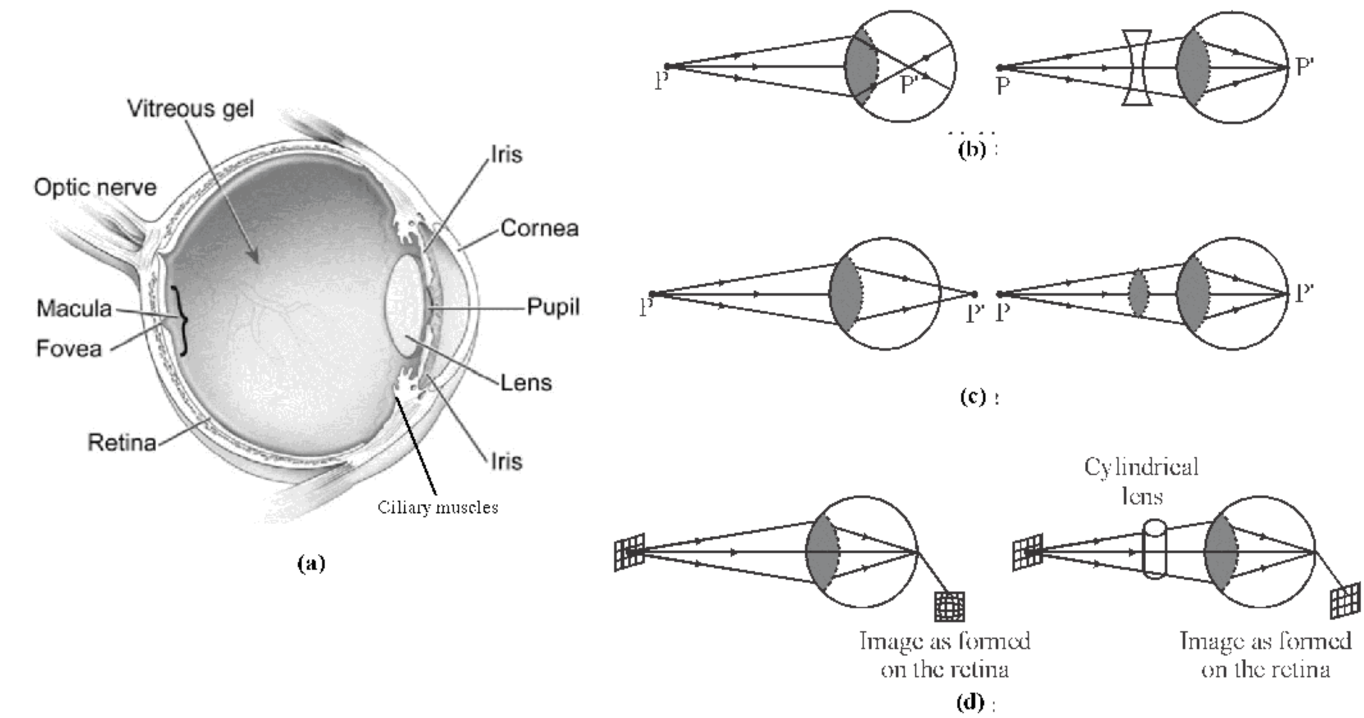


Fig. 11.27 (a) Construction of eye (b) Mopic eye and its correction (c) Hypermetropic eye and its corrction (d) Astigmatic eye and its correction

(ii) Near sightedness or Myopia : The person having this defect can't see clearly,the object placed at infinity. The far point of such eye is no longer infinity. It is somewhere nearer. Shifting of far point towards eye shortens the range of vision, this defect is called myopia. The lens in eye becomes more convergent, and its focal length can't be increased beyond certain limit by cilliary muscles. The clear image of the object at infinity is formed before the retina, the image formed at retina is blurred. A divergent (concave)lens of suitable focal length is used for correction.

(iii) Far sightedness or Hypermetropia : The person with this defect can't see the near objects. The clear image is formed by such eye, beyond retina. A converging/convex lens of suitable focal length is used for correction.

(iv) Astigmatism : In such defect the radius of curvature of the eye lens is different in different planes; same vission is not obtained in all directions. As shown in diagram 11.27 (d); if the object is like a grid, the image may of horizontal/vertical lines or a distorted grid. A cylendrical lens of appropriate focal length and orientation is used for correction.

11.11.1.1 Apparant Size

The size of an object depends on the size of image formed on retina,which depends on the angle subtended by the object on eye. This angle is called visual angle.

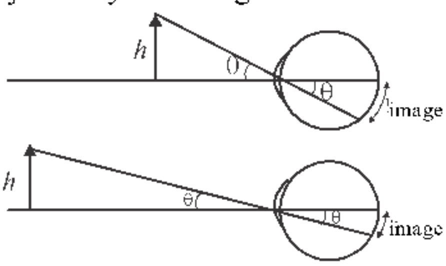


Fig. 11.28 Apparant size of an object

Size of object is h, but in fig. 11.28 (a) appear bigger than in fig. 11.28 (b) . In optical instruments this angle can be increased to get bigger and clear image.

11.11.2 Microscope

The visual angle formed by a very small object on eye is very small, so we can't see the object clearly. If an optical instrument can make its image bigger (magnified),the visual angle is increased and the object seems to be bigger and clearer. Such device is called micorscope.

11.11.2.1 Simple Microscope

It is also called magnifying lens or simply magnifier. It is actually a convex lens of small focal length. The object is placed between focus and the pole. The image formed is magnified, erect,virtual and in the same side between object and infinity.

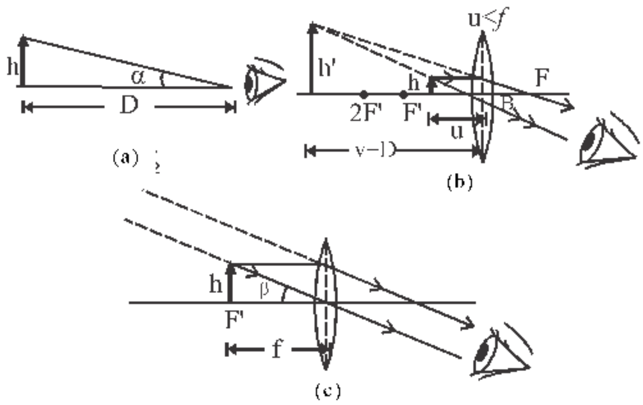


Fig. 11.29 Simple microscope

Magnification is given by

$$M = \frac{\text{Visual angle formed by image on eye}}{\text{Visual angle formed by object on eye}} = \frac{b}{a}$$

(When placed at same distance)

From fig. 1.29 (b) and (a)

$$\beta = \frac{h'}{v} = \frac{h}{u} \text{ and } \alpha = \frac{h}{D}$$

Because D = minimum distance for clear vission.

So magnification $M = \dfrac{h/u}{h/D} = \dfrac{D}{u}$

Normally there are two conditions in this case -

(i) If the image is at D (Near point of normal eye). Fig. 11.29 (a) then $v = -D$ and $u = -u$

from lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$;

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}; \quad -1 + \frac{D}{u} = \frac{D}{f}$$

So $M = \dfrac{D}{u} = 1 + \dfrac{D}{f}$ ... (11.46)

In this case since the image is formed at D,the eye is in most stressed condition.

(ii) If the image is formed at infinity (far point of normal eye)

In this case $v = -\infty$, So $\frac{1}{-\infty} - \frac{1}{-u} = \frac{1}{f}$

here $u = f$,

$$M = \frac{D}{u} = \frac{D}{f} \qquad \ldots (11.47)$$

In the second case although the magnification is less by a factor 1 comparing the first case; but the eye of the observer remain relaxed or unstrained.

11.11.2.2 Compound Microscope

A practical simple microscope has small magnification (<10). To increase magnification we use two lenses. Such micorscope is called compound micorscope. Both lenses are coaxial. The lens which is towards the object is called objective lens or field lens, it is denoted by L_o. The other lens which is towards eye is called eye-piece or ocular, it is denoted by L_E. Their focal lengths are taken as f_0 and f_e respectively. Aparture and focal length of objective lens are smaller compared to that of eye-piece. Smaller apparture reduces sperical abration while small focal length increases magnification.

Formation of image (ray diagram) fig. 11.30 shows the ray diagram of a compound microscope.

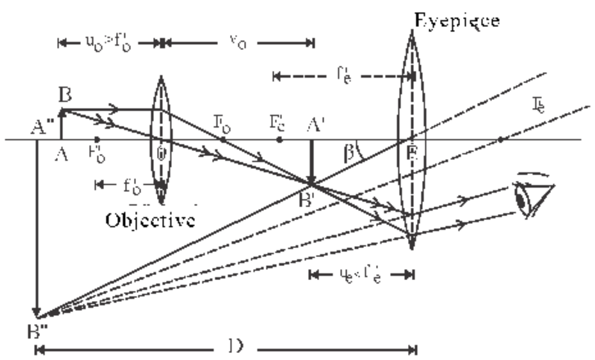


Fig.11.30 Compound microscope

An object AB is placed between F_0' and 2 F_0' of the objective lens. $u_0 > f'_0 (\approx f_0)$ fig. 11.30. Objective lens forms image A'B' on other side between 2F and infinity at a distance v_0. The image A'B' situated between eye-piece and its first focus, behaves like a virtual object, $\left[u_e < f'_e (\approx f_e)\right]$. The image formed by eye-piece A"B" is virtual, erect (comparing A'B') and very large. The final image A"B" of the object AB is very large, virtual and inverted (in comparison to AB). A"B" may be situated anywhere between D and infinity.

Magnifying power: Angular magnification of a compound microscope is given by -

$$M = \frac{\text{Visual angle formed by image on eye}}{\text{Visual angle formed by object on eye (when object is directly seen)}} = \frac{b}{a}$$

Because eye is very near to eye - piece, we take visual angle formed by A"B" on eye-piece as the visual angle formed on eye. Since object is small, α and β are very small we can approximate

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad \text{(for small angle } \theta \approx \tan\theta\text{)}$$

$$= \frac{\frac{A'B'}{EA'}}{\frac{AB}{D}} = \frac{A'B'}{AB}\left(\frac{D}{EA'}\right)$$

(Max. visual angle formed by obejct on eye is $\alpha = \frac{AB}{D}$)

If the object distance and image distance for objective lens are u_0 and v_0; using sign convention we get

$$M = \frac{v_0}{-u_0}\left(\frac{-D}{-u_e}\right) = -\frac{v_0}{u_0}\left(\frac{D}{u_e}\right)$$

$$\left[\because \frac{A'B'}{AB} = \frac{+v_0}{-u_0},\ EA' = -u_e \text{ and D is negative}\right]$$

$$\ldots (11.48)$$

There are two situation in this case

(i) When the final image is formed at D (near point of normal eye).

$v_e = -D$, hence for eye-piece

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \text{ or } \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$\frac{D}{u_e} = \frac{D}{f_e} + 1 \text{ and } u_e = \frac{f_e D}{f_e + D}$$

Substituting these values in equation 11.43 we get

$$M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) \qquad \ldots(11.49)$$

(ii) When final image is formed at infinity

here $v_e = -\infty$, so for eye-piece

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{here } u_e = f_e \text{(maximum value)}$$

other wire $u_e < f_e$

$$\text{hence } M = -\frac{v_0}{u_0}\left(\frac{D}{f_e}\right) \qquad \ldots(11.50)$$

Again the magnification in second case less by a factor 1,but the eye of the observer remain relaxed or unstrained as compared to the first case.

11.11.3 Astronomical Telescope

The distant objects like aeroplane, planets, stars appears us very small, because they subtain small visual angle on our eyes. With help of proper lenses we obtain their image near our eyes, which produces large visual angle on eyes and the object appears enlarged.

11.11.3.1 Refracting Astonomical Telescope

Construction of such telescope is shown in fig. 11.31.

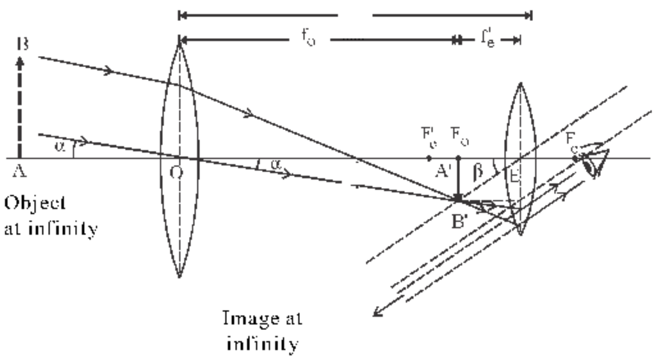


Fig. 11.31 Refracting astonomical telescope

If consists of two acromatic coaxial lenses whose principle axis coinside. A metallic pipe is fitted with a lens of large focal length and large apparture at one end. On the other end there is another small pipe which can move inside the larger pipe with help of rack and pinion arrangement. The smaller pipe is fitted with an eye-piece of smaller focal length.

The objective lens of focal length f_0 produces an inverted, real and bright image A'B' at its second focus,of the distant object, AB.The image A'B' acts as a real object for acromatic eye-piece of small focal length,which produces a magnified in image A"B" at infinity this image is inverted with respect to the object AB. The eye of the observer is relaxed (unstrained). For such telescope -

$$M = -\frac{f_0}{f_e} \text{ and } L = f_0 + f_e \text{ ; L = length of the tube}$$

11.11.3.2 Reflecting type of telescope

To obtain a bright image of a distant object,and to increase resolving power of refracting telescope, we have to use a lens of larger apparture. Obtaining such a lens is difficult. Moreover such lens creats so many problems, mechnical as well as optical such as spherical abration of the image.

To avoid all these problems reflecting telescope is used.Instead of lens a parabolic mirror of larger apparture is used intead of objective lens.

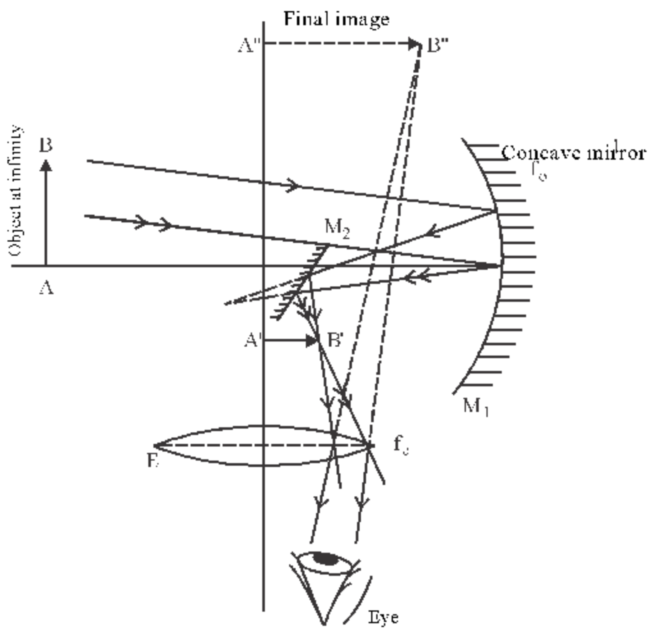


Fig.11.32 A reflecting telescope

As shown in fig 11.32 a parabolic mirror of large apparture is fitted at the end of a large tube. A plain mirror M_2 is fitted in the tube, inclined at 45^0. There is another smaller tube,which is fitted with an acromatic lens of smaller focal length, which works as eye-piece.

The incident parallel rays coming from distant object fall on parabolic (or spherical concave) mirror. The reflected rays fall on mirror M_2, and after reflection from M_2 these rays are recieved by the lens of eye-piece. We get a final magnified image A"B". Actually mirror M_2 form a real image A'B' of the object,this image act as a real object for eye-piece, which produces final virtual image A"B".

Example11.23 : Far point of a person is 5 m which statement is correct about the vission of the person?

(a) He suffers from hypermatropia,and needs a convex lens for correction.

(b) He suffers from hypermatropia, and needs a convex lens for correction.

(c) He suffers from myopia and needs a convex lens for correction.

(d) He suffers from far sightedness and needs a convex lens for correction.

Solution : Far point is infinity for a normal eye. For far point of the above person is 5 m, hence he suffers from myopia. For correction of myopia a concave lens of appropriate focal length is used.

Example 11.24 : An astronomical telescope is to be fabricated for a magnification of 50. If the length of the tube is 102 cm, find powers of objective and eye-piece.

Solution : Given $m = 50 = \frac{f_0}{f_e}$;

$$f_0 = 50 f_e \qquad \ldots \text{(i)}$$

also $f_0 + f_e = L = 102\,\text{cm} \qquad \ldots \text{(ii)}$

From these relations we get

$f_o = 100$ cm and $f_e = 2$ cm

Hence $P_o = 1$ D and $P_e = 50$ D

Example 11.25 : Magnification of a simple microscope is 11. The image is formed at the near point of clear vission. Find focal length of the lens.

Solution : Given M = 11; D = 25 cm

for a simple microscope when image is at D

$$M = 1 + \frac{D}{f} \text{ OR } 11 = 1 + \frac{25}{f} \text{ solving we get}$$

$$f = 2.5\,\text{cm}$$

Example 11.26 : Magnification of a telescope is 9. When it is set for parallel rays, the distance between objective and eye-piece is 20cm. Find the focal length of the two lenses.

Solution : As given,when the telescope is set for parallel rays, the final image is formed at infinity and magnification

$$M = -\frac{f_0}{f_e} = -9\ ;\ f_0 = -9 f_e\ ;$$

since $L = f_0 + f_e = 20$ (given)

$$9 f_e + f_e = 20\ ;\ f_e = 2\,\text{cm}\ ;$$

$L = f_0 + f_e = 20$ from we get

$f_o = 18$ cm

Important Points

1. In ray optics, the path of light is taken as straight line.
2. Laws of reflection

   (i) $\angle i = \angle r$

   (ii) The incident ray, reflected ray and the normal lie on the same incident plane.
3. Spherical mirror. Focus and focal length - for a concave mirror it is the point where incident rays parallel to principle axis meet after reflection. For convex mirror it is the point from where they seem to be originating after reflection. The distance between focus and pole is called focal length. Cartseian sign convetion - (1) All the distances are measured from pole. (2) The distance measured in the direction of incident rays is taken as positive and vise-versa. The distance upwards from principle axis is positive and vice-versa.

   Shperical mirror obey the following relation

   $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ and } f=\frac{R}{2}$$
4. Refraction - Bending of rays at the interface of two media when ray goes from one medium to another is called refraction.

   (i) Incident ray, refracted ray and the normal lie in the same plane.

   (ii) $\frac{\sin i}{\sin r}=\frac{n_2}{n_1}$ or $n_1 \sin i = n_2 \sin r$ (Snell's law)
5. Refractive index of a medium

   $$n=\frac{\text{speed of light in vaccum}}{\text{speed of light in that medium}}=\frac{c}{v}$$
6. Total internal reflection : (1) Angle of incidence $i > \theta_c$, the ray suffers total internal reflection. The critical angle $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ (2) They should go from the denser to rarer medium.
7. Refraction through prism - A prism deviates a ray of light passing through it. The angle of deviation remain same even if $\angle i$ and $\angle e$ are exchanged. For $\delta = \delta_m$; $i = e$, $r_1 = r_2 = A/2$ and

   $$n=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)}$$

   for small (in radian) $\delta_m = (n-1)A$
8. Refraction from a spherical surface. Shperical interface of the two media obey the relation

   $$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$
9. For thin lens : There are two focii, the second focus is that where the incident rays mparallel to prinicple axiseet after refraction. The rays originating from first focus go parallel after refraction. For a thin lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{ or } v=\frac{uf}{u+f}$$

and $$\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

The linear magnification by a lens is given by -

$$m=\frac{\text{height of image}}{\text{height of object}}=\frac{h'}{h}=\frac{v}{u}=\frac{f}{u+f}$$

10. Power of lens- It is ability to deviate the incident ray. It is given by $P=\frac{1}{f}$; (f in metre) and its unit is (dioptre).

The resultant focal length of combination of lenses is given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots+\frac{1}{f_n}$ and resultant power $P=P_1+P_2+P_3+\ldots P_n$ and the net magnification $m=m_1\times m_2\times m_3\times\ldots\times m_n$

11. Dispersion by a prism - dividing of light into its consistuant colours is called dispersion. White light breaks up into seven colours (VIBGYOR). The angle between two extream colours is called angle of dispersion dispersion $\theta=\phi_v-\delta_R=(\mu_v-\mu_R)A$

$$\omega=\frac{\theta}{\delta}=\frac{\delta_v-\delta_R}{\delta_y}=\frac{n_v-n_R}{n_y-1}$$

$$\omega=\frac{(n_v-n_R)}{(n-1)}$$

12. Microscope : It is a device which gives magnified image of small object.

(i) Simple microscope (ii) Compound microscope

(1) For simple microscope (i) $M=1+\frac{D}{f}$ (when image is formed at D)

(ii) $M=\frac{D}{f}$ (when image is formed at infinity)

(2) For compound microscope

(i) $M=-\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$ (when image is at D)

(ii) $M=-\frac{v_0}{u_0}\times\frac{D}{f_e}$ (when image is at infinity)

13. Telescope - device use to see distant object resolved.

Magnification $M = -\frac{f_0}{f_e}$ when image is at infinity.

Questions for Practice

Multiple Choice Questions -

1. Only paraxial rays are considered for the formation of image in spherical mirrors because -
   (a) It is geometrically easy to use
   (b) Most of the intensity lies in them
   (c) They form point image of a point object
   (d) They show minimum dispersion

2. An object is placed at 30 cm from the concave mirror of focal length 20 cm, the nature of image and magnification will be -
   (a) Real and -2 (b) Virtual and - 2
   (c) Real and + 2 (d) Virtual and + 2

3. The reflective index for infrared rays is -
   (a) Equal to that for UV rays
   (b) Equal to that for red light
   (c) Less than that for UV rays
   (d) More than that for UV rays

4. Total internal reflection will occur if -
   (a) They ray enters denser medium from a rarer medium and $i > i_c$
   (b) The ray enters rarer medium from a denser medium and $i > i_c$
   (c) The refractive index of two media are nearly same
   (d) The refractive index of two media are different but $i > i_c$

5. An object is placed at a distance of 20 cm from a concave lens, the image formed is small which statement is certainly correct?
   (a) Image is inverted
   (b) Image may be real
   (c) Image distance is > 20 cm
   (d) Focal length of the lens may be < 20 cm

6. A convex lens of + 6 D is kept in contact with a concave lens of power - 4 D. The focal length and nature of combination lens will be -
   (a) 25 cm, concave (b) 50 cm, convex
   (c) 20 cm, concave (d) 100 cm convex

7. A ray passes through an equilateral glass prism such that $i = e$, which is 3/4 of the prism angle. Then deviation will be -
   (a) 45^0 (b) 70^0
   (c) 39^0 (d) 30^0

8. The image formed by a compound microscope will be -
   (a) Virtual and magnified (b) Virtual and small
   (c) Real point image (d) Real and magnified

9. A double convex lens of refractive index 1.47 is immerged in a liquid. It behave like a plane sheet. It means the refractive index of the liquid is -
   (a) Greater than refractive of glass
   (b) Less than the refractive index of glass
   (c) Equal to the refractive index of glass
   (d) Less than 1

10. The angle of minimum deviation for a prism would be equal to prism angle if the refractive of the medium of prism is -
    (a) Between $\sqrt{2}$ and 2 (b) Less than 1
    (c) More than 2 (d) Between $\sqrt{2}$ and 1

11. A ray of light falls normal to a plane mirror, the reflection angle will be -
    (a) 90^0 (b) 180^0
    (c) 0^0 (d) 45^0

12. An object is placed at 20 cm from a concave mirror of focal length 20 cm, its image will be formed at -

(a) $2f$ (b) f

(c) 0 (d) Infinity

13. As observed from earth, stars seems to be twinkling, the reason is -

(a) It is true that stars do not emit light continuously

(b) Absorption of frequency by the atmosphere of star itself

(c) Absorption of frequency by earth atmosphere

(d) Variation in the refractive index of earth atmosphere

14. If the yellow light is reflected by a prism at angle of minimum deviation then -

(a) $\angle i = \angle e$ (b) $i + e = 90^0$

(c) $i < e$ (d) $i > e$

15. The minimum and maximum distance for clear vission for normal eye is -

(a) 25 cm and 100 cm

(b) 25 cm and infinity

(c) 100 cm and infinity

(d) Zero and from zero to infinity

16. The length of a normal astronomical telescope is -

(a) Equal to the difference in focal length of the two lenses

(b) Half of the sum of the focal lengths

(c) Equal to sum of focal lengths

(d) Equal to the product of focal lengths

17. A virtual and magnified image can be obtained by -

(a) A convex mirror (b) Concave mirror

(c) Plane mirror (d) Concave lens

18. Final image obtained from a compound microscope is -

(a) Real and erect (b) Virtual and inverted

(c) Virtual and erect (d) Real and inverted

19. The objective used in reflecting telescope is -

(a) Convex lens (b) Convex mirror

(c) Prism (d) Concave mirror

20. The power of objective and eye-piece is 5 and 20 diaptor respectively and the image is formed at infinity. Magnifiying power of the telescope will be-

(a) 4 (b) 2

(c) 100 (d) 0.25

21. Power of a convex lens is -

(a) Negative (b) Positive

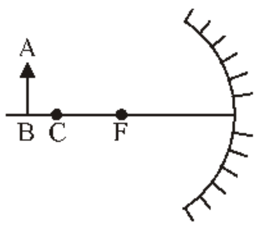
(c) Zero (d) Imaginary

Very Short Answer Questions -

1. Wha tis the focal length of a plane mirror?
2. Which lens has a magnification less than 1?
3. What is cause of refraction?
4. What is the cause of mirage seen in desert in summer?
5. For equal angle of incidence, refraction angle for these media. A, B and C are 15^0, 25^0 and 35^0 respectively. In which medium speed of light is minimum?
6. Write the working principle of an opitcal fibre.
7. What is relation between i and e when prism is in the position of minimum deviation?
8. Two lenses convex and concave of equal focal length are coaxially in contact. What will be the resultant focal length?
9. Why sun appears redish at sunrise and sunset?
10. What is the cause of rainbow?
11. What is myopia? Which lens is used for its correction?
12. On what factor, the intensity of scattered light depends?
13. Which type lens is used in a simple mirocscope?
14. How can you differentiate between a compound microscope and a telescope just by looking at it?

Short Answer Type Questions -

1. An object AB is placed is front of a concave mirror as shown in the given diagram -



(i) Draw ray diagram to form image.

(ii) If half of the apparture of mirror is blankend, how the position and intensity of image is effected.

2. Write uses of spherical mirrors.

3. Establish relation between focal length and radius of curvature for a spherical mirror.

4. (i) Why the sun appears redish at sun rise and sunset?

   (ii) For which colour, the refractive index of a medium is minimum and maximum?

5. (i) What is the relation between critical angle and refractive index of a medium?

   (ii) Do the critical angle depends on the colour of light? Explain.

6. On what factors the focal length of a lens depends?

7. How we can increase the magnifying power of a compound micorscope?

8. What do you understand by scattering of light? Write down its uses in daily life?

9. Define power of a lens and write down its unit. For two coaxially contacted lenses, derive the redeation -

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Esaay Type Questions -

1. Define a spherical mirror. Find relation between object distance, image distance and focal length for it.

2. Explain formation of images by convex and concave lenses for different object positive. Show the size, position and nature of image by ray diagrams.

3. What types of lenses are there? Establish relation between object distance, image distance and focal length of a lens.

4. Find relation between u, v and R for a convex refracting surface, when light enters a denser medium from rarer medium.

5. Draw a labled ray diagram of a compound micorscope showing image formation at near point of normal eye.

6. Draw a ray diagram for a monochromatic ray refracting from a glass prism. Write the expression for refractive index of glass in terms of prism angle and angle of minimum deviation.

7. Find relation between u, v and f for a lens assuming it to be surrounding by two spherical surfaces.

8. How many types of telescope are there? Obtain an expression for magnifying power of a refracting telescope, explaining its construction and working.

Answer MCQ

1. (c) 2. (a) 3. (b) 4. (b) 5. (d) 6. (b) 7. (d)
8. (d) 9. (c) 10. (a) 11. (c) 12. (d) 13. (d)
14. (A) 15. (b) 16. (c) 17. (b) 18. (b)
19. (d) 20. (a) 21. (b)

Very short answer type questions -

(1) Infinity (2) Concave

(3) Speed of light is different in different media

(4) Total internal reflection

(7) $\angle i = \angle e$

(9) Scattering (10) Dispersion

(12) Wavelength

(13) Convex of small focal length

(14) In compound micorscope apparture of the objective is smaller then eye-piece where in telescope apparture of objective is larger than its eye-piece.

Numberical Questions

1. An object is placed at a distance of 24 cm from a concave mirror of focal length 36 cm. Find image distance.

   (Ans : 72 cm, towards object)

2. Find the speed of ligth in a medium of refrative index 1.33 speed of light in air is $c = 3 \times 10^8$ m/s.

   (Ans : 2.25×10^8 m/s)

3. Radius of curvature of the two surfaces of convex lens of focal length 20 cm are 18 cm and 24 cm respectively. Find refractive index of the material of the lens.

(Ans : 1.514)

4. A ray of light is incident at an angle 50^0 on a glass slab. If the angle of refraction is 30^0. Find refractive index of glass.

(Ans : 1.532)

5. An object is placed at distance of 0.06 m from convex lens of focal length 0.10 m. Find the position of image.

(Ans : 15 m)

6. Find angular dispersion due to glass prism of refrating angle 6^0. Refractive index for red and violet light is given as 1.514 and 1.523 respectively.

(Ans : 0.054^0)

7. Find the resultant power of the combination of two lenses of power + 5 D and - 7 D will the combination be converging or diverging.

(Ans : - 2 D, diverging)

8. Focal lengths of objective and eye-piece of a compound microscope are 0.95 cm and 5 cm, and they are 20 cm appart from each other. Find magnification of micorscope, if the image is formed at 25 cm away eye-piece.

(Ans : 94)

9. A thin convex lens of glass ($n_g = 1.5$) has power of + 5.0 D. When this lens is placed in a liquid of refractive index n_e, its behave like a concave lens of focal length 100 cm. Find value of n_l.

(Ans : $n_e = 5/3$)

10. Find the angle of minimum deviation for a prism of refracting angle A and refractive index ($\cot A/2$).

(Ans : 180^0+2A)

Chapter - 12

Nature of Light

12.1 Nature of light

What is light? The question was of great concern for philophers and scientists for a long time. Descartes in 1637 gave particle (corpuscular) model of light, according to which light consist of tiny particles orignating from the source of light. This model was further developed by Isaac Newton. Newton was able to explain some optical phenomen, such as rectiliuar propogation of light, reflection and dispersion. In explaining refraction of light, Newton attributed imaginary properties to the refracting medium. He conculded that the speed of light in denser medium should be greater than the speed of light in the rarer medium. But the experiment in this regard by Foucault in 1850 proved converse of it.

In 1678, the Dutch physicist Christian Huyens a contemporay of Newton gave wave theory of light. This wave theory of light is the matter of concers to us in this chapter. The wave theory clearly expalined many optical phenomenon and moreover it was in agreement with Foucault's experimental results that speed of light in denser medium should be less than the speed of light in rarer medium. Even than the theory was not readily accepted, because of Newton's authority, and also that a wave always requires a medium for propogative, while light can travel in vaccum.

The interference experiment by Thomas Young in 1801 firmly established wave theory of light. The wave length of light was measured and the wave length of yellow light was found to be = 0.5 µm. Because of smallness of wave length, rectilinear propogation of light was asumed. After the experiment of Thomes Young, many experiments involing interference and diffraction were carried out which firmly established wave nature of light.

The only difficulty about the requirment of medium for propogation of light was removed by the elctromagnetic wave theory of light given by Maxwell. According to this light is an electro magnetic wave, and electromagnetic wave can travel in the vacuum.

12. 2 Huygen's Wave Theory and Wavefront

It was the Huygen who proposed the wave theory of light. Before using wave theory to explain the optical phenomenon let us first know some basic properties of a wavefront. According to wave theory, we first define a wavefront. If we drop a stone in still water, we see that circular rings spread outwards from the center (the point where stone hits the surface). We see that all the points on a ring oscillate in the same phase we can consider the snapshot of the ring as a wavefront oscillate in the same, phase and produces secondary wavelets.

A wave front can be defined as a locus of points which oscillate in the same phase. It is a surface of constant phase. All the points on a wavefront are source of secondary wavelets. The direction in which the wavefront travels is called a ray. Ray and wave front are always perpendicular to each other.

Let us discuss, the source and shape of wave front. If the source is a point source, than the shape of the wavefront will be spherical, as the medium is asumed to be homogenous (isotropic) and speed of light is same in all directions fig. 12 (a). If the source is a line source (a linear), the shape of the wave front will be cylendrical as shown in fig. 12.1 (b). The source is at a great distance, the rays are approimately parallel because the radius of curvature becomes very high and the wavefront is practically plane. Shown in fig. 12.1 (c).

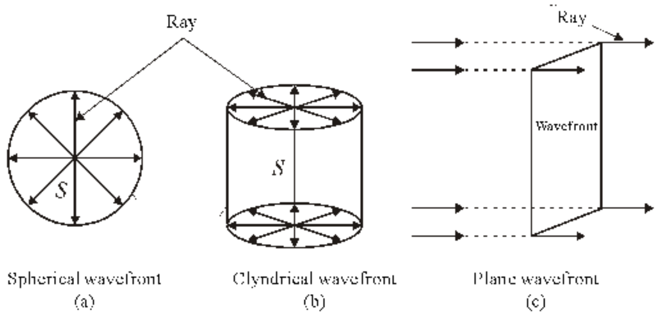


Fig. 12.1 (a) Spherical (b) Clyndcrical (c) Plane wavefront

To show the propagation of a wavefront, let us consider a part AB of a wave front. According to wave theory all the points on this wave front behave as secondary sources and produce secondary wavelets. If C is the speed of light, then $C\Delta t$ will be the radius of the secondary wavelet in time Δt. The envelops of all those secondary wavelets is the position of wave front at that instant. Fig. 12.2 B and B' shows the position of the wavefront at an instant t and $t + \Delta t$. For spherical and plane wavefront. The backwards motion was logically rejected by Fresnel, later on.

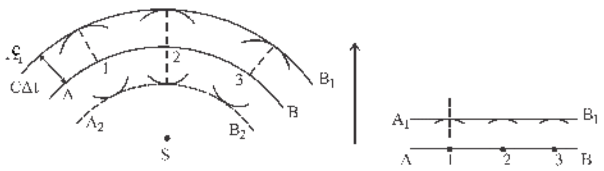


Fig. 12.2

12.3 Reflection and refraction at a plane surface

In the previous section we defined the construction and propagation of a wavefront based Huygen's principle. We will prove the experimental optical phenomenon of reflection and refraction and the laws govering them using Huygens theory.

12.3.1 Reflection at a plane surface

Fig. 12.3 shows MN as a reflecting surface, having a medium above it. Speed of wave is υ in this medium. AB is the incident wavefront and rays 1 and 2 are corresponding waves to it. All the points on the incident wavefront emits secondary wavelets. When the secondary wavelet from point B reaches the surface at C in time Δt ($BC = v\Delta t$), the secondary wavelet after reflection from A reaches point E. As the wavefront move forwards all the secondary wavelets on wave front AB get reflected and reach CE. Such that $AE = v\Delta t$.

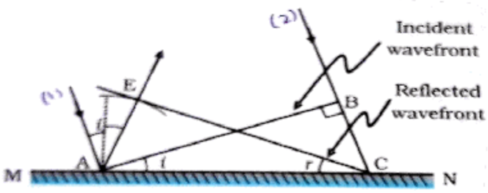


Fig. 12.3 : Reflection of the plane wave by a plane surface

Consider the Δ ABC and Δ AEC ; (i) AE = BC = $v\Delta t$, AC is common, and ∠AEC = ∠ABC = 90 (ray is always perpendicular to wavefront). The two triangles are congruant hence ∠i = ∠r. This is the law of reflection. Since incident ray, reflected ray and normal all lie in the same plane (plane of paper) II law of reflection is also varified.

12.3.2 Refraction at a Plane Surface

First we consider reflection from a rarer medium to a denser medium. Fig. 12.4 shows MN as an interface of two media velocity of the wave in medium 1 and medium 2 is v_1 and v_2 respectively. AB is the incident wavefront A'A is incident ray. ∠BAC = ∠i. All the points on incident wavefront AB emits secondary waves lets. While the secondary wave lets from A, enters second medium and reaches E (such that $AE = v_2 t$); the secondary wavelet from B still moves in first medium and reaches C (BC = $v_1 t$) during the same time interval t. The envelop of all secondary wavelets is CE (called refracted wavefront) and angle ACE = ∠r.

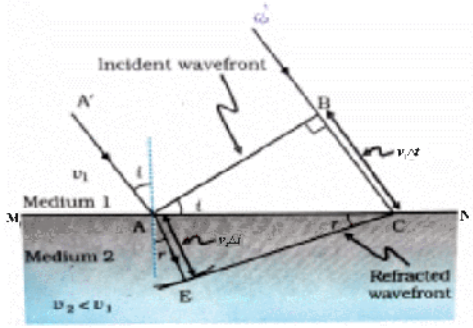


Fig. 12.4 : Refraction from a rarer medium to denser medium

Considering right angles ABC and AEC we get

$$\sin i = \frac{BC}{AC} = \frac{v_1 \Delta t}{AC} \qquad \ldots (12.1)$$

and

$$\sin r = \frac{AD}{AC} = \frac{v_2 \Delta t}{AC} \qquad \ldots (12.2)$$

dividing eq. 12.1 by eq. 12.2 we get

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1}{v_2} \qquad \ldots (12.3)$$

$n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$ (for first and second median respectively)

We get $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$ (where n_{21} is refractive index of medium 2, with respect to medium 1)

$$n_1 \sin i = n_2 \sin r$$

since $n_2 > n_1$; ∠i = ∠r. The refracted ray bends twoards the normal. From the geometry of the figure it is also verified (valid) for incident and refracted rays. The second law is also verified since the incident ray, refracted ray and the normal lie in the same plane as that plane of paper.

If λ_1 and λ_2 are the wave lengths in the media respectively - from $\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{\lambda_1}{\lambda_2}$ hence it is clear that during refraction, only wavelength changes due to change in velocity and frequency remains unchanged.

Now we consider Fig. 12.5 for refraction from denser medium to a rarer medium. From the diagram we see that $\angle i < \angle r$ hence the refracted ray bends away from the normal. In this case we see that

$$n_1 \sin i = n_2 \sin r \text{ and } \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

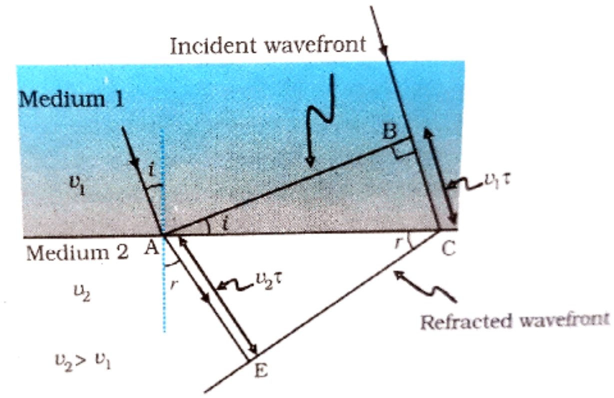
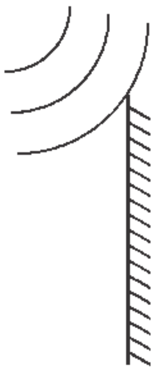


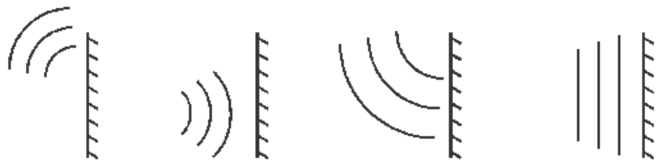
Fig. 12.5 : Refraction from a denser medium to rarer medium

If $r = 90^0$, $n_1 \sin i = n_2 \sin r$; at this angle no refrated ray will be obtained and $i = i_c$ (called critical angle for total internal reflection) hence

$$\sin i = \frac{n_2}{n_1} \sin r$$

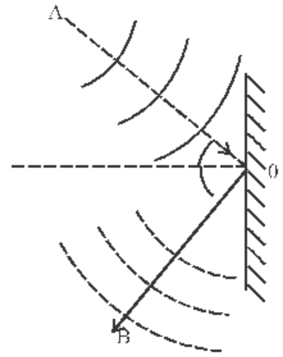
Example 12.1 : A spherical wavefront is incident on a reflecting surface. Which will be the refracted wavefront out of the 4 given.



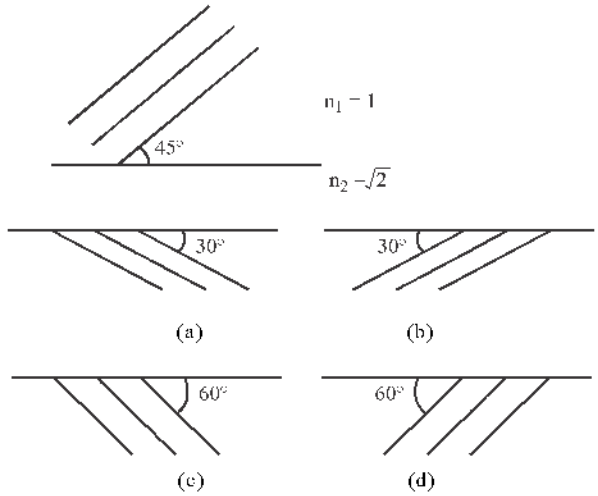


(a) (b) (c) (d)

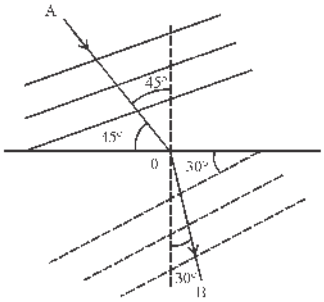
Solution : In a homogenous medium the wavefront is always perpendicular to the ray. In the above problem, take the centre of spherical wavefront as A and draw incident ray AO (perpendicular to wavefront) and normal at O. From geometry draw reflected ray (using $i = r$) OB. Now construct spherical wavefront taking O as a centre, which comes out as given in fig. (c).



Example 12.2 : Fig. shows the incident wavefront at the interface of two media. Which of the given four diagrams, represents the refracted wavefront.



Solution : The incident ray is given by AO. From Snell's law



$n_1 \sin i = n_2 \sin r$

$$1 \sin 45^\circ = \sqrt{2} \sin r$$

$$\frac{1}{\sqrt{2}} = \sqrt{2} \sin r$$

$$\sin r = \frac{1}{2}$$

$$r = 30^\circ$$

hence option (a) is the correct representation of the refracted wavefront.

12.4 Interference of Light and Coherent Sources

When two exactly similar wave meet (superpose) each other in space, the resultant effect on intensity is called interference. This phenomenon is based on principle of supperposition, according to which when two or more waves superpose each other at a point in space, then the resultant amplitude will be vector sum of individual amplitude. You have studied this effect for general waves in class XI.

Now we know that light energy also propagates in the form of waves, hence when two or more light waves superpose in space, the sustained effect of maximum and minimum intensity is observed. This phenomenon is called interference of light. The maximum effect is called constructive, while the minimum effect is called destructive interference.

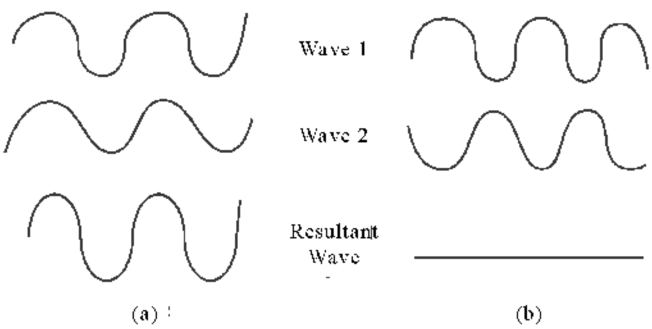


Fig. 12. 6 : To stages of super position of waves of equal amplitude

Fig. 12.6 shows the time dependance of two waves. Fig. 12.6 (a) shows that the two waves reach a point in same phase, i.e. crest of one meet the crest of other and trough of one meets the trough of other. This is possible when phase difference between the two waves is $\phi = 0, 2\pi, 4\pi ... \ 2n\pi$ Radian. It is constructive interference.

In fig. 12.6 (b) the two waves meet each other in opposite phase i.e the crest of one meet the trough of the other. Such type of effect is called destructive effect. For this effect the phase difference will be $\pi, 3\pi, 5\pi ...$ $(2n+1)\pi$ Radian.

The two waves are said to be coherent if their phase difference ϕ is not a function of time, which means ϕ should remain constant with time. This is the important condition for sustained effect, of interference.

Two separate sources like two candles or two bulbs (even of the same frequency operated by single switch) can never be coherent. Because the emission of light is an atomic phenomenon and it can't be synchronised. Emission of light from an atom takes place in 10^{-8} s. The different groups of atoms emitt light during theis period, which can't be synchronised with the emission from other source.

Fig. 12. 7 (a) Wave of finite length
Fig. 12.7 (b) Wave of infinite length

The phase of wave changes randomly and hence the phase difference between two waves also changes randomly.

Hence two independent sources can't produce interference. Intensity distribution is uniform in space.

It is not easy to obtain two coherent sources, unless we obtain two sources from a single source. Our such method is division of wave front into two, which is obtained in Young's double slit experiment.

Now a days coherent sources like LASER source are available which are monochromatic and intense sources. We can obtain interference using two such sources.

12.5 Necessary Conditions for Interference

For clear and sustained effect of interference the following conditions are there -

(1) The two sources should be coherent, i.e. the phase difference between the waves from the sources should remain constant with time.

(2) The two sources should have same freqeuncy.

(3) To obtain a better constrast between the maximum and minimum effect and that of average effect, the amplitude of the two wave should be nearly equal.

(4) Both the waves should move in the same direction and superpose.

(5) The two sources should be very near to each other.

(6) The slits used as a source should be narrow. Other wise a broad source can be treated as group of many point sources, which produce interference effect separately. And what we see is the average effect on screen which is uniform distribution of light and not interference.

(7) The path difference between the two waves should be very small, otherwise the waves from two sources that reach a point simultaneously to superpose, they may reach one after the other. The path difference must not exceed a few centimeters.

(8) If the light from the two sources is polarized, then they should have the same plane of polarization.

12.6 Young's Double Slit Experiment

In the orignal experiment by Thomas Young in 1801, the sunlight was passed through single pin hole and it is allowed to pass through two symetrical holes in a cord board. The interference pattern was obtained on a screen which was in the form of bright and dark areas. Here we will use a monochromatic source of light, a single slit and a double slit. The arrangement is shown in the figure 12.8.

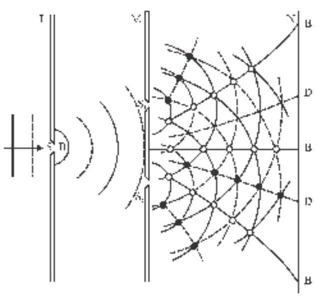


Fig. 12.8 : Young's double slit experiment

As shown in the above figure we have a screen L with a single slit. There is another screen M which has two slits very near to each other. On another screen N where we can see alternate dark and bright bands. The pattern of these bands are called interference and the bands are called dark and bright frings. Shown in fig. 12.9.

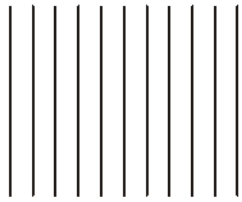


Fig. 12.9 : Interference fringes

A spherical wavefront which originate from a point source i.e. slit s spreads and reaches double slit screen. Since all the points on this wavefront are in same phase, the two slits S_1 and S_2 behave like two coherent sources having same phase. To show the superposition of the two waves in the space ahead of the double slit, we draw the waves in the form of bold line (showing crest) and dotted line (showing trough). The points where crest of one wave meet the crest of other, and where trough of one meet trough of other waves are shown as white circles and black dots indicate the points where a crest meets a trough, in fig. 12.8. The line joining these points are called nodal and antinodal lines. The colour of the bright strips is that of the monochromatic source used in the experiemnt.

12.6.1 Analytical Treatment of Interference

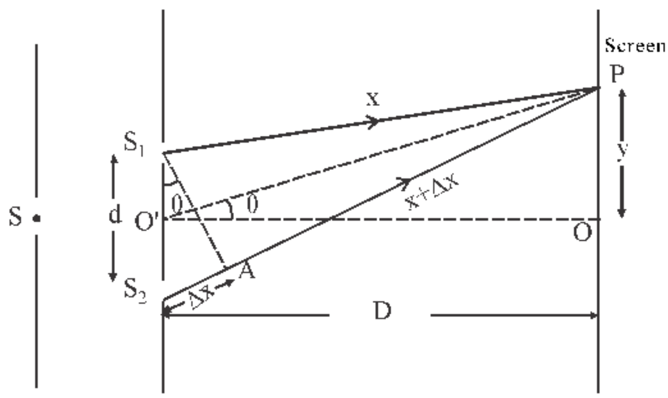


Fig. 12.10 Geometrical construction for anlytical treatment

For the analytical treatment of interference we take the light as an electromagnic wave, and only the electrical field vector E produce the whole visual effect. From fig.

12.10 we see that the light waves from the two sources S_1 and S_2 which reach a point P on the screen has covered two different path lengths x and $x + \Delta x$. The electric field due to these waves at P is E_1 and E_2 respectively given as

$$E_1 = E_{m1} \sin(kx - \omega t) \quad \ldots (12.7)$$

and
$$E_2 = E_{m2} \sin(k(x + \Delta x) - \omega t) \quad \ldots (12.8a)$$

$$= E_{m2} \sin(kx - \omega t + \phi) \quad \ldots (12.8b)$$

where
$$\phi = k\Delta x = \frac{2\pi}{\lambda}(\Delta x) \quad \ldots (12.9)$$

Here ω is angular frequency, λ = wave length and ϕ is the phase difference produced by path difference Δx.

From principle of superposition the resultant electric field at point is given by -

$$E = E_1 + E_2 = E_{m1} \sin(kx - \omega t) + E_{m2} \sin(kx - \omega t + \phi)$$

$$= E_{m1} \sin(kx - \omega t) + E_{m2} \sin(kx - \omega t)\cos\phi$$

$$+ E_{m2} \cos(kx - \omega t)\sin\phi$$

$$= (E_{m1} + E_{m2}\cos\phi)\sin(kx - \omega t) + (E_{m2}\sin\phi)\cos(kx - \omega t) \quad \ldots (12.10)$$

if
$$(E_{m1} + E_{m2}\cos\phi) = E_m \cos\alpha \quad \ldots (12.11)$$

and
$$E_{m2}\sin\phi = E_m \sin\alpha \quad \ldots (12.12)$$

$$E = E_m \left[\sin(kx - \omega t)\cos\alpha + \cos(kx - \omega t)\sin\alpha\right]$$

$$= E_m \sin(kx - \omega t + \alpha)$$

Hence the resultant wave will also be a sine curve of freqeuncy ω, whose amplitude is given by

$$E_m = \sqrt{E_{m1}^2 + E_{m2}^2 + 2E_{m1}E_{m2}\cos\phi} \quad \ldots (12.13)$$

and the phase angle

$$\alpha = \tan^{-1}\frac{E_{m2} + \sin\phi}{E_{m1} + E_{m2}\cos\phi} \quad \ldots (12.14)$$

From definition the intensity is equal to square of the amplitude i.e. $I \propto E_m^2$ or $I = KA^2$ where K is a constant hence the intensity

$$I = K E_m^2 = K\left[E_{m1}^2 + E_{m2}^2 + 2E_{m1}E_{m2}\cos\phi\right]$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \ldots (12.15)$$

where I_1 and I_2 are intensities from the sources S_1 and S_2 and $\frac{I_1}{I_2} = \left(\frac{E_{m1}}{E_{m2}}\right)^2$. From eq. 12.15 it is clear that the resultant intensity at P is different from the sum of individual intensities ($I_1 + I_2$) and depends on ϕ (the phase difference).

The resultant intensity will be maximum when $\cos\phi = 1$ i.e. $\phi = 0, \pm 2\pi, \pm 4\pi$................

or
$$\phi = \pm 2n\pi \quad n = 0, 1, 2 \quad \ldots (12.16)$$

or
$$\frac{2\pi}{\lambda}\Delta x = \pm 2n\pi$$

or λ the the path difference $\Delta x = \pm n\lambda$ where

$$n = 0, 1, 2 \quad \ldots (12.17)$$

and the maximum intensity

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \ldots (12.18)$$

If the amplitude of the waves are equal i.e.

$$I_{max} = \left(2\sqrt{I_0}\right)^2 = 4I_0$$

Hence if the phase difference and path difference between the rays reaching point P is $\pm 2n\pi$ and integral multiple of λ the intensity will be more than ($I_1 + I_2$) and these points are called maxima. And if $I_1 = I_2 = I_0$ the intensity at maxima will be $4I_O$.

The resultant intensity will be minimum when $\cos\phi = 1$

or $\phi = \pm\pi, \ \pm 3\pi, \ 5\pi$

$$\phi = \pm(2n - 1)\pi \quad \text{where } n = 1, 2, 3 \quad \ldots (12.19)$$

or path difference

$$\Delta x = \pm(2n - 1)\frac{\lambda}{2} \quad \text{where } n = 1, 2, 3 \quad \ldots (12.20)$$

and the minimum intensity will be -

$$I_{min} = \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right) = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \quad \ldots (12.21)$$

hence when the path difference and phase difference between the waves reaching point are odd multiple of (λ/2) and ϕ = 1 (2 n - 1) π respectively, the resultant intensity will be minimum. Moreover when $I_1 - I_2 - I_0$ the resultant intensity will be zero. The equation 12.15 can be written as -

$$\cos\phi = 2\cos^2(\phi/2) - 1$$

$$\therefore \quad I = 4I_0 \cos^2(\phi/2) \qquad I_1 + I_2 \qquad \ldots (12.22)$$

(using identity $\cos\phi = 2\cos^2(\phi/2) - 1$)

From the above analysis we can conclude that all the maxima are equidistant, distance between them and intensity $I_{max} = \left[\left(\sqrt{I_1} + \sqrt{I_2}\right)^2\right]$. The same is true for the minima *i.e.* $I_{min} = \left[\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2\right]$. The maxima and minima are alternatively situated because $\Delta x = 0, \pm\lambda, \pm 2\lambda \ldots$ for maxima.

Where as for minima, $\Delta x = \pm\frac{\lambda}{2}, \pm 3\frac{\lambda}{2} \pm 5\frac{\lambda}{2} \ldots$

Fig. 12.11 shows the graph between phase difference ϕ and the position of maxima and minima. It also show that I_{max} is greater than $I_1 + I_2$ by an amount $2\sqrt{I_1 I_2}$ and for I_{max} it is less than ($I_1 + I_2$) by the same amount.

The law of energy conservation holds goods in this phenomenon, the amount of energy gained at maxima is exactly equal to the energy lost at minima. Energy is only redistributed.

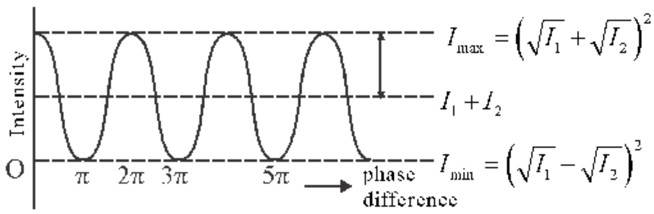


Fig. 12.11 : Intensity distribution in interference (for $I_1 = I_2$)

Here we see what happens when the sources are in coherent, i.e. ϕ when changes with ture. In this case $<\cos^2\phi/2> = 1$ (i.e. average for one cycle for $<\cos^2\phi/2> = 1/2$). From eq. 12.22 . $I = 4I_0 \times \frac{1}{2} = 2I_0$ Which shows that intensity at all points will be sum of intensity of each other.

12.6.2 Expression for Fringe Width

As we have seen earlier that the dark and bright fringes are equidistanly situated. The distance between the two consicutive bright or two dark finges is called fringe width.

We cosider Fig. 12.10 for determination of fringe width. If d = separation of two slits and D = distance of the screen from double slit. We find central bright fringe at point O, since path difference in this case is zero. Now consider the fringe at point P on the screen situated at a distance y from central bright fringe. The path difference between the waves originating from S_1 and S_2 and reaching P will be $\Delta x = S_2P - S_1P$, since D >> d, S_1P and S_2P can be taken as parallel. And S_1A is a perpendicular on S_2P; S_1A can be taken as perpendicular to O'P also

hence $\angle S_2S_1A = \angle OO'P = \theta$. From the diagram 12.10; from $\Delta OO'P$, $\tan\theta = \frac{OP}{OO'} = \frac{y}{D}$ and from ΔS_1S_2A $\sin\theta = \frac{S_2A}{S_1S_2} = \frac{S_2A}{d} = \frac{\Delta x}{d}$ since θ can be taken as very small and for very small angle in radian $\sin\theta \approx \theta$.

We get $$\frac{y}{D} = \frac{\Delta x}{d} \qquad \ldots (12.23a)$$

and $$y = \Delta x\left(\frac{D}{d}\right) \qquad \ldots (12.23b)$$

If there is n[th] bright fringe at P, then $\Delta x = n\lambda$

and $$(y_n)_{Br} = n\lambda\left(\frac{D}{d}\right) \qquad \ldots (12.24)$$

Similarly for $(n-1)^{th}$ bright fringe

$$(y_{n+1})_{Br} = (n+1)\lambda\left(\frac{D}{d}\right) \qquad \ldots (12.25)$$

Hence the fringe width of a dark fringe will be

$$\beta = y_{n\ 1} - y_n = \frac{D\lambda}{d} \qquad \ldots (12.26)$$

similarly for a dark fringe at P -

$$(y_n)_{dark} = (2n-1)\frac{\lambda}{2}D \qquad \ldots (12.27)$$

$$(y_{n-1})_{dark} = (2n+1)\frac{\lambda}{2}D \qquad \text{. . . (12.28)}$$

Again the fringe width for a bright fringe will be -

$\beta = \frac{D}{d}\lambda$ we conclude that all the fringes are of the same width and independent of n.

The angular width of a fring is given by -

$$\theta_o = \Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d} \text{ (independent of D)}$$

We can draw some important conclusion from, the above result.

(i) For a constant value of D and d; $\beta \propto \lambda$; $\lambda_{red} > \lambda_{blue}$ hence $\beta_{red} > \beta_{blue}$

(ii) $\beta \propto \frac{1}{d}$ (for D and λ constant) which indicates that d is kept small to get wide fringes as discussed in see. 12.5 point 5.

(iii) $\beta \propto D$ show that increases with screen distance. But it D is too large the intensity of the fringes diminishes and observation of interference pattern will be difficult.

(iv) We can find wave length of a monochromatic source by this method.

(v) If the whole experiment set up is placed in a medium of reflective index n. Then λ will be reduced to λ / n and $\beta' = \frac{\lambda' D}{d} = \frac{1}{n}\left(\frac{\lambda D}{d}\right) = \frac{1}{n}\beta_{air}$; we can find n by measuring β_{air} and β' .

12.6.3 Interference Fringes Produced by White Light

White light consists of the wavelengths ranging from 3800 Å to 7800 Å. The interference pattern obtained consits of central bright white fring followed by bands of colours in the sequence red to violet. Beyond violet a constant illumination is seen. This property is used to locate central bright fringe in the experiment using monochromatic light where location of central bright fringe is impossible. We just replace the monochromatic source with white light and mark the central bright fringe.

Example 12.3 : In Young's double slit experiment the ratio of the amplitude of the sources is 3 : 2. Find (a) Ratio of amplitude (b) Ratio of intensity for bright and dark fringes.

Solution : The resultant amplitude for maxima is $E_{max} = E_1 + E_2$ and for minima $E_{min} = E_1 - E_2$

$$\text{given } \frac{E_1}{E_2} = \frac{3}{2} \text{ hence } \frac{E_1 + E_2}{E_1 - E_2} = \frac{3+2}{3-2} = 5$$

$$\text{and } \frac{I_{max}}{I_{min}} = \left(\frac{E_{max}}{E_{min}}\right)^2 = 25$$

Example 12.4 : In Young's double slit experiment d = 0.2 mm for λ = 8000 Å . Find fringe width on a screen placed at 1 m from double slit.

Solution : $\beta = \frac{\lambda D}{d}$;

$\lambda = 8000\,Å = 8\times10^{-7}$ m, $D = 1$ m ,

Substituting we get $d = 0.2mm = 2\times10^{-4}m$

$$\beta = \frac{8\times10^{-7}\times1}{2\times10^{-4}} = 4\times10^{-3}m = 4\,mm$$

Example 12.5 : In Young's double slit experiment 60 fringes are seen in a given space when λ = 6600 Å is used. What will be be number of fringes in that area when the source is replaced by λ = 4400 Å .

Solution : If n fringes of width β are situated in area w then $w = n\beta$

$$w = n\beta = \frac{n\lambda D}{d} \quad \text{hence}$$

$$\text{and } n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60\times6600}{4400} = 90$$

Example 12.6 : A dichromatic source of wavelengths 4200 Å and 4800 Å is used in young's double slit experiment in which the separation between the slit is 2.0 mm and the screen is at 1 m from double slit. Find the minimum distance from central maxima, where the bright fringes of both the wavelengths coincide.

Solution : The distance of n^{th} bright fringe from central maxima is given by $y = \frac{n\lambda D}{d}$. If the two bright fringes of different colours coincide at a point then

$$y = \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \quad \text{hence} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4800}{4200} = \frac{8}{7}$$

hence n = 7 for 4200 Å and n = 8 for 4800 Å

$$\text{hence } y = \frac{8 \times 4800 \times 10^{10} m \times 1.0m}{2 \times 10^{-3} m}$$

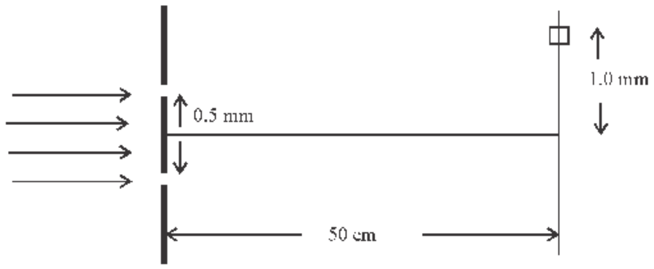
$$= 19200 \times 10^{-7} m$$

$$= 1.92 \times 10^{3} m = 1.92mm$$

Example 12.7 : In choherent white light (400 mm to 700 mm) is used in a double slit experiment. The separation of the slits is 0.5 mm and screen is at a distance 50 cm from slits. There is hole on the screen at a distance 1.0 mm from the central line.

(a) Which wavelength will be absent in the light passing through the hole?

(b) Which wavelength (wavelengths) passing through the hole will have maximum intensity?



Solution : (a) The wavelength which cannot pass through the hole, has destructive interference at that point -

$$y = \frac{(2n-1)\lambda D}{2d}$$

$$\lambda = \frac{2dy}{(2n-1)d} = \frac{2(0.5 \times 10^{-3} m) \times 10^{-3} m}{(2n-1) \times 50 \times 10^{2} m}$$

$$= \frac{2000}{(2n-1)} nm$$

$$n = 1 \quad \lambda_1 = \frac{2000}{1} = 2000$$

$$n = 2 \quad \lambda_2 = \frac{2000}{3} = 667nm$$

$$n = 3 \quad \lambda_3 = \frac{2000}{5} = 400nm$$

$$n = 4 \quad \lambda_4 = \frac{2000}{7} = 286nm$$

Similarly λ_1 and λ_2 are not a part of incident light, hence the absent wavelength are 667 nm and 400 nm.

(b) The wavelength which has maximum intnesity will have a constructive interference at the hole. For this -

$$y = \frac{n\lambda D}{d}$$

$$\lambda = \frac{yd}{nD} = \frac{0.5 \times 10^{3} \times 10^{3}}{n(0.5)} = \frac{1000nm}{n}$$

$n = 1 \qquad \lambda_1 = 1000nm$

$n = 2 \qquad \lambda_2 = 500nm$

$n = 3 \qquad \lambda_3 = 333.33nm$

Only the length λ_2 = 500 nm present in the incident light hence it will have maximum intensity in the outgoing light from the hole.

12.7 Differaction

Diffraction is a characterstic property of the waves. When light passes through an obstacle it bends at the edges of the obstacle. The rectilinear propogation of light seems to fail in this case. When light passes through an obstacle, the part of the wavefront which passes, spreads in the geometrical shadow or the incident rays bends at the edges of the obstacle in geometrical shadow. This phenomenon of bending of the rays or spreading of the wavefront passing through an obstacle is called diffraction.

The effect is pronounced if the condition is λ = d is fulfilled i.e. size of the obstacle should be of the order of wavelength. In daily life this condition is fulfilled for sound waves as the size of doors and windows are of the order of wavelength of sound. That is the reason that we can hear the conversation in the room, even if we are out of it. But diffraction of light is not commonly observed in daily life, because the size of the obtascle (hole) should be of the order of 0.0005 mm.

The detailed explanation of diffraction was given by

Fresnel. It would be very interesting to note an event in the history of science. A top apponent of wave theory. Poision ridiculed Fresnel; that if the explanation of Fresnel is correct, then we should find a bright spot at the center of the shadow of an opaque disc. The experiment was performed by poision and the Fresnel was proved right.

12.7.1 Composition of Diffraction of Sound and Light

As mentioned in the previous section it is easier to observe diffraction of sound in daily life, since the basic condition of clear diffraction is fulfilled.

To observe light diffraction in daily life the order of obstacle is = 10^{-7}m. For ultrasonic waves the required obstacle is = 1 cm. For diffraction of short radio waves, medium waves, the obstacle size is very large, hence these waves can diffract and bend by buildings and hills. For diffraction of X-rays, an obstacle of the order of 1Å is required. This is the dimension of crystal lattice and atomic spacing, this is the basis of crytallograhic study.

It can be understood by Huygen's principle that, if the size of an opening/obstacle is very large compared to wave length of light, the light wave will pass through it without bending which shows the rectilinear propagation of light. And if $\lambda = d$, the most part of the incident wave front is stopped by the obstacle and the small part passing through it will emittit secondary wavelets, and the wavelet spreads in geometrical shadow. (Actually it is the diffraction that occur at the slits in double slit experiment).

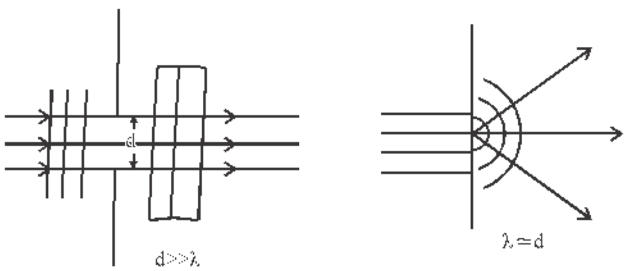


Fig. 12.12 Explanation of diffraction by Hutgen's principle

12.7.2 Types of Diffraction : Fresnel and Fraunhofer Diffraction

The study of diffraction can be devided into two classes.

(a) Fresnel's diffraction : When the source and the screen are at finite distance from the obstacle/apparture, then it is called Fresnel's diffraction. In Fresnel's diffraction the incident wavefront is either spherical or cylinderical.

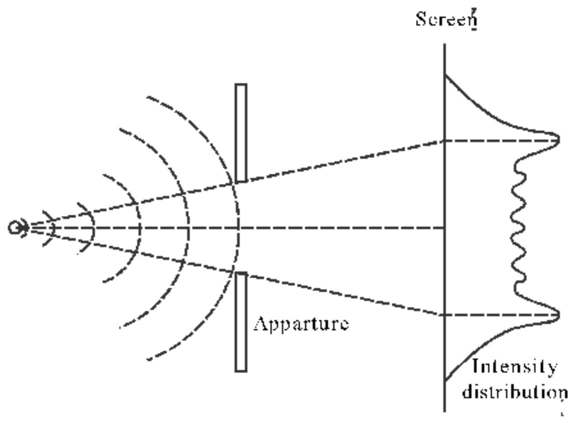


Fig. 12.13 Fresnel's diffraction

(b) Fraunhofer Dffraction : If the effective distances of source and the screen is infinite, the diffraction is called Fraunhofer diffraction. The incident and diffracted wavefronts are plane.

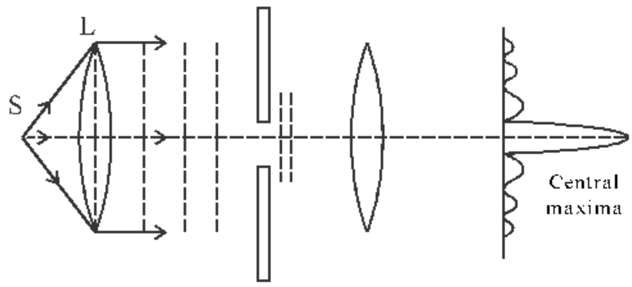


Fig. 12.14 Fraunhofer diffraction

Fresnel's explanation is based in certain hypothesis and the result is somewhat approximate. While in Fraunhofer diffraction the explanation is simpler because of use of plane wave front. We will confine ourself to Fraunhofer diffraction in this chapter.

12.8 Fraunhofer Diffraction Due to Single Slit

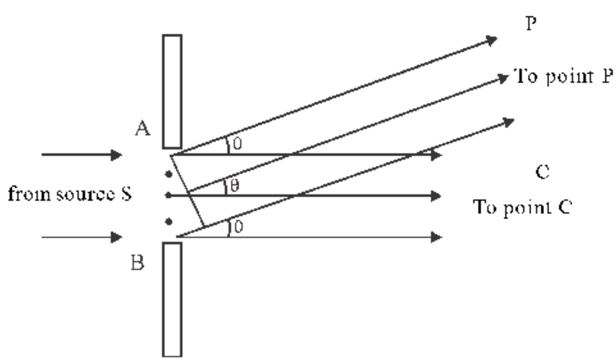


Fig. 12.15 Diffraction by a single slit

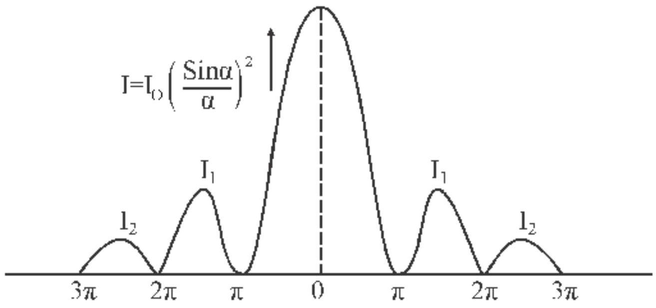


Fig. 12.16 The intensity distribution in Fraunhofer diffraction

When the size of the obstacle is of the order of wave length of light and the incident wavefront is plane, we obtain a broad pattern. This pattern consist of central bright spot and alternate dark and bright bands of decreasing intensity. This pattern is called Fraunhofers diffraction.

To understand it consider fig. 12.15. A plane wave front is incident on a slit AB of size a. M is mid point of AB. MC is a perpendicular to slit, where C is the center of the screen. All the secondary wave lets from different part of incident wavefront AB meet at C in the same phase, hence it is a bright spot of high intensity. To find the intensity on any other point on screen, we point to P from M, the angle between MP and MC gives the angular position of point P. Draw two lines from A and B, which are parallel to MP. The path difference between the two rays starting from A and B and meeting at P is

$$BP - AP = AQ = a\sin\theta \qquad \ldots(12.30)$$

(can be taken as $sin\theta \approx \theta$ for small angles)

The path difference between the above two rays reaching P may be taken as a θ. But there are large number of secondary wavelets on wavefront AB, which contribute differently at P due to their different path difference. To understand it we devide the wave front AB into two parts AM and MB. A wave starting from A and reacting P will be destroyed by a wave originating from M. So all the waves from AB which contribute at P; are destroyed by the same number of waves originating from MB and reaching at P, by one to one mapping. Hence the net intensity at P will be zero. We get a dark band at P. For this condition the path difference $a\sin\theta = \lambda$.

$$\sin\theta = \frac{\lambda}{a} \qquad \ldots(12.31)$$

Similarly for n^{th} minima we get

$$\sin\theta_n = n\left(\frac{\lambda}{a}\right) \qquad \ldots(12.32)$$

The above equation gives the angular position of n^{th} minima.

If the path difference between two waves originating from A and B and reaching a point on the screen is odd multiple of $\lambda/2$; say $= \frac{3\lambda}{2}$. In this case the wavefront AB can be devided in 3 - parts. All the secondary wavelets from one part, that reach a point Q on the screen, are canncelled/destroyed by the same number of wavelets orginating from the next second part of AB and reacting Q. Now the contribution of the next third part of AB will produce some intensity on Q. Hence Q will be a maxima, but of reduced intensity.

In this way for n^{th} order bright band

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad \ldots(12.33)$$

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a} \qquad \ldots(12.34)$$

Equations 12.32 and 12.34 gives the angular positions of minima and maxima; which is symetrical about the central maxima, width of central maxima will be greater than a (due to diffraction). The intensity of a minima is never zero, but extreamly small.

12.9 Difference between Interference and Diffraction

Now we will compare the pattern obtained in young's double slit experiment (interference pattern) and pattern by a single slit (diffraction pattern).

(i) In interference there is superposition of wave originating from two narrow slits. The diffraction pattern is due to super position of group of waves originating from each point of a single slit.

(ii) In terference pattern there are many bright and dark bands of same intensity and equidistant. In diffraction pattern there is central maxima whose width is twice in comparison to other maximas.

The intensity falls sucessively as we move to other maxima on either side of centre.

(iii) In identical experimental set up of the points at which there is maxima in interference, there is minima in diffraction. Its opposite is also correct.

(iv) The intensity at minima in interference is zero where as in diffraction the intensity at minima is non zero.

Example 12.8 : In single slit diffraction pattern the second order bright fringe is at a distance 1.4 mm from centre of central maxima. Screen is at a distance 80 cm from a slit of width 0.80 mm. Consdering monochromatic incident light find out its wave length.

Solution : Here $y_2 = 1.4\,mm = 1.4\times10^{-3}\,m$

$$D = 80\,cm = 0.8\,m$$

$$a = 0.80\,mm = 8\times10^{-4}\,m$$

For second order bright fringe

$$y_2 = \frac{5}{2}\frac{\lambda D}{a} \Rightarrow \lambda = \frac{2y_2 a}{5D}$$

$$= \frac{2\times1.4\times10^{-3}\times8\times10^{-4}}{5\times0.8}$$

$$= 5.6\times10^{-7} = 560\,\text{nm}$$

Example 12.9 : In single slit diffraction experiment the first order minima for red colour (λ = 660 nm) concides with first order maxima for their colour whose wave length is λ'. Find out λ'.

Solution : The position of minima in single slit diffraction experiment is

Given by $\sin\theta = \dfrac{n\lambda}{a}$

For red colour the position of first minima

$$\sin\theta_1 = 1\left(\frac{\lambda_R}{a}\right)$$

For position of nth maxima $\sin\theta_n = (2n+1)\dfrac{\lambda}{2a}$

For wavelength λ' the position of first maxima

$$\sin\theta_1' = \frac{3\lambda'}{2a}$$

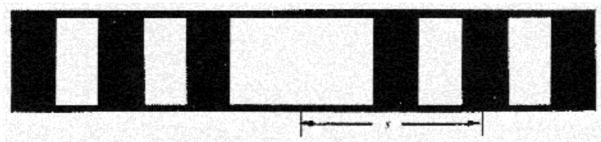
For condition given question $\sin\theta_1 = \sin\theta_1'$

$$\frac{\lambda_R}{a} = \frac{3\lambda'}{2a}$$

$$\lambda' = \frac{2}{3}\lambda_R$$

$$\lambda' = \frac{2}{3}\times660 = 440\,\text{nm} = 4400\,\text{Å}$$

Example 12.10 : Light of wave length 600 mm is incident single slit of width 4×10^{-4} m. Diffraction pattern is observed on a screen placed at 2 m from slit which is shown in fig. Find out from the fig.



Solution : In fig. the distance if second order minima from central maxima is 'S'.

$$\text{Hence } s = \frac{2\lambda D}{a} = \frac{2\times600\times10^{-9}\times2}{4\times10^{-4}} = 0.006\,\text{m}$$

12.10 Resolving Power

The smallest angular separation done by an apparatures is called its resoltuion. Reciprocal of angular resolution is called resoltuion power. Generally the apparture of camera, microscope and telescope is circular. Hence bright circular pattern due to diffraction is observed around central maxima by detailed analysis of circular apparture it is found that the minimum angular separation between images of two sources of just resolution is

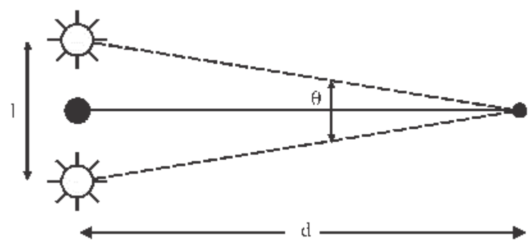


Fig. 12.17 Resolution Power

$$\theta_{min} = \frac{1.22\lambda}{D} \qquad \ldots(12.35)$$

(minimum angle of resolution for circular aperture)

Here d is diameter of aperture and θ_{min} in radian

If distance between two stars is ℓ and telescope is at a distance d from them than angle subtended by both points on objective lens of telescope is $\theta = \frac{\ell}{d}$.

For resolution

$$\frac{\ell}{d} = \frac{1.22\lambda}{D} \qquad \ldots (12.36)$$

For a microscope it is easy to take actual separation (S) between two points because points near to focal point of objective lens hence approximately

$$\theta_{min} = \frac{s}{f} \text{ and } s = f\theta_{min}$$

Here f is focal length of lens than using in equation 12.36 we have

$$s = \frac{1.22\lambda f}{D} \qquad \ldots (12.37)$$

(limit of resolution for a microscope)

Resolving Power

$$= \frac{1}{\text{resolving limit}} = \frac{D}{1.22\lambda f} \qquad \ldots (12.38)$$

$$\text{Resolving Power} \propto \frac{1}{\lambda}$$

From above equation it is clear that the resolving power of an optical instrument is reciprocal of wave length. Hence resolving power is more for light of short wave length.

Example 12.11 : Diameter of objective lens of telescope situated at mount Palomar is 5.00 m. Find our minimum angle of resolution for light of 600 nm wavelength.

Solution : Diameter of lens $D = 5.00\, m$

and $\lambda = 600\text{ nm} = 6\times10^{7}$ m

$$\theta_{min} = \frac{1.22\lambda}{D} = \frac{1.22\times6\times10^{7}}{5} = 1.46\times10^{-7} \text{ rad}$$

Example 12.12 : Distance between two narrow holes is 1.525 mm which are placed in front of a light source of wave length 5.00×10^{-5} cm. They are seen by a telescope having objective lens of diameter 0.400 cm. Find out the maximum distance from telescope so that holes are just resolved.

Solution : $\theta_{min} = \frac{\ell}{d} = \frac{1.22\lambda}{D}$ (From eq. 12.36)

here $\ell = 1.525$ mm, $\lambda = 5.00\times10^{5}$ cm, $D = 0.400$ cm

$$\frac{\ell}{d_{max}} = \frac{1.22\lambda}{D}$$

$$d_{max} = \frac{\ell D}{1.22\lambda}$$

$$= \frac{1.525\times10^{3}\times0.4\times10^{2}}{1.22\times5\times10^{-7}}$$

$$= 0.1\times10^{12} \text{ m} = 10 \text{ m}$$

Example 12.13 : Two paints seperated by 0.1 mm are just seen by a micorscope when light of 6000 Å wave length is used. If light of 4800 Å is used than what is the limit of resolution.

Solution : Limit of resolution for a microscope is

$$s = \frac{1.22\lambda f}{D} \propto \lambda$$

$$\text{Hence} \quad s_2 = s_1\frac{\lambda_2}{\lambda_1} = 0.1\times\frac{4800}{6000} = 0.08 \text{ mm}$$

12.11 Polarisation of Light

We know that in transverse wave, displacement is perpendicular to the direction of propogation of wave. If we think about a wave formed in a string, the vibration of string always remains in a plane it is called plane polarised wave. Hence polarisation is intrinsic property of transverse wave.

In year 1864 James Clark Maxwell showed theoritcally that light waves are electromagnetic waves. Electric and magnetic field vectors have wave motion in same phase in perpendicular planes to the direction of propogation of wave. In electromagnetic wave electric field vector E, magnetic field vector B and direction of propogation of wave all three are mutually perpendicular hence light waves are trasverse waves. In electrocmagnetic wave the plane which contains the plane of vibrations of electric field vectors and direction of propogation of wave is called plane of vibration. the electric field vector of electro magnetic wave is mainly responsible for all optical events. Hence electric vector E is called light vector.

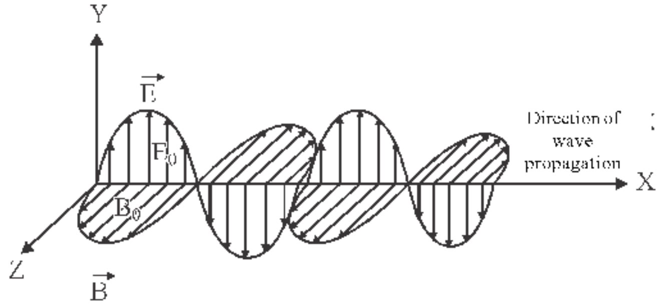


Fig. 12.18 Electromagnetic wave

Emission of ligth is due to transition of electrons in atoms from excited state to lower states. Hence in light vibrations of electric vectors of different waves are found in all possible directions perpendicular to direction of propogation. This type of light ray is called unpolarised light. It is nature of normal light. In this type of ray the vibrations of electric field are perpendicularly symmetrical to the direction of propogation. If direction of propogation is taken perpendicular to the plane of paper than unpolarised light can be shown by fig. 12.19 (a, b).

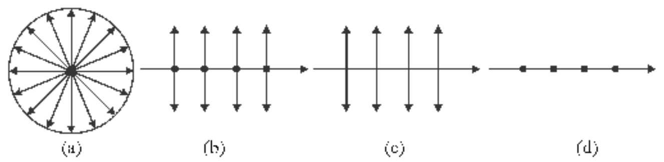


Fig. 12.19 (a,b) Unpolarised light
(c,d) Polarised light

If by some method the vibration of electric field vector in light ray are confine in a definite direction or directions than this event is called polarisation of light and light is called polarised light.

This type of light is shown in fig. 12.19 (c). In this electric field vectors are confine parallel to plane of paper. If vibration are confine perpendicular to plane of paper than they are shown by dot (•) as shown in fig. 12.19 (d). In plane polarised light the plane in which electric field vectors and directions of propogation of wave both are situated is called plane of vibration.

A plane perpendicular to plane of vibration in which there is only direction of propogation of wave and components of electric field vectors are zero is called plane of polarisation.

Hence plane of polarisation and plane of vibration are perpendicular to each other. In fig. 12.20 plane ABCD and EFGH are plane of vibration and plane of polarisation respectively.

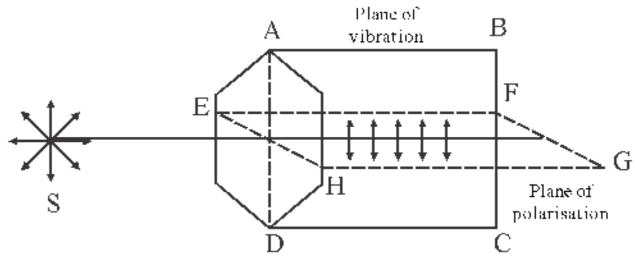


Fig. 12.20 Plane of vibration and plane of polarisation

12.12 Methods of Production of Plane Polarised Light

Following are the methods for production of plane polarised light

(i) By reflection (ii) By refraction

(iii) By double refraction (iv) By Dichroism

(v) By scattering

12.12.1 Polarisation of Light by Reflection and Brewster's Law

Scientist Brewster found that when unpolarised light is incident on a transparent medium (glass, water, etc.) at an specific incident angle i_p than reflected light is completely plane polarised. In this situation the reflected light and refracted light is perpendicular to each other. It is called Brewster's law and angle incident i_p is called Brewsters angle.

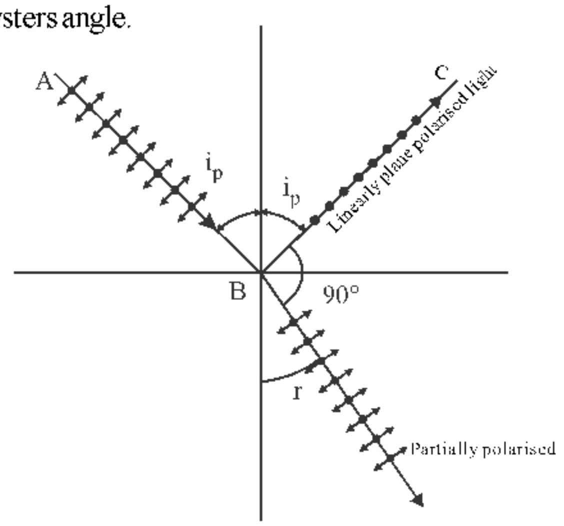


Fig. 12.21 Polarisation by reflection

According to fig. 12.21 when angle of incidence is equal to Brewsters angle i_p then angle of refraction

$$r = 90^\circ - i_p$$

from Snell's law

$$n = \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\sin\left(90^\circ - i_p\right)}$$

or
$$n = \frac{\sin i_p}{\cos i_p}$$

$$n = \tan i_p$$

$$i_p = \tan^{-1}(n) \qquad \ldots (12.39)$$

from eq. (12.39) it is clear that Brewsters angle only depends on refractive index of reflecting surface.

12.2.2 Polarisation of Light by Refraction

When unpolarised light is incident at Brewser's angle on a parallel glass plate (slab) than reflected light from upper and inner surfaces of plate is totally polarised but the refracted and emergent light is partially polarised. If many such same glass plates are placed parallel to each other and if unpolarised light is incident on first plate at Brewster's angle than reflected part of unpolarised light after reflection from the plates is totally polarised but amount of polarisation gradually increases in refracted part as it advance through the plates. If number of plates is large than emergent light is plane polarised such type of arragement of plates is called pile of plates.

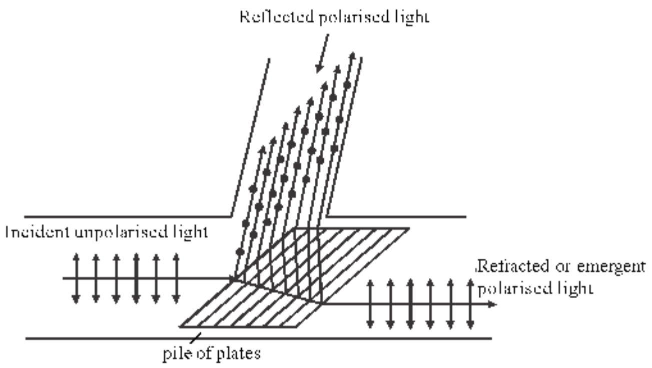


Fig. 12.22 Polarisation by refraction

12.12.3 Polaridsation by Double Refraction

When a light ray is incident on calcite or iceland spar crystle than after refraction two refracted rays are observed. Such type of action of light is called double refraction and crystle is called double refractive crystle. Light rays from double refractive crystle is plane polarisation.

To understand the process of double refraction we make ane ink point on white paper and a calcite crystle is placed above the point. On seeing from above two points are seen instead of one.

If crystle is rotated around the direction of incident light than one of the image remains stationary and second image revolve around the stationary image. The stationary image is formed according to general laws of refraction. The ray due to which stationary image is formed is called ordinary ray or O - ray and the image is called O-image or ordinary image ordinary ray follows the general laws of refraction hence always remains in incident plane and inside crystle its velocity remains same in all directions.

But the moving image is formed by extra ordinary ray or E-ray hence it is called extra ordinary image E-image . This ray does not follow the general laws of refraction and its velocity inside the srystle is different in different directions hence it is called E - ray or extra ordinary ray. E and O rays are plane polarised and vibrations of E and O ray are perpendicular to each other. For separation of E and O ray Nicol prism is used which is made up of calcite crystle.

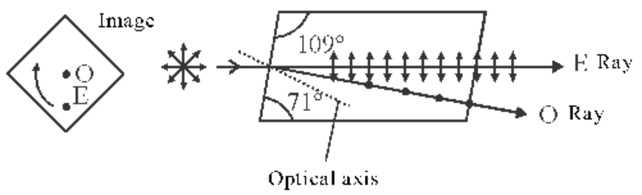


Fig. 12.23

12.12.3.1 Nicol Prism

It is an optical applience by which plane polarised light can be produced and also used for its analysis. Nicol prism works on the property of double refraction. The ordinary ray obtained by double refraction in nicol prism is reflected and separated by total internal reflection and only extra ordinary ray is allowed to emerge out of crystle which is plane polarised . Hence we get plane polarised light from Nicol prism.

Construction : For construction of Nicol prism a piece of calcite crystle is taken whose length is there times of its width. Its corner faced are cut such that the angles in the principle section becomes 66^0 and 112^0 in place of 71^0 and 109^0. The crystle is then cut diagonally into two parts. The surfaces of these parts are grinded to make optically flat and than they are polished. The polished surfaces are joined together with a special cement known as Canada balsam as shown in fig. 12.24. The upper and lower surfaces of this crystal are painted black.

Working : According to fig. 12.24 unpolarised light incident on surface AB of Nicol prism splits in O ray and E ray. For O ray the refractive index of calcite 1.658 is greater than the refractive index of Canada balsam 1.55. Hence O-ray travels from denser medium calcite in rarer medium Canada balsam. Because the incident angle for O-ray on Canada balsam surface is greater than critical angle hence this ray is reflected by total internal reflection and absorbed by black surface.

The refractive index of calcite for E-ray is 1.468 which is less than the refractive index of Canada baslam hence this ray passes through Canada baslam medium and emerges as plane polarised light from Nicol.

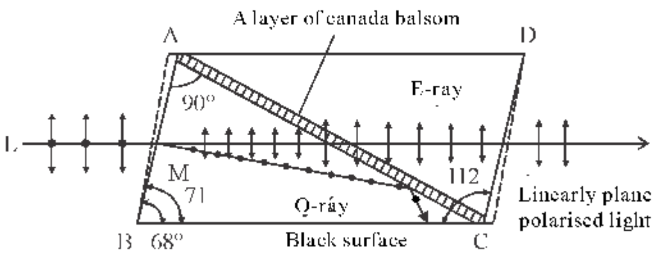


Fig. 12.24 Nicol Prism

12.12.4 Polarisation by Dichroism

When unpolarised light is incident on tourmaline crystle it splits in two plane polarised rays. Tourmaline cristal relectively absorbs one of the ray out of two refractive rays and other ray emerges out of crystal without absorption. Above action of crystle is called dichroism.

Hence transmitted light from tourmaline crystal is plane polarised. Like tourmaline crystle the property of dichroism is found in some or gave come pounds. For commerical use polaroid are made from organic compound on the basis of dichroism.

12.12.4.1 Polaroid

Polaroid is an cheap device to produce plane polarised light on commercial basis which work on dichroism. For construction of polaroid film micro crystal of organic compound herphethite or Iodosulfate of quinine are spread over a thin film of nitrocelulose and fixed such that their optical axis are parallel to each other. These crystals are highly dichroic and absorbs completely one of the two double refracted rays. This film is secured between two glass plates. It is a polaroid.

Working : When unpolarised light passes through a polariod it splits in two plane polarised rays. In which vibration of electric vector in one of the ray are parallel to the axis of herphethaite srystal and in other ray perpendicular to axis. According to fig. 12.25 the ray whose vibrations are perpendicular to the axis of herphethaite crystal is completely absorbed. The ray whose vibrations are parallel emerges from the polaroid. Hence emergent light is totally plane polarised light.

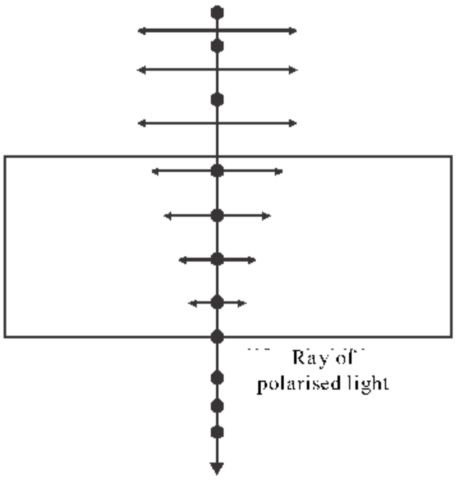


Fig. 12.25 Polarisation by polaroid

Uses of Polaroids

(i) Polaroid are used to produce and analyse plane polarised light.

(ii) A polaroid layer is fix on wind screen galss of car or other vehicle. So the intensity of reflected light from road and other surfaces is reduced for driver.

(iii) They are used to see three dimensional pictures.

(iv) Polaroid are used in head lights of car, truck etc. so that the intensity of bright light from front vehicles which falls on the eyes of driver can be reduced.

(v) Polaroids are used to study the optical properties of metals and structure of optically active materials.

(vi) Polarimeter is used to measure the concentration of optically active mateials like sugar syrup.

12.13 Identification of Plane Polarised and Unpolarised Light

By normal eye we can not study wheather a ray is polarised, partically polarised or unpolarised. To find wheather light is polarised or unpolarised we need calcite crystal, tourmaline crystal, nicol prism or polaroid.

(i) Unpolarised Light : We see emergent E ray from nicol prism or polaroid. When crystal is rotated, if there is no change in intensity of emergent say in any position than light ray is un polarised, because in unpolarised light vibrations are found in all possible directions. Hence intensity remains same in each situation.

(ii) Partially Polarised Light : On rotating the nicol prism or polaroid if intensity of emergent light changes but it is not zero in any situation than light is partially polarised.

(iii) Plane Polarised light : When the polariod in rotated, and the intensity of the emergent light in maximum at one position of the polaroid and zero when polariod in rotated by 90^0 from the position of maximum intensity. Then the incident light is plane polarised.

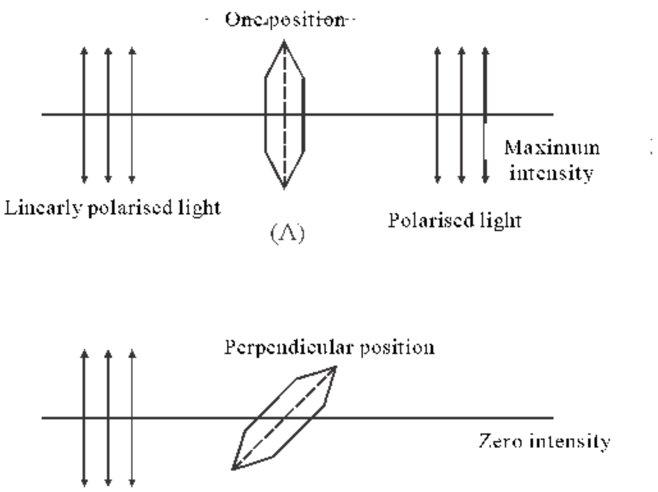


Fig. 12.26 : Detection of polarised light

12.13.1 Malus Law

When light ray is seen from two nicol prism or two polaroids when their axis are parallel than intensity of emergent light is maximum. And when polarised are is crossed position than intensity of emergent light is zero. First nicol prism or polaroid is called polariser and second nicol prism or polaroid is called analyser. For intensity of emergent ray scientist Malus gave a law which is called Malus law.

According to Malus law - When unpolarised light is transmitted through polariser and analyser than intnesity I of light transmitted by analyser is directly proportional to the square of the cosin of angle between the transmission axis of the analyser and the polariser.

$$I \propto \cos^2\theta$$

here θ is angle between axis of polariser and analyser.

$$\text{or } I = I_0 \cos^2\theta \qquad \ldots (12.40)$$

here I_O is maximum intensity of emergent ray and is equal to the intensity transmitted by polariser (incident on analizer)

(i) If $\theta = 0^\circ$

$I = I_0$ maximum value (parallel arrangement)

(ii) $\theta = 90^\circ$

$I = 0$ minimum value (crossed arrangement)

Example 12.14 : If critical angle for a material is 45^0 than calculate its angle of polarisation.

Solution : Critical angle $\theta_c = 45^\circ$

refractive index of material

$$n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 45^\circ}$$

Hence $n = \sqrt{2}$

From Brewster's law

$$\tan i_p = \mu = \sqrt{2} = 1.414$$

$$i_p = 54.7^\circ$$

Example 12.15 : For a transparent material slab when incident angle is 60^0 than reflected light is totally polarised. Find out refractive index and angle of refraction for material.

Solution : Here angle of polarisation $i_p = 60^\circ$

From Brewster;s law

$$n = \tan i_p = \tan 60^\circ$$

$$n = \sqrt{3} = 1.732$$

$$i_p + r = 90^\circ$$

angle of refraction

$$r = 90^\circ - i_p = (90^\circ - 60^\circ) = 30^\circ$$

Example 12.16 : When sunlight is incident at an angle 37^0 the reflected ray is completely plane polarized. Find the (i) refractive index of water and (2) angle of polarization.

Solution : When light is incident at 37^0 on surface of water -

$$\theta_p = 90 - 37 = 53^\circ$$

also $\quad n = \tan\theta_p = \tan 53^\circ = \frac{4}{3}$

also $\quad \theta_p + r = 90$

$$r = 90 - 58 = 37^\circ$$

Example 12.17 : Two polaroids are oriented in such a way that their planes are perpendicular to incident light. Their axis are at 30^0 with each other. What part of the incident unpolarized light will pass through?

Solution : Let the intensity of incident light be I_O. The intneisty after passing through first polarizer will be $I_O/2$. The out going intensity after passing through second polaroid will be -

$$I' = \frac{I_0}{2}\cos^2(30) = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0$$

$$\frac{I'}{I_0} = \frac{3}{8} = 37.5\%$$

Example 12.18: When the axis of polarizer and analyser are parallel the emergent (out going) intensity is I_O. If the analyser is rotated by 45^0, what will be the intensity of emergent light?

Solution : From Mallus law

$$I = I_0\cos^2\theta$$

$$\theta = 45^\circ$$

$$I = I_0\cos^2 45^\circ = I_0\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{2}$$

Important Points

1. Huygens principle - Each point situated on wavefront behaves as a source of new disturance. It is called source of secondary disturbance. Each point source due to its vibrations emit shperical waves in all directions.

2. Inteference - When two or more than two light waves of same frequency and constant phase difference with time, propogates in same direction, they super impose with each other, the resultant intensity in the super imposion region is different from the sum of intensity of each wave due to superposition region the change in the distribution of light intensity in spuperposition region in called inteference.

3. Young's double slit experinment - Conditions for constructive interference - two waves are in same phase at a point if their path difference is zero or integral multiple of wave length then there is constructive interference between the waves.

   Position of bright fringe $y = \frac{n\lambda D}{d}$ condition for distructive interference -

   When path difference between waves at a point is old multiple of half wave length the waves are in opposite phase at the point then there is distruction interference between them.

   Position of dark fringe is given by

   $$y = \frac{(2n-1)\lambda D}{2d}$$

   fringe width $\beta = \frac{\lambda D}{d}$

   angular fringe width $\omega = \frac{\beta}{D} = \frac{\lambda}{d}$

4. Coherent source and stable interference pattern - For stable interference pattern phase difference between the waves must remain same. In this situation the sources are called coherent. The frequency of coherent sources is same and phase difference between them remains constant with time.

5. Intensity distribution in Young's double slit experiment - If two waves of amplitude E_1 and E_2 and intensities I_1, I_2 respectively interference with each other, than resultant amplitude and intnesity are as follows -

   $$E_{max} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\phi}$$

   and $$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

   For constructive interference the resultant amplitude and intensity aremaximum and their expressions are as follows -

   $$E_{max} = E_1 + E_2$$

   $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad [\text{for } \phi = 2n\pi]$$

   For distructive interference the resultant amplitude and intensity are minimum and given by following expressions-

   $$E_{min} = |E_1 - E_2|$$

and $\quad I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \quad$ [for $\phi = (2n-1)\pi$]

Ratio of maximum and minimum intensities

6. Differection - Bending or spreading of light from the edges of an obstacles is called diffraction. For sharp diffraction pattern the size of obstacle should be in the range of incident wave length.

7. Fraun hofer diffraction through single slit - Minima in diffraction pattern - The angular position of minima on screen in diffraction pattern is given by following formula -

$$\theta = \sin^{1}\left(\frac{n\lambda}{a}\right) \approx \frac{n\lambda}{a} \quad \text{(for smaller values of } \theta\text{)}$$

Maxima in diffraction pattern -

For mixima in diffraction pattern

$$\sin\theta = (2n-1)\frac{\lambda}{2a} \quad \text{or} \quad \theta \simeq (2n-1)\frac{\lambda}{2a}$$

8. Resolving power of telescope and microscope. The smallest angular separation done by an apparatures is called resolution that apparatus and inverse of angular resolution is called resolution power.

$$\theta_{min} = \frac{1.22\lambda}{D} \quad \text{(For circular operture minimum angle of resolution)}$$

Here D is diameter of aperture and θ_{min} in radian.

9. Polarisation - If the vibration's of light are confined in a specific plane than light is called plane polarised.

10. Methods to produce polarised light -

Following are the methods to produce plane polarised light - (i) By reflection (ii) By refraction (iii) By double refraction (iv) By dichromatism (v) By scattering

11. Brewster's law - When unpolarised light is incident on a reflecting transparent surface then at a specific incident angle, reflected light is completely plane polarised. In this situation

$\tan\theta_P = n$ and $\theta_p = \tan^{1} n$ and

12. When unpolarised light transmitts through polariser and analyser than intensity of emergent light is directly propotional to square of cosine of angle between the pass axis of polariser and analyser.

Questions For practice

Multiple Choice Type Questions -

1. For demonstration of interference we require two sources which emits readiations of -

(a) Equal frequency and constant phase difference
(b) Approximately same frequency
(c) Equal frequency
(d) Different wave length

2. Monochromatic source of light is used in Young's double slit experinment. Shape of interference fringes on the screen will be -

(a) Straight line (b) Parabola
(c) Hyperbola (d) Circle

3. In an experiment for interference light of 700 nm wavelength is used. There is third bright fringe at a point on screen. For getting fifth bright fringe at the same point what is the required wavelength of light-

(a) 210 nm (b) 315 nm
(c) 420 nm (d) 490 nm

4. In Young's double slit experiment ratio of widths of slits is 4 : 9than what is the ratio of intensity of maximum and minima -

(a) 196 : 25 (b) 82 : 16

(c) 25 : 1 (d) 9 : 4

5. Light of two different wave lengths is used in young's double slit experiment. Position of third fringe for yellow-orange colour (> = 600 nm) concides with the position of fourth bright fringe for other colour. What is wave length of other colour-

(a) 5000 n m (b) 450 n m

(c) 225 n m (d) 350 n m

6. In Young's double slit experiment maximum intensity of light is I_{max}, what is the intensity when path difference is $\lambda/2$.

(a) I_{max} (b) $I_{\max}/2$

(c) $I_{\max}/4$ (d) Zero

7. Which of the following statement given more correct understanding that in most of the situations possibility of diffraction of sound is more than diffraction of light-

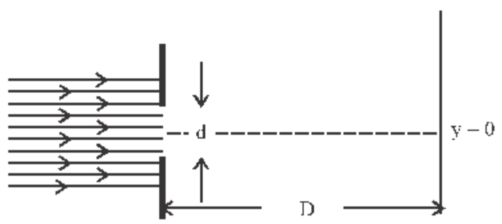
(a) Medium is necessary for second transmission

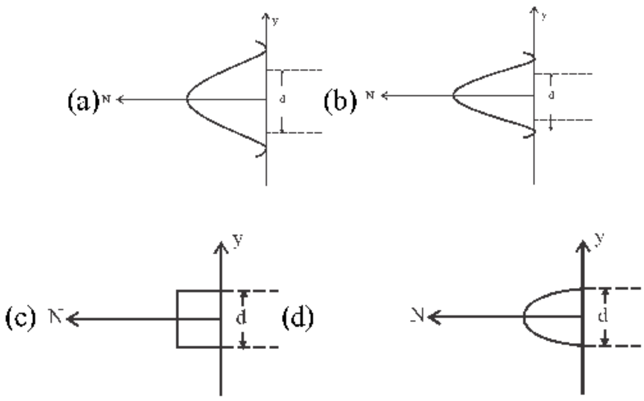
(b) Sound waves are longitudinal but light waves are transverse

(c) Wavelength of light is very short 1 n comparison to second

(d) Velocity of sound is very low than velocity of light

8. According to figure in an experiment electron's passes through a narrow slit of width 'd' which is of the range of its D' Broglie wave length and are detected on the screen at distance D from the slit. The intensity pattern on the screen is -





9. Light of 5000 Å wavelength is incident on a sreen. In diffraction pattern fifth minima is formed at 5 mm from the central maxima if distance between screen and slit is 1 m than width of slit is -

(a) 0.1 mm (b) 0.3 mm

(c) 0.5 mm (d) 0.8 mm

10. A beam of microwaves of wave length 0.052 m is propogating toward a rectangular hole. Resultant diffraction pattern is formed on a wall at a distance 8.0 m from the hole. What is the distance between first and second order outer fringes -

(a) 1.3 m (b) 1.8 m

(c) 2.1 m (d) 2.5 m

11. Aperture of astronomical telescope is large -

(a) To remove defect of spherical aberration

(b) For high resolution

(c) To increase thearea of observation

(d) For low dispersion

12. Two white points are 1 mm apart from each other on a black paper then are seem from pupil of eye of 3 mm diameter what is the maximum distance between them so that they are just resolve by eye. (wave length of light = 500 nm)

(a) 6 m (b) 3 m

(c) 5 m (d) 1 m

13. Electromagnetic waves are transverse in nature. Its proof is -

(a) Polarisation (b) Interference

(c) Reflection (d) Diffraction

14. The angle of incidence when reflection is from air to glass and reflected light is fully polarised is given

by (refraction index n)

(a) $\tan^{-1}(1/n)$ (b) $\sin^{-1}(1/n)$

(c) $\sin^{-1}(n)$ (d) $\tan^{-1}(n)$

15. A beam of unpolarised light is incident on four polarised plates. Plates are arranged so that each plate direction is at an angle 30^0 with its previous one. What is the intensity of light transmitted by each polarisar -

(a) 50% (b) 20 %

(c) 50 % (d) 21 %

16. Two nichol prisms are arranged so that the angle between their principle planes is 60^0. What percentage of incident unpolarised light passes through the system -

(a) 50 % (b) 100 %

(c) 12.5 % (d) 37.5 %

Very Short Answer Type Questions -

1. Line perpendicular to wave front gives direction of which quantity?
2. Which physical quantities affect the width of Young's fringes?
3. Write expression of Huygen's principle for diffraction of light?
4. Which type of wavefront emerges from

   (i) Point source (ii) Far light source
5. Which is the most important condition for interference of two waves?
6. How the angular separation between fringes changes in single slit diffraction experiment when distance between slit and screen doubled?
7. For clear diffraction of waves what should be the range of size of obstacle or hole?
8. Write expression of two physical events which proves wave nature of light?
9. Why light seems to propogate in straight line although it is of wave nature?
10. In an experiment for diffraction through hole, which light waves super imposes?
11. Write down mathematical form of Malus law?

Short Answer Type Questions -

1. Give Huygen's principle for light waves?
2. Define interference of waves.
3. What do you mean by Coherent sources?
4. What you understand by diffraction of light? Compare diffraction of light and sound waves?
5. Define resolving power of microscope. How it is affected -

   (i) When wave length of incident radiations is reduced.

   (ii) When diameter of objective lens is reduced. Give reason for your answer.
6. Frings are formed on screen due to interference of light from two thin slits. If distance between slits rises four times and distance of screen from slits is halws than how many times will be the fringe width?
7. Explain construction of polariod.
8. What do you mean by double refraction?
9. Write the difference between interference and diffraction?
10. Write down main difference between Fresnel and Fraunhofer diffraction.

Essay Type Questions -

1. On the basis of Huygen's secondary wave principle explain refraction of light and derive Snell's law.
2. On the basis of Huygen's wave principle of give intepretation for reflection of light.
3. Write down analytical explanation of interference and write down condition's for construction and destructive interference.
4. What do you mean by diffraction of light? Why diffraction of sound waves is easily observed than diffraction of light waves? Compare Fresnel's and Fraunhofer diffraction.
5. Interpret Fraunhofer diffraction through single slit.
6. What is polarisation? Interpret polarisation with the help of electric vector. Make it clear why it is a property of transverse wave only?
7. Write down name of four methods for producing polarised light? Define double refraction and explain it.
8. By reflection how we can get plane polarised light? What is Brewster's law? Prove that if light is

incident on plane transparent slab at angle of polarisation than reflected and refracted rays are perpendicular to each other.

9. Define plane of vibration and plane of polarisation give explation of Malus law and explain parallel and crossed arrangment?

Answer (Multiple Choice Questions)

1. (a) 2. (b) 3. (c) 4. (a) 5. (d) 6. (d) 7. (b) 8. (b)
9. (a) 10. (c) 11. (a) 12. (b)

Short Answer Type -

1. Direction of rays
2. Wave length of light, distance between sources, medium, distance of screen
4. (i) Spherical (ii) Plane
5. Both sources should be Coharent
6. It changes to half
7. Range of the wave length
8. Interference, diffraction, polarsation
9. Their wave length is very short
10. Between the waves from different sources at mid portion of the hole

Numberical Questions -

1. For two waves of same shape the ratio of amplitude is 2 : 1. Find out maximum and minimum ratios of amplitudes and intensities of vibrations within interference region.

(Ans : 3 : 1 and 9)

2. In an experiment for interference two sources of intensities I and 4 I are used. Find out the intensities an those points where the phase difference between the waves from two sources interfering each other is (a) zero (b) $\pi / 2$ (c) π

(Ans : 9 I, 5 I, I)

3. Find out distance between two holes which forms fringes of 1 mm width on the screen placed at 1 m distance, the wavelength of light is 5000 Å.

(Ans : 0.5 mm)

4. Light of wave length 5500 Å is incident perpendicularly on a linear hole of width 22 x 10^{-5} cm. Find out angular poision of first two minima situated on both sides of central maixma.

(Ans : $\theta_1 = 0.25\,\text{rad}$, $\theta_2 = 0.50\,\text{rad}$)

5. Two polaroides are placed in position so that intensity of emergent light is maximum. If one of the polaroide is rotated by 30^0, 90^0 relative to other than under new position the intensity of emergent light is how much part of maximum intensity?

(Ans : 3/4, 0)

6. When sun is at 37^0 with horizon than reflected light from surface of water is totally polarised. Find out refractive index of water.

(Ans : 1.33)

7. The polarising directions of two polariser plates are parallel hence the intensity of emergent light is maximum. What is the minimum rotation of one of the plates so that intensity of emergent light remains one fourth of maximum intensity?

(Ans : I 60^0)

Chapter - 13

Photo-Electric Effect and Matter Waves

In the previous chapter you have studied about wave nature of light. Optical phenomena like reflection, refraction, interference, diffraction and polarization can be explained using wave theory of light. Maxwell propounded electromagnetic waves theory in 1887 and then experimental demonstration of the existence of electromagnetic waves by Hertz very well established wave nature of light. You will read more about electromagnetic waves in chapter 17.

By nineteenth century the wave nature of light was well established, but at the same time some observations like photoelectric effect, Compton effect and later Raman effect could not be explained on the basis of wave theory. To explain them quantum (photon) theory was used which was related to corpuscular theory of light. In this chapter we will mention about photoelectric effect, its experimental results and Einstein's photo-electric equation for their explanation. You will learn more about Compton effect and Raman effect in higher classes. After that we will study about dual nature of light. After knowing about dual nature of light the question arises that if light beam which is supposed to have wave nature generally behaves like a particle in some cases then does a beam of particles or a beam of electrons in certain circumstances behave like waves? Later in this chapter we will be able to answer this question when we study about matter wave hypothesis of de Broglie (pronounced as de Broy in French) and its experimental confirmation. Towards the end of this chapter we will learn about Heisenberg's Uncertainly Principle.

13.1 Photo-electric effect and matter waves

In 1887 while doing experimental study on electromagnetic waves Hertz observed that when ultraviolet light from an Arch lamp falls an cathode the discharge between electrodes becomes easier. These observations led Hertz to propose that when ultraviolet light falls on cathode charged particles are emitted which makes discharge easier. In 1888 scientist Hallwachs connected a negatively charged zinc plate to an electroscope and observed that when ultraviolet light falls on negatively charged plate it loses its negative charge. When an uncharged plate is used and when it is illuminated by ultraviolet light it becomes positively charged. If a positively charged plate is used it gains more positive charge or it remain unaffected. From these experiments it was concluded that when a negatively charged or an uncharged plate is illuminated with ultraviolet light it emits negatively charged particles which makes the negatively charged plate neutral or uncharged plate becomes positively charged. In 1897 after the discovery of electron and on measuring the e/m of particles it was established that these particles are electrons which are emitted when light falls on the plate. Hallwachs and another scientist Leonard observed that when light of frequency lower than a fixed frequency falls on the plate no electrons are emitted. This lowest fixed frequency is known as threshold frequency and its value depends upon the nature of the material of the plate or any photosensitive surface.

We know that why a metal is good conductor is that it has large number of free electrons. These electrons move freely inside the metal but cannot come out of the surface. If they could come out of the surface the surface would have become positively charged and would have attracted electrons back inside. Obviously the elctrons near the surface face resistance. Hence they need some additional energy to come out of the surface and this can be given only by an external source. The minimum amount of energy needed by electrons to just come out of the surface (with zero kinetic energy) is called work function of the metal. The work function is generally denoted by ϕ_0 and is generally measured in eV (electron volt) (1eV = 1.602×10^{-19}J). Different metals have different work functions and this depends on the nature of their surface. When the free electrons of the metal get energy equal to or more than the work-functioon from the incident light, electrons are emitted. Because of their generation by light such electrons are called photoelectrons. Due to these emitted electrons the current flowing in a closed circuit is called photoelectric current. Hence (to summarise) "When light of a specific frequency or more than that frequency falls on a metal surface or optically sensitive surface the electrons are emitted. This phenomenon is called photoelectric effect."

Table 12.1 shows work functions of a few metals. The presence of impurities in the surface changes these values.

Table 13.1 : Work functions of some metals

Metal	Work function eV	Metal	Work function eV
Cesium Cs	2.14	Aluminium Al	4.28
Potassium K	2.30	Mercury Hg	4.49
Sodium Na	2.75	Copper Cu	4.65
Calcium Ca	3.20	Silver Ag	4.70
Molybdenum Mo	4.17	Nickel N	5.15
Lead Pb	4.25	Platenum Pt	5.65
Fe	4.7	Au	5.1
Ir	5.27	Os	4.38
Ta	4.25	W	4.55
Rh	4.98	Ru	4.7

It is observed from experiments that Alkali metals (like Lithium Sodium, Potassium etc.) show photo-electric effect for visible light also while Zinc, cadmium, Magnisium and other similar metals show this effect only for high frequency ultraiolet waves. For emission of electrons from metals there are other methods also. When metals are heated adequately electrons come out because they get extra heat energy. This method is known as thermionic emission. Electrons can also be emitted by applying strong electric field (of the order of 10^8 V/m). This method is known as field emission. Besides this if electrons having high kinetic energy are incident on the metal it is possible that electrons may be ejected. This method is called secondary emission.

13.2 Experimental results of photoelectric effect and their interpretations

To study photo-electric effect in the laboratory a simple experimental arrangement is shown in figure 13.1. It has a glass or quartz tube which has vacuum inside. This tube has two electrodes C and A. C is called cathode (or emitter), is a light sensitive metallic plate. Plate A is called Anode (or collector). Light from source S crosses a window and falls on plate C. The potential difference between C and A is maintained by a battery and it can be varied by means of a current controller in the circuit. The potential of A relative to C can be kept positive or negative through a commutator in the circuit. Photo-electrons emitted by C when light is incident on it are attracted towards A if A is positive with respect to C. The electrons collected by A flow through the microammeter, battery etc. in the external circuit and reach C and thus current flows in the circuit which is called photo-electriccurrent. The voltmeter V in the circuit measures potential difference between A and C and photo-electric current is measured by microammeter. For observing the effect of intensity of light on photo-electric current the intensity can be changed by changing the distance between source S and emitter C. Different sources of light with different frequencies can be used to see the effect of frequency on photoelectric current. Alternatively filters can be kept between source S and plate C to get light of requiredfrequency.

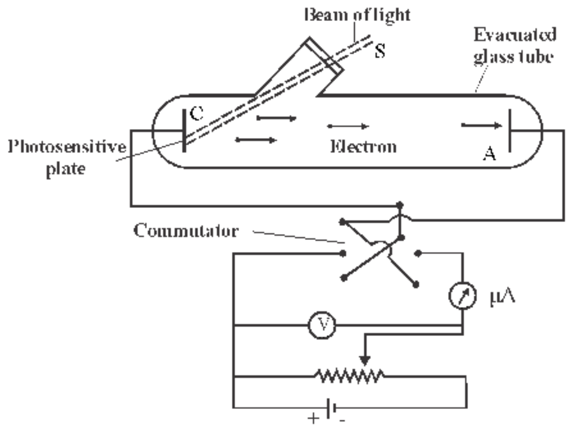


Fig. 13. 1 : Experimental arrangement for studying photoelectric effect.

The following results are obtained through the study of the photoelectric current.

13.2.1 Effect of potential on photoelectric current.

First of all the frequency of light and Intensity I are kept constant and light is incident on C. suppose A is at zero potential relative to C. In this condition all the electrons emitted by C cannot reach A. At any given time electrons emitted by C remain near it and form space charge. This negatively charged space charge repels electrons emitted by C. Even then some electrons reach A and thus photo-electriccurrent begins to flow. When

Anode A is given some positive potential relative to C some more electrons are attracted towards A and space charge is decreased and photoelectric current increases. Thus current depends upon the potential of Anode. Figure 13.2 shows variation in the current with variation in potential of A. If potential of A is gradually increased a stage comes when the effect of space charge becomes negligible and all the electrons emitted by Cathode are able to reach Anode. Then current becomes constant. This current is known as saturated current. This is shown by part bc in fig. 13.2. If potential of A is still increased no change in photo-electric current takes place.

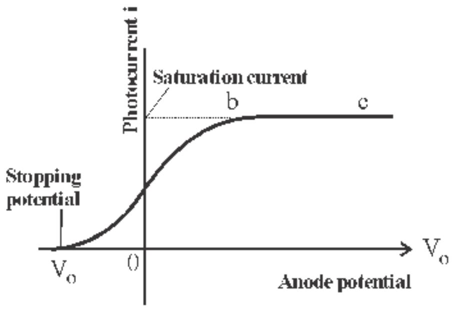


Fig. 13.2 : Graph between photocurrent and Anode potential at constant potential and intensity

If potential of Anode becomes negative relative to Cathode some electrons return to cathode due to repulsion by Anode and current is decreased. If negative potential on A is increased current decreases rapidly. Negative potential is also known as retarding potential. On a certain value V_0 of negative potential the current becomes zero. For a definite frequency of incident light the negative potential V_0 on Anode for which photo-electriccurrent becomes zero is known as cut off potential or stopping potential. This depends upon frequency of incident light.

Stopping potential has direct relation to maximum kinetic energy of emitted electrons. The kinetic energies of all the electrons emitted by cathode are not equal. For current to be zero we have to ensure that an electron with maximum kinetic energy (or fastest moving electron) is not able to reach the Anode. In this situation the maximum kinetic energy of the emitted electron K_{max} is equal to work done against the repelling force of stopping potential V_0 or

$$K_{max} = \frac{1}{2}mv_{max}^2 = eV_0 \qquad \text{. . . (13.1)}$$

where m = mass of electron, e = charge of electron v_{max} = maximum speed of an emitted electron

13.2.2 Effect of intensity of light on photo-electriccurrent

If we repeat the above experiment with a definite frequency but different intensities we will find that with increase in intensity the value of saturated current also increases. (If frequency of light is constant).

In fig 13.3 graphs between photoelectric current versus Anode potential for 3 different intensities have been displayed. Here $I_3 > I_2 > I_1$ It is evident that saturated current increases with increasing intensity. It means that on increasing intensity the electrons emitted by cathode per second and electrons reaching anode per second increase and current is also increased. It is to be kept in mind that stopping potential does not depend upon intensity. If the stopping pointical for a certain metal and for a certain frequency of light from a 1W source is -1.0 then for the same metal and same frequency of light from 5W source the stopping potential will remain - 1.0V.

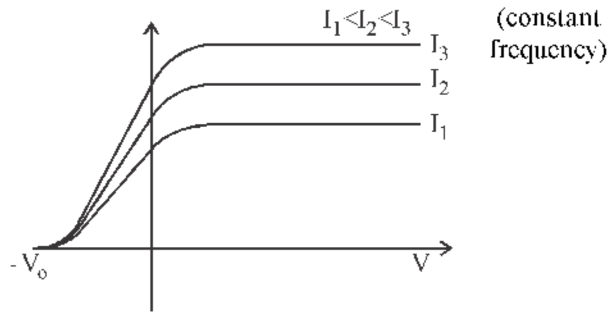


Fig. 13.3 : Graph between photo electric-current and Anode Potential for incident light of different intensities

When intensity of incident light is increased the saturated photo-electriccurrent increases linearly as shown in fig 13.4. Because photo-electric current is directly proportional to emitted electrons per second therefore the number of electrons emitted per second is also directly proportional to intensity of incident light. If the source of light is a point source then intensity of light I will be inversely proportional to distance (d) between cathode & source. Hence photo-electric current will also follow the same principle i.e.

$$i \propto \frac{1}{d^2}$$

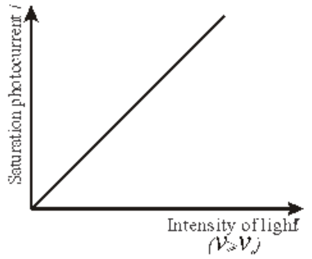


Fig. 13.4 : Graph b/w and intensity

13.2.3 Effect of Frequency on Photo-Electric Current

For a given photo sensitive plate and keeping the intensity of radiation constant if frequency of incident light is changed and for each frequency the corresponding stopping potential is measured, it can be observed that as the frequency is increased the stopping potential will be increased proportionately.

But saturated current will remain the same. In fig 13.5 graph between Anode Potential V and corresponding photo-electric current for three frequencies ν_1, ν_2 and ν_3 is displayed (where $\nu_3>\nu_2>\nu_1$) Here it can be seen that $V_{03}> V_{02}> V_{01}$. Because stopping potential is an indicator of maximum kinetic energy of electrons it can be said that as the frequency of incident light is increased the maximum kinetic energy of electrons will increase proportionaly.

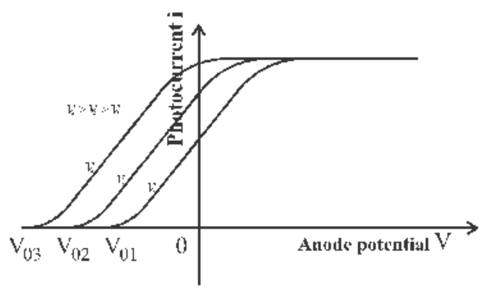


Fig. 13.5 : Graph between Anode Potential V and photoelectric current for different frequencies of incident light of a definite Intensity

If a graph is plotted between the frequency of incident light and corresponding stopping potential for different metals it will be a straight line as shown in fig 13.6 for two metals A and B. From this figure it is clear that for frequeney ν_{0A} for metal A and for frequency ν_{0B} for melal B the stopping potential for emitted electrons (i.e. maximum kinetic energy of electrons) is zero. Hence for every metal there is a definite cut off or threshold frequency ν_o for which the corresponding stopping potential is zero. If the frequency of the incident light is less than cut off value ν_o no photo electrons will be emitted whatever be the intensity. The wavelength λo corresponding to ν_o is called threshold wavelength $(\lambda_0 = c/\nu_0)$.

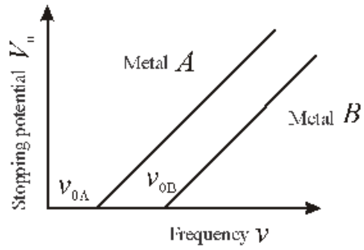


Fig. 13.6 : Graph between stopping potential or frequency for different metals

It should be borne in mind that if the frequency of incident light is more than the threshold frequency then there is no time lag between incidence of light and emission of electrons. Immediately after the incidence of light emission of electrons starts (within 10^{-9} s or less than that time).

The results of the above mentioned experiments can be summarised as follows :

1. When light of high frequency (or adequately short wavelength) falls on any metallic surface the metal emits electrons. This emission takes place immediately without any time lag.
2. For a given metal there is a definite threshold frequency ν_o . If incident light is of less frequency photo-electriceffect will not be observed how so high the intensity may be.
3. Photo electrons may have any Kinetic energy starting from zero to a maximum value K_{max}.
4. By keeping Anode at a negative potential relative to cathode emission of photo-electrons can be stopped. The minimum nagative potential of Anode for which photo-electric current becomes zero is called stopping potenial. The stopping potential v_o and maximum kinetic energy of emitted electrons are related by the equation, $K_{max} = e\,v_o$.

5. Stopping potential does not depend upon intensity of light. That means the maximum kinetic energy of electrons is independent of the intensity. But stopping potential has a linear relationship with frequency of incident light.

6. The saturated photo-electric current increases when intensity of light is increased.

13.2 4 Failure of wave theory to explain Photo-Electric Effect.

According to electromagnetic wave theory of light the distribution of energy takes place wherever the waves are present. Because waves have energy it seems that by absorbing energy from waves the electrons of the metal can come out. But on this basis photo-electriceffect can not be explained adequately.

1. Dependence of kinetic energy of photo-electrons on intensity of light. According to electromagnetic nature of light when light is incident on a metal a periodic force due to oscillatory electric field of the wave should act. If electric field of electromagnetic wave has an amplitude E_0, its intensity $I \propto E_0^2$ and effective force acting on the electron $F=eE_0$. When Intensity increases the force on the electron increases and therefore its acceleration and kinetic energy should increase. In ther words according to wave theory the continuous absorption of energy by electrons goes on and hence electrons will absorb more energy from high intensity beam of ligh and they should come but with greater energy. But according to experimental observations the maximum kinetic energy of electrons does not depend upon intensity of light.

2. Dependence of emission of electrons on frequency of light waves :

   According to wave theory photo-electric effect should be observed for all frequencies of light subject to intensity being adequately high so that electrons get needed energy to come out of the metal

   This is not observed in experiments as light of less frequency than the threshold frequency does not show photo-electric effect how so high the intensity may be.

(iii) Time Lag: In any wave energy distribution takes place at the wave front and hence all the electrons in the illuminated area should absorb energy. If the intensity is low the electron should take finite time to gain energy to come out of the metal. It means there should be measurable time lag between incidence of light and emission of electrons. According to wave theory this time could be even a few hours. But the experimental observation is that emission of electrons takes place immediately after incidence of light (within 10^{-9}s or even less time).

(iv) Dependence of kinetic energy of photo-electrons on frequency of light.

According to wave theory the frequency of light and kinetic energy of photo-electrons should have no relation while the experimental fact is that maximum kinetic energy of emitted electrons increases with increasing frequency of light.

Thus we conclude that wave theory of light cannot explain experimental results.

13.3 Concept of Photons:

To explain energy distribution in Black body spectrum Planck in 1900 proposed that emission of radiation by a body or absorption of the same does not take place continuously. But it happens by means of discrete bundles (called quanta). Agreeing to views of Planck, Einstein in 1905, proposed that light energy (universally known as electromagnetic radiation) is quantised. It means radiation energy is made of discrete units which are called quanta of radiation energy. We now call them photons. During mutual interaction with matter radiation behaves as if it is made of particles known as photons. Some important characteristics of Photons are given below :-

1. In vacuum every photon always travels with the velocity of light (3 x 108 m/s)

2. Every photon has a definite energy and a definite momentum. A photon of radiation (light) of frequency ν has energy equal to $h\nu = hc/\lambda$ and a momentum equal to $p = h\nu/c = h/\lambda$. Here λ is the wave length of electro magnetic radiation (light). h is a universal constant which is known as Plank's constant. Its value is $h = 6.63 \times 10^{-34}$ Js = 4.1 x 10^{-15} eVs.

3. Photon can collide with a particle (like electron). In such a collision the total energy and total

momentum are conserved. Photon can be absorbed also during a collision and /or new photons can be formed. It is not necessary that the number of photons remains unchanged.

4. Photons are electrically neutral and are not deflected by electric and magnetic fields.

5. If the intensity of light of a given frequency is increased the number of photons passing through a given area in a given time also increases. Energy of every photon remains the same.

6. Rest mass of a photon is zero.

Example 13.1 : For a photon of wavelength 4000Å find (a) Frequency (in Hz) (b) energy in eV and (c) momentum [h = 6.63 × 10^{-34} J.S and c = 3×10^{8} m/s]

Solution : (a) For light, $c = \nu\lambda$

$$\therefore \quad \nu = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{4000\times10^{-10}\,m}$$

= 7.5 × 10^{14} Hz

(b) energy of Photon $E = h\nu$

= (6.63 × 10^{-34} Js) (7.5 × 10^{14} s^{-1})

= 4.97 × 10^{-19} J

$$= \frac{4.97\times10^{-19}}{1.6\times10^{-19}} = 3.10\,eV$$

(c) Momentum of photon

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{4000\times10^{-10}}$$

= 1.66 × 10^{-27} N.s

Example 13.2 : A monochromatic source of light operating on 100W emits 4x10^{20} photons per second find the frequency of light [h = 6.63 × 10^{-34} J.s and c = 3 × 10^{8} m/s]

Solution : If number of photons emitted by source per second is n and every photon has energy E and Power of source is P

Then $P = nE$

$$\therefore \quad E = \frac{P}{n} = \frac{(100\,Js^{-1})}{4\times10^{20}}$$

= 2.5 × 10^{-19} J

Hence wavelength of Photon $\lambda = \frac{hc}{E}$

$$= \frac{(6.63\times10^{-34}\,Js)(3\times10^{8}\,m/s)}{2.5\times10^{-19}}$$

= 8.0 × 10^{-7} m = 8000Å

13.4 Einstein's Photo-electric equation and explanation of experimental results of photoelectric effect on the basis of this equation

As mentioned in the previous section in order to give theoretical explanation of photoelectric effect Einstein in 1905 considered light (Electromagnetic radiation) as made up of quanta. In that research paper Einstein used the word Quanta but we are using the word Photon.

Through this revolutionary theory proper explanation of experimental results about Photoelectric effect was given for which Einstein got Nobel Prize in 1921.

After striking the metallic surface photons collide with the free electrons of the metal. In a certain collision Photon can transfer all its energy E to free electron. If this energy is more than the work function of the metal the electron can come out of the metal. It is not necessary that if the energy given to electron is more than ϕ_0 it must be emitted. Deep inside the metal the electrons that have gained energy collide with the ions and may loose their energy. Near the surface of the metal the electron after getting additional energy may move towards the surface and come out. If such electrons after getting energy E from photon come towards the surface without further collision and come out their Kinetic energy will be $E - \phi_0$. If before coming out the electrons have further collisions their kinetic energy may be less than $E - \phi_0$. Thus the photoelectrons emitted by the metal may have any energy between zero and maximum value $E - \phi_0$. If this maximum value is denoted by K_{max} we have

$$K_{max} = E - \phi_0 \qquad \text{. . . (13.2)}$$

But energy of a photon having frequency is given by $E = h\nu$, hence

$$K_{max} = h\nu - \phi_0 \qquad \text{. . . (13.3a)}$$

or

$$h\nu = K_{max} + \phi_0 \qquad \text{. . . (13.3b)}$$

Equation 13.3 is known as photoelectric equation of Einstein. In reality this is the statement of law of conservation of energy about work function ϕ_0 and absorption of a single photon. If the mass of the ejected electron is m and maximum velocity is v_{max} we have

$$h\nu = \frac{1}{2}mv_{max}^2 + \phi_0 \qquad \text{. . . (13.4)}$$

If stopping potential is V_0 then $K_{max} = eV_0$

Hence $h\nu = eV_0 + \phi$ . . . (13.5)

Equation 13.4 and 13.5 are alternative forms of Photo-electric equation

Now we will explain experimental results of photoelectric effect using photoelectric equation.

(i) According to equation 13.3 the maximum energy of photoelectrons K_{max} varies linearly with the frequency of incident light and does not depend upon intensity. It tallies with the experimental observations.

(ii) By definition kinetic energy can never be negative. Hence in equation 13.3 it is provided that photoelectric effect can be observed only when

$h\nu > \phi_0$.

or $h\nu > h\nu_0$

where $\nu_0 = \dfrac{\phi_0}{h}$ . . . (13.6)

Thus threshold frequency comes into existence Light having a frequency less than threshold cannot eject electrons whatever be its intensity. It also tallies with the observations.

It is also evident from eq. 13.6 that threshold frequency must be more in case of metals having more work function ϕ_0.

(iii) Experimental observations regarding dependence of photoelectric current on intensity in case of frequency of light being more than threshold can be explained using the concept of photons. Intensity of light is proportional to the number of photons per unit area per unit time. If more photons are incident they will eject more photoelectrons i. e. photoelectric current will also be more. Hence for $\nu > \nu_0$ the photoelectric current will be proportional to intensity.

(iv) In propounding the photoelectric equation the basic consideration was absorption of a photon by an electron which can be considered as a collision. Time taken in collision is negligible hence absorption process is almost instantaneous. Hence in photoelectric effect there is no time lag between incidence of light and emission of electrons. This also is in agreement with experimental observations. Decreasing the intensity of light does not delay the emission of electrons because basic process is absorption of a photon by an electron. As given in (iii) intensity affects the magnitude of current only. In addition the rate of emission of electrons does not depend upon frequency of incident light because according photoelectric equation increasing results in increase of maximum kinetic energy of electrons but not the number of electrons emitted.

Using the relation $\nu = c/\lambda$ the photoelectric equation can be written in terms of wavelength also.

Equation 13.5 can also be written as :-

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e} \qquad \text{. . . (13.7)}$$

which is similar to linear equation $y - mx \quad c$. It means graph between stopping potential V_0 and frequency should be a straight line whose slope is h/e which does not depend upon the nature of the matter. such a graph has been depicted in fig 13.6 Since e is a known constant and slope h/e can be measured. Hence the value of h can be determined.

During 1906 to 1916 Millikan studied photoelectric effect using sodium metal and by measuring the slope of the straight line determined the value of Planck Constant h which agreed with the values determined through other methods. Thus Millikan established photoelectric equation of Einstein experimentally.

Example 13.3 : For a certain metal the work function is 2.2eV. Determine the maximum wavelength of light that can show photoelectric effect for this metal.

[$h = 4.14 \times 10^{-15}$ eV. s, $c = 3 \times 10^8 ms^{-1}$]

Solution :

Threshold frequency $\nu_0 = \dfrac{h}{\phi_0}$

But $\nu_0 = \frac{c}{\lambda_0}$ where λ_0 is corresponding threshold frequency.

$$\therefore \quad \lambda_0 = \frac{hc}{\phi_0}$$

$$= \frac{(4.14\times10^{-15} eVs)\times(3\times10^{8} m/s)}{2.2eV}$$

$$= \frac{12.42\times10^{-7} eV.m}{2.2eV}$$

$$= \frac{1242 eV\,nm}{2.2eV} = 564.5nm = 5645 \text{ Å}$$

Note : In this example it can be seen that hc = 1242 eVnm If we can remember this result it will help us in the solution, of those questions (specially objective type questions) where the values of h and c are not given or in the question energy is given in eV and wavelength λ is to be determined or vice-versa.

Example 13.4 : In an experiment on photo-electriceffect light of 200 nm is incident on lithium metal 0 = 2.5eV determine (a) maximum kinetic energy of photoelectrons in eV and (b) stopping potential

Solution : The maximum kinetic energy is given by

$$K_{max} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$

$$\therefore \quad K_{max} = \left(\frac{1242 eV.nm}{200nm}\right) - 2.5eV$$

= 6.21 eV – 2.5 eV = 3.71 eV

For practice get this result using h = 6.63 × 10^{-34} J.S and c = 3 × 10^{8} ms^{-1}. You will get in joules which you will have to convert in eV.

(b) Stopping potential is given this way -

$$V_0 = \frac{K_{max}}{e}$$

$$= \frac{3.71eV}{e} = 3.71V$$

Example 13.5 : Certain metal surface is illuminated first by light 2000Å wave length and then by light of 6000 Å wave length. It was observed that the ratio of maximum velocities of photo-elecrons emitted in these two cases is 3 : 1. Find out the work function of the metal.

Soloution: According to Einstein's photo-electric equation

$$\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv_{max}^2$$

For $\lambda_1 = 3000\mathring{A} = 300nm$

$$\frac{hc}{300nm} = \phi_0 + \frac{1}{2}m(3v)^2 \qquad \text{... (i)}$$

For $\lambda_2 = 6000\mathring{A} = 600nm$

$$\frac{hc}{600nm} = \phi_0 + \frac{1}{2}mv^2 \qquad \text{. . .(ii)}$$

Multiplying eq (ii) by 9 and then subtracting eq (i) from it we get

$$8\phi_0 = hc\left\{\frac{9}{600nm} - \frac{1}{300nm}\right\}$$

$$= \frac{1242 eV.nm\times7}{600nm}$$

$$\therefore \quad \phi_0 = \frac{1242\times7}{8\times600} = 1.81eV$$

= 2.89 J

13.5 Dual Nature of Light

What is the nature of light? Is it a wave or has a particle nature. Searching answers to these questions has an interesting background history. In the process of discovery there was very important advancement is knowledge and understanding in the area of Physics which became the basis of Quantum mechanics, upto 17th century some of the known properties of light were: 1 Motion along a straight line path, 2 Reflection of light from plane and curved surfaces, 3 Refraction at interface of two medium and 4 Colour dispersion of light about which you have read earlier.

Great Scientist Newton in his Corpuscular theory of light treated light to be composed of little corpuscles. Corpuscles emitted by a source travelled in straight lines in the absence of external forces. Not going into details of this theory we will mention that corpuscular theory successfully explained linear propagation of light, formation of shadow behind an obstacle and reflection of light. Refraction of light could also be partially explained but the conclusion of this theory that if the ray of light bends towards the normal in the second medium its speed in this medium should be more than the speed of light is against the experimental evidence.

Contemporary of Newton scientist Huygens proposed wave theory of light in 1678. You have learnt about it in detail in the previous chapter. After the successful explanation of reflection of light, refraction etc. the wave theory of light could also successfully explain other phenomena of light e.g. interference, diffraction and polarization which could not be explained by corpuscular theory. Wave theory gained more support when Maxwell in 1860 established mathematically the existence of electromagnetic waves by using electromagnetic equations. These waves move with the speed of light in vacuum and their transverse nature was in agreement with polarisation which is exclusive properly of transverse waves. In 1887 the experimental work done by Hertz regarding origin and detection of electromagnetic waves paved the way for universal acceptance of wave theory of light.

It is ironical that during discoveries done by Hertz the photo-electriceffect was discovered which put a question mark on completeness of wave theory. Compton effect and Raman effect observed later could be explained by photon, (quanta) model like photo-electric effect. In order to explain specific heat of solids. Debye put forward a hypothesis that lattice vibrations are quantised.

Thus we find that in some optical phenomena like reflection, refraction, diffraction, polarisation etc light has a wave nature and they cannot be explained on corpuscular theory of light. On the other side the phenomena like photo-electriceffect, Compton effect and Raman effect can be explained only on the basis of quantum (photon model) theory and not on wave theory. Hence it is an open question even now whether light is a wave or a particle. At present there is a general agreement that light has a dual nature. It has characteristics of waves as well as particles. In some event it behaves like a wave and in another behaves like a particle. You can note that a beam of light behaves like a particle in some event and the same beam behaves like a wave in another event. It can be said that particle model and wave model of light are complementary. It may be mentioned that in any single experiment light does not behave as a wave and as a particle simultaneously.

13.6 De-Broglie Hypothesis and wavelength of matter wave

As has been mentioned earlier in this chapter that usually behaving like a wave, light in some circumstance behaves as a particle (photon). Naturally the question arises whether particles of matter like electron, proton and Neutron can behave like a wave. In view of the similarities of different kinds in nature a French Physicist de-Broglie in 1924 put forward a hypothesis that as light (electromagnetic radiation) energy exhibits dual nature, matter should also show dual nature i.e. in some circumstances matter should behave like waves de-Broglie suggested that the formula $p = h/\lambda$ for photon should also be applicable on a particle. A particle of momentum p can be associated with a wave length as given below-

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{...(13.8)}$$

Where m is the mass and v the speed of the particle λ is called the wavelength of matter wave or de-Broglie wave length. Equation 13.8 which is called de-Broglie eq also specifies dual nature of matter. On the left the wave length λ is a characteristic of wave and on the right side momentum p is characteristic of a matter particle. To estimate the order of λ we consider a ping pong ball of mass 10gm (10^{-2} kg) moving with a speed of 2 m/s. Then-

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,Js}{(10^{-2}\,kg)(2\,m/s)} = 3.31\times10^{\ 34}\,m$$

Obviously the measurement of such a short length is not possible with the present apparatus. This is why the matter waves associated with bulky bodies cannot be observed. We will later see that wavelengths associated with subatomic particles like electrons and protons having sufficient energy are of the order of wavelengths for x-rays can be measured.

If the kinetic energy of a particle of mass m is K we have

$$K = p^2/2\text{ m} \ ; \quad p = \sqrt{2mK}$$

Using this relation in 13.8 we have

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \ldots(13.9)$$

If kinetic energy of a particle is known we can determine the de-Broglie wavelength associated with it.

13.7 Now we will find wavelengths associated with different kinds of particles

(a) Wavelength of charged particle accelerated by potential difference of V volts :

If a particles of mass m and charge q is accelerated by potential difference V its kinetic energy will be $K = qV$. Hence from eq (13.9)

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

For electron $m = m_e = 9.1 \times 10^{31}$ kg and $q = e = 1.6 \times 10^{19}$ C

$$\lambda_e = \frac{6.63 \times 10^{-34} \; J.s}{\sqrt{2 \times 9.1 \times 10^{-31} kg \times 1.6 \times 10^{-19} CV}}$$

$$= \frac{12.27}{\sqrt{V}} \times 10^{-10} m = \frac{12.27}{\sqrt{V}} \mathring{A} \quad \ldots.(13.9)$$

For proton $m_p = 1.67 \times 10^{-27}$ kg and $q = e = 1.6 \times 10^{19}$ C

$$\lambda_p = \frac{6.63 \times 10^{34}}{\sqrt{2 \times 1.67 \times 10^{27} \times 1.6 \times 10^{19} V}}$$

$$= \frac{0.286}{\sqrt{V}} \quad \ldots(13.10)$$

similarly for dutron $m_d = 2\, m_p$ and $q = e$

$$\lambda_d = \frac{0.0202}{\sqrt{V}} \mathring{A} \quad \ldots(13.11)$$

And for α – particle $m_\alpha = 4m_p$ and $q = 2e$ के लिए

$$\lambda_\alpha = \frac{0.101}{\sqrt{V}} \mathring{A} \quad \ldots(13.12)$$

(b) For in charged particles, Neutrons and gas molecules :

If particles of mass m e.g. neutron or gas molecules are in thermal equilibrium at Absolute temperature T, Then by equipartition of energy their average kinetic energy k=3/2 KT, where $k = 1.38 \times 10^{-23}$ J, K is Boltzman constt and from eq. (13.9)

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{3\, mkT}} \quad \ldots(13.13)$$

Putting the values of m, k and T in the above equation the wavelength of the concerned particle can be determined

Hence for one neutron or proton $m_n = m_p = 1.67 \times 10^{-27}\, kg$

$$\lambda = \frac{25.2}{\sqrt{T}} \text{Å}$$

Sometimes thermal energy is of the order of kT

$$\lambda = \frac{h}{\sqrt{2mkT}} \quad \ldots(13.14)$$

which is the maximum possible energy. Hence K=kT

In this situation wavelength of Neutron is

$$\lambda = \frac{30.8}{\sqrt{T}} \text{Å}$$

Example 13.6

Find out the ratio of wavelengths of proton and λ particles which are accelerated by the same potential difference.

Solution : From formula for de-Broglie wavelength of charged particles

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For same V

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p q_p V}} \frac{\sqrt{2m_\alpha q_\alpha V}}{h}$$

$$= \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

because $m_\alpha = 4m_p$ and $q_\alpha = 2q_p$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Example 13.7

Find out the wavelength of an electron accelerated by a potential difference of 100V.

Solution : The formula for de-Broglie wavelength of an electron accelerate by a potential difference of V is

$$\lambda_e = \frac{12.27}{\sqrt{V}} \mathring{A}$$

$$\lambda_e = \frac{12.27}{\sqrt{100}} \mathring{A} = 1.227 \mathring{A} \simeq 1.23 \mathring{A}$$

Here we can see that this wavelength is of the order of wavelength of x-ray.

Example 13.8

An α particle and a proton enter an equal magnetic field such that their velocity vectors are perpendicular to the field. α–particles and proton travel in a way such that their radius of curvature are equal. Determine the ratio of their de-Broglie wavelengths.

Solution : We know that when a particles of mass m and charge q enters a magnetic field B such that B and v are perpendicular then it moves in a circular path. If the radius of the path is r we have

$$Bqv = \frac{mv^2}{r}$$

or $\quad mv = qBr$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{h}{qBr}$$

hence $\quad \frac{\lambda_\alpha}{\lambda_p} = \frac{q_p}{q_\alpha}$ (given B = r)

$$= \frac{1}{2} \quad \{ \text{because } q_\alpha = 2q_p \}$$

13.8 Davisson and Germer Experiment and its conclusion

This experiment verified wave nature of electrons for the first time. We know that diffraction is a characteristic of a wave. This experiment showed that diffraction in electron beam is possible. We know that for clear diffraction it is essential that the size of the diffractor is of the order of the wavelength of wave. The wavelength of matter waves associated with moving electrons is of the order of wavelength of x-rays.

To see the diffraction of X-rays a crystal is used as a diffractor. Davisson and Germer thought if particles like electrons have wave property then they should also be diffracted by the crystal like x-rays.

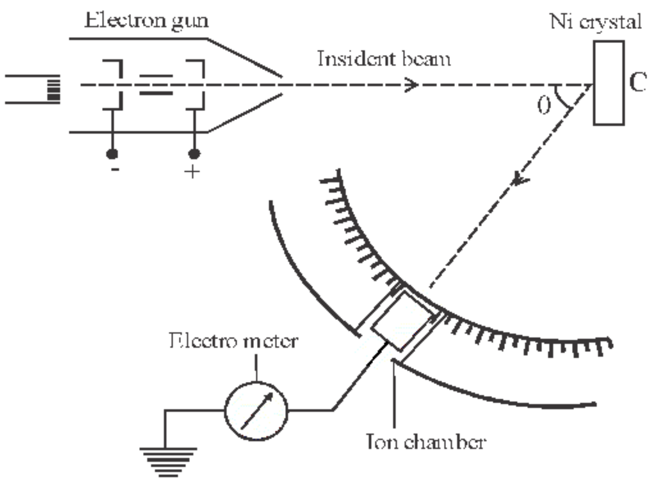


Fig 13.7 : Experimental setup of Davisson and Germer Experiment

Fig13.7 shows the experimental arrangement of Davisson and Germer. A beam of high energy electrons is obtained from an electron gun. In this current flows through a tungsten filament which emits electrons due to thermionic emission. These electrons pass through holes in linear diaphrams. Diaphrams are kept at a high positive potential relative to filament. This accelerates electrons. Thus a narrow beam of accelerated electrons is produced. By changing the potential difference between the filament and the Diaphrams the energy of the electron beam can be changed.

The electron beam obtained from the electron gun is incident on the Nickel crystal normally. Electrons scattered from the crystal are collected in a detector. The detector is a device for identifying the particles and in measuring their energies. In this experiment an ionization chamber was used as a detector. When electrons diffracted from the crystal enter the ionization chamber the gas is ionized The number of ions depends upon the energy of diffracted electron beam. A current flows in the galvanometer because of ions. By moving the detector in a circular path the angle between incident electron beam and scattered beam can be changed. If electrons are treated as particles then according age-old theory of scattering the intensity of scattered electron beam should have very little change due to variation of θ but the results of this experiment are different.

In the experiment polar graphs are drawn between intensity of diffracted beam and angle of diffraction from various values of accelerating voltage, In fig 13.8 graphs are shown for accelerating voltages 44*v*, 54*v* & 64*v*. For definite values of θ the length of the radius vector is a measure of the intensity of the diffracted electron beam. From these graphs it is clear that when accelerating voltage is 54v the intensity in the detector is maximum at 50°. For voltage more than or less than this value the peak disappears. The formation of sharp peak at 54 Volts indicates that the electrons are being diffracted.

Fig 13.8 shows the diffraction of electrons by the Nickel Crystal. In this experiment such lattice surfaces are chosen where the distance between two atoms is less than 2.15A. If diffraction of electrons beam and X-rays is compared we have for diffraction of X-rays the path difference is given by Bragg's law

d sin u = nλ

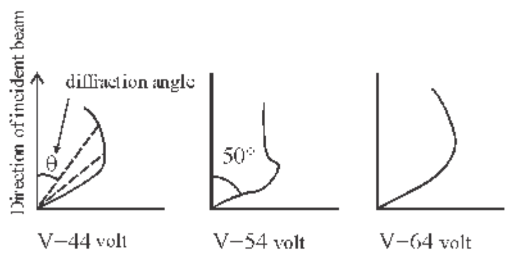


Fig 13.8 : Polar diagram for diffrent voltages

where, n is the order of diffraction Hence for d= 2.15, θ=50 ° and n= 1 the value of wave length is given by

$$\lambda = 2.15 \times \sin 50^\circ$$
$$= 2.15 \times 0.766$$
$$= 1.65 \mathring{A}$$

For accelerating voltage 54 volts the theoretical value of de Broglie wave length of electron is given by

$$\lambda = \frac{12.27}{\sqrt{V}} \mathring{A} = \frac{12.27}{\sqrt{54}} \mathring{A} = \frac{12.27}{7.348} \mathring{A} = 1.67 \mathring{A}$$

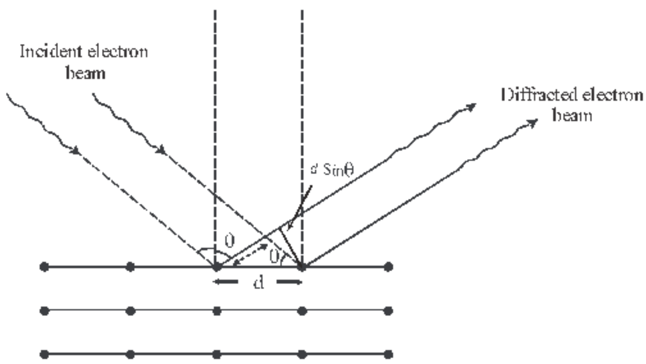


Fig 13.9 : Diffraction of electrons
by lattice surfaces of the crystal

In this way the experimental and theoretical value of wave lengths from Davissan and Germen experiment are very near to each other. Hence it is concluded from Davisson and Germen experiment that diffraction of electrons is possible. Because diffraction is a property of waves it is proved that waves are associated with electrons or the hypothesis of de Broglie regarding matter waves is verified. Apart from this the experiment of G.P Thomson also confirms matter waves. Like light the interfrence for electrons also has been observed by two slit experiment. An important theoretical application of de Broglie hypothesis is that it explains the second postulate of Bohr's Atomic Model about which you will learn later. Diffraction of electrons and Neutrons gives information regarding crystal structure. In addition to it the electron microscope was developed on the basis of wave properties of electrons.

13.9 Heisenberg's Uncertainty Principle

In 1923 Heisenberg's propounded the uncertainly Principle. According to it "At any instant (time) the position of a particle and its momentum cannot be determined completely and accurately at the same time and in the same direction. The product of uncertainty in the position of the particle, Δx and uncertainty in the x component of the momentum Δp_x can never be less than $\hbar/2$

Mathematically, according to this theory

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \ldots (13.15)$$

Here $\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} J \cdot s$

It should be borne in mind that Δx and ΔPx are not the errors due to measurement by instruments. These errors will not end even if we use very sensitive instruments. In reality Δx and ΔPx are such inherent errors which follow Heisenberg's uncertainly principle because of the matter waves associated with particles of matter.

Position in the x direction, x and momentaum in the same direction Px are cannonically conjugate quantities. Like 13.15 the uncertainly principles can also be written:

$$\Delta y \Delta p_y \geq \frac{\hbar}{2} \qquad \ldots (13.16)$$

$$\Delta z \Delta p_z \geq \frac{\hbar}{2} \qquad \ldots (13.17)$$

Similarly energy and time are also cannonical conjugate quantities, hence for them the uncertainly principle is

$$\Delta E \Delta t \geq \frac{\hbar}{2} \qquad \ldots (13.18)$$

This means energy of a particle and its time coordinate can not measured with unlimited accuracy. All measurements of energy will have inherent uncertainty unless you have infinite time for measurement. For example the energy of the atom in its original stationary condition is well defined because the atom remains in this condition for infinite time, while the excited conditions of atoms are not well defined as it remains in this condition only for $\Delta t = 10^{-8}$s. After that it reverts back to lower energy level. Hence there is uncertainty in the energy in the excited condition or the energy level is not sharply defined. It should have some breadth given by equation

$$\Delta E \sim \frac{\hbar}{\Delta t}$$

Heisenberg's Uncertainty principle is for both micro as well as macro particles. The size of macro particles is too big so that uncertainty in its position is negligible and since mass is more therefore uncertainty in macro objects is not observed.

Example 13.9 : If the uncertainty in the position of a particle is 0.1 nm calculate the uncertainty in its momentum

Solution : According to Uncertainty principle of Heisenberg

$$\Delta x.\Delta p_x \geq \frac{\hbar}{2}$$

If minimum value of the product of uncertainties is taken we have

$$\Delta x.\Delta p_x \simeq \frac{h}{2}$$

Therefore uncertainty in momentum

$$\Delta p_x \simeq \frac{h}{2\Delta x} = \frac{1.054\times10^{-34}}{2\times0.1\times10^{9}}$$

$$= 0.53\times10^{-24}\ kg\cdot m/s$$

Example 13.10 : In an atom the time period for excited energy level is 1.0×10^{-8}s. Find the minimum uncertainty in the frequency of emitted photon in transition.

In the question $\Delta t = 1.0 \times 10^{-8}$ s

Hence according to uncertainty principle of Heisenberg

$$\Delta E = \frac{h}{4\pi\Delta t}$$

$$= \frac{6.63\times10^{34}}{4\times3.14\times10^{-8}} = 0.53\times10^{26}\ J$$

$\therefore$ Uncertainty in frequency

$$\Delta v = \frac{\Delta E}{h}$$

$$= \frac{0.53\times10^{-26}}{6.63\times10^{34}} = 8\times10^{6}\ Hz$$

Important Points

1. **Work Function** The minimum energy required by free electrons with maximum Kinetic energy on the surface of a metal to come out of the surface of the metal is called work function of the metal. The value of work function for different metals is different and depends upon the impurities present on the surface of the metal.

2. **Photoelectric Effect** When light of a specific frequency or more than that frequency falls on a metalic surface it emits electrons. This phenomenon is called photoelectric effect.

3. **Threshold Frequency** The threshold frequency for a metal is that minimum frequency of light below which the light cannot eject photons from the metal.

4. **Threshold Wavelength** The wavelength corresponding to threshold frequency is called Threshold wavelength.

5. Threshold frequency and threshold wavelength for a light sensitive substance (metal) depends upon the metal and nature of its surface.

6. The number of electrons emitted per second depends upon the intensity of light and not its energy.

7. The maximum kinetic energy of photoelectrons depends upon the frequency of incident light and not on its intensity

8. **Stopping Potential** The negative potential of the collector (Anode) plate which makes photoelectric current zero is called stopping potential. Its value depends upon the frequency of incident light.

9. **Photon** It is a quanta of electromagnetic energy such that its energy is proportional to frequency of light and can be calculated by the formula $E = h\nu$. The rest mass of Photon is zero.

10. It is not possible to explain photoelectric effect on the basis of electromagnetic wave theory. Einstein explained it on the basis of quantum theory (photon).

11. Einstein's photoelectric equation is $K_{max} = h\nu - \phi_0$ or $K_{max} = h\nu - h\nu_0$

12. de Broglie Hypothes is every moving particle has a wave associated with it which is called matter wave. Like light matter also has dual nature.

    The wavelength of matter wave is inversely proportional to the momentum of the particle and can be calculated by the following formula

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

13. Formula for de-Broglie wavelengths for different particles :

$$\lambda_e = \frac{12.27}{\sqrt{V}} \mathring{A} \quad ; \quad \lambda_p = \frac{0.286}{\sqrt{V}} \mathring{A}$$

$$\lambda_\alpha = \frac{0.101}{\sqrt{V}} \mathring{A} \quad ;$$

Wave length of a particle in thermal equilibrium at absolute temp T, $\lambda = \frac{h}{\sqrt{3mkT}}$

14. De Broglie Hypothesis is verified by Davisson and Germer experiment. This experiment also proves that diffraction of matter particles is possible

15. Heisenberg's Uncertainty Principle

    $\Delta x \Delta p_x \geq \hbar / 2$

    At any instant the position of a particle and its momentum in the same direction can not be determined with cent percent accuracy simultaneously. According to it,

    $\Delta x \, . \, \Delta Px \geq \hbar /2$

    Time Energy uncertainty relation is $\Delta E \Delta t \geq \hbar / 2$

Questions For practice

Multiple Choice Questions

1. A Photon of ernergy 40ev is incident on a metal surface. Due to this an electron having kinetic energy 37.5eV is emitted. The work function of metal surface is

   (a) 2.5eV (b) 57.5eV

   (c) 5.0eV (d) zero

2. For light having frequency more than threshhold frequency the number of electrons emitted in photo-electric effect experiment is proportional to

   (a) Their Kinetic energy

   (b) Their potential energy

   (c) Frequency of incident light

   (d) The number of photons incident on the metal

3. The energy of a photon of light beam A is twice that of photon of onother light beam B. Then ratio of their momenta P_A / P_B is

   a) ½ (b) ¼

   (c) 4 (d) 2

4. The emission of electrons from a metal begins when green light is incident on it The emission of electrons will be possible for which of the folio wing group of colours ?

   (a) yellow, blue, red, (b) violet, red , yellow

   (c) violet , blue, yellow (d)violet, blue, indigo

5. The de-Broglie wavelength associated with an electron emerging from an electron-gun is 0.1227 A^0. The value of the accelerating Voltage applied on the gun is

   (a) 20 kV (b) 10 kV

   (c) 30kV (d) 40 kV

6. If the energy of a non-relativistic free electron is doubled the frequency of the matter wave associated with it will be changed by which factor?

   (a) $1/\sqrt{2}$ (b) ½ (c) $\sqrt{2}$ (d) 2

7. If the position of a particle is determined with cent percent accuracy the uncertainty in its momentum according to uncertainty principle will be

   (a) zero (b) ∞

   (c) -h (d) nothing can be said

8. Which property of electrons asssoeiated with waves was demonstrated by Davisson and Germer experiment

   (a) Refraction (b) polarisation

   (c) Interference (d) diffraction

9. The de-Broglie wavelength associated with an electron having kinetic energy 10 eV is

   (a) 10Å (b) 12.27Å

   (c) 0.10 Å (d) 3.9 Å

10. An electron and a proton are contrained to remain in a linear box of dimension 10 Å . The ratio of uneertainties in their momenta is

    (a)1:1 (b) 1:1836

    (c) 1836:1 (d) insufficient information

Very Short Type Questions

1. Write Einstein"s photo-electric equation.
2. The stopping potential dependa upon what ?
3. To observe photo-electric effect the frequency of incident light should be more than which Frequency ?
4. What is the name given to a quanta of electronagnetic energy ?

5. Write the formula for wavlength of a matter wave according to de-Broglie hypolheies

6. Write down the relation betweeen the uncertainties in the position of a particle and its associated momentum accerding to Heisenberg.

7. Write the name of an experiment that establishes matter wave theory of de- Broglie.

Short Answer type questions

1. What is photo-electric effect?
2. What do you mean by Threshhold frequency?
3. Write down the definition of work function?
4. State the objective of Davissone Germer experiment.
5. Write down the hypothesis of de-Broglie about the dual nature of matter waves.
6. Define Uncertainty Principle.

Essay Type questions

1. Explaining photo-electriceffect describe experimental obervation associated with it.
2. Why it is not possible to explain photo-electriceffect on the age old wave theory? Explain.
3. Explain the explanation given by Einstein about photo-electric effect. What is meant by threshhold frequency?
4. Explaining the concept of photon describe its various properties.
5. Mention de-Broglie hypothesis and describe in detail the experiment of Davisson and Germer for its experimental verification.
6. Extablish the formulae for finding de-Broglie wavelengths of electrons, proton and $\propto$ particles.

Answers

Multiple Choice Questions.

1. (a) 2. (d) 3. (d) 4. (d) 5. (b)

6. (d) 7. (b) 8. (d) 9. (d) 10. (a)

Very Short Answer Questions

1. $h\nu = \frac{1}{2}mv^2_{max} + \phi$
2. On the frequency of incident light.
3. Greater than the threshhold frequency of the material.
4. Photon
5. $\lambda = \dfrac{h}{mv}$
6. $\Delta x \cdot \Delta p \geq \dfrac{\hbar}{2}$
7. Davission and Germer Experiment.

Numerical Questions

1. The thresh hold frequncy for copper is 1.12×10^{15} Hz. Light of wavelength 2537Å falls on its surface. Find the work function and stopping prtential $h=6.63\times10^{-34}$ Js (Ans. 4.64 ev, 0.24v)
2. For a certain metal threshhold frequency is 5675 Å Calulate the work function of the metal $h=6.63\times10^{-34}$Js (Ans. 2.20 eV)
3. Calculate the difference between kinetic energies of photo-electrons emitted by radiations of wavelengths 3000 Å and 6000 Å.

   (Ans. 2.07 eV)
4. Calculated the de-Broglie wavelengths associated with an electron and an $\propto$ particale accelerated by equal potential difference of 100v. (Ans. 1.227 Å, 0.010 Å)
5. A 20 watt bulb is emitting light of frequency 5×10^{14} Hz. Find the number of photons emitted by the bulb in one second. (Ans. 6×10^{19})
6. First order diffraction is observed in Davisson and Gemer experiment, Accelerating voltage in 54 volt. If the distance between reflecting surfaces of the used Ni crystal is 0.92 Å find out the angle of diffraction. (Ans. 65°)
7. If the uncertainty in the x component of the momentum of a moving electron is 13.18×10^{-30}

Kg m/s. find out the uncertainties in the x component of positon and velocity. (Ans. $0.40x10^{-5}$ m, 14.48 m/s)

8. Find out the ratio of de-Broglie wavelengths of a proton and an $\propto$- particle having equal energies. (Ans. 2:1)

9. The period of electromagnetic vibration is 0.30 ms. Find out the uncertainly in the energy of photon. (Ans. $1.76x10^{-31}$J)

10. The work function for sodium is 2.3ev. Find out the maximum wavelength of light that can emit photo electrons from sodium. (Ans. 539nm)

11. When a metallic surface is illuminated with light of frequency $8.5x10^{14}$ Hz the maximum kinetic energy of emitted electrons is 0.52ev. When the same surface is illuminated by light of frequency $12.0x10^{14}$ Hz the maximum kinetic energy of emitted electrons is 1.97 ev. Find out work function of the metal. (Ans. -3eV)

12. At room temperature T=300K Neutrons are in thermal equilibrium. Find out their de-Broglie wavelengths. (Ans. 1.45 Å)

Chapter - 14

Atomic Physics

The concept that matter is composed of tiny particles has been a subject of study for the philosophers from the ancient times. The author of the book Vaisheshik Darshan and Indian philosopher Maharshi Kanad considered matter as made up of tiny individual particle called parmanu. Greek philosopher Democretes also put forward a similar hypothesis. The first scientific theory about atom was given by scientist Dalton in 1803 which you have learnt in previous classes. The basic idea in all such thoughts was that atom was indivisible and it has no internal stucture of its own. The experiments done towards the end of nineteenth century and in the beginning of twentieth century had put a question mark on this idea. In experiments on discharge through gases at low pressure, cathode rays were discovered which were made of negative charged particles. These particles were later on named as electrons by british scientist J.J. Thomson in 1887 considered as an essential component of all atoms. In this connection Thomsons experiment for the measurement of e/m of the electron and Millikans oil drops experiment of electronic charge are worth mentioning. Atom is electrically neutral hence same amount of positive charge should be there in the atom as it has negative charge due to eletrons. Naturally the question arose about the distribution of positive and negative charges in an atom i.e. what is the sttucture of atom? The same question became the basis of all propositions about atomic models. It was well established that an atom is stable, hence, in all atomic models it was essential to explain how inspite of attraction between positive and negative charges they provide stability to the atom without eliminating each other.

Apart from this it was also known in the beginning of nineteenth century that at low pressure when current flows through atomic gases or vapour or heating by a flame, the rarified gases emit electromagnetic radiations of specific frequencies which give line spectrum. In rarified gases the distances between atoms is much more, the emitted radiations are not because of mutual interaction of atoms but because of individual atoms. Every element has its characteristic spectrum. Apart from it this was also known that when metals are heated to a high temperature they emit electrons. Upto first three decades of twentieth century efforts to provide theotetical explanation to experimental evidences became the reason for the development of different models of the atom. In this chapter first we will study about Thomson's atomic model, Rutherford's atomic model and also about their success and failure. After it we will study about Bohr's model of Hydrogen atom which in itself was a revolutionary idea. After explanation of Hydrogen spectrum by Bohr's model we will mention some of its weaknesses. In the end we will see how de Broglie's matter wave theory successfully explains postulates about orbital quantisation.

14.1 Thomson Model of the Atom

In 1838 Thomson proposed that an atom is positively charged sphere of radius 10^{-10}m. Throughout the volume of this sphere positive charge is uniformly distributed. To balance the positive charge negative charges in the form of electrons are embedded in the sphere. In Thomson model the negative electrons are embedded in the same way as fruit pieces are kept in the pudding to increase its beauty and taste. Alternatively the arrangement of electrons is supposed to be like seeds in the water melon.

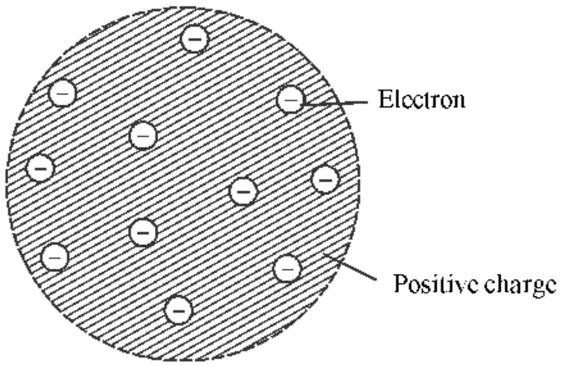


Fig 14.1 : Thomson Model

The experimental evidences till that time could be successfully explained by Thomson model. This model was able to explain successfully, stability of atom, ionisation of gases, thermionic emission of electrons. To explain emission of electromagnetic radiation from atom

Thomson supposed that after receiving energy from external sources electrons begin to oscillate about their mean positions. Because of oscillary motion the electrons are in accelerated motion and as accelerated charge emits electromagnetic radiation hence atom emits electromagnetic radiation. Thomson assumed that frequency of radiation is the same as the frequency of oscillation of the electron. If this idea is applied to hydrogen atom which has only one electron the spectrum of hydrogen should have only one line. Experimental observations indicate there are series of spectral lines in the spectrum of hydrogen which we shall study later. In this way this model could not explain hydrogen spectrum. This model got a serious blow when this model could not explain experimental observations of Rutherford's α scattering experiments.

14.2 Alpha Ray Scattering Experiment and Rutherford Model of Atom

In 1901 Rutherford and his associates Geiger and Marsden did an important experiment which showed that Thomson Model of atom is not correct. In this experiment α rays striked a thin foil of gold and the deviated particle after passing through the foil were measured at different angles. α particle is doubly ionised Helium atom whose mass is about four times the mass of hydrogen atom (7000 times the mass of electron) and has charge +2e. α particles are emitted on their own by radioactive nuclei like polonium, thorium and uranium etc.

Fig 14.2 shows the experimental arrangement for a-particle scattering experiment. A beam α -particles from radioactive Polonium passed through lead bricks to obtain a callimated beam. This linear beam striked 10^{-7}m thick foil of gold. A scintillator counter was used to detect scattered electrons at different angles. It has a screen of Zinc Sulphide which produces scintillations when α -particle strike it which can be seen using a microscope and in this way the numbers of particles scattered at different angles is measured.

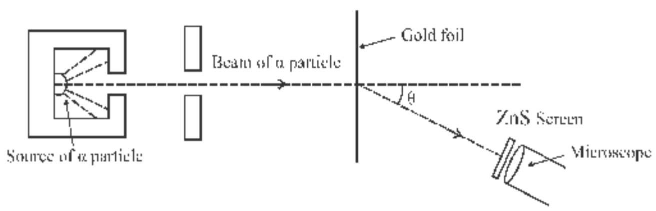


Fig 14.2 : Experimental arrangement of α scattering Experiment

Rutherford, Griger and Marden found that most of the α -particles passed through the foil undeviated or were deviated by very small angles. Rutherford concluded that the atom has lot of empty space. Hence Thomson's atomic model, which assumed atom to be uniformly distributed solid sphere, is not correct.

A more interesting result was that only a very few α -particles were deflected by 90^0 or more. It was observed that one of two particles out of 8000 returned in a direction opposite to the direction of incidence. In the word of Rutherfor "This is an event which I have never seen earlier in my life. It is like throwing a 15" cannon on a piece of paper which struck you back. It is impossible to explain large angle scattering by Thomson's model. Since α -particles are too heavy in comparison to electron the deviation in their path due to collision with electron will be negligible. For large angle scattering of α -particle it is essential that some large repelling force acts on it. In Thomson's model the positive charge is uniformly distributed in the solid sphere and hence it is not possible for a feeble positive charge to deflect α -particle by a large angle. Hence there is nothing in Thomson model which can explain the returning of α -particle. Rutherford argued that for a very large angle of scattering a great repulsive force should act on α -particle which is possible only when all the positive charge of the atom and almost all its mass is concentred at the centre of the atom called nucleus (instead of being distributed uniformly in the whole volume). When an α -particle comes very near to the nucleus without entering into it, a large repulsive force acts on it to scatter it by a large angle.

Fig 14.3 shows the path of some α -particles while crossing the atoms of gold foil. It can be easily seen that most of the α -particles either do not deviate or deviate through a very small angle. Only one or two α -particles that are reaching very near to nucleus are being deflected by large angle. Rutherford estimated that nucleus is of the order of 10^{-15} which is 10^5 times shorter than the atom. For comparison, if an atom is of the size of a large stadium the nucleus is like a housefly sitting in the centre of the stadium. In this way the whole volume of atom is mostly hollow. Since the atom is mostly hollow it becomes easy to explain how an α -particle comes out of the gold foil without deviation. As the thickness of the metal foil is very small it can be assumed that an α -particle is not scattered more than once while crossing the foil. Since nucleus of gold is almost 50 times heavier than an α -particle it can be assumed that gold nucleus remains stationary during scattering process.

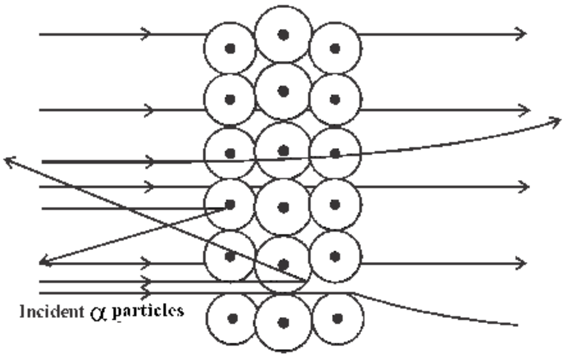


Fig 14.3 : Scattering of α particles

At what angle an α-particle deviates depends on how much near the centre of the atom it passes. α-particle passing very near to the centre are scattered by obtuse angles. In 1911 on the basis of α -particle scattering experiment Rutherford presented a new model in respect of atomic structure which is called nuclear model of atom. According to it :-

(i) All the positive charge of the atom and nearly all the mass is confined to a very small space of 10^{-15} m radius at the centre. This small space is called nucleus.

(ii) Out side the nucleus electrons are distributed in the hollow space of radius 10^{-10}m approximately. Hence most space in atom is hollow. All the positive charge in the nucleus is equal to the negative charge of electrons present in the atom.

(iii) If the electrons were at rest they would have fallen into the nucleus due to Coulumbian attraction force. Hence Rutherford assumed that electrons revolve around the nucleus and in circular paths and Coulombian attractive force provides needed centripetal force for the circular motion and only changes the direction of velocity.

In this way it is clear that Rutherford's model explain electric neutrality of atom and also hollow space in the most of the part. This model also explains emission of electrons from the atom. The nucleus is very heavy in comparison to electrons therefore when atom gets energy from external sources nucleus is not affected.

In Rutherford model electrons were supposed to be moving in circular paths. But this motion created problems for Rutherford model. An electron moving in a circular path is accelerated and according to electromagnetic theory accelerated electron should continuously emit electromagnetic radiation. It should happen at all temperatures. Due to emission of radiation the energy of electrons will decrease continuously and the radius of its path will also decrease and the electron will move in a spiral path towards the nucleus and will ultimately fall into it, (Fig 14.4), Such an atom cannot be stable. This is failure of Rutherford model.

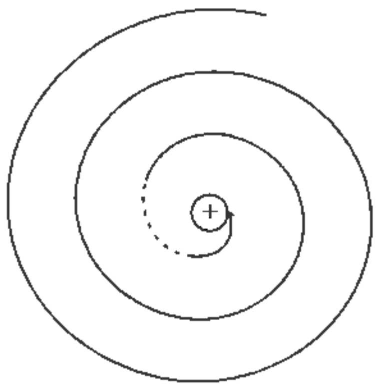


Fig 14.4 : Spiral path of electron according to Rutherford model

As mentioned above electron moving in a circular path should emit electromagnetic radiation at all temperatures the wavelength of radiation is related to frequency of revolution. Since, the radius of revolving electron is continuously decreasing the frequency of revolution should also change. Hence, an electron should emit radiations of continuously changing wavelength till it falls into the nucleus. Hence emission spectrum of the atom should be continuous. The experimental evidence is against it. The atom is not only stable but its characteristic spectrum is made up of definite wavelengths i.e. are line spectrum.

Example 14.1 : Find out the distance of closest approach of an α -particle of energy 2.5 MeV which is being scattered by a gold nucleus (Z-79)

Solution : The distance of closest approach is possible only when after scattering an α -particle is deviated by 180^0 and returns in the reverse direction.

This is the case of head on collision. At distance of closest approach the whole kinetic energy of α -particle K equals electrical potential energy of α -particle nucleus system and it comes to rest for a moment and then returns in the reverse direction due to Coulombian force of repulsion, Hence at this distance

$$K = \frac{(Ze)(2e)}{4\pi \in_0 d}$$

$$\therefore \qquad d = \frac{(Ze)(2e)}{4\pi \in_0 K} = \frac{2Ze^2}{4\pi \in_0 K}$$

For gold nucleus Z = 79,

$K = 2.5 MeV = 2.5 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 4.0 \times 10^{-13}$ J

On putting the values

$$d = \frac{2 \times 79 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4.0 \times 10^{-13}}$$

$= 9.10 \times 10^{-14}$ m

14.3 Bohr Model for Hydrogen Atom and Hydrogen like Ions

The scientist of Denmark Niel's Bohr did serious thinking about the problem of stability of atom and problem of continuous spectrum in Rutherford model. Although Bohr knew that according to classical physics electron orbits can not be stable but atom is stable. Therefore validity of widely accepted electromagnetic theory in atomic processes has to be reconsidered. It was also known at that time that for visible light, hydrogen atom cannot emit (or absorb) radiation of all wavelengths but hydrogen atom emits only 4 special wavelengths for visible light. Balmer, empirically gave a formula from which these wavelengths could be calculated.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \text{here } n = 3, 4, 5, 6 \ldots \qquad (14.1)$$

Where R is a constant later called Rydberg constant.

But neither Balmer nor anybody else could give theoretical explanation for the establishment of this formula. Bohr after seeing this formula in 1913 felt that he could explain the stability of atom and the formula provided some postulates were propounded. Bohr mixed classical physics with early quantum concepts and put forward his theory in the form of three postulates for hydrogen atom. Initially, this theory was for hydrogen atom but it could be applicable for hydrogen like ions like He+, Li++ because in these ions only one orbital electron exists. The postulates of Bohr's theory are given below

(i) In an atom electron revolves in some specific orbits of definite redii, while revolving in these orbits the electrons do not emit radiations (against the electromagnetic theory) These specific orbits are called stationary orbits. The centripetal force require for revolution in these orbits comes from Coulomb's attractive force.

(ii) The second postulate of Bohr defines stationary orbits. According to it the electron can revolve only in those orbits in which its angular momentum L is a multiple of $h/2\pi$ where h is Plank's constant. If nth orbit has a radius r_n, the velocity of electron is v_n and angular momentum is L_n then mathematically

$$L_n = mr_n v_n = n\frac{h}{2\pi} \qquad \ldots\ldots. 14.2$$

Where $n = 1, 2, 3 \ldots$ n is called the priciple quantum number and often $\hbar$ is written in place of $h/2\pi$

$$L_n = mr_n v_n = n\hbar \qquad \ldots (14.2(a))$$

The condition imposed by Eq. 14.2 is called Bohr's quantum condition.

(iii) For a given stationary orbit the energy is constant. Transition of an electron from one stationary orbit to another stationary orbit can take place. If the transition of electron from high energy orbit En_2 to lower energy orbit E_{n1} takes place the frequency of emitted photon (radiation) is given by Einstein equation

$$E_{n2} - E_{n1} = h\nu = hc/\lambda \qquad \ldots (14.3)$$

The electron can absorb energy from an external source in a lower energy orbit and jump to high energy orbit.

Now we will derive expressions for radius of stationary orbit, the velocity, momentum kinetic energy and total energy of electron moving in it, according to Bohr's model.

14.3.1 Radius of Electron Orbits

Now, we assume that charge in the nucleus is Ze (Z is the number of protons in the nucleus and for Hydrogen Z=1) and electron is moving at uniform speed of v_n in a circular path of radius r_n with its centre at the centre of the nucleus. For this type of motion the centripetal force is provided by the Coulombian attraction force between the nucleus and the electron i.e.

$$\frac{mv_n^2}{r_n} = \frac{Ze^2}{4\pi \in_0 r_n^2} \quad ...(14.4)$$

According to Bohr's quantum condition (eq 14.2)

$$mv_n r_n = \frac{nh}{2\pi}$$

or $$v_n = \frac{nh}{2\pi m r_n} \quad ...(14.5)$$

Putting the value of v_n from eq 14.5 in equation 14.4

$$\frac{m}{r_n}\left\{\frac{nh}{2\pi m r_n}\right\}^2 = \frac{Ze^2}{4\pi \in_0 r_n^2}$$

or $$r_n = \frac{\in_0 n^2 h^2}{\pi m Z e^2} \quad ...(14.6)$$

Ze for an ion similar to hydrogen is fixed $\in_0$, h, π, m and e are constants hence we see that permitted radii are proportional to n^2 or $r_n \propto n^2$. For every value of n there is a corresponding permitted orbit.

For n = 1 we have first orbit (minimum radius),

n = 2 we have second orbit and so on. For Hydrogen the radius of the first orbit is given by

$$r_1 = \frac{\in_0 h^2}{\pi m e^2}$$

Putting the values of other constants we have $r_1 = 0.529\ \AA \simeq 53\ pm$. The radius of the first orbit of hydrogen is also called Bohr's radius and it is denoted by a_0. Hence

$$a_0 = \frac{h^2 \in_0}{\pi m e^2} \quad ...(14.6\ (a))$$

From equation (14.6) and (14.6a)

$$r_n = \frac{n^2 a_0}{Z} \quad ...(14.7)$$

and for hydrogen

$$r_n = n^2 a_0 \quad ...(14.7(a))$$

Hence it is clear $r_n \propto 1/Z$ and $r_n \propto n^2$

14.3.2 Orbital speed of Electron

Putting the value of r_n from equation (14.5) in equation (14.4)

$$v_n = \frac{nh}{2\pi m\left\{\frac{\in_0 n^2 h^2}{\pi m Z e^2}\right\}}$$

or $$v_n = \frac{Ze^2}{2 \in_0 nh} \quad ...(14.8)$$

or

$$v_n = \frac{1}{4\pi \in_0}\frac{Ze^2}{n(h/2\pi)} = \frac{1}{4\pi \in_0}\frac{Ze^2}{nh}$$

From equation 14.8 it is clear that $v_n \propto Z$ and putting Z = 1 for $v_n \propto 1/n$ hydrogen we get -

$$v_n = \frac{e^2}{2 \in_0 nh} \quad ...(14.9)$$

and for velocity of electron in the first orbit of hydrogen atom

$$v_1 = \frac{e^2}{2 \in_0 h} = 2.189 \times 10^6 \text{ m/s}$$

or $$v_1 = \frac{c}{137} \text{ m/s} \quad ...(14.10)$$

Where c is the speed of light in vaccum.

Note : The ratio of speed of electron in first Bohr orbit (n=1) and the speed of light in vacuum c is called fine structure constant and is denoted by α

$$\alpha = \frac{v_1}{c} = \frac{e^2}{2 \in_0 hc} = \frac{1}{137} = 7.2397 \times 10^{-3} \quad ...(14.11)$$

14.3.3 Orbital frequency of Electron

Orbital frequency of electrons in n[th] orbit (or number of revolutions per second)

$$f_n = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi}\left(\frac{Ze^2}{2\in_0 nh}\right)\left(\frac{\pi m Z e^2}{\in_0 n^2 h^2}\right)$$

$$= \frac{mZ^2e^4}{4\in_0^2 h^3}\frac{1}{n^3} \qquad \ldots (14.12)$$

and the time period $\quad T_n = \frac{1}{f_n} = \frac{4\in_0^2 h^3}{mZ^2e^4}n^3$

clearly $\quad f_n \propto Z^2$ and $f_n \propto \frac{1}{n^3}$

14.3.4 Total Energy of Electron in n^th orbit

Total energy of electron E_n is equal to the sum of its kinetic energy K_n and potential energy U_n.

$$K_n = \frac{1}{2}mv_n^2$$

$$mv_n^2 = \frac{Ze^2}{4\pi\in_0 r_n}$$

$$\therefore \qquad K_n = \frac{Ze^2}{4\pi\in_0 (2r_n)} \qquad \ldots (14.13)$$

and

$$U_n = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r_n} = -\frac{1}{4\pi\in_0}\frac{Ze^2}{r_n}$$

$$\ldots (14.14)$$

Potential energy at infinity is Zero

Hence total energy of electron

$$E_n = K_n + U_n = \frac{1}{4\pi\in_0}\frac{Ze^2}{(2r_n)} - \frac{1}{4\pi\in_0}\frac{Ze^2}{r_n}$$

$$E_n = -\frac{1}{4\pi\in_0}\frac{Ze^2}{2r_n} \qquad \ldots (14.15)$$

$$E_n = -\frac{mZ^2e^4}{8\in_0^2 h^2}\left(\frac{1}{n^2}\right) \qquad \ldots (14.15A)$$

Front this equation it is clear that total energy of electron is quantized.

Putting the values of various constants

$$E_n = -\frac{2.18\times 10^{18}}{n^2}Z^2 J \qquad \ldots (14.16)$$

Atomic energies are generally denoted in eV instead of Joule.

Because 1eV = 1.6x10^{-19}J hence eq (14.16) reduces to

$$E_n = -\frac{13.6}{n^2}Z^2\, eV \qquad \ldots (14.17)$$

For hydrogen (Z=1) equation (14.15) to (14.17) can be written as

$$E_n = -\frac{me^4}{8\in_0 h^2 n^2} = -\frac{2.18\times 10^{18}}{n^2}J = -\frac{13.6}{n^2}eV$$

$$\ldots (14.18)$$

The total energy of electron in the orbit is negative hence it shows that electron is bound to the nucleus. Hence, to eject electron from the atom energy has to be given to the electron from an external source.

For hydrogen atom and for n=1 the total energy is

$$E_1 = -13.6 eV$$

Hence it can also be written that

$$E_n = -\frac{E_1}{n^2}$$

Similarly for $n = 2$

$$E_2 = \frac{E_1}{4} = -3.4 eV$$

for orbit $n = 3$, $\quad E_3 = \frac{E_1}{9} = -1.5 eV$

Fig 14.5 shows the energy levels of various stationary orbits of hydrogen. It is to be noted that total energy is negative, hence more magnitude means lower energy. The zero value is corresponding to $n = \infty$ whose physical meaning is that distance between electron and nucleus is infinite. The minimum energy level of the atom $(n = 1)$ is called ground level and all higher energy levels are called excited states.

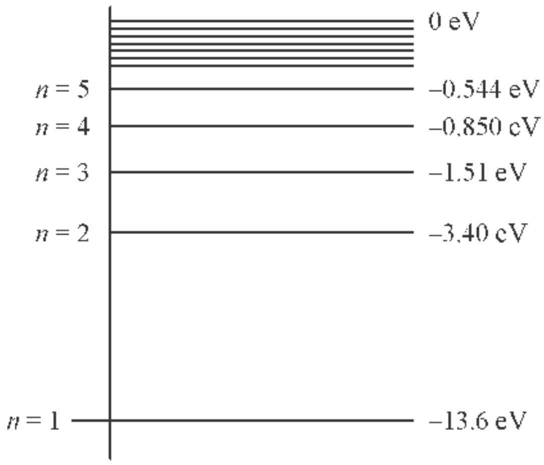


Fig 14.5 : Different energy levels for hydrogen atom

From the above discussion it is evident that K_n is always positive but U_n and E_n are always negative. Also

$$K_n = |E_n| = \frac{1}{2}|U_n|$$

Hence Kinetic energy of electron is numercally equal to magnitude of its total energy and half of the magnitude of its potential energy.

Example 14.2 : Find out the radius of the first orbit of Li which is similar to the atom of hydrogen.

Solution : For hydrogen like ions the formula for radius is

$$r_n = \frac{n^2 a_0}{Z}$$

Here Bohr radius $a_0 = 53$ pm

Z=3 for Li^{++} and for first orbit n = 1

$$r_1 = \frac{(1)^2\, 53}{3}\, pm$$

$$= 17.66\, pm$$

Example 14.3 : Assuming Bohr's atomic model to be true find out the expression for magnetic field of electron moving in the first orbit at the position of the nucleus of hydrogen atom in terms of fundamental constant.

Solution : For an electron moving in a circular path of radius r and having centre at the nucleus

$$\frac{mv^2}{r} = \frac{r^2}{4\pi \in_0 r^2}$$

or $$v^2 r = \frac{e^2}{4\pi \in_0 m} \quad \ldots \text{(i)}$$

For $n = 1$ according to Bohr's condition $mvr = h/2\pi$

or $$vr = \frac{h}{2\pi m} \quad \ldots \text{(ii)}$$

from equations (i) & (ii)

$$r = \frac{\in_0 h^2}{\pi m e^2} \quad \ldots \text{(iii)}$$

and $$v = \frac{e^2}{2 \in_0 h} \quad \ldots \text{(iv)}$$

The equivalent current for an electron moving with speed in a circle of radius r will be $i = \frac{ev}{2\pi r}$ such a current is like a current carrying circular loop which produces a magnetic field B at the centre.

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{4\pi r^2}$$

Putting the values of v and r from equations (iii) and (iv)

$$B = \frac{\mu_0 e}{4\pi} \frac{e^2}{2 \in_0 h} \times \frac{\pi m^2 e^4}{\in_0 h^2}$$

$$= \frac{\mu_0 e^2 \pi m^2}{8 \in_0 h^3}$$

Example 14.4 : For an atom the energies at levels A,B and C are E_A, E_B and E_C where $E_A < E_B < E_C$. If the wavelength of radiations due to transition of electrons from C to B, B to A and C to A are λ_1, λ_2 and λ_3 then prove that

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Solution : We know that $E_{n2} - E_{n1} = \frac{hc}{\lambda}$

Hence according to Question $E_C - E_B = \frac{hc}{\lambda_1}$ . . . (i)

$$E_B - E_A = \frac{hc}{\lambda_2} \qquad \text{. . . (ii)}$$

and $$E_C - E_A = \frac{hc}{\lambda_3} \quad \text{. . . (iii)}$$

Adding (i) and (ii)

$$E_C - E_A = hc\left[\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right] \qquad \text{. . . (iv)}$$

Compering equations (iii) and (iv)

$$\frac{hc}{\lambda_3} = hc\left[\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right]$$

or $$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

or $$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Example 14.5 : In hydrogen atom when an electron jumps from $n = \infty$ to $n = 3$ what will be the wavelength of the emitted radiation?

Solution : For hydrogen atom $E_n = -\frac{13.6}{n^2} eV$

$\therefore \quad E_\infty = 0$

and $$E_B = -\frac{13.6}{3^2} = -\frac{13.6}{9} = -1.51 eV$$

Hence wavelength emitted because of transition

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_\infty - E_3}$$

$$= \frac{1242 eV\ nm}{1.51 eV} = 822.51 nm$$

14.4 Line Spectrum of Hydrogen and its explanation

When hydrogen gas is filled in a sealed tube and is heated or it is filled in a tube at low pressure and current is passed through it, radiation is emitted. Radiation obtained in this way is analysed by a spectrometer and it is found that only a few specific wavelength are there. Such a spectrum is called emission line spectrum and shining lines on a black background are visible. A part of the spectrum of atomic hydrogen gas is shown in fig 14.6

In the visible part of this spectrum four main lines can be seen and the corresponding wavelength are 656.3nm, 486.1nm, 434.1 nm and 480.2nm respectively. In 1885 Swiss teacher Johan Balmer on the basis of this experiment found that wavelength of these lines and lines in the same range can be expressed by the following equation.

$$\lambda = \frac{364.56 n^2}{n^2 - 4} \quad \text{here } n = 3, 4, 5...$$

The lines obtained from the above formula are expressed in nm. The series of wavelengths so obtained is called Balmer series. Some lines of Balmer series are found near ultraviolet region. After a few years the scientist called Rydberg expressed the formula of Balmer in a simpler way as follows

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \text{here } n = 3, 4, 5... \qquad \text{. . . (14.19)}$$

Where R is a constant which is now called Rydberg constant.

If the wavelength λ is expressed in units of metres, then the value of R is

$$R = 1.09737 \times 10^7\ m^{-1} \simeq 10^7\ m^{-1}$$

The formula of both Balmer and Rydberg were correct for wavelength of Balmer series but were not based on a theoretical model as they were formulae based on experience. On the basis of these formula it was not possible to explain the reason for the presence of specific wavelength in the line spectrum of hydrogen. It was discovered later that besides Balmer series there are other series also in the line spectrum of hydrogen which were named after their original discoverers such as Lyman series, Paschen series, Bracket series and Pfund series. In fig 14.6 Lyman series and Paschen series are also shown besides Balmer series. It can be seen that towards lower wavelength side of each series, the interval between spectral times goes on decreasing and in the end a continuum is visible.

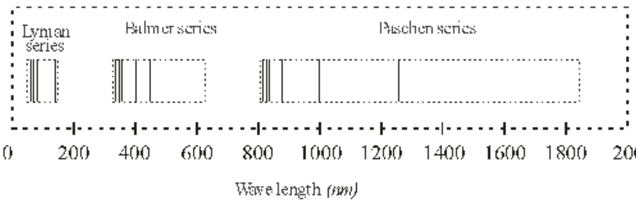


Fig 14.6

In fig 14.6 some portion of the line spectrum of hydrogen is depicted in which Lyman series, Balmer series and Paschen series are shown. The other two series, Bracket series and Pfund series which are found in far infra red region could not be shown in view of their large wavelengths.

These series can be expressed by the following formulae : -

Lyman series $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) n = 2, 3, 4\ldots$ . . .(14.20)

Paschen series $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) n = 4, 5, 6\ldots$ (14.21)

Bracket series $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) n = 5, 6, 7, 8\ldots$ (14.22)

Pfund series $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) n = 6, 7, 8\ldots$ . . .(14.23)

14.4.1 Explanation of Hydrogen spectrum by Bohr's Theoty

At room temperature atomic hydrogen does not emit emission spectrum because nearly all the hydrogen atoms are in their ground states *(n=1)* at such temperatures and no states of lower energy are available for transition of electrons. Hence atoms do not emit radiation. When gas is given energy by heat or electric discharge or any other source, transition of electron to high energy levels *n=2, n=3* etc. take place. When electron returns to lower energy levels atoms emit electromagnetic radiation. According to 3rd postulate of Bohr when eletron jumps from high energy level E_{n2} to lower energy level E_{n1} the emitted energy is given by eqn. 14.3, which is written below

$$E_{n2} - E_{n1} = h\nu = \frac{hc}{\lambda}$$

$$\therefore \quad \frac{1}{\lambda} = \frac{E_{n2} - E_{n1}}{hc} \qquad \ldots (14.23)$$

According to Bohr's theory the total energy of electron in nth orbit is given by equation (14.15a) according to which

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\frac{1}{n^2}$$

$$\therefore \quad E_{n2} = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\frac{1}{n_2^{\,2}}$$

and $$E_{n1} = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\frac{1}{n_1^{\,2}}$$

Putting the values of E_{n2} and E_{n1} in equation (14.24), we have

$$\frac{1}{\lambda} = \frac{mZ^2e^4}{8\in_0 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \ldots (14.25)$$

The quantity $\frac{1}{\lambda}$ is also called wave number and is denoted by $\bar{\nu}$

The frequency of radiation

$$\nu = \frac{c}{\lambda} = RCZ^3\left[\frac{1}{n_1^2} - \frac{1}{n_1^2}\right] \qquad \ldots (14.26)$$

where $$R = \frac{me^4}{8\in_0 h^3 c} \qquad \ldots (14.27)$$

is called Rydberg constant

Putting the values of constants in the expression for R, its value obtained is $1.097373 \times 10^7 \text{ m}^{-1}$. In terms of Rydberg constant the energy of electron in nth orbit is given by

$$E = -\frac{Rhc^2Z^2}{n^2} \qquad \ldots (14.28)$$

It is useful to remember that $Rhc = 13.6$ eV

Many times the energy of the atom is expressed in Rydberg unit, 1 Rydberg or 1 R = -13.6 eV

For hydrogen atom (Z=1) equation (17.25) gives

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \ldots (14.29)$$

It may be noted that in above relations $n_2 > n_1$

If in equation (14.29) $n_1 = 2$ and $n_2 = n$ where n=3,4,5 etc.

then we get

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad n = 3, 4, 5$$

This equation is exactly the same as Balmer and Rydberg mentioned on the basis of experience. (see equation 14.19). This result is a very big achievement of Bohr's theory. In this way by putting values of n_1 and n_2 in equation (14.29) we can get expression for different series, eg.

$n_1 = 1$ and $n_2 = 2,3,4$ .... we get expression for Lyman series

$n_1 = 2$ and $n_2 = 3,4,5$ .... we get expression for Balmer series

$n_1 = 3$ and $n_2 = 4,5,6$ .... we get expression for Pascher series

$n_1 = 4$ and $n_2 = 5,6,7$ .... we get expression for Bracket series

$n_1 = 5$ and $n_2 = 6,7,8$ .... we get expression for Pfund series

These have already been given by equation (14.20) to (14.23)

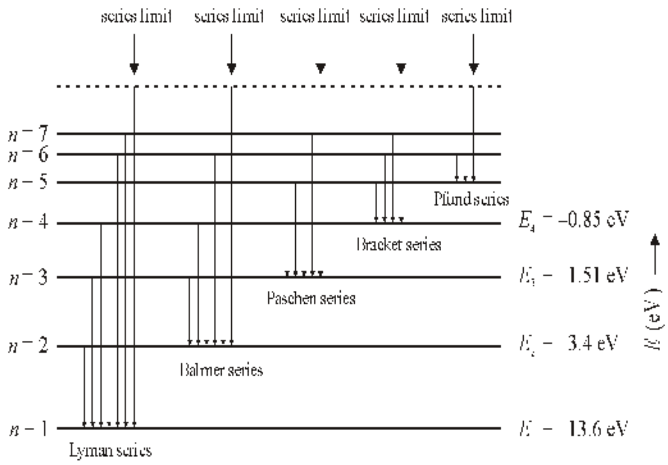


Fig 14.7 : Differents engry levels and corrosponding trasitions for hydrogen atom for different series

Fig 14.7 shows the different energy levels and electron transisons for different series of hydrogen spectrum.

Lyman series is obtained when an electron jumps from high energy levels $(n_2 = 2, 3, 4, ...)$ to ground energy state $n_1 = 1$

In equation (14.20) putting $n = 2$ and $n = \infty$ we get the longest wavelength 1216Å and lowest wavelength 912Å for this series. The shortest wavelength of any series is called series limit. The limit for Lyman series is 912Å. Lyman series is in the ultraviolet region of electromagnetic spectrum.

For Balmer series the electron jumps from high energy levels (n_2=3, 4, 5.....etc to n_1=2) . This series is found in ultraviolet and visible region of electromagnetic spectrum. In this the longest wavelength is 6563Å and shortest wavelength series limit is 3646Å.

For Paschen series the transition of electron takes place from high energy levels n_2=4,5,6 etc. to third energy state n_1=3. The lines of this series are found in infrared region of the spectrum. In this the longest wavelength is 18751Å and the shortest wavelength is 8220Å.

For Bracket series transition of electron takes place from high energy states n_2=5,6,7 etc to fourth energy state n_1=4. These lines are also found in infrared region. The longest and the shortest wavelengths of this series are 40477Å and 14572 Å, respectively

For Pfund series, transition of electron is from high energy states n_2=6,7,8....etc to 5th energy state n_1=5. The lines of this series are also in infrared region. The longest and the shortest wavelengths are 74515Å and 22768Å, respectively.

When radiation (Photon) whose energy is just equal to energy required by an electron to take it from lower energy level fall on an atom to higher energy level then absorption of that photon takes place. If radiation of continuous frequencies after passing through a rarified gas are analysed by a spectrometer then a series of dark absorption lines in the continuous spectrum are seen which correspond to those wavelength which have been absorbed. Nearly, all the atoms of hydrogen lie in the ground state (n=1) hence in hydrogen absorption from $n = 1$ to high energy levels is only possible.

Hence in absorption spectrum of hydrogen generally, lines of only Lyman series are present. The absorption transition of Balmer series should begin from n=2 but generally there are no electrons in n=2 state hence such absorption transition are not possible. Hence in absorption spectrum of hydrogen atom Balmer series as well as Paschen, Brackett and Pfund series are not present. If the temperature is too high as in the sun then many atoms of $n-2$ are present and their transition from $n-2$ to higher states is possible. Thus Balmer series is present in absorption spectrum of Sun.

14.5 Ionisation and Excitation Potential

Generally, electrons are present in ground state in an atom but if energy is given to an atom then transition of electron to high energy levels takes place. For example, the electron in hydrogen remains in an orbit defined by principal quantum number n=1 where energy is -13.6eV. Now, if electron is given energy more than 13.6 V it means total energy of electron is now positive. Actually zero total kinetic energy corresponds to that condition where separation between electron and nucleus is infinite. In such a condition electron is not bound to nucleus and is free to go anywhere. In this condition atom is ionised. The minimum energy needed to ionise an atom is called ionisation energy. The potential difference which accelerates an electron so that it receives ionisation energy is called ionisation potential. The ionisation energy for hydrogen is 13.6eV and ionisation potential is 13.6eV. The energy required by an atom to go from ground state to an excited state is known as excitation energy and potential difference corresponding to it is known as excitation potential. For example the energy needed to excite a hydrogen atom from its ground state (n=1) to first excited state, where total energy is -3.4eV, is -3.4-(-13.6) = 10.2eV. This is excitation energy for orbit n=2 and the is excitation potential is 10.2 Volt.

Bohr atomic model propounded the presence of discrete energy levels in an atom. Bohr's concept was found correct by experimental confirmation of discrete energy levels in an atom by Frank Hertz's experiment in 1904. The explanation of hydrogen spectrum given by Bohr model was very helpful in the development of modern quantum theory. For these achievements Bohr was awarded Noble Prize.

Example 14.6 : If initially the electron is excited in principal quantum number 3 energy level how many wavelength will be observed due to transition of electron to lower energy levels?

Solution : Electron present in the n^{th} state can undergo transition to $(n-1)^{th}$, $(n-2)^{th}$ ....2^{nd}, 1^{st} energy levels state. Thus transition from nth to $(n-1)^{th}$, (n-1) to (n-2)th etc. is possible. Similarly transitions to lower energy levels can be considered. Suppose N is the total number of possible transitions

$$N=(n-1)+(n-2)+(n-3)+...+2+1$$

$$=\frac{n(n-1)}{2}$$

Example 14.7 : Find out the wavelength in the emitted radiations when an excited electron in n=4 state returns to ground state.

Solution : As given in example 14.6 the total number of emitted waves, for n = 4

$$N=\frac{(n)(n-1)}{2}=\frac{4\times 3}{2}=6$$

These will be corresponding to transitions from n=4 to n=3, n=4 to n=2, n=4 to n=1, n=3 to n=2, n=3 to n=1 and n=2 to n=1

Energies for n=1,2,3 and 4 are

$$E_1=-13.6eV$$

$$E_2=-\frac{13.6eV}{4}=-3.4eV$$

$$E_3=-\frac{13.6eV}{8}=-1.51eV$$

and $$E_4=-\frac{13.6eV}{16}=0.85eV$$

due to transition from $n=4$ to $n=1$ the wavelength of emission will be

$$\lambda=\frac{hc}{\Delta E}=\frac{1242eV\cdot nm}{(13.6-0.85)eV}=97.4nm$$

due to transition from $n=4$ to $n=2$ the wavelength of emission will be

$$\lambda=\frac{hc}{\Delta E}=\frac{1242eV\cdot nm}{(3.4-0.85)eV}=487nm$$

due to transition from $n=4$ to $n=3$ the wavelength of emission will be

$$\lambda=\frac{hc}{\Delta E}=\frac{1242eV\cdot nm}{(1.51-0.85)eV}=1881nm$$

Simillary for transion from $n=3$ to $n=1$ the wavelength will be

$$\lambda = \frac{hc}{\Delta E} = \frac{1242}{(13.6-1.51)} \frac{eV \cdot nm}{eV} = 103\,nm$$

The wavelength obtained as a result of transition from n=3 to n=2 is 654 nm

and for n=2 to n=1 is 122 nm

Hence different wavelengths found are 97.4 nm, 487nm, 1881nm, 103 nm, 654 nm and 122 nm

Example 14.8 : For hydrogen atom the wavelength for second line in Balmer series is 4861Å. Find out the wavelength of 4th line in this series.

Solution : The comprehensive formula for wavelength of Balmer series is

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

Second and fourth lines will be obtained when electron jumps from n=4 and n=6 to n=2. If the wavelength of corresponding lines are λ_2 and λ_4 respectively we have

$$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}R \quad \ldots \text{(i)}$$

$$\frac{1}{\lambda_4} = R\left[\frac{1}{2^2} - \frac{1}{6^2}\right] = \frac{8R}{36} \quad \ldots \text{(ii)}$$

From equ (i) and (ii)

$$\frac{\lambda_4}{\lambda_2} = \frac{3R}{16} \cdot \frac{36}{8R} = \frac{27}{32}$$

$$\therefore \quad \lambda_4 = \frac{27}{32} \times \lambda_2 = \frac{27}{82} \times 4861\mathring{A} = 4101.5\mathring{A}$$

Example 14.9 : If in the spectrum of hydrogen atom the wavelength of first line of Lyman series is 1215Å, find out the wavelength of second line of Balmer series.

Solution : for first line of Lyman series the transition will be from $n_2 = 2$ to $n_1 = 1$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \quad \ldots \text{(i)}$$

The transition for 2nd line of Balmer series will be from $n_2 = 4$ to $n_1 = 2$

Suppose its wavelength is we have λ_2'

$$\frac{1}{\lambda_2'} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16} \quad \ldots \text{(ii)}$$

from equation (i) and (ii)

$$\frac{\lambda_2'}{\lambda_1} = \frac{3R}{4} \times \frac{16}{3R} = 4$$

$$\therefore \quad \lambda_2' = 4\lambda_1 = 4 \times 1215\mathring{A} = 4860\mathring{A}$$

Example 14.10 : The wavelength of first line of Lyman series of hydrogen is equal to wavelength of second line of Blamer series of hydrogen like ion X. Calculate the energies of first two states of X.

Solution : For hydrogen like ion he wavelength is given by

$$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The wavelength of first line of Lyman series $(n_1 = 1,\ n_2 = 2)$ of hydrogen atom (Z=1)

The wavelength line

$$\frac{1}{\lambda_H} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R$$

According to question

$$\lambda_X = \lambda_H$$

$$\therefore \quad \frac{3}{4}R = \frac{3}{16}Z^2 R$$

or $\quad Z = 2$

This ion is ionised Helium

Also $(E_X)_n = Z^2 (E_H)_n = 4(E_H)_n$

ground state of hydrogen $n = 1$, $E_H = -13.6$

$$\therefore \quad (E_H)_n = -\frac{13.6}{n^2}$$

and $\quad (E_X)_n = -4\dfrac{13.6}{n^2}$

For first state of x

$$(E_X)_1 = -4(13.6) = -54.4\,eV$$

and for second state of x

$$\left(E_X\right)_2 = -4(13.6) = -13.6\,eV$$

Example 14.11 : A group of energy levels in a hydrogen like atom X wavelength are emitted due to all possible transitions. The group has energies between -0.85eV and -0.544eV (including these two values). Find out the atomic number of the atom (ii) calculate the minimum wavelength emitted due to these transition. (given hc=1242eVnm and ground energy of hydrogen atom = -13.6eV)

Solution : The energy of nth state of an atom of atomic number Z

$$E_n = -Z^2 \frac{13.6}{n^2} eV$$

For 6 transitions four consecutive energy levels are necessary. Suppose their quantum numbers are n, n+1, n+2 and n+3 then

$$-Z^2 \frac{(13.6)}{n^2} = -0.85\,eV \qquad \ldots (i)$$

and

$$-Z^2 \frac{(13.6)}{(n+3)^2} = -0.544\,eV \qquad \ldots (ii)$$

Dividing eqn. (i) by eqn. (ii)

$$\frac{(n+3)^2}{n^2} = \frac{0.85}{0.544} = 1.5625$$

$$\frac{n+3}{n} = \sqrt{1.5625} = 1.25$$

hence $n = 12$

Putting the value of n in eqn (i)

$$-Z^2 \frac{(13.6)eV}{144} = -0.85\,eV$$

or $$Z^2 = \frac{0.85 \times 144}{13.6} = 9$$

or $$Z = 3$$

(ii) The wavelength emitted by transition between two energy levels having a difference ΔE is given by

$$\lambda = \frac{hc}{\Delta E}$$

For λ to be minimum ΔE should be maximum

$$\therefore \quad (\Delta E)_{max} = E_{n+3} - E_n = -0.544\,eV - (-0.85\,eV)$$

$$= 0.306 \text{ eV}$$

$$\therefore \quad \lambda_{min} = \frac{hc}{(\Delta E)_{max}} = \frac{1242\,eV.nm}{0.306\,eV}$$

$$= 4503 \text{ nm}$$

14.6 Limitations of Bohr Model

Inspite of Bohr model's achievement like explanation of stability of atom and line spectrum of hydrogen it had some limitations that are described below : -

(i) This model was valid for hydrogen and hydrogen like ions. It can not be applied to atoms having more than one electron, not even to helium. In Bohr's model there was no provision for mutual interactions of electrons in atom having more than one electron.

(ii) In Bohr's model electrons were supposed to be moving in circular orbits but did not emit radiation even though they are in accelerated motion. For this postulate there was no theoretical explanation available in Bohr's model.

(iii) If spectral lines of hydrogen are observed by a microscope having high resolving power it is observed that each line observed earlier is a set of many closely spaced lines. This fine structure can not be explained by Bohr's model.

(iv) This model does not give any information about intensity of spectral lines.

(v) This model failed to explain splitting of spectral lines due to magnetic field, (Zeeman effect) and splitting due to electric field (Stark effect).

(vi) Generally, motion due to centripetal force is in elliptical orbits but in Bohr's model orbit are circular. It is worth mentioning that both position and momentum of electron can not be simultaneously determined accurately due to Heisenberg's uncertainity principle. In Bohr's model position and speed of electrons are simultaneously discussed.

14.7 Explanation of Bohr's second postulate by matter waves

According to second postulate of Bohr's theory only those orbits are permitted in which the angular momentum of electron is quantised i.e. $mvr = nh/2\pi$. As has been mentioned earlier no theoretical explanation has been given. In 1923, de Broglie on the basis of matter wave concept, proposed that in Bohr's model the electron moving in circular orbits should be treated as a matter wave and such a wave should be considered as a stationary wave for which it is essential that the circumference has integral number of wave lengths. Hence the circumference of the orbit is an intergral multiple of wavelength. Mathematically, an electron moving in a circle of radius r should fulfil the following relation

$$2\pi r = n\lambda \qquad n = 1, 2, ... \qquad (14.30)$$

For electron the wavelength of matter wave

$$\lambda = h/mv$$

Hence putting this value of λ in eq. (14.30) we have

$$2\pi r = \frac{nh}{mv}$$

or $$mvr = \frac{nh}{2\pi}$$

This is Bohr's second postulate.

In figure (14.8) for circular orbit, n=4 a stationary wave has been shown where the circumference has four de Broglie waves. Hence $2\pi r = 4\lambda$

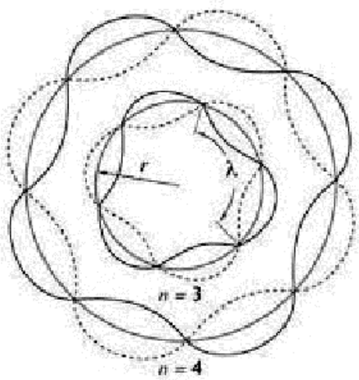


Fig. 14.8 : The stationery wave accordingly to deBroglie principle for n=4

To understand de Broglie argument in a better way let us consider the physical phenomenon of resonance. For resonance, in a specific length whole number of waves are present. For example when in a closed pipe, along the length of the pipe, should waves are spread then due ot reflections at the ends of the pipe constructive interference takes place and stationary waves to high amplitude are formed and resonance occurs. In fig (14.8) if an integral number of waves are not present then after some revolutions wave will have opposite phases and will have in significant amplitude. This is the reason why de Broglie proposed an integral number of waves in the circumference (closed path)

Example 14.12 : The energy of hydrogen atom from ground state is -13.6eV. Calculate de Broglie wavelength of electron in this condition. For n=1 find out the circumference and compare it with de-Broglie wavelength. What do you conclude from this? (given : Bohr's radius a = 53pm)

Solution : From Bohr's theory we know that the kinetic energy of electron K is numerically equal to its total energy.

$$K = |E|$$

Hence in first orbit the kinetic energy of electron will be K = 13.6eV. The electron gets this energy on being accelerated by 13.6eV. Hence the de-Broglie wavelength of this electron

$$\lambda = \frac{12.27}{\sqrt{V}}\mathring{A} = \frac{12.27}{\sqrt{13.6}} = 3.32\,\mathring{A}$$

Given $a_0 = 53\ pm = 53 \times 10^{-6}\ m$

Hence circumference of first orbit

$$= 2\pi a_0 = 2 \times 3.14 \times 53 \times 10^{-6} = 3.32\,\mathring{A}$$

This is equal to the wavelength of matter wave of electron. Hence in orbit n=1 one full de-Broglie wave length will be present.

Important Points

1. An atom has a whole is electrically neutral. The cathode rays obtained by discharge through gases at low pressure are actually negatively charged particles called electrons which are essential component of every atom. For electrical neutrality of atom it is necessary that it has positive charge the same amount as it has negative charge due to electrons.

2. In Thomson's atomic model an atom is supposed to be a positively charged sphere in which electrons are embeded inside it. This model could satisfactorily explain the stability of atom, ionisation of gases and thermal emission and was not able to explain the line spectrum of hydrogen. It was unsuccessful to explain the experimental observations of α -particle scattering experiment of Rutherford.

3. On the basis of α-particle scattering experiment Rutherford concluded that the atom has mostly empty space inside it. According to nuclear model of Rutherford the whole positive charge and nearly all mass of the atom is confined to a very small space called nucleus. Electrons revolve around it. The size of nucleus is about 1/10000th of the size of the atom.

4. The main short comings of in Rutherford's model are-

(i) This model can not explain stability of the atom, because electrons revolving around nucleus are in accelerated state and should emit electromagnetic radiation and should move in spiral path and ultimately fall into the nucleus.

(ii) This model can not explain the characterstic line spectrum of hydrogen.

5. To explain the stability and line spectrum of hydrogen atom Bohr proposed a model for hydrogen and hydrogen like ions whose three important postulates are

(i) In an atom the electrons revolve in definite orbits without emitting radiation. These orbits are called stationary orbits.

(ii) Stationary orbits are those orbits in which angular momentum of electron is an integral multiple of $h/2\pi$

$$L_n = mv_n r_n = \frac{nh}{2\pi} \quad n - 1,2,3....$$

Where n=1,2,3...... and n is called principal quantum number.

(iii) When an electron jumps from a stationary higher energy orbit to a lower energy orbit a photon is emitted whose energy is equal to the difference of energy levels of initial and final stationary orbits i.e.

$$hv = E_{n2} - E_{n1}$$

Where v is the frequency of emitted photon.

6. For hydrogen like atoms the radii of permitted orbits are given by

$$r_n = \frac{\in_0 n^2 h^2}{\pi m Z e^2}$$

and energy of electrons in these orbits is given by

$$E_n = -\frac{mZ^2e^4}{8\in_0 h^2}\left(\frac{1}{n^2}\right) = -\frac{13.6}{n^2}Z^2\left(eV\right)$$

For hydrogen (Z=1) the energy of electron in ground state (n=1) is -13.6eV

7. For the characteristic spectrum of hydrogen, the formula for different series and corredponding wave

length are

(i) Lyman series $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$ $n = 2, 3, 4, ...$

(in ultraviolet region)

$\lambda_{min} = 912\,Å$ $\lambda_{max} = 1216\,Å$

(ii) Balmer series $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n = 3, 4, 5, ...$

(in ultraviolet and visible region)

$\lambda_{min} = 3646\,Å$ $\lambda_{max} = 6563\,Å$

(iii) Paschen series $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ $n = 4, 5, 6, ...$

(Infrared region) $\lambda_{min} = 8107\,Å$ $\lambda_{max} = 18751\,Å$

(iv) Brackett series $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ $n = 5, 6, 7, ...$

(Infrared region)

$\lambda_{min} = 14572\,Å$ $\lambda_{max} = 40477\,Å$

(v) Pfund series $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$ $n = 6, 7, 8, ...$

(Infrared series) $\lambda_{min} = 22708\,Å$ $\lambda_{max} = 74515\,Å$

8. Bohr's model could explain line spectrum of hydrogen. For wavelength of emitted lines the comprehensive formula is

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where $n_2 > n_1$

By choosing proper value of n_1 and n_2 the wavelength of different lines of different series can be found.

9. Bohr's model is valid only for hydrogen like atoms. It can not be applied to even two electron atom like Helium. Even for hydrogen, this model can not explain relative intensities of different spectral lines, fine structure of spectral lines, Zeeman effect and Stark effect etc. In adition to this it does not provide any theoretical basis for second postulate also.

10. The second postulate of Bohr regarding quantisation of angular momentum can be explained on the basis of de-Broglie matter wave concept. Circular orbits correspond to stationary waves where the circumference is an integral multiple of de-Broglie wave length i.e.

Where $2\pi r = n\lambda$ $n = 1, 2, ...$

Questions For Practice

Multiple choice question

1. The energy of hydrogen atom in ground state is -13.6eV. Its energy n=5 level will be -

(a) -0.5eV (b)-0.85eV

(c) -5.4eV (d) -2.7eV

2. Energy of n^{th} orbit of hydrogen atom is $E_n = -\frac{13.6}{n^2}eV$. The energy needed to send an electron from first orbit to second orbit will be-

(a) 10.2eV (b) 12.1eV

(c) 13.6eV (d) 3.4eV

3. In hydrogen atom if an electron jumps from third to second orbit the wavelength of emitted radiation

(a) $\frac{5R}{36}$ (b) $\frac{R}{6}$

(c) $\frac{36}{5R}$ (d) $\frac{5}{R}$

4. In which part of the electromagnetic spectrum Lyman series of hydrogen is found : -

(a) Ultraviolet (b) Infrared

(c) Visible (d) X-ray region

5. The number of spectral lines emitted by hydrogen atom excited to n=4 energy level is

(a) 2 (b) 3 (c) 4 (d) 6

6. For Lyman series of hydrogen the minimum and maximum wave lengths are

(a) 909 Å and 1212 Å

(b) 9091 Å and 12120 Å

(c) 303 Å and 404 Å

(d) 1000 Å and 3000 Å

7. For some atom, when electronic transition takes place from 2 E to E the emitted photon's wavelength is λ. Wavelength of emitted photon will be when transition takes place from 4 E/3 energy state to E energy state : -

(a) $\lambda/3$ (b) $3\lambda/4$

(c) $4\lambda/3$ (d) 3λ

8. In an excited hydrogen atom if angular momentum according to Bohr' quantum condition is $\left(\frac{2h}{2\pi}\right)$ its energy will be -

(a) -13.6eV (b) -13.4eV

(c) -3.4eV (d) -12.8eV

9. What will be the principal quantum number of the excited state from which hydrogen atom jumps to its ground state by emitting a photon of wave length λ

(a) $\sqrt{\frac{\lambda R}{\lambda R-1}}$ (b) $\sqrt{1-\lambda R}$

(c) $\sqrt{\frac{\lambda}{\lambda R-1}}$ (d) $\sqrt{\frac{1-\lambda R}{R}}$

10. Which one of the following variables remains the same in all hydrogen like ions in their ground state.

(a) orbital speed of the electron

(b) radius of the orbit

(c) angular momentum

(d) energy of the atom

11. The energy of a hydrogen like ion in its ground state is -54.4eV it can be

(a) He+ (b) Li++

(c) Deuterum+ (d) Be+++

12. In a hydrogen atom on increasing the value of principal quantum number 'n' the potential energy will -

(a) decrease (b) increase

(c) remains the same

(d) potential energy decreases and increases alternately

13. Transition of hydrogen atom takes place from n=4 to n=1, recoil momentum of hydrogen atom (in unit of eV/c) is : -

(a) 13.60 (b) 12.75

(c) 0.85 (d) 22.1

14. The magnetic moment due to orbital motion of an electron in n^{th} orbit of hydrogenin term of (angular momentum = L)

(a) $\frac{-neL}{2m}$ (b) $\frac{-eL}{2m}$

(c) $\frac{-eL}{2mn}$ (d) $\frac{-eLm}{m}$

15. When hydrogen atom goes from ground state to first excited state the angular momentum increases by

(a) $6.63 x 10^{-34}$Js (b) $1.05 x 10^{-34}$Js

(c) $41.5 x 10^{-34}$Js (d) $2.11 x 10^{-34}$Js

Very short answer type questions : -

1. Which experiment indicated that the whole positive charge of an atom is confined to a very minute space in the centre?
2. Write two short comings of Rutherford model regarding structure of matter.
3. In hydrogen atom if the angular momentum of electron has the value h/π in which orbit it is situated?
4. In which region of the electromagnetic spectrum of hydrogen the Lyman series is present?
5. The energy of electron in first orbit of a hydrogen like atom is -27.2eV. What shall be its energy in third orbit?
6. What is the ratio of radii of different orbits in hydrogen atom?
7. What is the potential energy of electron in eV in the first orbit of hydrogen atom?
8. If the radius of first Bohr orbit of hydrogen atom is 0.5A, write dowm the value of radius of fourth Bohr orbit.
9. Write down the wavelength of the last line of Balmer series.
10. Write down the formula for quantisation of angular momentum in Bohr's theory.
11. Write down the name of the series whose few lines fall in visible region of hydrogen spectrum.
12. On what hypothesis it is possible to explain second postulate of Bohr theory?

Short Answer type Questions : -

1. Write the short comings of Thomson's atomic model.
2. Mention the main consideration in Rutherford's atomic model.
3. Explain briefly how the Rutherford atomic model is not able to explain stability of the atom.
4. Write the short comings of Bohr's Theory.
5. Hydrogen atom has only one electron but there are many lines in its emission spectrum. Explain briefly how it is possible.
6. Explain how element can be identified by studying line spectra?
7. In a sample of hydrogen gas most of the atom are in n=1 energy level. When visible light passes through it some spectrum lines are absorbed. Lines of which series (Lyman or Balmer) are most absorbed and why?
8. According to Bohr's theory what is meant by stationary orbit and what is the condition for it?
9. Balmer series was observed and analysed before other series. Can you give some reason for it?
10. In Bohr model the total energy in n^{th} orbit is E_n and angular momentum is L_n. What is the relation between them?

Essay type Question

1. Describe briefly the α-particle scattering experiment of Rutherford. How was nucleus discovered by it?
2. What were the short comings of in Rutherford model? Explain in detail how Bohr removed them in his model.
3. Write Bohr's postulates for hydrogen atom. Derive a formula for the total energy of the electron in nth orbit.
4. Explain line spectrum of hydrogen atom on the basis of Bohr's atomic model.
5. Write the short comings of Bohr model. Explain how quantisation of orbital angular momentum

can be explained on the basis of de-Broglie matter wave theory.

6. Derive the formula for the radii of stationary orbits of hydrogen atom according to Bohr's model and prove that the radii of stationary orbits of hydrogen atom are in the ratio of 1:4:9.............

Answers

Multiple type questions

1. (a) 2. (a) 3. (c) 4. (a) 5. (d) 6. (a) 7. (d)

8. (c) 9. (a) 10. (c) 11. (b) 12. (a)

13. (b) 14. (b) 15. (b)

Very short answer questions

1. Rutherford α particle scattering experiment
2. (i) Failure to explain stability

   (ii) Failure to explain line spectrum
3. second orbit
4. In ultraviolet region
5. -3.02eV
6. 1:4:9..........
7. -27.2eV
8. $r_4 = n^2 r_1 = 16 \times 0.5 = 8.0$ Å
9. 3048Å
10. $L_n = nh/2\pi$ or $mv_n r_n = nh/2\pi$
11. Balmer series
12. de-Broglie matter wave theoty

Numerical Questions

1. Find out radius of Bohr's second orbit, speed of electron in it and its total energy for hydrogen atom.

   (Given : mass of electron $m = 9 \times 10^{-31}$ kg] $e = 1.6 \times 10^{-19}$ C] $h = 6.6 \times 10^{-34}$ Js)

   (Ans : 2.116 Å, 1.1×10^6 m/s, –3.4 eV)

2. If the wavelength of first line of Lyman series is 1216Å. Find out the radii of first lines of Balmer and Paschen series.

   (Ans : $\lambda_{B1} = 6566.4$ Å , $\lambda_{P1} = 18761.1$ Å)

3. In an atom for transition from energy level A to C the wavelength of emitted photon is 1000Å and for transition from B to C the photons of wavelength 5000Å are emitted. What will be the wavelength of photon when transition from A to B energy levels?

   (Ans : 1250Å)

4. Doubly ionised Lithium atom is hydrogen like whose atomic number is 3 then -

   (i) find out the wavelength of radiation required to excite an electron from first orbit to third orbit.

   (ii) how many lines will be observed in emission spectrum of the excited system?

   (Ans : 114Å, 3 lines)

5. First line of Balmer series has wavelength 6564Å, Find out Rydberg constant and wave number

   (Ans : $R = 1.097 \times 10^7$ m^{-1}, $\bar{v} = 15 \times 10^5$ m^{-1})

6. A hydrogen like ion emits radiation of frequency 2.467×10^7 Hz in transition from n=2 to n=1. Find the frequency of emitted radiation in transition from n=3 to n=1.

   (Ans : 2.92×10^7 Hz)

7. A monochromatic radiation of wavelength λ is incident on sample of hydrogen whose atoms are in ground state. Hydrogen atoms absorb the radiation and then emit waves of six different wavelengths. Find out the value of λ (Given : hc = 1242eV.nm, ground state energy of hydrogen E = 13.6eV)

   (Ans : $\lambda = 97.5$ nm)

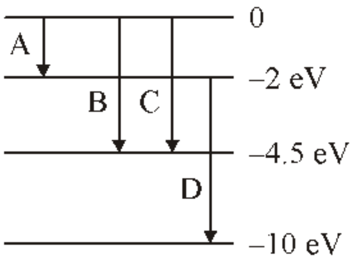
8. Light corresponding to transition from n=4 to n=2 in hydrogen atoms incident on a metal having a work function 1.9eV. Find out the maximum kinetic energy of emitted photo electrons.

   (Ans. : 0.65 eV)

9. A sample of hydrogen is in a specific excited state A. By absorption of photons of energy 2.55eV it reaches another excited state B. Find out the principal quantum number for states A and B.

   (Ans : $n_A = 2$; $n_B = 4$)

10. The energy levels for an atom are shown in the following diagram. Find out the wavelength of photons corresponding to transitions from B and D.



(Ans : 2750Å, 1550Å)

11. For hydrogen atom find out the maximum angular speed of the electron in a stationary orbit.

(Ans : $1.4\times10^{16}\ rad/s$)

12. What is the recoil momentum of hydrogen atom after emitting a photon due to transition from n = 5 state to n = 1 state.

(Given : $R = 1.097\times10^{7}\ m^{\;1}$
$h = 6.63\times10^{\;34}\ Js$ and mass of hydrogen atom = $1.67x10^{-27}$Kg)

(Ans : $6.98\times10^{\;27}\ kg\ m/s$)

Chapter - 15

Nuclear Physics

In 1931, for explanation of experimental obersations of α ray scattering experiments Rutherford proposed that nearly all of its mass and the entire charge of an atom is concentrated into a very small region called the "nucleus" situated at the centre of atom. You have studied in detail about this experiment and the nuclear atom model in previous chapter. Now it is a natural question to ask whether the nucleus has also some internal structure like internal structure of an atom. In this chapter we will make efforts to answer this question by discussing about nuclear constitution, nuclear size and nuclear forces.

Prior to the discovery of the nucleus it was known that some heavy elements like uranium, thorium etc, decay spontaneously by emitting certain particles called α, β and γ radiations. This phenomenon discovered by Becqueral in 1936 is called the radioctity. We will see later that the radioctity too is a nuclear phenomenon. After discussing the laws of radioactive decay and related definitions we will discuss about nuclear fission and nuclear reactors which at present play an important role in fullfilling our energy needs. Towards the end of this chapter we will discuss about fusion which is the source of energy generation in sun and other stars, and promises to be a pollution free source of energy in future.

15.1 Nuclear Structure

With the exception of normal hydrogen the nuclei of all other atoms are formed of two components called neutrons and protons. The nucleus of ordinary hydrogen contains only a single proton and no neutron. Two other forms of hydrogen (istopes) known as deuterium and Tritieum contains respectively 1 and 2 neutrons in addition to 1 proton. The proton is positively charged with magnitude of charge equal to the electronic charge while the neutron is electrically neutral. The masses of protons and neutrons are respectively as

$$m_n = 1.67626231 \times 10^{-27} kg$$

$$m_p = 1.6749286 \times 10^{27} kg$$

(Later on we will describe masses in terms of another unit (u) and equivalent energy units). Electron is a fundamental particle of nature but neutrons and protons are not fundamental particles in a true sense. These are supposed to be made up of other particles called 'quarks'. In this chapter our emphasis is primarily on those properties of the nucleus which are not related to the internal structure of proton or neutron thus we will not be discussing about quarks any further.

The number of protons present in a nucleus of an element is called its proton number and it is also called as atomic number and is denoted symoblically by Z. This number is equal to the number of electrons present in a neutral atom of the said element. The number of neutrons present in a nucleus is termed as the neutron number and is denoted by N. If we ignore the difference of charge (q = + e for proton and q = o for neutron) the neutrons and protons are very nearly identical particles. Their masses are very nearly the same and inside the nucleus these are subjected to identical nuclear force. For these reasons we often classify neutrons and protons taken together as 'nucleons'. The number of nucleons (= Z + N) is called as the mass number of nucleus and is denoted by A. By specifying Z and A (and hence N) we can identify a particular nuclear species or a nuclide. As per convention a nuclide is symbolically denoted by ${}^A_Z X$ or ${}^A X_Z$ where

X = chemical symbol of element

Z = atomic number of element which is also the number of protons in the nucleus

A = mass number of nuclide which is equal to the number of nucleons in the nucleus

Thus, ${}^4_2 He$ represents a helium nucleus which contains 2 protons and 4 nucleons and therefore 2 neutrons. Similarly ${}^{107}_{46}$Ag represents a silver nucleus containing 46 protons and 107 nucleons therefore 61 neutrons.

15.1.1 Some important Definitions

Isotopes : These are atoms having same number of protons Z in their nuclei: but having different mass number A i.e the nuclei of different isotopes of same element contains same number of protons but different number of neutrons. For example consider three isotopes of oxygen $^{16}_{8}O,\ ^{17}_{8}O,\ ^{18}_{8}O$. As the chemical properties of any elements is determined by Z, the number of electrons in it, so all the isotopes of a given elements show identical chemical properties and they occupy same place in periodic table. They cannot be separated by chemical analysis but can be done so by mass-spectrograph.

Isobars : These are nuclei having same mass (nucleon) number A but different atomic (proton) number Z, and neutron number N. For example 1 $^{14}_{6}C$ and $^{14}_{7}N$ are isobars for each A = 14 but Z and N are different. As the atomic numbers are different these belong to different chemical elements and occupy different positions in periodic table. They can be separated by chemical means but not by mass spectrograph.

Isotones : These are nuclei belonging to different elements having same neutron number N but different atomic number Z and different mass number A. e.g $^{13}_{6}C$ and $^{14}_{7}N$ are isotones for each of which N = 7. These belongs to different elements and can be separated by both the chemical means and mass spectrograph.

Mirror Nulcei : In such nuclei the mass number A is same but proton number and neutron number are interchanged i.e number of neutrons in one equals the number of protons in other and Vice-Versa. e.g $^{7}_{4}Be(Z=4,\ N=3)$ and $^{7}_{3}Li(Z=3,\ N=4)$

Isomers : For these nuclei each A and Z are same but their radioactive properties (like half lives, nuclear energy states) are different. Isomers are represented by same chemical symbol with a marked as superscript to differentiate it with nucleus in ground state.

15.2 Nuclear Size

In previous chapter while analysing the α particle scattering experiments we have calculated the distance of closest approach for on α particle for a given nucleus. For α scattering experiment involving 5.5 MeV energy α particles and gold nucleus, Rutherford found this distance to be nearly 4.0×10^{14} m . At such a distance α particle retraces its original path after stopping momentarily due to the Coloumb repulsion of the nucleus. From this Rutherford concluded that the size of gold nucleus must be smaller than 4.0×10^{14} m . Rutherford also found the distance of clost approach for silver nuclei to be nearly 2.0×10^{14} m . Thus it is obvious that if we assume nucleus to be spherical its radius must be of the order of 10^{14} m .

In modern experiments for determining the nuclear radius high energy electrons or neutrons are utilised. In electron scattering experiments electron beams having energy 200 MeV or more are used for their de-Bragile wavelength is short enough for them to act nuclear structure sensitive probe. In effect such experiments diffraction pattern of scattered electrons from which shape of target (nucleus) is determined. It is worth noting that electron scattering experiments provides information regarding the charge distribution in nucleus while neutrons scattering experiments determines the distribution of matter (mass) in nucleus. From these experiments although we found that the nucleus has no sharply defined surface but majority of the nuclei are very nearly spherical with some of them having ellipsoidal surfaces. However, there is a general agreements that one can define an average or a mean radius for a nucleus as follows

$$R = R_0 A^{1/3} \qquad \text{. . . (15.1)}$$

where A is the mass number of the nucleus and R_0 is constant with a value about 1.2×10^{15} m .

A convenient unit for measuring nucleur radius and nuclear distances is femtometer also called fermi and abbreviated as fm, such that

1 femtometer = 1 fermi = 1 fm = 10^{15} m, thus in this unit $R_0 = 1.2\,fm$. As discussed in previous chapter it is to be noted that nuclear radius is smaller by a factor of 10^4 compared to the atomic radius.

15.2.1 Nuclear Volume

If we asume that the nucleus to be spherical with a radius R then its volume is

$V = 4/3\ \pi R^3$

or $\quad V = (4/3)\ \pi R_0^{\ 3} A \qquad$ . . .15.2

Therefore the volume of a nucleus is proportional to its mass number. This in turns means that the density of the nuclear matter is independent of its mass number and is same for all nulcei.

Example 15.1 Determine the radius of $^{27}_{13}\mathrm{Al}$ nucleus.

Solution : The nuclear radius is given by

$R = R_0 A^{1/3}$ where $R_0 = 1.2\, fm$ and as per question A = 27, thereofre

$$R = 1.2 \times (27)^{1/3}$$

$$= 3.6\, fm = 3.6 \times 10^{-15}\, m$$

Example 15.2 Determine the potential energy due to electrical repulsion between two $^{27}_{13}\mathrm{Al}$ nuclei when they just touch each other at the surface.

Solution : As per solution obtained in example 15.1 above the radius of each $^{27}_{13}Al$ nucleus is $R = 3.6 \times 10^{\ 15}$ m. When they just touch other at the surface the separation between their centres is $d = 2R = 7.2 \times 10^{-15}$ m . So the potential energy associated with this pair will be

$$U = \frac{q_1 q_2}{4\pi \in_0 d}$$

Here each nucleus contains 13 proton, so

$$q_1 = q_2 = 13 \times 1.6 \times 10^{-19}\, C$$

$$\therefore \quad U = \frac{\left(9\times10^9\, N\, m^2 / C^2\right)\left(13\times1.6\times10^{\ 19}\, C\right)^2}{7.2\times10^{-15} m}$$

$$= 540.8 \times 10^{24} \times 10^{-38}\ \mathrm{Nm}$$

$$= 540.8 \times 10^{\ 14}\ \mathrm{J} = \frac{540.8\times10^{-14}}{1.6\times10^{\ 19}}\mathrm{eV}$$

Example 15.3 Estimate the numerical value of nuclear density for a nucleus of mass number A.

Solution : The mass of proton and neutrons are very nearly equal say m, then the mass of a necleus of mass number M = mA. From equation 15.2 the nuclear volume $V = \frac{4}{3}\pi R_0^3 A$

the density of nuclear matter ρ =

$$= \frac{m}{4/3\pi R_0^3} = \frac{3}{4}\frac{m}{\pi R_0^3}$$

Which is independtent of the mass number A. Taking $R_0 = 1.2 \times 10^{-15}$ m we obtain

$$\rho = \frac{3}{4\times3.14} \times \frac{1.67\times10^{\ 27}}{\left(1.2\times10^{-15}\, m\right)^3}$$

$$= 2.3 \times 10^{17}\, kg / m^3$$

From the above example it is clear that the density of nuclear matter is indepentent of the mass number and is quite high of the order of $10^{17}\, \mathrm{kg/m^3}$. This is expected as the nuclear matter is confined to a very small volume. If we compare the density of nuclear matter with the density of water $\left(\rho_w = 10^3\, kg / m^3\right)$ then we find it to be greater than by a factor of 2.3×10^4 . Matter with such a high density is found in neutron stars.

15.3 Atomic mass unit

The atomic and nuclear masses are of the order $10^{\ 25}$ kg to $10^{\ 27}$ kg. In practice it is not convenient to use such smaller quantities, therefore these are expressed in another smaller unit called unified atomic mass unit (u) [earlier this unit was called as atomic mass unit (amu)]. This unit is selected such that when expressed in this unit the mass of a 12 C atom (not the nucleus) is exactly 12 u.

$$\text{So} \quad 1u = \frac{^{12}_{6}\mathrm{C}\ (\text{mass of carbon atom})}{12}$$

$$= \frac{1.992647\times10^{\ 26}\, kg}{12}$$

$$= 1.66054 \times 10^{-27}\ \mathrm{kg}$$

Note that the atomic masses refers to the masses of neutral atoms and note of bare nuclei. Thus an atomic mass always includes the masses of its Z electrons. Exact measurements of atomic masses is done by mass spectrograph. When expressed in u, atomic masses of many elements are found to be very nearly equal to integral multiples of the atomic mass of hydrogen atom. However, there are a few exceptions e.g the atomic mass of chlorine is 35.46 u.

On using the Einstein's famous mass-energy equivalence relation, $E = mc^2$ we can obtain energy equivalent to 1μ mass as follows

$$m = 1u = 1.66050\times10^{-27}\,kg$$

$\therefore$ Equivalent energy $E = (1u)c^2$

$$E = (1.6605\times10^{-27})(2.9979\times10^{8})^2 \text{ kg m}^2/\text{s}^2$$

$$= 1.4924\times10^{\;10}\text{ J}$$

$$= \frac{1.4924\times10^{\;10}}{1.602\times10^{-19}}\text{eV}$$

$$E = 931.5\,MeV$$

This suggests that one can write 1 μ = 931.5 MeV / c^2 or one can determine energy equivalent to a given mass difference expressed in μ or vice versa. In table 15.1 the masses of proton, neutrons electron and ordinary hydrogen atom are mentioned in different mass unit.

Table 15.1 Masses of proton, neutron, electron and hydrongen atom ($_1{}^1$H)in various mass units.

Particle	Mass		
	kg	**u**	**MeV / c^2**
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	$1.6750\times10^{\;27}$	1.008665	939.29
Electron	9.1095×10^{-31}	0.0005486	0.511
$^1_1$H atom	$1.6736\times10^{\;21}$	1.007825	938.79

Although to be exact 1 u = 931.5 MeV but for ease of numerial calculations in what follows we shall take 1 μ = 931 MeV .

15.4 Mass Defect and Nuclear Binding Energy

Except hydrogen (^1_1H) nucleus all other nuclei are composed of neutrons and protons. Thus it is quite natural to expect that the mass of a nucleus M must be equal to the sum of the masses of its constituent nucleons $\sum m$. However, the mass of the nucleus M as measured experimentally is alwyas found to be smaller than $\sum m$. This difference in mass is called the mass defect and is denoted by ΔM i.e

$$\Delta M = \sum m - M$$

If a nucleus of mass number A, consists of Z protons and N neutrons with m_p and m_n as mass of proton and neutrons respectively then

$$\Sigma M = Zm_p + Nm_n$$

then accordingly

$$\Delta M = Zm_p + Nm_n - M \qquad ...(15.3)$$

and as

$$N = A - Z$$

So one can also write

$$\Delta M = Zm_p + (A - Z)m_n - m \qquad ...(15.4)$$

The theoretical explanation of mass defect lies in the Eintein's mass-energy relationship. According to it the energy equivalent to mass defect $\Delta E_b = \Delta Mc^2$ is the binding energy of the nucleus. The necleons in a nucleus are bound together and to pull them apart from each other so that these are separated from each other by long distances the energy is to be given to the nucleus (Fig 15.10). This energy is called the binding energy of the nucleus. Alternatively if intially the nucleons are well separated from each other and are brought together to form a nucleus this much amount of energy is going to be released in the process. (Fig 15.1 (b).

One cannot make or break a nucleus in the manner suggested above but still the binding energy of the nucleus gives us an idea about how well the nucleons and bound together in a nucleus.

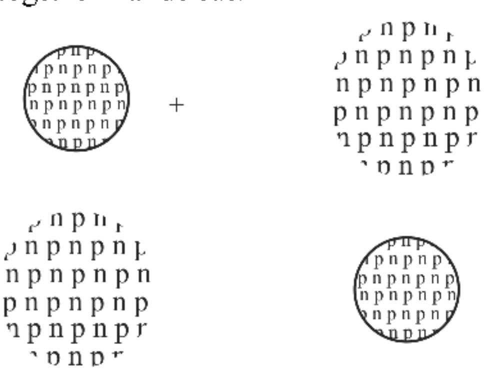


Fig 15.1 (a) The energy equal to the binding energy is to be given to the nucleus to break it into its constituents

nucleons. Each such nucleons is at rest and well separated from the other nucleons.

Fig 15.1 (b) Construction of a nucleus from its constituents nucleons, binding energy is released in the process

Now accroding to the mass energy relation

$$\Delta E_b = \Delta M c^2$$

So on substituting for from equation 15.3

$$\Delta E_b = (Zm_p + NM_n - M)c^2 \qquad \ldots(15.5)$$

So on substituting for M from equation if instead of nuclear masses we use atomic masses (as usually atomic masses are reported rather than the nuclear masses) then the above equation assumes the following from

$$\Delta E_b = \left(ZM_H + NM_n - {}^A_Z M\right)c^2 \qquad \ldots(15.5a)$$

Where M_H refers to the mass of ordinary hydrogen $\left({}^1_1\text{H}\right)$ atom and ${}^A_Z M$ is the mass of the neutral atom of the nucles under consideration. Here it can be seen that Z hydrogen atoms contain Z electrons and atomic mass of atom ${}^A_Z M$ also includes the mass of Z electrons and hence the masses of electrons cancel out in above equation. (However, such cancellation may not take place in process of β decay discussed latter). Their is a slight difference between the binding energy calculated from equations 15.5 and 15.6 owing to the binding energy of electron in atomic masses, However as the atomic binding energy is of the order of a few eV while the nuclear binding energy is of MeV so the difference is quite small and is to be negleted.

Example 15.4 Calculate the binding energy for the following nuclei

(i) Deutron $\left({}^2_1\text{H}\right)$ (ii) ${}^{120}_{50}Sn$ Given that $m_p = 1.007u$, $m_n = 1.008u$ mass of deutron nucleus $M_d = 2.013u$ and the mass of S_n nucleus $M_{Sn} = 119.902u$ $(1u = 931 MeV/c^2)$

Solution : The formula for binding energy is

$$\Delta E_b = \left[Zm_p + (A-Z)m_n - M\right]C^2$$

(i) For deutron ∵ $Z = 1 \quad A = 2$

$$\text{So } \Delta E_b = \left[1m_p + 1m_n - M_d\right]c^2$$

$$= [1.007 + 1.008 - 2.013]uc^2$$

$$= [2.015 - 2.013] \times 931\ MeV$$

$$= 0.002 \times 931 = 1.862\ \text{MeV}$$

(ii) For Sn nucleus Z = 50, A = 120 So A - Z = 70

$$\Delta E_b = [50 \times 1.007 + 70 \times 1.008 - 119.902] \times 931\ \text{MeV}$$

$$= [50.35 + 70.56 - 119.902] \times 931\ \text{MeV}$$

$$= [120.91 - 119.902] \times 931\ \text{MeV}$$

$$= 1.008 \times 931 MeV = 938.448 MeV$$

In above example rather than taking the exact masses for m_p, m_n and nuclei for the sake of simplicity in calculation we have taken their approximate values, still we can note that the binding energy is in MeV range much higher than the atomic binding energy (a few eV). Also note that the binding energy of an intermediate mass nucleus like ${}^{120}_{50}S_n$ is quite large compared to a lighter mass nucleus like ${}^2_1\text{H}$.

15.4.1 Binding Energy per Nucleon

The quantity obtained on dividing the binding energy ΔE_b of a nucleus by its mass number A is termed as the binding energy per nucleon. It is denoted by ΔE_{bn} or $\overline{\Delta E_b}$ i.e.

$$\Delta E_{bn} = \frac{\Delta E_b}{A} \qquad \ldots(15.6)$$

It is a very useful concept. Higher is the value of ΔE_{bn} more stable is the nucleus. If a graph is plotted between binding energy per nuleon for various nuclei and corresponding mass numbers then a curve as shwon in Fig 15.2 is obtained.

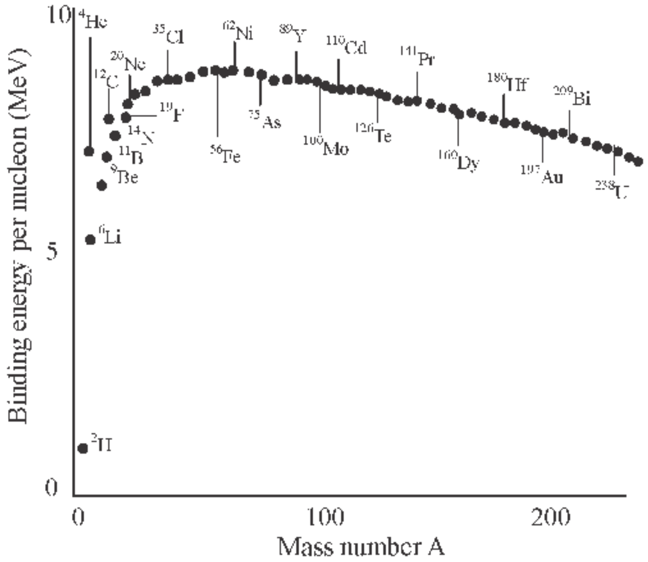


Fig 15.2 : Variation of binding energy per nucleon for some representative nuclides with corresponding mass number A.

The binding energy is maximum (8.8 MeV) for ^{62}Ni nucleus amongest all known stable nuclides. Also note that nuclei like $^{4}_{2}He$, $^{16}_{18}O$, etc are more stable than their neighbours becasue of their higher binding energy per nucleon.

From the study of this curve following conc、ulsions are drawn.

(i) Initially the value of ΔE_{bn} increases, attains a maximum and then decreases slowly.

The nuclei for which the nucleon number is a multiple of 4 i.e A = 4, 8, 12, 16 ..... have higher values of ΔE_{bn} compared to their immediate neighbours. So these are relatively more stable (this suggests a shell structure for nucleons in a nucleus, like the shell structure for electrons in atoms. Inert gases having a completely filled outermost shell are more stable than other elements, like wise nuclei with nucleon numbers suggested above also have completely filled nuclear shells. You will learn more about this in higher classes).

(ii) The elements with A ~ 50 to A ~ 80 are most stable. For them average $\Delta E_{bn} \sim 8.7\,\text{MeV}$ per nucleon. For both A < 50 and A > 80, ΔE_{bn} decreases. The binding energy per nuclear is maximum near A~60, (therefore having A nearing this value) like F_e, N_i and C_o are very stable E_{bn} is maximum = 8.8 MeV for ^{62}Ni nucleus.It is a bit smaller for $^{60}_{26}Fe$ nucleus. This is the reason why molten Ni and Fe are most abundent in earth core.

(iii) For intermediate mass numbers (30 < A < 170) the binding energy per nucleon can be assumed to be practically constant at about 8 MeV. In this range ΔE_{bn} does not vary significantly with A, indicating short range and saturation property of nuclear forces about which we shall learn in brief in following section.

(iv) As mentioned above, nuclei having intermediate mass numbers are relatively more stable compared to those having higher mass numbers. Therefore if a heavy nucleus breaks into two nuclei of inermediate masses, the total binding energy increases while the rest mass energy decreases. Thus energy is released in the process in the form of kinetic energies of the fragments and (or) in some other forms. This process called nuclear fission will be discussed in a latter section of this chapter.

(v) Likewise it can be imagined that two light nuclei ($A \leq 10$) can be combined to form a relatively heavier nucleus. As the binding energy per nucleon for the lighter nuclei is smaller compared to the middle mass nuclei there is a possibility of release of energy in the process. This process called fusion will be discussed in detail towards the end of this chapter.

Some others conclusions which we can not infer directly from the curve shown in Fig 15.2 are as follows. Nuclei with even A, even Z are usually stable and mass abundant, Nuclei with odd A, even odd Z are unstable, in general. Nuclei with odd Z and even A are also unstable with exceptions like ^{2}H, ^{6}Li, ^{10}B, ^{14}N which are stable.

15.5 Nuclear Forces

Two protons (positive charges) which are so near in a nucleus repels each other with such a large electrostatic force, then what keeps a nucleus from breaking? Clearly there should be some strong attractive force operating with in the nucleus which helds the nulceons bound together as nuclei of many of the elements available in nature are stable. This force can not be gravitational as due to very small masses of nucleons and small value of G, even for inter nuclear distance gravitational forces between two nucleons is so week to counteract the repulsive electrostatic force. In fact the gravitational forces are completly ignored in the domain of nuclear physics. Therefore, there must be some other kind of force operating between the nucleons inside a nucleus to held the nucleons bound together. This force is called as the nuclear force. The overall effect of the nuclear force is that it is much stronger than the repulsive Coloumb force operating between two protons and thus the nucleus stays bound.

Unlike electrostatic or gravitational force there is no simple single mathematical expression to determine the nuclear force operating between two nucleons. In fact, many details of the nuclear force are yet to be understood. Some of the qualitative features of the nuclear forces are as follows.

(i) Nuclear forces are independent of charge. For a given separation the nuclear force between two protons is the same as that between two neutrons or between a neutron and a proton. [electrons are not affected by the nuclear forces that is the reason why in electron scattering experiments electrons are scattered by nuclear charges and therefore electron scattering experiments provides information about the distribution of nuclear charges]. Likewise in neutron scattering experiment there is no role of nuclear charge but of nuclear force. Thus neutron scattering experiments provides information about distribution of mass in a nucleus].

(ii) Nuclear forces are short range. The range upto which the nuclear force acts is called nuclear range and it is of the order of a few femto meter, however with in this range the nuclear force is much larger than the electrostatic force (50 ~ 60 times larger). Outside the nuclear range nuclear forces are not effective.

(iii) Nuclear forces are non central in nature. In addition to separation between the nuleons the force between a pair of nucleons also depends on relative orientations of spins of the neuleons.

(iv) Along with the short range of nuclear forces the fact that the density of nuclear matter is constant and the binding energy per nucleon for middle mass nuclei is roughly constant indicates that each nucleon in a nucleus does not interact with every other nulceons in the nuleus. It interacts only with a few neighbouring nucleons. (Consider a nucleon in a nueleus of mass number A. If it would interact with all other nucleons then we would be having A(A–1)/2 such an interactions. In such a case the binding energy would be proportional to A(A-1) and for A >> 1 this would mean A^2 i.e. not a constant). This property of nuclear force is called saturation of the nuclear force. This is different from electrostatic force. (A proton in a nucleus interacts with all other electrons and number of such interactions $\frac{Z(Z-1)}{2} \sim Z^2$

(v) Nuclear forces are attractive in general. However for separations less than 1 fm the nuclear force between a pair of nucleons tends to be repulsive. A detail discussion of this property is beyond the level of the present study.

15.6 Radioactivity

At the beginning of this chapter we mentioned the discovery of radioactivity by Becquerel in 1896 which indicated spontaneous disintegration of heavy elements like uranium, thorium etc by emission of particles or radiation. During the process new atoms (elements) are formed which may themselves be radioactive and the process continues till a stable element is formed.

Becquerel discovered radioactivity accidentally when he found that uranyl potassium sulphat crystals emitted an invisible radiation that could darken a photographic plate when the plate was covered to exculde light. From a series of experiments he also found these radiations capable of ionisation of gases. The most significant investigations of the phenomenon were conducted by polish scientists Marie curie and Piere Curie. After several years of laborious chemical separation on tons of pitch belnde, a radio active ore, the Curies discovered two previously unknown elements both of which were radio active. These were named Polonium and radium. Experimental work by Ruther ford showed that radio active radiations was of three types which he called alpha, beta and gamma rays. Later experiments showed that alpha rays are helium nuclei, β rays are electrons or positrons and gamma rays are high energy photons. Experinments also predicted that the radioactivity is a nuclear phenomenon which involves decay or disintegration of an unstable nucleus. Some important facts regarding radioactivity are as follows.

(i) Radioactivity is not influenced by external parameters like pressure, temperature, phase of radioactive material (solid, liquid, or gas). Radioactivity is not affected by chemical reactions or chemical combination (e.g both uranium or its salts (compounds) are radioactive). As outer atomic electrons are involved in chemical reaction, therefore electronic configuration of an atom plays no role in the phenomenon of radioactivity. Also the emission α of particle, energetic β particles or high energy γ ray photons is not possible from external part of an atom thus radioactivity is purely a nuclear phenomenon.

(ii) In radioactive decay of any nucleus, conservation laws like mass-energy conservation, linear and angular mommontum conservation along with conservation of nucleon number must be obeyed.

(iii) A nucleus X shall be unstable for α or β decay in principle if its mass is more than the sum of the masses of decay products.

(iv) The energy realeased per atom in radioactive decay is few MeV while that in chemical reactions is few eV only.

15.6.1 Rutherford-Soddy Law of Radioactive Decy

Radioactive decay is a random process. It is a statistical phenomenon that obeys the laws of probability (In fact it is the phenomenon of radioactivity which provided the first evidence that the laws governing the subatomic world are statistical in natural). In reference to the decay of atoms present in a sample of radioactive material each decay is an independent event. There is absolutely no way to predict whether any given atom in a radioactive sample will be among the small number of nuclei that decays during the next second. All have the same probability. The radioactive decays law was given by Rutherford and Soddy. According to this law the rate of decay of nuclei (-dN/dt) at some given instant is proportional to the number of nuclei N present at that instant i.e.

$$-\frac{dN}{dt} \propto N$$

or $$\frac{dN}{dt} = -\lambda N \qquad \ldots (15.7)$$

The negative sign indicates that N decreases as t increasses. λ is a constant called decay ot disintegration constant. It has a characteristics value for every radioative nuclide and its SI unit is inverse second (s^{-1}). To understand equation 15.7 the logic is as follows. Assume that at certain instant the number of radioactive nuclei is N. How many of them are going to decay in the next small interval dt? This number will be proportional to both N and dt. For each nucleus there is a chance of decay in interval dt. So more the number of nuclei present at instant t more will decay in next time interval dt. Likewise, if dt is made slightly longer more nuclei will decay because each nuclei will hare more chance of decaying. Hence

$$-dN \propto Ndt \quad \text{or} \quad dN = -\lambda N\, dt$$

which is same as equation 15.7. On rearranging equation 15.7

$$\frac{dN}{N} = -\lambda dt$$

and than integrating both sides, obtaining

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

or $$\ell n N - \ell n N_0 = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda t} \qquad \ldots (15.8)$$

Here N_0 is the number of active nuclei at $t = 0$. From the equation 15.8 it is obvious that the number of active nuclei decreases exponentially with time. This has been shown graphically in Fig 15.3.

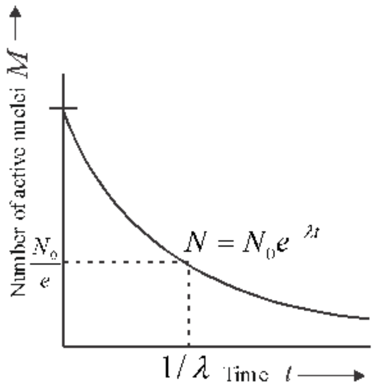


Fig 15.3 : The exponential decay of radioactive atoms

The number of nuclei that have decayed in time t is

$$N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0\left(1 - e^{-\lambda t}\right) \quad \ldots (15.9)$$

Thus, the fraction of nuclei that have decayed in time *t* is

$$\frac{N_0 - N}{N_0} = 1 - e^{\lambda t}$$

Decay constant

From equation 15.7

$$\lambda = \frac{|(dN/dt)|}{N}$$

hence, the decay constant is the rate of decay of radioactive atoms per atom

$$\text{Also } \lambda = \left(-\frac{dN}{N}\right)\frac{1}{dt}$$

Thus, the decay constant is the probability of decay per unit time. In addition to this if in equation 15.7 we take $t = 1/\lambda$, then

$$N = N_0 e^{-1} = \frac{N_0}{e} = 0.368\, N_0$$

Thus, the decay constant is the reciprocal of that time in which the fraction of active atoms reduces to 1/e or 0.368 i.e 36.8% atoms remains active or about 63.2% of the atoms decays. This can be calculated from fig 15.3.

Activity : We are often interested in determining the number of nuclei decaying per second than N (as it is more convinient to measure than N). This is called the activity of the sample. It is also called decay rate of sample and by defination it is a positive quantity, denoted by R.

$$\text{Activity } R = \left|\frac{dN}{dt}\right| \quad \ldots (15.10)$$

From equation 15.11

$$\left|\frac{dN}{dt}\right| = \lambda N$$

$$\therefore \quad R = \lambda N = \lambda N_0 e^{\lambda t}$$

$$= R_0 e^{-\lambda t} \quad \ldots (15.11)$$

$$\text{Where} \quad R_0 = \lambda N_0 \quad \ldots (15.12)$$

is initial activity of the sample. The SI unit for activity is the becquerel (Bq) and

1 Bq = 1 disintegration/second

However, a traditional unit of activity, the curie (Ci) still in common use is defined as the activity of 1 g of radium (^{226}Ra) with a value.

$$1C_i = 3.7 \times 10^{10} \text{ disintegration/s}$$

$$= 3.7 \times 10^{10} \text{ Bq}$$

The graphical representation of equation 15.12 is similar to that shown in Fig 15.3 however, we have to take R in place of N on y axis.

15.6.2 Half Life

The time in which the number of active nuclei present in a sample of a radioactive element raduces to half of its initial value is called as the half life of that radioactive element. If we denote it by T then from equation 15.8 for t=T we have $N = N_0/2$ i.e

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\text{or } e^{\lambda T} = 2$$

$$\text{or } \lambda T = \ell n 2$$

$$\therefore \quad T = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda} \quad \ldots (15.13)$$

Therefore the half life of a radioactive material is inversely proportional to its decay constant. It is contant for a given material and is not affected by external parameters like pressure, temperature etc. As the activity also decays exponentially with time with a decay constant λ thus the activity of a sample decays to half of its initial value in on half life period. Some radioactive nuclides have half-lives which are only a millionth of a second, while for others halflives are billions of years. Thus half life varies in a wide span. e.g an isotope of polonium ${}^{214}_{84}P_0$ half-life is only 10^{-15} s while for Uranium ${}^{238}_{92}U$ half life is 4.5×10^9 years. In fig. 15.4 the radioactive decay is depicted in terms of number of half-lifes.

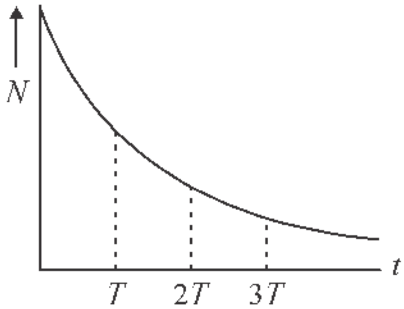


Fig 15.4 Exponenetial decay of a radioactive material, after each half life the number of active atoms reduces to half the value present at the begining of proceeding half life.

On using equation (15.13) equation 15.8 can be rewritten at

$$N = N_0 e^{-\lambda t} = N_0 e^{-(\ell n 2)t/T}$$

$$= \frac{N_0}{\left[e^{\ell n 2}\right]^{t/T}} = \frac{N_0}{2^{t/T}} \qquad \left[\because\ e^{\ell n 2} = 2\right]$$

$$\text{or } \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \qquad \ldots (15.14)$$

Likewise, we can show that

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T} \qquad \ldots (15.15)$$

The above relations are very useful in calculations of T and λ as we shall see shortly in numerical examples to follow.

15.6.3 Average life

According to the radioactive decay law, number of active atoms decreases exponentially with time however complete disintegration of a sample is possible only in infinite time i.e the life of individual active atoms can have any value between O and? The average life of a radioactive substance is defined as the ratio of the total life time of all the radioactive atoms to the total number of such atoms in it. It is denoted by τ then

$$\tau = \frac{\text{Sum of the ages of all atoms}}{\text{Number of atoms}} \qquad \ldots (15.16)$$

Consider a sample containing N_0 radioactive atoms initially $(t = 0)$ and after time t the number of active atoms is N. Then the number of atoms undergoing decay in a very small next time interval dt is dN. Since dt is very small therefore we are safe in assuming that the each of these dN atoms has a life time t. Then the sum of lives of these dN atoms is tdN. As stated earlier the life time span of atoms in a sample is in between O and therefore the sum of lives of all N_0 atoms (say S) present in the sample will be

$$S = \int_0^\infty t\, dN$$

From equation 15.17

$$S = \int_0^\infty t\, dN = \int_0^\infty \lambda\, Nt\, dt = \int_0^\infty \lambda\, N_0 e^{-\lambda t} \cdot t\, dt$$

$$\tau = \frac{\int_0^\infty t\, dN}{N_0} = \int_0^\infty \frac{\lambda N_0 e^{-\lambda t} \cdot t\, dt}{N_0}$$

$$= \lambda \int_0^\infty t\, e^{-\lambda t} dt$$

On solving above integral we obtain

$$\tau = \frac{1}{\lambda} \qquad \ldots (15.17)$$

Thus, the average life is reciprocal of decay constant. Recall that in a time $t = 1/\lambda$ the number of active reduces to $1/e$ of its initial value. Therefore the average life time can also be defined as that time in which number of active atoms reduced to $1/e$ of its initial number.

From equation (15.13) and (15.17) it can be noted that

$$T = \frac{\ell n2}{\lambda} = \tau \ell n2 = 0.693\tau \qquad \ldots (15.18)$$

Note that all the equations derived above are of statistical nature. They do not predict the exact behaviour for each individual atom. In one half life-half the initially active atoms will decay but which of the atom will decay in this half life period can never be predicted. Also note that these equations will work out well only if N is sufficiently large.

Example 15.5 Consider a radoactive sample of 1000 atoms of half life T. Then how many atoms remain active after time T/2.

Solution : Use $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

given $t = T/2$ so $\frac{t}{T} = \frac{1}{2}$

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$$

or $\quad N = \frac{1}{\sqrt{2}} N_0 = (0.707) \times 1000$

$= 707$ atoms

Example 15.6 The activity of a radioactive sample drops to 1/32 of its initial value in 7.5 h. Find the half life of atoms of the sample.

Solution : Given $\frac{R}{R_0} = \frac{1}{32}, t = 7.5h$

Therefore, using $\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T}$

We have,

$$\frac{1}{32} = \left(\frac{1}{2}\right)^{7.5/T}$$

or $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{7.5/T}$

or $5 = \frac{7.5}{T}$

$$\therefore \quad T = \frac{7.5}{5} = 1.5\,h$$

Example 15.7 What is the activity of a 10 kg sample of ^{235}U if the half life of uranium ^{235}U is 7.04×10^8 years. [Take 1 year = 3.15×10^7 s and atomic mass of $^{235}U = 252$ g/mol]

Solution : For a sample of mass M containg N atoms each of atomic mass M, N is given by

$$N = \frac{m}{M} N_A \text{ where } N_A \text{ is Avogadro number}$$

on substituting relevent values

$$N = \frac{10 \times 10^3}{235}\left[6.02 \times 10^{23}\right] = 2.56 \times 10^{25}$$

therefore activity $R = \lambda N = \frac{(\ell n2)N}{T}$

$$= \frac{0.693 \times (2.56 \times 10^{25})}{7.04 \times 10^8 \text{ year}}$$

$$= 2.52 \times 10^{16} \text{ disintegration/year}$$

$$= \frac{2.52 \times 10^{16}}{3.15 \times 10^7}$$

$$= 8.0 \times 10^8 \text{ Bq}$$

Example 15.8 ^{214}Bi nucleus can decay by two channels. In one of decay channels it decays by α emission with a decay constant λ_1, according to

$$^{214}Bi \xrightarrow{\lambda_1} {}^{210}T\ell + {}^4_2He$$

or it decays via β emission with a decay constant λ_2 according to

$$^{214}Bi \xrightarrow{\lambda_2} {}^{214}Po + {}^0_{-1}e$$

If the life time corresponding to the two channels are T_1 and T_2 and in a sample of ^{214}Bi some atoms decays via first and other via second channel. Then obtain an expression for effective half life for such a sample.

Solution : According to question decay constants for first and second processes are λ_1 and λ_2. The probability that an active nuclei decays by the first process in time dt is $\lambda_1 dt$. Similarly the probability that ir decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ this probability is also equal to λdt. Therefore

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or effective decay constant, $\lambda = \lambda_1 + \lambda_2$ ... (i)

and as $\lambda_1 = \dfrac{0.693}{T_1}, \ \lambda_2 = \dfrac{0.693}{T_2}$

and if the effective halflife is T then $\lambda = \dfrac{0.693}{T_2}$ on substituting for λ_1, λ_2 and λ in equation (i), we obtain

$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

or $$T = \frac{T_1 T_2}{T_1 + T_2}$$

Example 15.9 In some radioactive process assume that a nucleus A is transforming into a nucleus B with a decay constant λ_A. The nucleus B so formed is itself radioactive and is decaying in to another nucleus C with a decay constant λ_B. Let N_A and N_B be the number of nuclei of A and B at time *t*. Find the condition for which number of nuclei of B becomes constant.

Solution : The number of nuclei of A decaying in a small time interval *t* and *t+dt* is $\lambda_A N_A dt$. This is also the number of nuclei B produced in this interval. For decay of B into C the number of nuclei of B decaying in the same time interval is $\lambda_B N_B dt$. The number of nuclei of B wil be constant if their rate of production is equal to their decay rate. i.e

$$\lambda_A N_A dt = \lambda_B N_B dt$$

or $\lambda_A N_A = \lambda_B N_B$

Example 15.10 ^{238}U, decays into ^{206}Pb with a half life of 4.47×10^8 y. In a smaple of rock 1.19 mg of ^{238}U and 3.09 mg of ^{206}Pb are found. Assuming all lead to be formed from uranium, estimate the age of rock.

Solution : Let, here N_F = number of product ^{206}Pb nuclei at time t.

N_I = number of ^{238}U nuclei at time t.

m_u = mass of U in sample

m_{pb} = mass of Pb in sample

M_{pb} = atomic mass of Pb

M_u = atomic mass of U

and N_A = Avogadro number, then

$$N_F = \frac{m_{pb}}{M_{Pb}} N_A, \ N_I = \frac{m_U}{M_U} N_A$$

$$\therefore \quad \frac{N_F}{N_I} = \frac{m_{Pb}}{m_U} \frac{M_u}{M_{Pb}}$$

$$= \frac{3.09(mg)}{1.19(mg)} \frac{238 g/mol}{206 g mol} = 3$$

If N_0 = initial number of ^{238}U nuclei, then

$$N_F + N_I = N_0$$

$$3N_I + N_I = N_0$$

or $N_I = \dfrac{N_0}{4}$

Thus, at time t the number of uranium nuclei reduces to 1/4 of its initial value (value at the time of rock formation). Therefore

$$t = 2T = 2 \times 4.47 \times 10^3 = 8.94 \times 10^8 y.$$

Example 15.11 How much time it will take to reduce a radioactive sample to reduce to 10% due to decay. The half life of materials is 22 years.

Solution : Use $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

$$\frac{10}{100} = \left(\frac{1}{2}\right)^{t/22} \quad \text{or} \quad \frac{1}{10} = \left(\frac{1}{2}\right)^{t/22}$$

or $(2)^{t/22} = 10$

On taking log of both sides

$$\frac{t}{22}\log 2 = \log 10$$

$$\frac{t}{22} \times 0.301 = 1$$

$$t = \frac{22}{0.301} = 73y$$

15.7 α, β and γ rays and their properties

α, β and γ rays emitted in the process of radioactive decay process are collectively known as the nuclear radiations. In this section we will study main properties of these radiations.

15.7.1 Properties of α particles

The important properties of α rays are as follows.

(i) α particles are positively charged particles. In fact these are doubly ionsied helium atoms i.e helium nuclei (^4_2He) having mass four times the mass of a proton and change twice the protonic charge.

(ii) As these are charged particles so these are deflected by both electric and magnetic fields. Owing to their relatively higher mass deflection of alpha particles is comparatively smaller than β particles for a given electric or magnetic field.

(iii) The velocity of α particles is in the range $1.4 \times 10^7 \sim 1.7 \times 10^7$ m/s i.e $v_\alpha \sim 0.05c$ (where c is the velocity of light in free space).

(iv) On passing through gases alpha particles collide with gas atoms to knockout electron from them so gases are ionised. Their ionisation power is 100 times more than that of β rays and 10,000 times more than γ rays. This is due to relatively higher charge and relatively smaller velocity of α particles.

(v) The distance covered by α particles in air at N.T.P is called range of α particles i.e distance after covering which the penetration power of α particles is no more is called range. The range in air is small 2.7 cm to 8.6 cm. According to Gieger and Nuttal law the decay constant λ and energy E of α particles are related as $\ell n\ \lambda = A_1 + B_1 \ell n\ E$. The graph between $\ell n\ \lambda$ and ℓn E is a straight line. Future the range depends on energy $R_\alpha \propto E^{3/2}$ or $R_\alpha \propto v^3$ (Vα = Velocity of α particle). Using it, the Giger- Nuttal relation becomes $\ell n\ \lambda = A + B\ell n\ R$ so graph between $\ell n\ \lambda$ and $\ell n\ R$ is a straight line. The value of B is same for various radioactive series (mentioned later in this chapter).

(vi) The penetration power of α particles is much smaller comapred to β and γ rays. These are stopped by piece of a card board or 0.1 mm thick aluminium sheet. The reasons lies in their relatively large mass and ionisation power so while passing through a material the energy of α particle decreases rapidly (in comparision to β or γ particles). On stopping by materials α particles produce heating effect.

(vi) They effect photographic plates and produces fluorescence in Zns or barium platinocynoide.

(viii) After emission of an α particle the atomic number Z of nucleus decreases by 2 while mass number decreases by 4 and size of nucleus reduces.

(ix) The energy spectrum of α particles is a discrete line spectrum which is indicative of presence of discrete energy states for a nucleus.

15.7.2 Properties of β rays

β radiations consists of charged particles. For β^- decay, these radiations are electrons coming out of nucleus (this happens when neutrons are converted into protons inside the nucleus). For β^+ decays these are positrons. β decay usually implies β^- decays. Main properties of β particles are as follows

(i) β^- rays being electrons have charge

$= +e = -1.6 \times 10^{-19}$ C while

β^- rays being positrons have charge $= -e = -1.6 \times 10^{19}$ C

positron is the antiparticle of electron

(ii) As β rays are charged these are deflected by both electric and magnetic fields. Deflection is larger compared to that for α particles. For β^- particles direction is opposite to that for α particles while for β^+ particles direction is same as to that for α particles.

(iii) Velocity of β particle range from 1% to 99% of the velocity of light. Also the velocity of β particles emitted from the same source differs very much so is the kinetic energy.

(iv) The kinetic energy of β particles emitted from a radioactive materials is distributed continuously from zero to a maximum value, hence energy spectrum of β particles is continuous. For this reason range of β particles varies but is much large than of α particles in air it is tens of centimeter.

(v) β rays produces medium ionisation in gas through which β radiation pass. Their ionisation power 1/100th of α particles but 100 times larger compared to γ particles.

(vi) Their penetration power is 100 times larger than α particles but smaller by the same factor compared to γ rays. They can penetrate an aluminium sheet of thickness 10 cm.

(vii) These also affects the photographic plate produce fluroscence in ZnS barium plantinocyanide, calcium tungustate, willemite etc.

(viii) In β decay atomic number Z changes by unity while the mass number A and size of the nucleus are unaffected.

15.7.3 Properties of γ rays

The main properties of γ rays are as follows -

(i) γ rays are electromagnetic radiations (photons) of very high frequency or very small wave lengths (wavelength range is from 10^{-4} Å to 1 Å). These are uncharged and the rest mass of γ ray photons is zero.

(ii) Being uncharged these are not deflected by electric or magnetic fields.

(iii) The velocity of γ ray photons is same as that of velocity of light.

(iv) These produce a very weak ionisation in gases as these are uncharged and move with very high velocity.

(v) The range of γ ray photons is very large in air it is several hundred meters.

(vi) Their penetration power is much larger compared to α and β rays. γ rays may penetrate a 30 cm thick iron plate. If γ rays of intensity I_0 enters some material then after passing through x thickness of material the intensity is given by

$$I = I_0 e^{-\mu x}$$

where μ is called absorption coefficient, μ depends on nature of material and wavelenght of γ radiation $(\mu \propto \lambda^3)$.

(vii) These affects photographic plates and produce flourescence in ZnS, barium platinocyahide etc.

(viii) Like X rays, γ rays are also diffracted by crystals. However their sources of origin are different. Emission of X rays is an atomic property due to electron transition between atomic energy levels, while γ rays are emitted when an excited nucleus makes a transition to a lower energy state or ground state thus emission of γ ray is as nuclear property.

(ix) Depending on energy of γ rays, their interaction with matter results in phenomenon of photo electric effect, compton effect and pair production.

All the above radiation produces heating effect when absorbed in a medium. The human body when exposed to α, β and γ radiation suffers incurable burns. Excess exposure may lead to cancer.

15.8 α, β and γ Decay

So far more than 1000 nuclides are known, however a majority of them are unstable. An unstable nucleus changes its composition by emitting some particle while a stable nucleus does not. The two main processes by which a nucleus decays are α decay and β decay.

Often after α or β decay the product nucleus is formed in its excited states and emittes γ ray photons while returning to in the ground state. In all these decay processes conservation laws like mass-energy conservation, linear and angular momentum conservation, charge conservation must be obeyed, and nucleon number must also be conserved. It is also ncessary that mass of the nuleus under going decay must be greater than the sum of the masses of decay products. In this section we will discuss α, β and γ decay.

15.8.1 α – Decay

When a nucleus undergoes α decay, it transforms into a different nuclide by emitting an α particle (a helium nucleus $_{2}^{4}\mathrm{He}$). Therefore the α emission reduces the mass number by four and atomic (proton) number by two. Since the atomic number is changed the product nuclide belongs to a different element. Nucleus that undergoes decay is called parent nucleus and the product nucleus is called daughter nucleus. Thus the α decay of parent nucleus $_{Z}^{A}\mathrm{X}$ can be written symbollicaly as

$$_{Z}^{A}\mathrm{X} \longrightarrow {}_{Z-2}^{A-4}\mathrm{Y} + {}_{2}^{4}\mathrm{He} \quad \ldots (15.20)$$

From above equation note that both the nucleon number and charge are conversed. An example for the α decay is

$$_{92}^{238}\mathrm{U} \longrightarrow {}_{90}^{234}\mathrm{Th} + {}_{2}^{4}\mathrm{He} \quad \ldots 15.21$$

α decay occurs for all the nulcei with mass number A > 210. We can see that leavy nuclei are unsatble because of large Coulomb repulsion force between their constituent protons. α emission reduces mass number and size of the parent nucleus so it tends towards attaining stability. As required, for decay process the total mass-energy of decay products must be less than mass-energy of the parent nucleus. The difference in mass-energy of the parent and sum of mass-energy of decay product is called distintegration energy or Q value of the process. If the masses of parent atom X, daughter nucleus γ and α particle are denoted by M_x M_y and M_x respectively then

$$Q = \left(M_x - M_y - M_\alpha\right)c^2 \quad \ldots (15.22)$$

The distintegration energy Q represents the decrease in the binding energy of system and appears in the form of kinetic energies of daughter nucleus and α particle. If initally the parent nucleus is at rest then from conservation of linear momentum the daughter nucleus and α particle must have equal and opposite momentum. However as $M_y >> M_\alpha$ therefore, speed of a α particle is much more than the of daughter nucleus. Therefore, Q is mostly associated with the kinetic energy of α particle. From conservation of momentum and energy it can be shown that the kinetic energy of α particle K_α is related to the mass number A of parent nucleus and Q value, according to

$$K_\alpha \simeq \frac{A-4}{A}Q \quad \ldots (15.23)$$

Since A > 210, So K_a is only slightly less than Q.

According to equations (15.22) and (15.23) α particles must be emitted with a discrete energy K_a. However, experiments suggest that α particles are emitted with a set of discrete energies, with the maximum value given by equation 15.20. This occurs because the energy of nucleus is quantized, like quantized energies in an atom. In equation (15.20) we assume that the daughter nucleus is formed in its ground state. If the daughter nucleus is formed in one of its excited states, however, less energy is available for the decay and α particle is emitted with less than the maximum energy. The fact that the alpha particles have a discrete set of energies is a direct evidence of energy quantization in nucleus.

It is a natural question to ask that why a nucleus can not decay by neutron or proton emission. This does not happen because in such a case sum of masses of decay product $Y+n$ or $Y+p$ exceeds the mass of parent X. In such a case $Q<0$ and process is not energetically favourable to proceed spontaneously. In fact, in α decay the binding energy per nucleon for α particles is high enough (~ 7.1 MeV) so as to reduces masses of product Y + α that much for α decay to be possible.

If the nucleus that reults from a radioactive decay is itself radioactive then it will also decay and so on. The sequence of decays is known as a radioactive decay series. As in alpha decay the mass number decreases by 4 so if the mass number of parent nucleus is 4n (n is integer) than the mass number of daughter and other successive nuclei in the decay series will also have mass numbers equals to 4 times an integer. Similarly, it the mass number of the original nucleus is 4n+1, where n is an integer all the nuclei in the decay chain will have mass number given by 4n+1 with n decreasing by unity at each decay. We can see, therefore that there are four possible α decay series depending upon wherer A equals to 4n, 4n+1, 4n+2 or 4n+3 where n is an integer. All but one of these decay series are found in nature. The 4n+1 series is not found because it longest member (other than the stable end product 209 Bi) is ^{237}Np which has a half life of 2×10^{6} y. Because this is much less thant the age of the earth, this series has disapeared. These four series are shown in table 15.2 below.

Table 15.2 : Different Radioactive series

Mass number	Series	Parent nucleus	Stable end products
4n	Thorium	$^{232}_{90}Th$	$^{208}_{82}Pb$
4n+1	Neptunium	$^{237}_{93}Np$	$^{209}_{83}Bi$
4n+2	Uranium	$^{238}_{82}U$	$^{206}_{82}Pb$
4n+3	Actinium	$^{235}_{82}U$	$^{207}_{82}Pb$

The half live values are different for different emitters. e.g for ^{238}U for α decay the half 4.47×10^{9} y life is while it is 550s only for ^{228}U. It is a natural question to ask that through energy is released in each α decay but why is such a huge variation in half life of various α emittes. Newtonian mechanics does not provide answer to such questions, from the point of view of newtonian mechanics even it is not possible for the α decay process to take place at all. Such questions can be answered only with the help of quantum mechanics details of which cannot be possible to discuss here. For the sake of knowledge we try to give a brief account of Gammow theory for α decay in very simple terms.

According to this theory α particles assumed to exist in nucleus prior to α decay. In Fig 15.2 the potential energy function for an α particle and the residual nucleus is shown as a function of separation r between them. This energy is a combination of

(i) The energy associated with attractive nuclear forces inside the nucleus ($r<R_1$) and

(ii) The energy associated Coulomb repulsion between residual nucleus and α particles (outside the nucleus after decay) ($r>R_1$).

From figure it is apparent that a Coulomb energy barrier is present at the surface of nucleus.

The line marked by Q_α depicts the distintegration energy for α decay (which is nearly equal to the kinetic energy of α particle). From figure it is clear that in region $R_1<r<R_2$ the energy of α particle E is less than potential energy suggesting its kinetic energyto be negative which is impossible. Thus in realm of newtonian mechanics α decay is not possible.

However, if we think of α particle as a matter wave then according to quantum machanical consideration there is a smal but finite probability for the matter wave to tunnel through this barrier. This is knwon as the tunnel effect meaning that α decay is possible. The tunnelling probability is a very sensititve function of barrier dimensions (barrier height and width). α decay processes for which Q_α is small and barrier height is quite high the tunneling probability is quite small. For nuclei like ^{238}U it is so. Calculation shows that if we considered the α particle to be repeatedly colliding with the nuclear barrier then for such nuclei the α particle has to collide 10^{38} times before coming out of the nucleus. Therefore such a nucleus will have large half life for α emission.

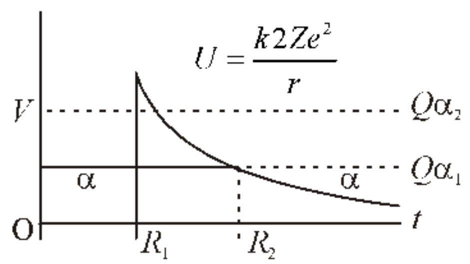


Fig 15.5 The potential energy function for α decay from the nucleus. The shaded region depicts the Coulomb potential barrier that opposes the decay process.

Next we consider an α decay for which disintegration energy is $Q'_\alpha\,(>Q_\alpha)$ from figure it is clear that for such α particles both the barrier height and width are comparatively small. Since tunelling probability increases rapidly with reduction in barrier height and width such α particles are emitted easily and consequently for corresponding nucleus the half life is much small.

15.8.2 β decay

When a nucleus decays by emitting an electron or a positron the decay process is termed as β decay. In β minus (β) decay the nucleus emits an electron and an antineutrino and daughter nucleus has the same mass number as that of parent but atomic number increases by unity. Symbolic form for β decay process is

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{-1}e + \bar{v} \quad ...(15.24)$$

and as an example we have

$$ {}^{234}_{90}\text{Th} \longrightarrow {}^{234}_{91}\text{Pa} + {}^{o}_{1}e + \bar{v}$$

Note that both the nucleon number and charge are conserved in the process. Antineutrino $\bar{v}$ is a neutral particle with practically no rest mass. It interacts so weakly with matter to make its detection very difficult. In above example the total charge before decay is + 90 e and after decay it is 91 e + (–e) + 0 = 90 e. Since both electron and antineutrino are not nucleon so nucleon number stays conserved at 234.

It may seen strange that a nucleus emits electron (positron) and antineutrino (neutrino) as nucleus contains only proton and neutrons. (There is ample evidence to suggest non existence of electrons in a nucleus). However in previous chapter we have seen that atom emits photons but we have never said that atom contains photons. What we say actually is that photons are formed at the time of emission (during transition of atom from excited state to ground state). Same is true for electron (positron) and anti neutrino (neutrino) in case of β decay. These are formed at the time of emission process. In negative beta (β^-) decay a neutron inside nucleus transforms itself into a proton according to the following equation

$$n \longrightarrow p + e^- + \bar{v} \qquad ...(15.25)$$

Such a transformations takes place under the influence of special type of weak nuclear forces (known in general as weak interaction) proton remains in nucleus. Since both protons and neutrons are nucleons so in β decay one nucleus is changing into other so nucleon number is uncharged.

For β decay, the disintegation energy can be calculated as follows. Let m_x and m_y are nulcear masses of X and Y and m_e is the mass of electron, then mass defect

$$\Delta m = m_x - \left[m_y + m_e \right] \qquad ...(15.26)$$

Where antineurino is assumed to have zero rest mass. Now if we add and substract Zm_e to to the right hand side of above equation, we obatin

$$\Delta m = \left(m_x + Zm_e \right) - \left[m_y + \left(Z+1 \right) m_e \right]$$

$$\text{or } \Delta m = \left(M_x \right) - \left(M_y \right) \qquad ...(15.27)$$

Where M_x and M_y are atomic masses of elements X and Y respectively then disintegration energy is given by

$$Q = \Delta mc^2 = \left(M_x - M_y \right) c^2 \qquad ...(15.28)$$

In above equations we have neglected the contributions due to binding energies of electrons in atoms X and Y which small enough to be ignored.

Disintegration energy Q appears in the form of kinetic energy of decay products. Because of relatively higher mass of residual nucleus it can be assumed that the energy Q is shared- in varying proportions between the emitted electron and the antineutrino. Sometimes the electron gets nearly all the emitted energy and sometimes the antineutrino does. In every case, however the sum of energies of electron and antineutrino gives the same value Q. Thus emitted electrons can have any value of energy between O and Q. Therefore the energy spectrum of electrons emitted in β^- decay is continuous between zero and Q (figure 15.6). Recall that the energy spectrum for α paricles is discrete.

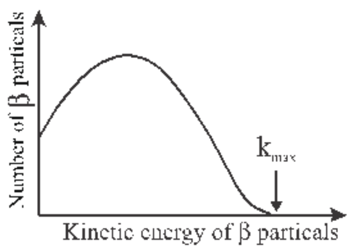


Fig. 15.6 Energy spectrum of β particles

In positive beta decay a positron (e) and a neutrino (υ) are emitted from the nucleus. The symbolic representation of such a decay is

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e \;\; + v \quad \ldots(15.29)$$

and an example of such a decay is

$$^{12}_{11}Na \longrightarrow {}^{12}_{10}Ne + {}^{0}_{1}e^{-} + v$$

Here, the mass number A is unchanged but atomic number Z is decreased by one. Like antineutrino a neutrino is uncharged and of negligible rest mass. Positron and neutron are formed at the time of decay as a proton inside nucleus changes into a neutron, positron and neutrino.

$$p \longrightarrow n + e^{+} + v \qquad \ldots(15.30)$$

Like β^{-} decay, in β^{-} decay, nucleon number and charge is conserved. For β^{+} decay the disintegration energy is $Q = (M_x - M_Y - 2\,m_e)\,C^2$ given by and energy of positron can have any value between zero and Q.

In some nuclei another from of β decay, called electron capture is observed. In such nuclides β decay is not energetically favoured, however, nucleus may capture an orbital electron (usually electron from K shell) which combines with a proton in nucleus to form a neutron. The neutron remains in nucleus and a neutrino is emitted in the process. The symbolic representation of such a process is

$$^{A}_{Z}X + {}^{o}_{-1}e \longrightarrow {}^{A}_{Z-1}Y + v \qquad \ldots(15.32)$$

and an example is

$$^{64}_{29}Cu + {}^{0}_{-1}e \rightarrow {}^{64}_{28}Ni + v$$

The Q value for the process is given by

$$Q = \left(M_x - M_y\right)c^2$$

As the nucleus captures an atomic electron there exists a vacancy in corresponding orbit. To fill this vacancy electrons from higher energy states make transition to this orbit as a result X rays are produced.

From β decay we found that neutrons and protons are not fundamental particles in nature. It can also be noted that the process $n \rightarrow p + e^{-} + \bar{v}$ is possible both inside and outside the nucleus i.e an isolated neutron can decay into a proton. However, the process $p \rightarrow n + e \;\; + v$ is not possible out side the nucleus. As mass of neutron is more than that of proton, an isolated proton can not decay in a proton.

15.8.3 The Neutrino Hypothesis

Prior to 1930, for explanation of β decay it was assumed that in decay process the decaying nucleus decays into a residual nucleus and an electrons (positron) only i.e β decay processes were assumed to be like

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{o}_{1}e \qquad \ldots(15.34)$$

or $$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{o}_{-1}e \qquad \ldots(15.34a)$$

However, there were several difficulties associated with such an assumption. If only an electron and residual nuleus are available as a result of the decay process then owing to the higher mass of the residual nucleus, all the disintegration energy must be available to electrons only making this energy to be unique. This is against the observed experimental fact that the β energy spectrum is a continuous one. It was apparent as if the principle of energy conservation was violated in the process. The smae was the difficulty with the conservation of linear momentum. If we assume the parent nucleus X to be at rest before the decay, then after decay electron and recoiled nucleus Y must move in opposite directions. However, experiments suggested that electrons could move at various angles to the direction of recoiled nucleus? Likewise angular momentum conservation seems to be violeted in the said process. Scientists were started thiking as β decay to be an exceptional process in which conservation laws like energy, momentum, angular momentum etc could not be obeyed.

To explain the apparent non conservation of energy and momentum Pauli in 1930 suggested that a third particle is also emitted in process. Later on Fermi named this particle as neutrino (neutrino means little neutral one). We have already seen how the neutrino and β particle share energy such that energy is always conserved. Although the rest mass of neutrino is zero it has a momentum due to energy. Thus the momentum conservation can be explained as the vector sum of the linear momenta of electron and neutrino must be equal and opposite to the that of recoiling residual nucleus.

(The neutrino is assigned a spin 1/2 to hold the conservation of angular momentum in β decay the details regarding this are not discussed here because of the level of study. Antineutrino is antiparticle of neutrino and is same as neutrino in every respect except for a property called helicity).

Neutrino interaeacts so weakly with matter to make their detection difficult. Neotrinos were first detected by Reines and Cowar in 1956.

15.8.4 γ Decay

A nucleus can exists in states having energies more than in ground state. This is similar to what we have seen in case of atoms with a difference that atomic energy states are in range of a few eV and keV while nucleus energy states range in MeV an excited nucleus is represented by putting a superscipt * on its symbol. When an excited nucleus returns to a lower energy state or ground state photons having energy equal to the difference in initial and final states are emitted. Such photos are called γ ray photons. Their energy is in MeV range. Often a nucleus is formed in its excited state after α or β decay therefore a γ decay follows. The symbolic representation of γ decay is as follows

$$ {}_{Z}^{A}X^{*} \longrightarrow {}_{Z}^{A}X + \gamma \qquad \ldots(15.35)$$

In γ decay as there is no change in A and Z so there is no transformation of one element into the other. In γ decay all known conservation laws are obeyed and γ ray energy spectrum is discrete. In Figure 15.7 decay of ${}_{12}^{27}Mg$ by successive β and γ emission into ${}_{13}^{27}\text{Al}$ is shown.

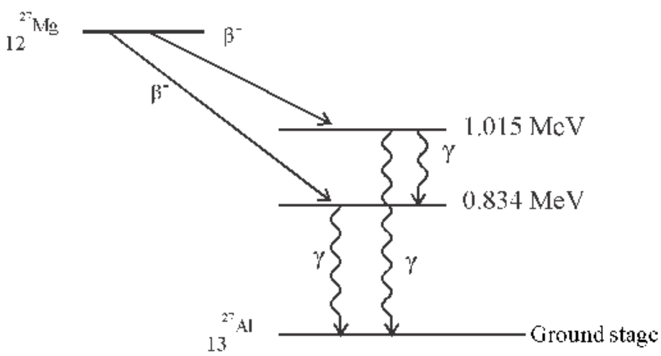


Fig. 15.7 Decay of ${}_{12}^{27}\text{Mg}$ after β decay to ${}_{13}^{27}\text{Al}$ emting γ rays

Example : Radioactive nuclide ${}_{90}^{228}\text{Th}$ by successive decays is ultimately converted into ${}_{83}^{213}\text{Bi}$. How many α and β particles are emitted in this process.

Solution : For the process

$$ {}_{90}^{228}\text{Th} \longrightarrow {}_{83}^{212}\text{Bi} $$

the net decrease in mass number and atomic number is as follows

$$\Delta A = 228 - 212 = 16$$

and $\quad \Delta Z = 90 - 83 = 7$

Since in β decay mass number is not changed, the change in ΔA must correspond to α decay. As in each α decay mass number changes by 4 so number of α particles emitted must be 16/4 = 4. However expected decrease in Z due to emission of 4 α particle = 4 x 2 = 8 therefore the final value of Z must be 82 but according to question the final value of Z is 83. This is only possible if $1\beta^-$ particle is also emitted in the process so as to obatin a final value of 83 instead of 82. Therefore 4 α and $1\beta^-$ particles are to be emitted in the given process.

Example 15.13 ${}_{97}^{238}\text{U}$ nucleus undergoes α decay with a half life of 4.5×10^{9} y. Write the decay equation and from the data given below estimate the kinetic energy of emitted α particles.

$$M\left({}_{92}^{238}\text{U}\right) = 238.0507\,\text{u}$$

$$M\left({}_{2}^{4}\text{He}\right) = 4.0026\,\text{u}$$

$$M\left({}_{90}^{234}\text{Th}\right) = 234.0435\,\text{u}$$

Take $\text{u} = 931\,\text{MeV}/\text{c}^2$ and assume the nucleus to be at rest initially.

Solution : The required decay equation is

$$ {}_{92}^{238}\text{U} \rightarrow {}_{90}^{234}\text{Th} + {}_{2}^{4}\text{He} $$

and for this process the Q value is given by

$$Q = \left[M\left({}_{92}^{238}\text{U}\right) - M\left({}_{90}^{234}\text{Th}\right) + M\left({}_{2}^{4}\text{He}\right)\right]c^2$$

on substituting values of various quantities

$$Q = [238.0507 - 234.0435 - 4.0026]c^2$$

$$= [0.0046] \times 931 = 4.28\,\text{MeV}$$

Assuming initially ${}^{238}_{92}U$ to be at rest, from conservation of linear momentum.

$$0 = \mathbf{p}_\alpha + \mathbf{p}_{Th}$$

$$\therefore \quad p_\alpha = p_{Th}$$

or $$\frac{K_\alpha}{K_{Th}} = \frac{p_\alpha^2 / 2m_\alpha}{p_{Th}^2 / 2m_{Th}} = \frac{m_{Th}}{m_\alpha} = \frac{A-4}{4}$$

(A is mass number of parent nucleus)

or $$K_{Th} = \frac{4}{A-4} K_\alpha$$

But $K_\alpha + K_{Th} = Q$

$$K_\alpha + \frac{4K_\alpha}{A-4} = Q$$

or $$K_\alpha = \frac{A-4}{A} Q$$

$$= \frac{238-4}{238} \times 4.28 = 4.20\,\text{MeV}$$

Example 15.14 For the decay scheme shown in the adjoining diagram calculate the maximum kinetic energy of emitted β particles and radiation frequencies in γ decay. Given

$$M\left({}^{198}_{79}Au\right) = 197.9682\,u,$$

$$M\left({}^{198}_{80}Hg\right) = 197.9667\,u$$

and / assume $\quad 1u = 931\,\text{MeV}/c^2$

Solution : In β^- decay if the daughter nucleus is formed in its ground state then the maximum kinetic energy available to β^- particle is equal to the Q value, which in this case is

$$Q = \left[M\left({}^{198}_{79}Au\right) - M\left({}^{198}_{80}Hg\right)\right]c^2$$

$$= [197.9682u - 197.9667u]c^2 \times 931\,\text{MeV}/c^2$$

$$= 1.396\,\text{MeV}$$

However, in question the nucleus being formed by emission of β particles indicated by β_1^- is in its second excited state which is 1.008 MeV above its ground state so the maximum kinetic energy available to such β particles will be

$$k(\beta_1) = 1.396 - 1.008 = 0.288\,\text{MeV}$$

Like wise for β particles indicated by β_2 daughter nucleus is being formed in an excited state at energy 0.412 MeV above the ground state, so for such β particles the maximum kinetic energy will be

$$k(\beta_2) = 1.396 - 0.412 = 0.984\,\text{MeV}$$

For various transitions shown in figure, the frequencies can be obtained using $\nu = \frac{\Delta E}{h}$ as follows

$$\nu(\gamma_1) = \frac{1.008 \times 10^6 \times 1.6 \times 10^{-19} J}{6.63 \times 10^{-34} J \cdot s} = 2.62 \times 10^{20}\,\text{Hz}$$

$$\nu(\gamma_2) = \frac{(1.008 - 0.412) \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.63 \times 10^{20}\,\text{Hz}$$

$$\nu(\gamma_3) = \frac{0.412 \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 0.99 \times 10^{20}\,\text{Hz}$$

Example 15.15 For β^- decay,

${}^{25}A\ell \rightarrow {}^{25}Mg + e^- + \nu$ calculate Q value. Given, $M\left({}^{25}A\ell\right) = 24.990u \quad M\left({}^{25}Mg\right) = 24.9858u$

Solution : For given β^- decay, the Q value is

$$Q = \left[M\left({}^{25}A\ell\right) - M\left({}^{25}Mg\right) - 2m_e\right]c^2$$

where m_e is mass of electron

$$\therefore \quad Q = [24.9904u - 24.9858u]c^2 - 2[0.511\,MeV]$$

[in last term of above expression, energy equivalent of rest mass of electron is used]

$$= [0.0046] \times 931 MeV - 1.022 MeV$$

$$= 4.282 - 1.022 = 3.26 MeV$$

15.9 Nuclear Energy

You are well aware of various forms of energy. In view of Einstein mass-energy relation you also know that matter itself is a concentrate of energy. But the energy that we need to perform different task in our day to day life is required in specific forms. For example, heat is required to cook food, boiling water while operation of appliances like fans, cooler, bulbs etc. requires electrical energy. Fuels like coal, natural gas, wood all contain internal energy but this internal energy can not be converted directly into heat. To obtain heat from fuel it is essential to burn them which involves a chemical reaction. In such chemical reactions we are tinkering with atoms of fuels rearranging their outer electrons in a more stable configuration. Likewise, we can also obtain energy from a nuclear system via different nuclear reactions. In our discussion about binding energy per nucleon we have seen that this quantity is more for intermediate mass nuclei to make them relatively more stable. To achieve stability heavy nuclei have a tendecny to fission into middle mass nuclei along with a release of energy. Also, lighter nuclei have a tendecny to fuse to form a middle mass nuclei and again energy is released. In both fission and fusion nucleons are beining rearranged to obtain a more stable configuration. However in chemical reaction energy released is of eV to keV order. Although in both combustion of fuel or fission of nuclear fuel like uranium the decrease in rest mass energy of fuel appears as energy. However in case of nuclear fuel a much larger fraction of rest mass energy is converted into the other forms of energy. For example, theoretically burning of 1 kg coal gives energy to operate 100 W bulb for 8hours. While fission of 1 kg ^{235}U gives energy to operate the same bulb for 3×10^4 y years. Like fission, fusion is also a promising source of energy. In future, about which we shall be discussing at the end of this chapter.

15.10 Nuclear Fission

A few years after the discovery of neutron in 1932, Fermi found that when various elements are bomarded with neutrons, new radioactive elements are produced. Neutron is a useful nuclear projectile, as it is electrically neutral it experieneces no Coulomb repulsive force when it reaches a nuclear surface. Even slowly moving thermal neutrons can enter a nucleus to interact with its nucleons. Thermal neutrons are neutrons in thermal equilibrium with matter at room temperature. At a temperature of T = 300K the average kinetic energy of thermal neutrons is

$$K_{av} = \frac{3}{2} kT = \frac{3}{2} \left(8.62 \times 10^{-5} eV/K\right)\left(300 K\right)$$

$$= 0.04 eV$$

These neutrons are useful for nuclear reactions.

In 1939 German Scientists Otto Hahn and Fritz Strassmann on bombarding uranium with thermal neutrons found that many new radioactive elements were produced one of them was similar to barium in chemical properties. Later on this element was positively identified as barium (Z=56). It was difficult for Hahn and Strassmann to explain that how could a middle mass element like barium is produced by bomardment of uranium (Z=92) with neutrons?

This difficulty was resolved by physicists Lee Meitner and Otto Frisch. They put forward a mechanism by which a uranium nucleus after absorbing a thermal neutron could spilt into two nuclei of intermediate masses (one of which might well be barium) along with the release of energy. They termed the process fission. In addition to fission fragments and energy, neutrons are also released in the fission process. It is worth noting here that fission products are not unique, which can be seen from few fission process illustrated below for the fission of ^{235}U

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{90}_{36}Kr + 2\,^{1}_{0}n$$

$$\rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$

$$\rightarrow ^{148}_{57}La + ^{85}_{35}Br + 3\,^{1}_{0}n$$

$$\rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^{1}_{0}n$$

$\rightarrow$ and many more

Fission products are two middle mass nuclei of different mass number. Usually the fission products themselves are radioactive and undergo a series of β decays till a stable end product is formed. An example is shown below

$$^{140}_{54}Xe \xrightarrow{\beta} ^{140}_{55}Ca \xrightarrow{\beta} ^{140}_{56}Ba \xrightarrow{\beta} ^{140}_{57}La \xrightarrow{\beta} ^{140}_{58}Ce$$

For fission of $^{235}_{92}U$ on an average 2.5 neutrons are obtained per fission event. These neutrons are called fast neutrons having nearly 2 MeV energy each. These neutrons are not capable of further fission of ^{235}U unless these are moderated to thermal speeds. In Fig 15.8 a graph is plotted between percentage yields of of different fission products and respective mass numbers for the case of fission of ^{235}U. More than 100 nuclides which belongs to 20 different elements are shown. For most fission product mass number lies between 90~ 100 and 135 ~100. The most propbable mass numbers are A = 95 and A = 140. The probability of having nearly equal mass numbers us small.

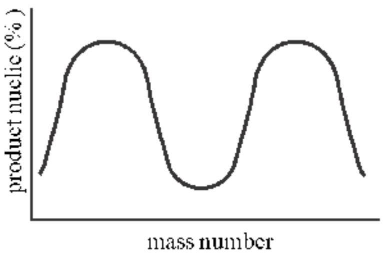


Fig. : 15.8 Graph between percentage yield of fission products and mass number

For fission energy released Q is much higher than in chemical reactions. To estimate the amount of energy released in fission we take the help of binding energy per nucleon curve. From this curve we can see that for heavy nuclides binding energy per nucleon E_{bn} is approximately 7.6 MeV while for middle mass nuclides it is approximately 8.5 MeV.

Next, assume that a high mass nuclide A = 240 undergoes a fission to yield two middle mass nuclei with A = 120 each. Then total binding energy for nucleus A = 240 is $\Delta E_{bi} = \Delta E_{bni} A$

$$\Delta E_{bi} = (7.6) \times 240 \text{ MeV}$$

and for two (A = 120) nuclei the total binding energy

$$\Delta E_{bf} = 2(\Delta E_{bnf}) A/2 = 2(8.5) \times 120$$

$$= (8.5) \times 240 \text{ MeV}$$

Therefore, the energy released in the process

$$Q = \Delta E_{bf} - \Delta E_{bi}$$

$$= (8.5 - 7.6) \times 240 = 216 \text{ MeV}$$

Therefore the energy per fission of ^{235}U is of the order of Q ~ 200 MeV. The most of this energy appears in the form of kinetic energies of fission products and partly in kinetic energy of neutrons and subsequent decay products.

The fission of a nucleus can be explained by a the liquid drop model developed by Bohr and wheeler. Here, we are giving a brief account of this model.

In liquid drop model a nucleus is treated like a spherical charged liquid drop which is in equilibrium under the effects of internal attractive forces and Coulomb repulsive forces. Fig 15.9 shows the process of fission for a ^{235}U nucleus. When such a nucleus absorbs a thermal neutron, the potential energy of the nucleus associated with nucleons gets converted into internal excitation energy. The excitation energy for this process is nearly 6.5 MeV (see example 15.12) and due to this excitation energy the nucleus starts vibrating violently [Fig 15.9 (b)]. Figure 15.9 (c) shows that the oscillating nucleus sooner or later develops a short neck and assumes a dumble like shape. If conditions are right then two globs part of dumble separates apart (fission) due to mutual electrostatic repulsion other wise the nucleus emits a γ ray to assume its original shape. According to calculations done by Bohr and Wheeler based on quantum mechanics the critical energy required to break the ^{235}U nucleus comes out to be nearly 5.3 MeV which is less than the excitation energy so fission of ^{235}U is possible by thermal neutrons.

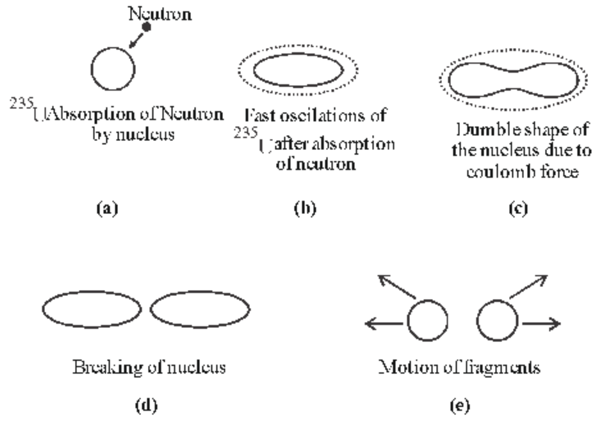


Fig. 15.9 : Process of neulear fission according to liquid drop model

(a) Absorption of a thermal neutron by ^{235}U nucleus

(b) Violent vibrations of compound nucleus formed after obsorption

(c) development of neck in the nucleus

(d) fission of nculeus

(e) formation of fission fragments and emission of fast neutrons

If the internal excitation energy of nucleus, after the absorption of thermal neutrons is less than the corresponding critical energy for breaking the nulceus the fission is not possible. That is why the fission of ^{235}U and ^{239}Pu is possible but ^{238}U and ^{244}Am are not fissonable by thermal neutrons. Such nuclei can be made to fission by fast neutrons.

Example 15.16 In process of nulear fission a ^{235}U nucleus absorbs a neutron to form a ^{236}U nucleus. Calculate the internal energy recieved by the nucleus in this process.

Given : $M\left({}^{235}U\right) = 235.0439\,u$

$M\left({}^{236}U\right) = 236.0455\,u$

$M\left({}^{1}n\right) = 1.0086\,u$

and take $1u = 931$ MeV

Solution : The given process can be described as

$${}^{235}U + {}^{1}n \longrightarrow {}^{236}U$$

Sum of the masses of reactants before this reaction

$$M_i = M\left({}^{235}U\right) + M\left({}^{1}n\right)$$

$$= 235.0439 + 1.0086 = 236.0525\,u$$

and mass of the product

$$M_f = 236.0455\,u$$

Clearly $M_i > M_f$ implying a mass defect meaning that the decrease in rest mass energy appears in the form of internal energy given by

$$E = \left(\Delta Mc^2\right) = \{236.0525 - 236.0455\}\,uc^2$$

$$= \{.0070\} \times 931\,\text{MeV} = 6.51\,\text{MeV}$$

Example 15.17 For the fission process

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n$$

Calculate the energy released

Given $M\left({}^{235}_{92}U\right) = 235.0439\,u$

$M\left({}^{1}_{0}n\right) = 1.00867\,u$

$M\left({}^{140}_{54}Xe\right) = 139.9054\,u$

$M\left({}^{94}_{38}Sr\right) = 93.9063\,u$

Solution : For the said reaction Q value is

$$Q = \Big[M\left({}^{235}_{92}U\right) + M\left({}^{1}_{0}n\right) - M\left({}^{140}_{54}Xe\right) - M\left({}^{94}_{30}Sr\right) - 3M\left({}^{1}_{0}n\right)uc^2$$

$$= [235.0439 + 1.00867 - 139.9054 - 93.9063 - 3(1.00867) \times 931\,\text{MeV}$$

$$= [0.22353] \times 931 \approx 208\,\text{MeV}$$

Example 15.18 Calculate the energy released in fission of 1 kg ^{235}U, assuming energy per fission to be 200 MeV.

Solution : One mole of uranium (atomic mass 235) meaning 0.235 kg mass contains number of atoms (nuclei) = Avogadro Number $N_A = 6.023 \times 10^{23}$

number of nuclei in 1 kg of $^{235}U = \dfrac{6.02 \times 10^{23}}{0.235}$

Energy obtained from fission of 1 kg uranium

(This energy is equivalent to energy obtained form the explosion of several thousand tons of TNT)

15.11 Controlled and Uncontrolled Chain Reactions

In the case of fission of a high mass nucleus like ^{235}U we have seen that on an average 2.5 neutrons are produced per fission. This means that if 100 uranium nuclei are under fission than we are obtaining nearly 250 neutrons more. These neutrons are called secondary neutrons and are fast. If these neutrons are slowed down to thermal energies then under appropriate conditions they can produce more fission in other ^{235}U nuclei present in the fissionable material. This further produces more neutrons which causes more fissions. This leads to a formation of chain of fission events and process is termed as a chain reaction. Such a chain reactions is depicted in Figure 15.10 where only two neutrons are shown to proceed the charge reaction.

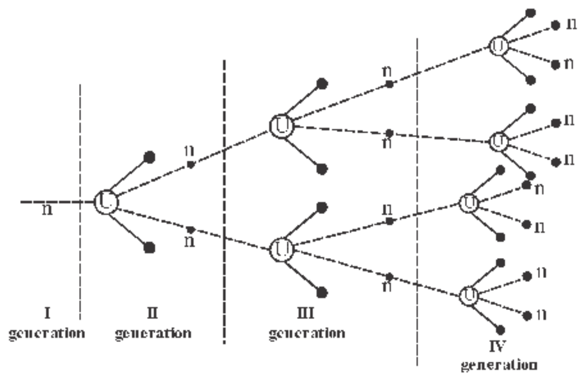


Figure 15.10 : A schematic representation of a chain reaction. Here shows fission fragments and n represents neutrons

For a chain reaction to be self sustained it is essential that at least one neutron produced during each fission, on the average, cause another fission. This condition is often expressed in terms of a parameter K called neutron multipication factor or reproduction factor and it represents the average number of neutrons obtained fission. It is also defined as follows

$$K = \frac{\text{number of neutrons present in a particular generation}}{\text{number of neutrons present at the begining of previous generation}} \quad \ldots (15.36)$$

For ^{235}U based chain reactions maximum possible value of K=2.5, however it is usually less than this, because (i) some of the neutrons may leak from the fissionable material and (ii) some of the neutrons may be absorbed by nuclei present in fissionable material without causing fission.

If $K < 1$ it is abvious that the chain reaction will not be substained because after each subsequent fission the number of neutrons will go on decreasing and ultimately chain reaction wil stop.

If $K > 1$ the rate of reaction will increase rapidly e.g if $K = 1.5$ and in some generation 100 neutrons are produced then in next generation 150 neutrons are obtained and so on. In such a case reaction is called uncontrolled and proceeds so rapdily producing a large amount of energy in the form of heat ultimately leading to explosion. In an atomic bomb such an uncontrolled reaction is required.

If $K = 1$ then reaction is self substained. For example if in certain generation 100 neutrons are obtained then in next generation again 100 neutrons are obtained. Once started the chain reaction will proceed at a constant rate. Such a reaction is called controlled chain reaction. In nuclear reactors $K \simeq 1$ is maintained.

In natural uranium two isotopes ^{235}U and ^{238}U are present in percentage abundance 99.3% and 0.7% respectively. ^{238}U is not fissionable by slow (thermal) neutrons, rather it captures thermal neutron to form ^{239}U which undergoes radioactive decay. ^{235}U is fissionable by thermal neutrons however because of its small percentage in natural uranium the fission probability is too small. So for increasing the probability of fission the content of ^{235}U is increased upto 3%. by artifical means. Such urnaium is called enriched. Even with enrich uranium there are following three difficulties in making a chain reaction 'go'.

(1) The neutron leakage problem : A certain percentage of neutrons obtained by fission will simply leak out of the fissionable material and lost to chain reaction. Some of the neutrons may also be absorbed by the shields enclosing the fissionable material and reaction will not proceed. If too many neutrons are leaked, the chain reaction will not proceed.

Leakage is a surface effect, its magnitude proportional to the square of dimension of fissionable material. For example if material is in the form of a sphere then leakage will be proportional to surface area $= 4\pi r^2$. Neutron production however, is a volume effect, proportional to the cube of a typical dimension (for spherical material volume $= 4/3\pi r^3$). The fraction of neutrons lost by leakage can be reduced by making the material volume large enough there by decreasing its surface to volume ratio ($= 3/r$ for a spherical material). Thus there must be a minimum size or minimum mass of the fissionable material for chain reaction to continue. This mass is known as critical mass.

(2) The neutron energy problem : Neutrons produced in fission are fast with kinetic energies about 2

MeV, but fission takes place due to slow neutrons. Therefore the fast neutrons must be slowed down to thermal energies (0.04 eV). This is done by using substances called moderators. A moderating substance should have the following properties (i) mass of its atom has to be small enough so that in collision of fast neutrons with atoms of the moderator there is an appreciable loss in the kinetic energy of neutrons (b) it does not absorb neutrons excessively to remove them from the fission chain. Water, heavy water (D_2 = 0) and graphite are commonly used moderators.

(3) The neutron Capture problem : The ^{238}U nuclei present in nuclear fuel are good absorbers of neutrons having energy in 1 - 100 eV range. As the 2 MeV fast neutrons produced in fission are slowed down in the moderator they must pass through the 1 - 100 eV energy interval in which they are highly likely probable to be captured by ^{238}U nuclei . To minimise this problem uranium fuel (usually in the form of rods) and the moderator are not intimatley mixed but are "clumped" remaining in close contact but arranged in such a manner that most of the neutrons comes into contact with fuel rods after moderation.

The controlled chain reaction is utilized in nuclear reactors.

15.11 Nuclear Reactor

In a nuclear reactor the energy obtained from nuclear fission is converted into electrical energy for power generation. In a nuclear reactor controlled chain reaction is used. Here we will discuss in brief about a nuclear reactor based on fission of by ^{235}U thermal neutrons. A simplified design of a nuclear reactor is shown in Figure 15.11. Uranium is taken in the form of cylinderical rods arranged in a regular pattern in the active reactor care. The volume of core is filled by moderatoring material like, water, heavy water or graphite etc.

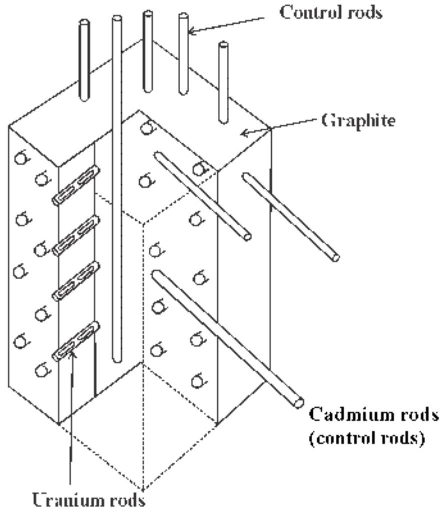


Fig. 15.11 : A simplified diagram of design of a nuclear reactor

When fission takes place in a urainum rod majority of the fast neutrons produced escape from rod and enters into the moderator. In moderator, these neutrons make collisions with moderstor atoms. After few collisions their energy decreases from 2 MeV to thermal energy range. The distances between rods is adjusted in such a manner that a neutron escaping from one rod is generally slowed down to thermal energies before reahing the other rod. This reduces the possibility of absorption of 1- 100 eV neutrons by ^{238}U present in rods. The geometry of core is designed such that the leakage of neutrons is limited to the condition that out of average of 2.5 neutrons produced per fission one neutron is avaiable for triggering next fission. In this condition multipication factor K = 1 and chain reaction proceeds at a constant rate. If by due to the leakage of neutrons is decreased then condition K > 1 is there which may lead to explosion. To control this cadmium rods are used which are inserted upto certain depth in the moderator. Cadmium is a good absorber of neutrons. In fact at the start of chain reaction K be kept greater than unity and then cadimium rods are quickly pushed into the moderator upto such depth that condition K = 1 is achieved. If necessity arises cadmium rods are pushed to full depth to achieve condition $K < 1$. In this condition reactor is shut off. This is done for safety or mainainence of nuclear reactor. Some liquid coolent like water at high pressure or molten sodium is circulated arround the reactor core to extract heat generated. The heat is used to produce steam from water. The steam so produced is used to run turbines and electric power generation.

The reactor region is surround by thick concrete walls so that the harmful radiations produced by highly radioactive fission products do no enter the environment. Thus safety of persons working in nuclear power plants and residents of neighbourhood is ensured.

In addition to power generation nuclear reactors are also used for producing radioactive isotopes used in various applications, and for obtaining neutron mean for research purpose. Although the nuclear power plants play on important role in electricity generation, the problem of disposal of hazadeous nuclear waste is very serious. There are a number of International rules and laws for safe operation of nuclear reactor which are to be adhered strictly. Even a small mistake in operation of a nuclear reactor may lead to destruction therefore a constant montioring is required. In 1986, at chernobyl in Ukraine (formely a part of USSR) the reactor core melted and large amount of radioactivity was released , which adversely affected of population of the adjoining area and East Europian countries. Indian atomic energy programme maintains a good safety record as per international norms.

Example 15.19 The energy obtained per fission of ^{235}U nucleus is about 200 MeV. If a nuclear reactor based on ^{235}U is producing 1000 kW power than how many ^{235}U nuclei are undergoing fission per second.

Solution : As per question, the energy generated per second = 1000×10^3 J

$$= \frac{10^6}{1.6\times10^{-19}}\text{eV} = 6.25\times10^{24}\text{eV}$$

So the number of fissions per second

$$= \frac{6.25\times10^{24}}{200\times10^6} = 3.12\times10^{16}$$

15.13 Nuclear Fusion

While disscussing the binding energy per nucleon curve we pointed out that large amount of energy can be released if light nuclei are combined to form nuclei of some what larger mass number, a process called fusion. The mass of final products is always less than sum of the masses of the reactants. The lost mass is converted into energy.

Fusion is a difficult process, for the two nuclei to fuse they must come close to one another with in the range of attractive nuclear force. This is a difficult task as this process is hindered by the mutual Coulomb repulsion that tends to prevent two positively charge nuclei from coming close together and fuse. To overcome the Coulomb barrier, the kinetic energies of the reacting nuclei most be large in the range 0.1 MeV to 1 MeV (see example 15.16). Such energies can be achieved through particle accelerators (like cyclotron). However in accelerators the probability that particles are scattered is much more than that of fusion. Also the energy input needed to accelerate one paticle (nucleus) for bomording on the other is higher than the energy released through fusion. So there is no hope of useful power generation through fusion in a bulk material using particle accelerators. The best hope for fusion to occur in bulk material is to raise the temperature of material in the form of gas to such values so that the particles accquire large speeds and come close enough during the collisions for fusion to take place. The process is called thermonuclear fusion and the temperature required is of the order of 10^9K.

As an example of fusion process, consider the fusion of two deuterons

$$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_1\text{H} + ^1_1\text{H} + 4.0\,\text{MeV}$$

The $\left(^3_1\text{H}\right)$ nucleus obtained in the process again combines with one deutron to form a helium nuclei

$$^3_1\text{H} + ^2_1\text{H} \rightarrow ^2_4\text{He} + ^1_0\text{n} + 17.6\,\text{MeV}$$

On combining above two equations the resultant reaction is

$$3\left(^2_1\text{H}\right) \rightarrow ^4_2\text{He} + ^1_1\text{H} + ^1_0\text{n} + 21.6\,\text{MeV}$$

Thus the over all result is combining of three deutrons to form a helium nuclei and 21.6MeV energy released.

Although the energy obtained from fusion mentioned above is small compared to 200 MeV energy obtained in fission of a ^{235}U nucleus, however if we compare the energy obtained from fusion of 1 kg of deuterium with that obtained from 1 Kg of ^{235}U, then it is much more. (see example 15.17). In addition to this fusion products are not radioactive while that obtained in fission are highly radioactive so fusion promises to be a clean source of energy.

Example 15.20 The deuteron ^2_1H has a charge + e and has a radius nearly 2fm. Two such deuterons are fired at each other with the same initial kinetic energy K. What must be the value of K if the two particles are brought to rest by their mutual Coulomb repulsion when the two are just touching. Also calculate the temperature corresponding to this kinteic energy?

Solution : Because, the two deuetrons are momentarily at rest when they just touch other, their total kinetic energy has been converted into electrostatic potential energy. If we treat them as point charges separated by a distance 2R (R = radius of each nucleus) then from conservation of energy

$$2K = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi \in_0}\frac{e^2}{2R}$$

or $$2K = \frac{1}{4\pi \in_0}\frac{e^2}{4R}$$

$$= \frac{\left(9\times10^9\ N.m/C^2\right)\left(1.6\times10^{-19}\ C\right)^2}{4\left(2\times10^{-15}\ m\right)}$$

$$= 2.7\times10^{-14}\ J \simeq 170\ keV$$

If T is the temperture corresponding to K them

$$K_{av} = \frac{3}{2}kT$$

or $$T = \frac{2}{3}\frac{K_{av}}{k}$$

$$= \frac{2}{3}\frac{170\times10^3\ eV}{\left(8.62\times10^{-5}\ eV/K\right)}$$

$$= 1.31\times10^9\ K$$

Example 15.21 From fusion of 3 deutrons approximately 21.6 MeV energy is released. Calculate the energy released from the fusion of 1 Kg of deuterium.

Solution : One mole of deuterium (0.002 kg) contains 6.02×10^{23} (Avogedro number) nuclei, hence number of nuclei in 1 kg of deuterium is

$$= \frac{6.02\times10^{23}}{0.002} = 3.01\times10^{26}$$

Since 3 deuteron fuses to give 21.6 MeV of energy, so energy corresponding to one deuteron

$$= \frac{21.6}{3} = 7.2\ MeV$$

$\therefore$ Energy released in fusion of 1 Kg deuterium is

$$= 3.01\times10^{26}\times7.2\ MeV$$

$$= 21.67\times10^{26}\ MeV$$

$$= 21.67\times10^{27}\times1.6\times10^{-19}\ J$$

$$= 34.67\times10^{13}\ J$$

In example 15.14 we have seen that the energy obtained from fission of 1 kg of uranium is about 8.19×10^{13} J so energy obtained from fusion is 4 times more. In example 15.16 we have determined the temperature required for initiating a fusion reaction. At such high temperature electrons completely detatched from atoms and the matter is in the form of completely ionised state called plasma consisting of nuclei and electrons. As we shall see in next subsection the thermonuclear fusion is the source of energy in stars. First fusion reaction was performed on earth in 1952 when a thermonuclear (Hydrogen) bomb exploded. To achieve temperature high enough for the fusion to take place an atomic bomb was exploded prior to the hydrogen bomb. Hydrogen bomb is an example of uncontrolled thermonuclear fusion and is very destructive.

The fuel used for fusion on earth is deuterium which is available in natural water and with oceans as almost unlimited source of water we are sure of fuel supply for several thousand years, so if controlled thermonuclear is possible on earth most of our energy demaded can be fulfilled. Fusion reactors are not designed and functional till date. There are a number of difficulties involved for harressing fusion energy for power generation. On of the difficulty is to obtain high temperature required for initiation of fusion. At present pulsed lasers are used to produce temperature of 10^8 K in laboratories. However, major problem is of confiment of plasma. For fusion reaction to occur in large number it is necessary to confine plasma at very high temperatures and high particle density. It is abvious that at such high temperatures plasma is not be confined to a solid container is the biggest problem in achieving the controlled fusion reaction on earth. Scientist are working on two techniques called magnetic confinment and inertial confinment. A device called toka mak has been designed in which magentic confinement is utilized and power levels of 1 MW for 1 second has been achieved . It is expected that by the mid of this century the controlled fusion reactors shell be made available for power production.

15.13.1 Thermonuclear Fusion in Sun and Stars

The sun radiates at the rate of 3.9×10^{26} W and has been doing so for about 4.5×10^{8} years. Prior to 1930, it was assumed that energy generation in sun was due to burning of carbon (coal) and oxygen in its interior part. However, the sun whose mass is 2.0×10^{30} kg was doing so it would have last only for a few thousand years. Another point of view was that , as due to cooling of core the pressure in the core could be reduced and it would have shrinked under the action of its strong gravitational forces. Thus gravitational energy could be transformed into internal energy resulting in increase of temperature of core making sun to radiate continuously. Calculations shows however, that the sun could have radiated from this cause for 10^8 years. The present composition of the sun's core is about 35% of hydrogen by mass, about 64% of helium and 1% of other elements. Due to absence of heavy elements fission cannot be the source of energy generation in sun.

In 1939 American Scientist Bethe proposed that energy generation in sun and other stars is due to thermonuclear fusion in which hydrogen nuclei are fusing to form helium nuclei. This process known is proton cycle is represented by the following set of reactions

$$^1_1\mathrm{H}+{}^1_1\mathrm{H}\rightarrow{}^2_1\mathrm{H}+e^-+\gamma+0.42\,\mathrm{MeV} \quad \ldots(i)$$

$$e^-+e^-\rightarrow\gamma+\gamma+1.02\,\mathrm{MeV} \quad \ldots(ii)$$

$$^2_1\mathrm{H}+{}^1_1\mathrm{H}\rightarrow{}^3_2\mathrm{He}+\gamma+5.49\,\mathrm{MeV} \quad \ldots(iii)$$

$$^3_2\mathrm{He}+{}^3_2\mathrm{He}\rightarrow{}^4_2\mathrm{He}+{}^1_1\mathrm{H}+{}^1_1\mathrm{H}+12.86\,\mathrm{MeV} \quad \ldots(iv)$$

For the fourth of the above reactions to take place it is essential that first three reactions should take place twice each so that two $^3_2\mathrm{He}$ nuclei needed for the fourth reaction are made avaiable. Thus considering first three reaction twice each and fourth reaction once, the overall effect is

$$4\left({}^1_1\mathrm{H}\right)+2e^-\rightarrow{}^4_2\mathrm{He}+2\nu+6\gamma+26.7\,\mathrm{MeV}$$

If in above equation, we add two electrons to each side, then above equation assumes the following from

$$4\left({}^1_1\mathrm{H}+e^-\right)\rightarrow\left({}^4_2\mathrm{He}+2e^-\right)+2\nu+6\gamma+26.7\,\mathrm{MeV} \quad \ldots(15.36)$$

Note that the quantities in the parantheses represents atoms of hydrogen and helium. The energy released in the reaction is 26.7 MeV, this can be varified as mass defect

$$Q=(m_i-m_f)c^2=\left[4M_{^1_1H}-M_{^4_2He}\right]c^2$$

Where $M\left({}^1_1\mathrm{H}\right)$ and $M\left({}^4_2\mathrm{He}\right)$ are atomic masses of hydrogen and helium having values 1.007825u and 4.002603u respectively

$$\therefore \quad Q=\left[4(1.007825)u-4.002603u\right]\times[931.5\,\mathrm{MeV/u}]$$

$$=26.7\,\mathrm{MeV}$$

As γ rays are massless and neutrinos are of negligibly small rest mass so these masses are not included in calculations.

The temperature of inner core of sun is estimated to be nearly 1.5×10^7 K . A little while ago we have seen that fusion requires temperature arround 10^9K then why fusion takes place in sun. Answer to this puzzle lies in that we have calculated the kinetic energy of nuclei from relation K = 3/2 ^{KT} ineffect corresponding to the energy kinetic energy of nuclei. Because of large mass of sun there exist nuclei in large number having energies far greater than the average energy, such nuclei are responsible for fusion in sun.

According to present estimate hydrogen in sufficient amount is available in the sun for the fusion to continue for next 5×10^9 year. After that sun core will consit of helium only. Due to gravitational forces sun's core will contract and its temperature will rise. Due to this outer envelope of the sun will expand to much possibly large enough to encompass the orbit of sun. In terminology of astronomy then sun will become a red giant.

Important Points

1. Nucleus, consits of protons (+ve charge +e) and neutrons (neutral) collectively knwon as nucleons. A nuclide is represented by ${}^A_Z X$ where X is chemical symbol of element corresponding to nucleus, A is mass number (nucleon number) and Z is proton (Atomic) number, neutron number N = A - Z. Ordinary hydrogen nucleus consits of a single proton only.

2. Nuclei are almost spherical with radii given as

   $R = R_0 A^{1/3}$ where $R_0 = 1.2\,fm$

   accordingly nuclear volume $V = \frac{4}{3}\pi R_0^3 A$

   i.e $V \propto A$.

   This means that the density of nucleus is independent of its mass number. Its value is about $2.3 \times 10^{17}\ kg/m^3$ which is same nearly for all nuclei.

3. Nuclear masses are expressed in unified atomic mass unit (u)

   $$1u = \frac{\text{mass of } {}^{12}C \text{ atom}}{12} = 1.66054 \times 10^{-27}\ kg$$

   also $\quad 1u = 931.5\,MeV/c^2$

4. Mass of a nucleus M is less than the sum of masses of its constituents nucleons $\sum m$.

   Mass defect is given by $\Delta M = \sum m - M$.

   Energy equivalent to mass defect is called binding energy ΔE_b i.e $\Delta E_b = \Delta Mc^2$. For a nucleus consisting Z protons and N neutrons

   $\Delta E_b = \left[Zm_p + Nm_n - M\right]c^2$

   where m_p and m_n are masses of proton and neutron respectively and M is mass of nucleus. IF in place of nuclear masses, atomic masses are used

   $\Delta E_b = \left[ZM_H + Nm_n - {}^A_Z M\right]c^2$

   Where M_H = mass of hydrogen atom, and ${}^A_Z M$ is mass of atom of the corresponding nucleus. Binding energy is a measure of nuclear stability. If we are able to separate a nucleus into its nucleons then this much of energy is to be supplied to the nucleus.

5. The binding energy per nucleon ΔE_{bn} for a nucleus is the quantity obtained on dividing its binding energy E_b by its mass number A

$\Delta E_{bn} = \Delta E_b / A$

A higher value of ΔE_{bn} indicates more stability of nucleus. A graph plotted between ΔE_{bn} and A suggests that middle mass nuclei are relatively more stable than either light mass or heavy mass nuclei. This curve also suggests the possibility of energy release by fission of high mass nucleus into middle mass nuclei or fusion of light nuclei into a middle mass nuclei.

6. Nuclear force: Nuclear force is a very strong attractive force which bounds nucleons together inside a nucleus. This is a very short range force (range ~ few fm), however, in this range it is 50 ~ 60 times greater than electrostatic force of repulsion between protons.

Nuclear forces are charge independent. For a given separation nuclear forces between n-p, p-p and n-n are very nearly same. Nuclear forces are non central they have the property of saturation. For separation between nucleons less than 1 fm the nuclear forces becomes repulsive.

7. Nuclei of heavy mass elements like uranium , thorium, radium, etc decay spontaneously by emitting α, βor γ radiations. This process is called radioactivity. This a nuclear phenomenon which is independent of external parameters like pressure, temperate, phase change and chemical combinations.

8. α particles are nuclei of helium (^4_2He) and are positively charged. β rays are stream of electron (e^-)or positron (e^-) . γ rays (neutral) are very high energy photons with wavelength smaller than that for X rays.

9. Radioactive decay law : Radioative is a random process and obeys laws of probaility (statistics). According to Ruther ford-Soddy decay law, the rate of radioative decay at a given instant is proportional to the number of active nuclei present at that instant i.e $-\frac{dN}{dt} = \lambda N$. The constant of proportionality λ is called as decay constant. Accordingly the number of active nuclei at time t is given by

$$N = N_0 e^{-\lambda t}$$

Where N_0 is number of active nuclei at t = 0. Thus number of active nuclei in a radioactive material decays exponentially with time.

10. $\lambda = \frac{dN/dt}{N}$ is rate of radioatice decay per atom, or decay probability per unit time. At $t = 1/\lambda$ the number of radioactive nuculei becomes 1/e of initial number N_0.

11. Activity R : This is number of decays per unit time

$$R = \left|\frac{dN}{dt}\right| = \lambda N \text{ and } R = R_0 e^{\lambda t},\ R_0 = \lambda N_0$$

R_0 is initial activity. Activity too decreases exponentially with time. Its SI unit is (Bq) and 1 Bq = 1 disintegration/s. The traditional unit for radioactivity is curie (Ci) which is activity of 1g of radium

$$1Ci = 3.7\times10^{10} \text{ disintegration/s} = 3.7\times10^{10}\ Bq$$

12. Half life : The time interval in which the number of active nuclei in a radioactive sample (or activity of sample) reduces to half its initial value N_0 (or initial activity R_0) is called as half life

$$T = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda}$$

the number of active atoms present at time t in terms of half life is given by

$$N = \frac{N_0}{(2)^{t/T}}$$

13. Average half life : This is the time (t) in which the number of activite atoms N and activity R both decay to 1/e of their initial values.

$$\tau = \frac{1}{\lambda} = \frac{T}{\ell n 2}$$

14. In α decay a heavy mass nucleus (parent) X changes into another nucleus Y by emitting an α particle

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

The atomic number of Y is 2 less than that of X while mass number is 4 less than A. The disintegration energy for the process is

$$Q = \left(M_x - M_y - M_\alpha\right)c^2$$

A major part of this is in the form of the kinetic energy of α particle. α particle energy spectrum is a set of discrete energies suggesting quantization of nuclear energy levels.

15. In β^- decay the parent nucleus X changes into daughter nucleus Y by emitting an electron and an antineutrino

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$$

The mass number of daughter Y is smae as that of X but atomic number increases by 1. Actually in β^- decay a neutron inside the nucleus changes into a proton, electron and an antineutrino

$$^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e^- + \bar{\nu}$$

proton remians inside the nucleus while eletron and antineutrino are emitted. For β^- decay disintegration energy is given by

$$Q = \left(M_x - M_y\right)c^2$$

electron and anti neutrino share this energy in varibale proportions.

16. In β^+ decay parent nucleus X, changes into daughter nucleus Y by emitting a positron $\left(e^+\right)$ and a neutrino (v).

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$$

The mass number of daughter Y is same as that of X but atomic number decreases by 1. In β^- decay a proton inside nucleus transforms into a neutron, an electron and a neutrino

$$^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + \nu$$

Above transformation is not possible out side nucleus. For β^+ decay the disintegration energy is given by

$$Q = \left(M_x - M_y - 2m_e\right)c^2$$

again this energy is shared between positron and neutrino.

17. In certain nuclei, electron capture process takes place instead of a β decay. In this process the nucleus capture an atomic electron belonging to some inner shell of atom which combines with a proton in nucleus to form a neutron.

$${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y + \nu$$

18. For β decay (both β^+ and β^-) energy spectrum of β particles is continuous which varies from zero to a maximum value Q. This is because of sharing of energy between β particle and neutrino (anti neutrino).

19. Neutrino and antineurino both are neutral particles of negligible rest mass. Neutrino hypothesis was given by Pauli for explanation of energy spactrum, momentum and angular momentum conservation in β decay.

20. γ decay : Often, after α or β decay the daughter nucleus is formed in one of its excited state, such a nucleus returns to a lower energy state or ground state by emitting photons having energy equal to the energy difference between final and initial states. Such photons are called γ ray photons

$${}^{A}_{X}Z^* \rightarrow {}^{A}_{X}Z + \gamma$$

In γ ray both the mass number and atomic number are uncharged. The γ ray energy spectrum is discrete.

21. In nuclear fission a high mass nuclei breaks into two middle mass nuclei and energy is released in the process. The fission in ^{235}U can be triggered by a thermal neutron. The energy realeased in fission

Q = (final binding energy ΔE_{bf}) - (Initial binding energy ΔE_{bi}). About 200 MeV is released per fission of ^{235}U. If the energy of fast neutrons emitted in fission is moderated to thermal energy then such neutrons can fission more ^{235}U nuclei. This leads to a chain reaction. Chain reaction may be controlled or uncontrolled. A chain reaction is called controlled if from neutrons obtain from fission of a nucleus only one is available for next fission. A controlled chain reaction is used in nuclear reactors for power generation. Uncontrolled chain reaction is destructive and used in atomic bomb.

22. Nculear reactor is a device in which a self sustained controlled chain reaction is employed for power generation. Usually enriched uranium 235 is used as nuclear fuel. Fast neutrons obtained in fission are slowed down to thermal energies by collisions form atoms of moderating substances like water, heavy water or graphite. Chain reaction is controlled by cadmium rods, cadmium is a good absorber of neutrons. Energy generated in form of heat in nuclear reactor is extracted by coolents like water, molten sodium etc. To shield environment from harmful nuclear radiations produced in a reactor it is surrounded by thick concrete walls.

Questions For Practice

23. In process of a nuclear fusion two light nuclei combines to form a nucleus of relatively high mass number and energy is released. For thermonuclear fusion very high temperature ~10^9 K is required. Thermonuclear fusion is the source of energy generation in stars. Fusion reactor is still under develoment.

Multiple Choice Questions

1. The radius of $^{64}_{30}Zn$ is about (in fm)

(a) 1.2 (b) 2.4

(c) 4.8 (d) 3.7

2. If the mass of $^{7}_{3}Li$ isotope is 7.016005 u and masses of H atom and neutron are respectively 1.007825 u and 1.008665 u then the binding of Li nucleus is

(a) 5.6 MeV (b) 8.8 MeV

(c) 0.42 MeV (d) 39.2 MeV

3. If at some given instant there are 1.024×10^{24} active atoms in some radioactive sample, then number remaining after eight half lives will be

(a) 1.024×10^{20} (b) 4.0×10^{17}

(c) 6.4×10^{18} (d) 1.28×10^{19}

4. It is found that in an archaeological specimen of wood the activity of ^{14}C was 10 dis/minute per gram of specimen while the activity in fresh wood is found to be 14.14dis/minute per gram. If the half life of ^{14}C is 5700 year the age of the sample approximately is

(a) 2850 years (b) 4030 years

(c) 5700 years (d) 8060 years

5. $^{238}_{92}U$ after a series of decays transforms to stable end product $^{206}_{82}Pb$. The number of α and β particles emitted in the process are

(a) 8, 8 (b) 6, 6

(c) 6, 8 (d) 8, 6

6. For deuteron the binding energy per nucleon is 1.115 MeV then for this nucleus the mass defect is

(a) 2.23 u (b) 0.0024 u

(c) 0.027 u (d) more information is required

7. Two proton are separated by 10 Å. Let F_n and F_e be the nuclear force and electrostatic force between them, so

(a) $F_n >> F_e$ (b) $F_e >> F_n$

(c) $F_n = F_e$ (d) F_n is slightly more than F_e

8. For a deuteron and an alpha paticle binding energies per nucleon are x_1 and x_2 respectively then the energy released Q in the fusion reaction is

$^{2}_{1}H + ^{2}_{1}H + \rightarrow ^{4}_{2}He + Q$

(a) $4(x_1 + x_2)$ (b) $4(x_1 + x_1)$

(c) $2(x_1 + x_2)$ (d) $2(x_2 - x_1)$

9. Of the following given below the nucleus having highest binding energy per nucleon is

(a) $^{238}_{92}U$ (b) $^{4}_{2}He$

(c) $^{16}_{8}O$ (d) $^{56}_{26}Fe$

10. In nuclear reactor of 40% efficiency 10^{14} dis/sec takes places. If energy per fission is 250 MeV then power output of the reactor is

(a) 2 kW (b) 4 kW

(c) 81.6 kW (d) 3.2 kW

11. Origin of β^- electrons emitted during decay is

(a) from inner orbits of an atom

(b) from free electron present in atom in nucleus

(c) from disintegration of a neutron in nucleus

(d) from a photon emitted from the nucleus

12. In an average life

(a) half of the nuclei decay

(b) more than half of the nuclei decay

(c) less then half of the nuclei decay

(d) all the nuclei decay

13. On increasing the mass number which of the nuclear properties is not changed

(a) mass (b) volume

(c) binding energy (d) density

14. Which of the following is electromagentic wave

(a) α rays (b) β rays

(c) γ rays (d) cathode rays

15. ^{23}Ne after energy absorption decays into two α particles and an unknown nucleus. The unknown nucleus is

(a) Oxygen (b) Boron

(c) Silicon (d) Carbon

Very Short Answer Questions

1. What is the number of protons and neutrons in $^{22}_{15}X$ nucleus.
2. Write energy equivalent (in MeV) of 1 μ mass.
3. A nucleus after β decay converts into its isotope or isober which?
4. For which, α or β rays the energy spectrum is discrete.
5. On what type of chain reaction the working of a nuclear reactor is based?
6. Write name of any one material used as moderator in nuclear reactors.
7. Write relation between half life (T) and decay constant (λ) for a radioactive substence.
8. What is the SI unit for activity.
9. After four half lives how much percentage of a radioactive substance remains?
10. Which nuclear reaction is responisble for energy generation in sun?
11. A radioactive element having mass number 218 and atomic number 84 emits β^- particles. What are the mass number and atomic number after decay?
12. Does there a loss in mass number after γ decay.
13. From which it is easier to take out a nucleon, iron or lead?
14. A nucleus undergoes fission into two enequal parts. Which of the two (lighter or heavier) parts will have more kinetic energy?
15. If the nucleons of a nucleus are well separated from each other total mass increases. From where this mass comes.

Short Answer Type Questions

1. An hydrogen molecule has two protons and two electrons. In discussing behaviour of the hydrogen molecule the nuclear force between these protons is always ignored, why?
2. A student claims that a hevier form of hydrogen decays by α emission. What will be your reaction?
3. Defined unified atomic mass unit (u).
4. Explain the meaning of nuclear mass defect.
5. Define Radioactivity.
6. Mention the Ruther Ford-Soddy decay law.
7. Give definitions of half life and mean life of a radioactive substance and write relation between then.
8. What is α decay? What is the type of α particle energy spectrum?
9. β ray energy spectrum is continuous? What is the meaning of this?
10. The neutrino hypothesis is helpful in describing which conservation laws in β decay process?
11. Write any two properties of the nulear forces?
12. What do you mean by binding energy per nucleon? How it is related with nuclear stability?
13. Define nuclear fission.
14. What is meant by critical mass in reference to nuclear chain reaction.
15. Heavy water is a good moderator in nuclear reactors? Why?

Essay Type Questions

1. Describe composition of the nucleus and discuss nuclear forces?
2. Explain mass effect and binding energy? Explain the main conculsions which can be drawn from the binding energy per nucleon versus mass number diagram.
3. Write law of Radioactive decay. Using the exponential decay law derive expressions for half life and mean life of a radioactive element.
4. What is meant by nuclear fission? Why a nuclear fission chain is not self substained? Explain what is to be done to make a chain reaction sustained.
5. Draw a simple diagram of a nuclear reactor and explain its working.
6. Explain β decay. Discuss the neutrino hypotheseis of β decay.

7. Discuss α decay from a radioactive nucleus. Explain that the α ray energy spectrum consists of a set of discrete energies.

8. How does the proton-proton cycle proceeds in fusion. Why such thermonuclear reactions can not be performed in laboratory.

Answer

Mutliple Choice Questions -

1. (C)	2. (D)	3. (B)	4. (A)
5. (D)	6. (B)	7. (B)	8. (B)
9. (D)	10. (C)	11. (C)	12. (D)
13. (D)	14. (C)	15. (D)	

Very Short Answer Questions

1. 15, 17
2. 931.5 MeV
3. Isobaric
4. α particle
5. controlled
6. graphite/Heavy water/water
7. $T = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda}$
8. 1 Bq =/ disintegration/sec
9. 6.25%
10. Thermonuclear fusion
11. 218, 85
12. No
13. Lead nucleus
14. Lighter nucleus
15. from binding energy of the nucleus

Numerial Questions

1. Radius of a nucleus of mass number 16 is 3×10^{15} m. What is the radius of a nucleus of mass number 128.

(Ans : 6×10^{-15} m)

2. Calculate binding energy for $^{56}_{26}Fe$ nucleus. [Given, atomic mass of $^{56}_{26}Fe = 55.9349u$, mass of neutron $= 1.00867u$, mass of proton $= 1.00783u$ and $1u = 931\ MeV/c^2$.

(Ans : 492 MeV)

3. The half life of a radioactive substance X is 3s. Initially a specimen of this substance contains 8000 atoms. Calculate (i) its decay constant (ii) time t at which 1000 atoms remain active in this sepcimen.

(Ans : 0.231 s^{1}, 9 s)

4. A radioactive nucleus undergoes decay as follows

$X \xrightarrow{\alpha} X_1 \xrightarrow{\beta^-} X_2 \xrightarrow{\alpha} X_3 \xrightarrow{\gamma} X_4$

If the mass number of X is 180 and atomic number is 72 calculate the mass number and atomic number of X_4.

(Ans : 172,69)

5. 200 MeV energy is obtained per fission of U-235. If a reactor with U- 235 as fuel generates 1000 kW power then calculate the number of disintegrations per second for nuclei in this reactor.

(Ans: 3.12×10^{16})

6. For the fusion reaction $^2_1H + ^2_1H \rightarrow ^3_2He + ^1_0n$ masses of deuteron, helium and neutrons are respectively 2.015 u, 0.017 u and 1.009 u. If 1 kg of deuterium undergoes complete fusion then calculate amount of energy released.

(Ans : 208 MeV)

7. Calculate the Q value for reaction $^{235}_{92}U + ^1_0n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^1_0n + Q$ Given

mass of $^{235}_{92}U = 235.0435$ u

mass of $^{14}_{54}Xe = 139.9054$ u

mass of $^{94}_{38}Sr = 93.9063$ u

mass of $^1_0n = 1.00867$ u

and $14 = 931\ MeV/C^2$

(Ans : 208 MeV)

8. Calculate the mass of ^{227}Th for 1mci activity, its half life is 1.9 y.

(Ans : 1.206×10^{6} g)

9. In some experiment the activity of a given radioactivite was found to be 6400 dis/min. On repeating the experiment after 6 days the activity

was found to be 400 dis/min. Determine the half life of element.

(Ans : 1.5 days)

10. An α particles is emitted by a $^{226}_{88}Re$ nucleus. If the energy of α particles is 4.662 MeV then what is the total energy released in the process.

(Ans : 4.746 MeV)

11. A nucleus ^{176}X, β transforms to ^{176}Y after β decay. If the atomic masses of X and Y are respectively 175. 9426944 and 175.941420 μ then determine the maximum energy of emitted β particles.

(Ans : 1.182 MeV)

Chapter - 16

Electronics

The success of science and technology in this era is due to a big contribution from electronics. Electronic devices and equipments are used in telecommunications, satellite communication, entertainment and areas like computers, weather forecasting, nuclear physics, etc. The working basis of electronic devices is the controlled flow of electrons. The period of electronics started after the construction of devices based on vacuum tubes. But today devices based on semiconductors have replaced the equipments using vacuum tubes. Semiconductor devices have not only made the electronic equipments smaller in size and increased their work efficiency but their low cost has also made them available for the usage of common man.

For example consider personal computers which are very small in size compared to ancient computers which were based on vacuum tubes having the size of a big room and were of limited use only for normal calculations. The device solar cell which converts solar energy into electric energy is also made from semiconductors. For understanding the behavior of semiconductors devices, it is utmost necessary to have knowledge of theoretical aspects of semiconductors. From this point of view we will study about the nature, conduction process in semiconductors and also gain knowledge about the uses of some common semiconductor devices like diode and transistor, in this chapter.

16.1 Energy Bands in Solids

From the study of atomic structure we know that in an isolated atom, the electrons are confined to well defined energy levels. Every electron in an atom stays in one of these discrete energy levels. The maximum number of electrons present in any energy level is decided by the Pauli exclusion principle. The outermost energy level in which the electrons are present in unexcited state of atom is called the valence energy level, and such electrons are called valence electrons. For example, the electronic configuration for sodium (atomic number 11) is $1s^2 2s^2 2p^2 3s^1$. Here, in 3s energy level there is one electron which is the valence electron.

Generally, most of the solid substances including metals are of crystalline nature. In crystalline solids the atoms are arranged in a regular periodic arrangement and the nearby atoms are apart by a very small distance called the 'lattice constant'. The value of lattice constant is different for different crystalline substances and is of the order of atomic size (Å). Clearly, it is seen that due to such small distance between the nearby atoms the electrons of any atom are not only affected by the coulomb force of its nuclei but nucleus and electrons of nearby atoms would also exert some coulomb force them. This can be summarised as the mutual interaction between the atoms of solids. This interaction is responsible for the bonding of various atoms leading to the formation of crystalline structure.

When the atoms in a crystal are interacting then they are not isolated. Hence in crystals the energy levels of the electrons in atoms are not same as the energy level of isolated atoms. Therefore, due to the interaction of the atoms of a crystal, energy levels for atoms in crystals are modified. To understand this transformation first, we discuss the interaction between two identical atoms. Initially, it is assumed that these atoms are so apart that there is no interaction between them (if the distance between two atoms is more (~ 50Å) than its linear dimension (~ 10Å) then it is possible to assume so). In this situation both atom can be treated as isolated. Therefore their energy levels will be same as the energy levels of isolated atoms. This is shown in the diagram 16.1 (a). When these two atoms are close enough to interact. Due to this interaction each energy level of both the atoms is splited into two energy levels, out of which one is a little higher and the other is a little lower than the original energy level. [figure (16.1(b)]. In other words, it can be said that for the system formed by the two atoms there are two energy levels corresponding to each level of an isolated atom. This change in energy levels is in accordance with the 'Pauli exclusion Principle'.

Now we will see the process of crystal formation. For convenience we assume a one dimensional crystal made up of N-atoms. When these atoms are brought closer then due to their mutual interaction energy level of every atom gets splited. This split in the energy levels is directly proportional to the number of interacting atoms. As a result, in a system of N atoms there would be 2 N energy levels in place of a single energy level. If the value of N is very high then these N energy levels will be so closely spaced that they can be considered as to form nearly continuous energy groups theses continuous energy groups are called energy bands. In an actual crystalline solid the value of N is very high approximately from 10^{22}-10^{23} atom/cm^3. Hence, in each energy bands same number of energy levels will be there. There would be a very small separation between such energy levels. For example if there is a difference of leV between the minimum and maximum energy of a band and there are 10^{22} energy levels then there would be 10^{-22}eV interval between two consecutive energy levels. For such a smalls energy interval it can be considered that in an energy band, energy is continuous. This process of energy band formation is shown in the figure 16.1 (c) It is worth noting that in the process of getting the atoms closer the electrons in the outermost orbit of all the atoms will be affected first and and the most. Due to this the energy bands associated with them will be of higher width. The internal orbits of the atoms are not much affected by the interaction hence the energy bands will be of smaller width. This is also shown in the figure 16.1(c). It is also clear from the figure that corresponding to 1s, 2s, 2p, 3s...........energy levels for atoms there are associated energy bands, in a crystal which are called 1s band, 2s band, 2p band 3s band etc.

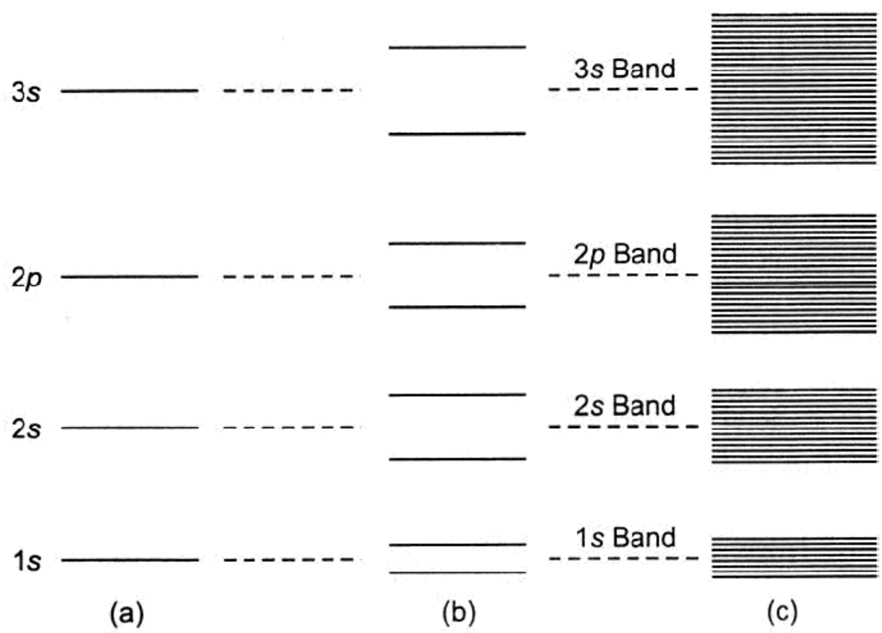


Fig. 16.1 : (a) Engergy level of single isolated atom (b) Split in energy levels due to the interaction of two atoms (c) N atoms (Band formation in crystal structure)

In an isolated atom the electrons first fill the minimum energy level, then to higher energy level then to still higher energy level; upto the valence levels similarly the electrons are filled in these bands. The band in which valence electrons are present is called the valence band. In an isolated atomt here exists an energy gap between discrete energy levels. Similarly in solids there are forbidden energy gaps between the energy bands with the gap representing a range of energies that no electron can have. Here it should be mentioned that the energy band formation is different for different soilds and the energy span of bands and value of forbidden energy gaps is different for different solids.

As in case of isolated atoms the electrons can move from lower energy level to higher energy level by gaining necessary energy. Similar transition of electrons is possible in solids. In solids these transition are of two types : (1) transition with in the same band of energy levels. (2) transition between of energy level belonging to different bands. For both the transition it is necessary that higher energy level should be empty. Since there is very little difference between the two consecutive energy levels of the same band, for such type of transitions very small energy is required. Whereas for transition between two consecutive bands the electrons will require energy equal to the forbidden energy gaps.

16.2 Classification of Soilds as Conductors, Insulators and Semiconductors

Electrical conductivity is a physical quantity whose span is very wide. On one hand we know about metals having very high electrical conductivity on the other hand there are insulators like quartz and mica whose electrical conductivity is very low. Substances are also known, whose conductivity is very small compared to metals at normal temperatures but very large than that of insulators. These are called semiconductors, examples being silicon and germenium.

The conductivity of semiconductors is not only intermediate to conductors and insulators but also the change in the conductivity with temperature is very interesting. Near absolute zero their behaviour is similar to insulators but as the temperature increases the conductivity also increases which is opposite to the observed behaviour for metals. The following questions are not answered by free electron theory for metallic conduction which you have read earlier:

(1) Why are the conductivities of solid substances different?

(2) Why does a substance show the behaviour of a semiconductor?

(3) Why is the change in conductivity with temperature different for metals and semi conductor?

The theory of energy bands in solids provides answer to these questions, and on the basis of their band structures materials are divided into conductors, insulators and semiconductors. Every solid substance has its own band structure which defines its electrical behaviour.

16.2.1 Conductors

Conductors are such solid substances in which, the velence band is partially filled or this band overless with next higher band to from a new band which is again partially filled. In both the situations there are empty energy levels available with in the band to which electrons can make transitions, provided they get energy from external electric field and get exicted.

For example we consider sodium which is a monovalent metal. For this the band structure is such that 1s, 2s and 2p bands are filled to their capacities with electrons and 3s band is half filled.

The reasons for this is that in an isolated sodium atom1s, 2s and 2p energy levels are completely filled but 3s energy level, whose capacity is to take two electrons, is filled only with one electron. Completely filled inner energy bands are not useful for electrical conductivity because the transition of electrons is not possible between electronic energy levels within such bands.

The transition of electrons is not possible between internal bands from 1s to 2s or 2s to 2p because in both the situations the empty energy levels are not available to the electrons. Opposite to this is 3s band which is half filled so half of the energy levels in this band are available for transitions of electrons.

Therefore, the electrical conductivity of sodium is due to this partially filled band. This band is shown in the figure 16.2 (a). The lower half part of this band is called the valence band and upper half part is called the conduction band, because after getting the energy from the electric field the electrons reach this part to start conduction.

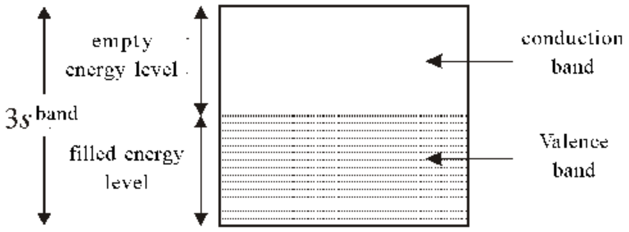


Fig. 16.2 (A) Half filled 3s band for sodium

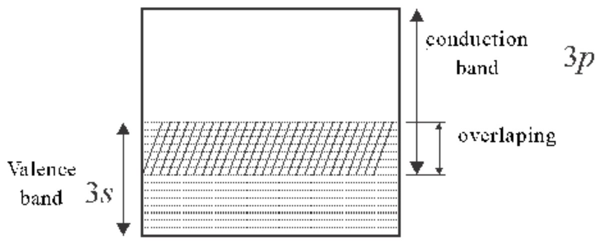


Fig. 16.2 (B) Overlaping of bands in magnesium

Like sodium other monovalent metals also have partially filled bands.

Magnesium, zinc and other substances like them, which are the members of the second group of the periodic table are called divalent substances and they also come in the category of metals. In these types of substances (in the solid state) there is overlapping of bands. For example magnesium's (atomic number 12) electrnic configuration is $1s^2\, 2s^2\, 2p^6\, 3s^2$, and in atomic state there is energy gap between 3s and 3p. But in the process of crystal formation the spliting in the energy levels is such that the fully filled 3s band gets overlapped with fully empty 3p band. Now electrons get the necessary empty levels for transition in this overlapped band. In this state if 3s band is called the valence band, then 3p is the conduction band, thus for such metals the valence band and the conduction band overlap. (figure 16.2(b)).

To conclude, in both of the above metals there is no interval between the maximum energy of valence band and minimum energy of conduction band because of this, the forbidden energy gap for a metal is zero.

Generally, the energy provided by conventionally used sources of electric current is in the range 10^{-4} to 10^{-8}eV; which is sufficient to move the electrons to the empty energy levels of this partially filled band. As mentioned already, there is very small energy difference between the energy level of any band. Thus if empty energy levels are available with in a band, the electrons after absorbing a very small energy reach to such states like free electrons. Hence these are also called free electrons. When electrons after getting a very small amount of energy, reach the empty energy levels then there is a drift of electrons due to which conduction is possible.

In metals, both the free electrons and the necessary energy levels for their transition are available in abundance. Due to this the electrical and heat conductivites of metals are very large. For normal temperatures, the value of electrical conductivity of metals is of the order of 10^6 to 10^8 mho/meter clearly supporting this fact. There is no significant change in the number of free electrons, due to thermal energy in a metal. The effect of heat energy will be to increase in number of collisions between the free electrons and the vibrating ions of the solids, due to which their conductivity will reduce with the increase in temperature.

16.2.2 Insulators

Insulator are solid substances in which the configuration of energy bands is such that the valence band is completely filled and the forbidden energy gap between the maximum energy E_v of valence band and minimum energy E_c of conduction band is very large. The forbidden energy gap is given by; (figure 16.3).

$$E_g = E_c - E_v$$

For insulators, the value of E_g is around 3 - 7 eV. As there are no electrons in fully empty band so it does not take part in electrical conduction. Similarly, in fully filled band the electrons are there but there are no empty energy levels available for the transition so again no electrical conduction is possible for such bands.

As it is clear that very less energy is gained from commonly employed energy sources, hence the electrons do not get enough energy to move from valence band to conduction band. Similarly at normal and even at high temperatures the electrons of valence band do not get energy equal to the forbidden energy gap. Due to this the electrons are not able to reach from valence band to conduction band. Hence, such solids are called insulators.

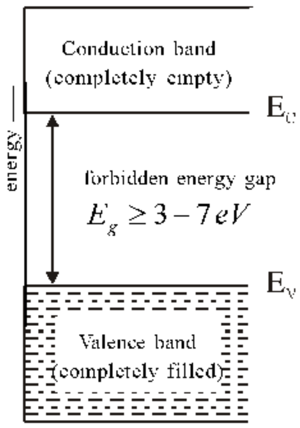


Fig. 16.3 Energy band diagram for insulator

For example, diamond (forbidden energy gap (~ 6eV), comes in the category of insulators. Generally, the conductivity of insulators is between 10^{-12} mho/meter to 10^{-18} mho/meter (meaning resitivity 10^{12} ohm-meter to 10^{18} ohm-meter).

16.2.3 Semiconductors

Semiconductors are such solid substances having band structure similar to that of insulators. The only difference is that the value of forbidden energy gap is less in comparison to that of insulators and is approximately of 1eV order. At absolute zero, the valence band is completely filled and the conduction band is fully empty. Due to this both these bands do not participate in conduction. This is the same bahaviour as that of insulators, thus, at absolute zero the insulators and semiconductors behave in the same way.

At room temperature or a temperature higher than that, the electrons in the energy levels of the upper part of the valence band get thermal energy and move to conduction band, to take part in the process of conduction. Same number of energy levels get emptied in the valence band in which the electrons of the same band make transition to take part in the process of conduction. (In valence band the process of conduction is explained in terms of a positive free charge (hole) which is explained in next section). Thus at room temperature the conductivity of semiconductors is more than that of insulators. However, as only a very small number of the electrons from the valence band reach to the conduction band due to thermal energy for participation in the process of conduction. In the case of conductors a large number of electrons are present in the conduction band. Thus the conductivity of semiconductors is much less, in comparison to conductors. The above discussion clearly suggests that the conductivity of semiconductors is intermediate between the conductivity of conductos and insulators. This is the reason why such solids are named as 'semiconductors'.

The conductivity of intrinsic semiconductors increases as the temperature increases, this is explained in next section. Silicon, germanium and gallium, arsenide are examples of some useful semiconductors. Apart from this lead sulphide indium antimonide, gallium phosphilde and silicon carbide are also semiconductors. The forbidden energy gaps for some semiconductors are given in the table (16.1) below.

Table 16.1

Semiconductors	Forbidden Energy Gap
Silicon (Si)	1.12 eV
Germanium (Ge)	0.7 eV
Indium antimonide (InSb)	0.17 eV
Gallium arsenide (GaAs)	0.33 eV

16.3 Intrinsic Semiconductors

The semiconductors having no impurity are called intrinsic semiconductors. In ideal state in this type of intrinsic semiconductors, there should be atoms of only that semiconductor. But in reality it is not possible to get such type of crystals. Therefore, if in a semiconductor material the number of inpurity atoms and the number of semiconductor atoms is in the ratio $1:10^8$ or less, then it is considered as an intrinsic semiconductor. To study the properties of intrinsic semiconductors, here we consider silicon and germanium.

Silicon and germanium both are the members of fourth group of the periodic table and their valency is 4. Their electronic configurations are as follows:

Ge (32) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^2$

Si (14) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$

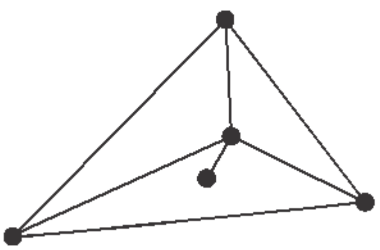


Fig. 16.4 : Tetrahadron structure for silicon (or germanium)

In the crystal of both these elements the atoms are in an ordered array in such a way that each atom is inside a regular tetrahedron and on the four corners there are other nearby atoms. Figure (16.4) shows one such tetrahedron unit. Every valence electron of the atom makes a covalent bond with the valence electron of the nearby atom. In this way every atom is connected with the nearby four atoms through covalent bonds. In every covalent bond there are two electrons. For convenience in study ; figure (16.5) shows the two dimensional form of crystal structure of germanium; which is also applicable for silicon.

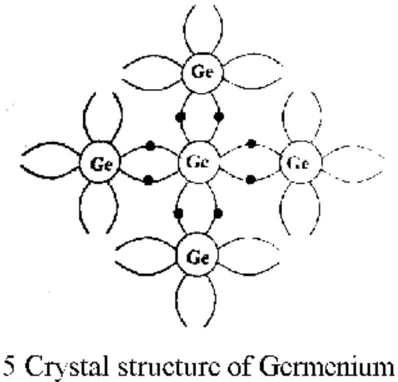


Fig. 16.5 Crystal structure of Germenium at 0 K

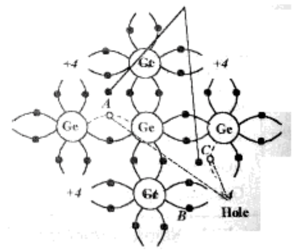


Fig. 16.6 Electron-hole pair in germenium

At absolute zero (0 K) the valence electrons are bonded in covalent bonds hence there is no free electron available for electrical conduction. Due to this, at absolute zero temperature, intrinsic semiconductors behave as insulators. When the temperature of the crystal is increased, some valence electron gain enough thermal energy and break their covalent bonds to become free. These free electrons move in the crystal freely and participate in electrical conduction, when external electric field is applied

When one electron moves out of the covalent bond a vacancy is created. This vacancy is called a hole. The absence of electron is equal to the presence of same magnitude of positive charge. Hence, the hole can be assumed as a positive charge having the same magnitude as that of an electron. As explained later, holes also participate in electrical conduction in semiconductors. In a semiconductor when a covalent bond breaks then an electrons hole pair is generated. At room temperature (300K) or a temperature close to it there are many such electron hole pairs available. The production of electron hole pair is shown in figure (16.6)

At normal temperatures, electrons and holes have random motion in semiconductors. The random motion of hole can be understood in figure (16.6).

Suppose due to thermal energy an electron gets free from a covalent bond at position A, hence at this place a hole is generated. From a nearby atom, the valence electron breaks the covalent bond (at place B) and have a transition to hole at position A. The reason for this is that electron is having transition from one bond to the other and all the electrons present in the bond are approximately are of same energies. As shown in the figure due to the movement of electron from C to B the hole will move from B To C, etc. In this way in any semiconductor the electrons and holes both do random motion. Since, hole can be taken as a free positive charge +e, hence in a semiconductor both holes and electrons act as charge carriers and participate in electrical conduction.

This process of hole electron pair production in intrinsic semiconductors can also be explained by band theory. The forbidden energy gap in semiconductors is of 1eV order. At absolute zero temperature, the valence band is completely filled and conduction band is fully empty. Hence, semiconductors behave like insulators. When the temperature of the crystal is increased electrons get enough thermal energy and are capable of crossing the forbidden energy gap. Such electrons reach the conduction band from the valence band and holes are generated in the valence band in place of electrons (figure 16.7) Due to transition of one electron from valence band to conduction band a hole is generated and here also the presence of electron hole pair can be understood. The electrons present in conduction band are called free electrons and do random motion, similarly in valence band holes do random motion.

In fact, the band describtion and covalent bond describtion for a semiconductor are equivalent. It can also be said that due to thermal energy the covalent bond breaks and the electron reaches the conduction band from the valence band. Therefore, for this process the necessary minimum energy would be equal to band interval energy E_g. Here this questions is logical that at room temperature ie. 300 **K** the average kinetic energy of the electron is kT = 1/40 = 0.025 eV, whereas the necessary energy for transition is about 1eV. Then, how does an electron reaches the conduction band from the valence band? Here it is worth noting that kT is the value of average kinetic energy and not that all electrons are of this energy. Very few electrons are there having energy of 1eV order capable of making transitions from valence band to conductrion band.

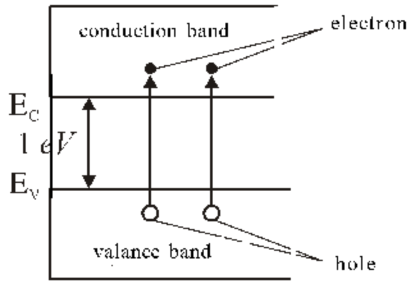


Fig. 16.7 : Effect of temprature on intrinsic semi conductor

It is clear from above discussion that in any intrinsic semiconductor at a finite temperatures the number of electrons and holes is same. If in an intrinsic semiconductor the concentrations of free electrons and holes are n and p respectively then

$$n = p = n_i$$

$$np = n_i^2 \qquad ......(16.1)$$

where n_i is called as the intrinsic charge carrier density.

When the temperature of an intrinsic semiconductor is increased compared to prior temperature, more covalent bonds break, meaning more electrons reach the conduction band from valence band and the number of electron hole pair increases. This means that n_i is dependent on temperature and increases with temperature. Like wise of two different semiconductors, (for example, silicon and germanium) are at same temperatures then since the value of forbidden energy gap E_g for them is different, hence, the semiconductor for which the value E_g is less will have more covalents bonds to break or relatively more electrons will make transition from valence band to conduction band. Clearly at same temperature, for two semiconductors for which the forbidden energy gap E_g are not same; the value of intrinsic charge carrier density, n_i will not be same but would be more for the semiconductor having less value of E_g. In this way in an intrinsic semiconductor, the value of intrinsic charge carrier density, n_i is dependent on (i) temperature (ii) and nature of semiconductor material. In mathematical form this dependency is shown by the following formula;

$$n_i \approx AT^{3/2} \exp-\left[\frac{E_g}{2kT}\right] \qquad ....(16.2)$$

Where T is absolute temperature, k is Boltzmann constant and A is another constant.

At same temperature, comparing intrinsic silicon (E_g = 1.1 eV) and germanium (E_g = 0.7 eV) there are more number electron hole pairs in intrinsic germanium than in intrinsic silicon.

16.3.1. Conduction in Semiconductors

If some electric potential is applied across a semiconductor, then drift motion of elctron and holes takes place. Electrons drift towards the positive terminal from the negative terminal whereas the holes drift from positive terminal to negative terminal. (figure 16.8). In other words, in a semiconductor the electrons move opposite to the direction of applied electric field and holes move in the direction of electric field. The current in the metallic wires in external circuit is due to the flow of electrons.

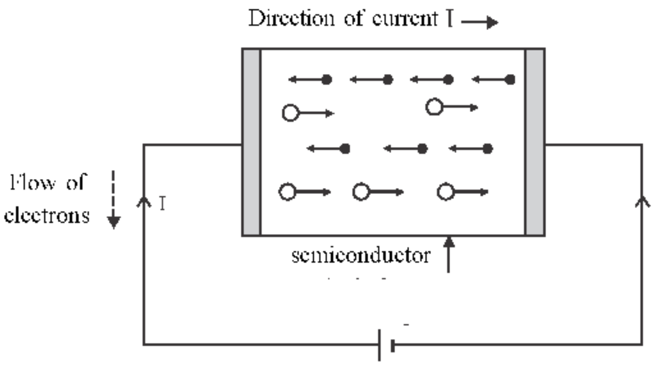


Fig. 16.8 : Current in intrinsic semiconductor

Generally, the drift velocity of electrons and holes for commonly employed electric fields is directly proportional to the applied electic field. If v_n and v_p are the drift velocities for electrons and holes respectively

then ; $v_n \propto E$ and $v_n = \mu_n E$ ....(16.3)

and $v_p \propto E$ or $v_p = \mu_p E$ .....(16.4)

Where μ_n and μ_p are the mobilities for electrons and holes respectively. The mobility is measured in $m^2 v^{-1} s^{-1}$. Drift velocites in vector form are given as

$$\vec{v}_n = -\mu_n \vec{E}$$

$$\vec{v}_p = +\mu_p \vec{E} \qquad ....(16.5)$$

beacause electrons move opposite to the field $\vec{E}$. In presence of some electric field if the current density due to electrons is denoted by $\vec{J}_n$ and that due to holes by $\vec{J}_p$ then total current density due to drift in a semiconductor is;

$$\vec{J} = \vec{J}_n + \vec{J}_p \qquad ....(16.6)$$

By the diffinition of current density ;

Current Density = The number density of number of charges $\times$ charge $\times$ drift velocity, therefore

$$\vec{J}_n = n(-e)\vec{v}_n = n(-e)(-\mu_n \vec{E})$$

or $\vec{J}_n = ne\mu_n \vec{E}$ ....(16.7)

and $\vec{J}_p = p(e)\vec{v}_p = pe\mu_p \vec{E}$ ....(16.8)

Hence, it is clear that although the drift velocities of both the electrons and holes are in opposite direction but the corresponding current densities are in the same direction (direction of external electric field). Therefore total current density;

$$\vec{J} = \vec{J}_n + \vec{J}_p$$

$$\vec{J} = ne\mu_n \vec{E} + pe\mu_p \vec{E}$$

$$\vec{J} = (n\mu_n + p\mu_p)e\vec{E} \qquad ....(16.9(a))$$

Since, in the above formula $\vec{J}$ and $\vec{E}$ are in the same direction. Hence, in scalar form:

$$J = (n\mu_n + p\mu_p)eE \qquad ....(16.9(b))$$

By difinition the electrical conductivity of semiconductor.

$$\sigma = \frac{J}{E} = e(n\mu_n + p\mu_p) \quad \text{..........(16.10)}$$

And resistivity of semiconductor;

$$\rho = \frac{1}{\sigma} = \frac{1}{e\left(n\mu_n + p\mu_p\right)} \quad \text{.........(16.11)}$$

The above discussion is true for both intrinsic and extrinsic semiconductors for an intrinsic semiconuctor $n - p - n_i$ respctively.

$$\sigma = n_i e\left(\mu_n + \mu_p\right) \quad ...(16.12)$$

$$\rho = \frac{1}{n_i e\left(\mu_n + \mu_p\right)} \quad ...(16.13)$$

Here, it is worth noting that the electron and hole have same charge magnitude but their mobilities are not same. In electric current, holes behaves as a positive charge having some effective mass. This mass is slightly more than the effective mass of an electron. Hence, the mobilities of electron and hole are different. (You will study about the effective mass in detailed in higher classes). The effective mass of electron and hole are different for different substances therefore mobilities of electrons and holes are different for different semiconductors. Table (16.2) shows the values of μ_n and μ_p for silicon and germanium at temperature T= 300 K.

Table 16.2

Semiconductor	μ_n(m^2/Vs)	μ_p(m^2/Vs)	$n_i(/m^{-3})$
Si	0.13	0.048	9.65×10^{15}
Ge	0.39	0.19	2.5×10^{19}

Since, in the flow of current in a semiconductor both negative and positive free charges (electron and hole) participate; the conduction in semiconductor is called bipolar conduction. Opposite to this is metals where only electrons participate in the flow of current. hence they have unipolar conduction.

16.3.2 Effect of Temperature on Electrical Conductivity of Interinsic Semiconductors

The equation (16.12) suggest that the conductivity of any intrinsic semiconductor is dependent on the number of charge carriers (n_i) present. Since, n_i is dependent on temperature and forbidden energy gap, hence, σ also dependent on them. For any given semiconductor (E_g = constant) if the temperature T increases then σ increases exponentially (or for resistivity, when temperature increases it decreases exponentially.) Also with the temperature increase the collisions between the free electrons and holes and vibrating atoms of semiconductors will be more frequent due to which the value of μ_n and μ_p decreases. However, this reduction is not that effective as the increases in the value of n_i due to increase in temperature and the increases in the value of σ. The increase in the conductivity of intrinsic semiconductors is a special property of intrinsic semiconductors which is not seen in metals. In metals the conductivity is reduced (or the resistivity is increased) with the increase in temperature, exhibiting $\sigma \propto T^{-1} (or\, \rho \propto T)$ behaviour at normal temperatures.

The reason for the difference in two is that in metals the free electron density n does not change with temperature but with the increase in temperature the collisions between the electrons and vibrating ions are more frequent in metals so conductivity decreases with temperature.

On the basis of above description, it can also be said that the temperature coefficient of resistance for intrinsic semiconductors is negative. For silicon its average value is $\approx -0.07/K$. Here it is important to note that at absolute temperature $n_i = 0$ for semiconductors hence $\sigma = 0$ (insulator behaviour).

Since the conductivity of intrinsic semiconductors is small and limited, there is no direct use of these in semiconductor devices. But their resistivity changes due to temperature or incident light energy hence these are used in as radiation sensitive resistance. or as LDR.

Example 16.1 : Calculate the resistivity of intrinsic germanium at 300 K temperature. Mobility of electrons and holes and n_i value for germanium are according to the table (16.2).

Solution : Resistivity formula for intrinsic semiconductor

$$\rho = \frac{1}{n_i e(\mu_n + \mu_p)}$$

Where $e = 1.6 \times 10^{-19} C$ and from table (16.2)

$$\mu_n = 0.39 m^2 / Vs.$$

$$\mu_p = 0.19 m^2 / Vs.$$

$$n_i = 2.5 \times 10^{19} / m^3$$

Substituting the value in above formula;

$$\rho = \frac{1}{1.6 \times 10^{-19} \times 1.5 \times 10^{19} (0.39 + 0.19)}$$

$$= \frac{1}{4 \times 0.58} = 0.43 \Omega m$$

Example 16.2 : For some intrinsic semiconductor the forbidden energy gap is E_g electrons volt. What maximum wavelength of light can be absorbed by this semiconductor.

Solution : If v is the frequency of incident light photon then the energy of this photon will be hv. If the energy of incident photons is higher than the forbidden energy gap of the semiconductor then the electrons present in the valence band will absorb these photons and reach the conduction band. The process is called photo electron hole pair production. Hence, minimum frequency of photon for absorption v_{min}, is given by -

$$hv_{min} = E_g.$$

If λ_{max} is the maximum wavelength for absorption of photon then;

$$\frac{hc}{\lambda_{max}} = E_g \quad \text{or} \quad \lambda_{max} = \frac{hc}{E_g}$$

As hc = 1242 eV nm and E_g when taken in eV unit, we have

$$\lambda_{max} = \frac{1242}{E_g} nm.$$

16.4 Extrinsic Semiconductors

As described earlier that intrinsic semiconductors have limited conductivity, hence they are not used as such. If in any intrinsic semiconductor impurities of suitable material are mixed in very small proportion then the conductivity of semiconductor so obtained is many times more than that of intrinsic semiconductors. To increase the conductivity of a semiconductor impurities of suitable kinds are added to it in very small quantity. This process is called doping, and the semiconductors so obtained is called extrinsic semiconductors. The suitable impurities for silicon and Germanium are either the elements of Vth group of the Periodic Table (like arsenic (As), antimony (Sb), phosphorous (P), etc). or the element of group (III) (like, aluminium (Al), gallium (Ga), indium (In), boron (B) etc). Due to the impurities of group V element in Si or Ge, the number of free electrons increases while for the impurities of Group III elements the number of holes increases.

Therefore the extrinsic semiconductors are of two types.

(i) N-type extrinsic semiconductors.

(ii) P-type extrinsic semiconductors.

The conductivity of any extrinsic semicounductor is controlled by the quantity of impurity mixed. The quantity of impurity element is very small. One atom of impurity is mixed with approximately 10^6 atoms of an intrinsic semiconductor. Due to this there is no significant change in the original crystal lattice but the increase in conductivity is very large. For example, if 1 impurity atom is mixed in 10^9 germanium atoms then its conductivity increases 10^3 times in comparison to intrinsic semiconductor. Generally, all semiconductor devices are made form extrinsic semiconductors.

16.4.1 N-Type Semiconductors

When a very small amount of impurity of pentavalent element is mixed in in tetravalent intrinsic semiconductor then N-type of extrinsic semiconductors is obtained. To understand the effect of such kind of impurity element on the semiconductors, we take the example of mixing phosphorus (impurity) in intrinsic silicon. In the process some silicon atoms are replaced by the phosphorous atoms. Since, the impure materials are mixed in very small quantity, hence, they are surrounded by silicon atoms. Phosphorous atom has five valence electrons to make the bonds with neighbouring Silicon atoms having only four valence electrons. Hence, pentavalent phosphorous makes covalent bonds with the four nearby silicon atoms but its fifth electron remains unbound, [figure (16.9)]. In comparison to the four other electrons, this fifth electrons is lousely bound its bonding energy is approximately 0.05 eV. This energy is equal to the mean thermal energy of the atoms, hence the electron gains this energy and is freed. At room temperature (300 K) each impurity atom can provide one free electron to the crystal. In other words, every impurity atom gives a free electron to the semiconductor. Due to this, the impurity atom is called donor atom and such type of impurities are called donor impurities. The number density of electrons so obtained depends on the number of donor atoms.

On the other hand the energy required for getting free electron by breaking the covalent bonds of silicon is 1.1eV. Hence, a very small number of electrons get free. Therefore, at normal temperature only 1 atom out of 10^{10} silicon atoms is able to generate electron hole pair. Now if one impurity phosphorous atom is present per 10^6 silicon atoms, then every phosphorous atom denotes one electron to the crystal and per 10^{10} atoms one electron is produced by breaking of covalent bond. Hence, when comparison is done then a ratio of $10^4 : 1$ is obtained from impurity and thermal generation respectively. Clearly, with a very small impurity the number of free electrons and conductivity increases by a multiple of 10^4. In such type of semiconductors the number of free electrons is more than the number of holes. Therefore here electrons are majority charge carriers and holes are minority charge carriers. Such type of semiconductors are called N-type of semiconductors which clearly illustrates the fact that here the negative free charge carriers electrons are in abundance.

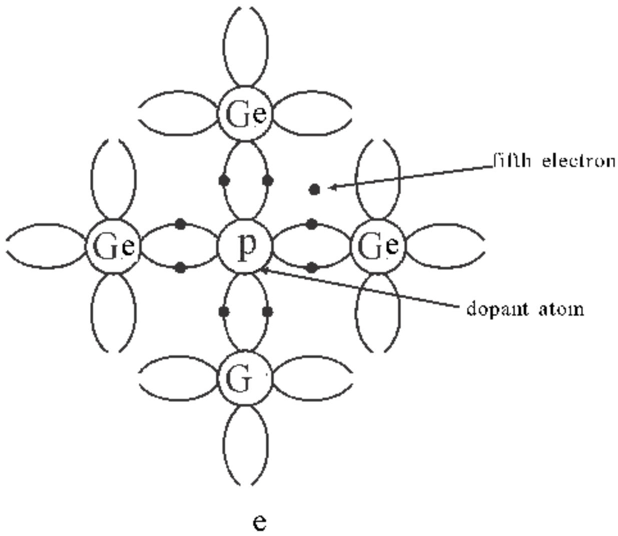


Fig. 16.9 : Dopping of phospores in germanium

Although in a N-type semiconductor, the negative charge carriers are in abundance as compared to holes but over all, the N-type semiconductors is electrically neutral. Donor atom gives an electrons and becomes a positive ion. The number of free electrons given to the crystal by the donor atoms, is equal to the number of positive donors ions so produced. Thus electrical neutrality of crystal is maintained. Positive ions remains fixed at their place in the crystal, because their remaining electrons are bonded with the other atoms. Hence, they do not contribute in electrical conduction.

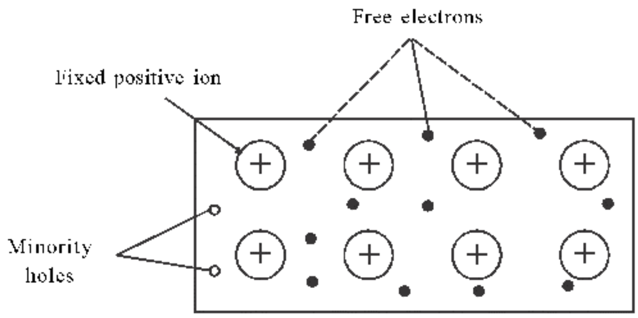


Fig. 16.10 N type semiconduction

16.4.1 Energy Band description for N-type Semiconductors

When donor impurities are mixed in a intrinsic semiconductors then such impurity atom create energy levels a little below the bottom of conduction band in band gap. These levels are called impurity levels. (Fig 16.11). The difference between minimum energy of conduction band E_c and the energy of these energy levels E_c - E_d, is very small (for phosphorus in Si it is ~ 0.05 eV and for Germanium it is ~ 0.01 eV). Hence, the electrons of these donor levels, gain energy easily form the crystal's thermal vibrations and reach the conduction band to participate in conduction.

Apart from this, due to thermal energy some electrons from valence band also reach conduction band as in the case of intrinsic semiconductor. In such a process holes are generated in valence band. But now the number of holes in valence band is less then the number of electrons in conduction band; because the conduction band is getting electrons both from thermally generated electrons and from donor energy levels.

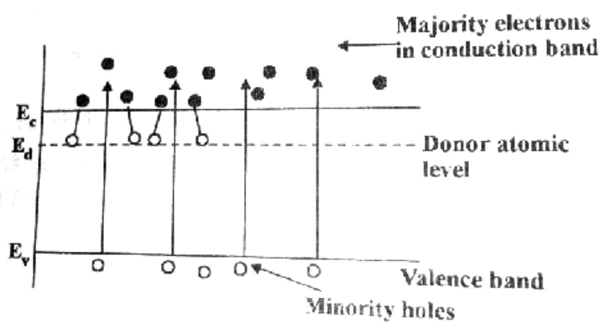


Fig. 16.11 : Band model for N-type semiconductor

16.4.2 P-Type Semiconductors

When a trivalent impurity (element) like (aluminium, boron, indium or gallium) is mixed in very small quantity with tetravalent intrinsic semiconductors, then P-type semiconductors are obtained. For example, if indium is mixed as an impurity element in intrinsic silicon then indium atom replace silicon atom in the crystal lattice. Its there valence electrons form covalent bonds with the nearby silicon atoms but fourth silicon atom's covalent bond lackes one electron . Hence, a vacancy is created which generates a hole. At room temperature due to thermal energy, an electron from a near by silicon atom may move towards the vacant position of this impurity atom to complete the bond. [Figure 16.12 (a)]. In this process, the impurity atom now becomes a negatively charged ion. On the other hand a hole is generated in this atom. Here, every impurity atom gains an electrons and provides a hole to the semiconductor. These holes are in addition to the thermally generated holes. The impurity atoms are called acceptors because they recieve electron from the bonds of intrinsic semiconductors. The number density of the holes depend mainly on the quantity of impurity.

In such type of semiconductors the number of holes is much more than the number of free electrons. Hence, here the holes are the majority charge carriers and the electons are the minority charge carriers. Since, holes behave as positive particles, such type of semiconductor is called P-type semiconductors.

P-type semiconductors can also be understood, on the basis of band structure. Due to the doping of acceptor atoms, acceptor energy levels are generated in forbidden energy gaps a little above the maximum free energy E_V of the valence band [Figure 16.12(b)].

If E_a denotes the energy of these energy levels then in Silicon E_a E_V ~ 0.05 eV and for germanium E_a $-E_V$ ~ 0.01 eV. This energy is easily acquired by the thermal vibrations of the crystal and the electron make transition form the valence band to acceptor energy levels; due to which a hole is generated in the valence band.

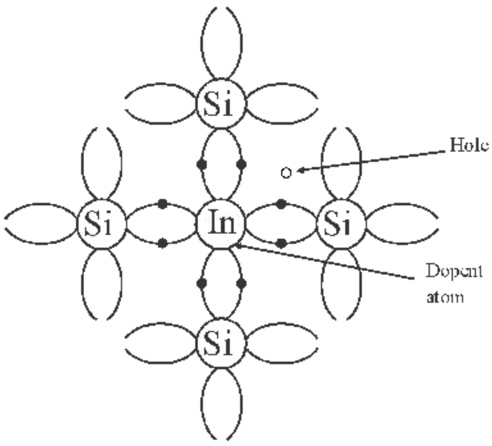


Fig. 16.12 (A) Bond diagram for P-type semicondutor

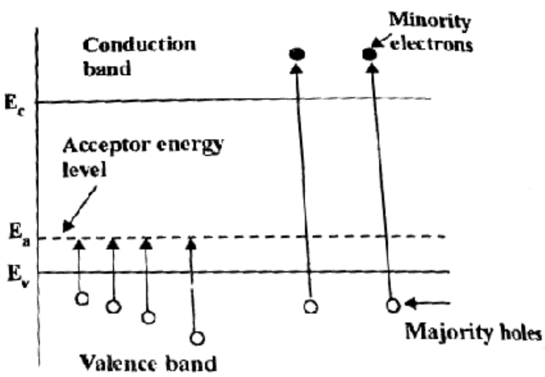


Fig. 16.12 (B) Energy band model for P-type semiconductor

Similar to N-type semiconductor, P-type semiconductors is also electrically neutral. Here as many holes are generated from impurty atoms, same number of negative acceptor ions are also produced. Also the total sum of positive and negative charges is zero. Negative ions are fixed in the lattice hence, they do not contribute to electric current.

16.5 P-N Junction

When a P-type semiconductor is joined atomically to a N-type semiconductor so that the continuity of crystal structure is maintained at their contact surface (interface) ; then this contact surface is called P-N junction and the device so formed is called PN junction diode. Generally, in a single crystal of silicon or germanium, the impurities are so added by special techniques that its one side becomes P-type semiconductor and the other side becomes N-type semiconductor, the boundary between these two regions is called P-N Junction. A PN junction cannot be formed by pressing a P-type semiconductor over a N-type semiconductor because in this case, the crystal structure is not continous across the contact surface.

As soon as a junction is formed, as there is a large concentration of holes on P side and a very small concentration of electrons on N side there will be a diffusion of holes from P side to N side. Similarly because of difference in concentration electrons diffuse from N side to P side. This motion of charge gives rise to diffusion current across the junction. Note that diffusion current due to electrons and holes is from P side to N side. When an electron diffuses from N side to P side it leaves behind an ionised donor on N-side. This ionised donor (positive) charge is immobile as it is fixed in crystal lattice. As the electrons continue to diffuse form N to P, a layer of positive charge (or positive space-charge-region) is formed on N side of the junction. Similarly, when a hole difuses form P to N, it leaves behind an ionised acceptor (negative charge) which is also immobile. As the holes continue to diffuse, a layer of negatively charged ions (or negative space-charge region) is formed on the P-side of the junction . Thus a space charge layer is formed near the junction having negative immbolie ions on P-side and positive immbolie ions on N-side. This region is devoided of free charge carries and is called the deplection layer. (Fig. 16.13)

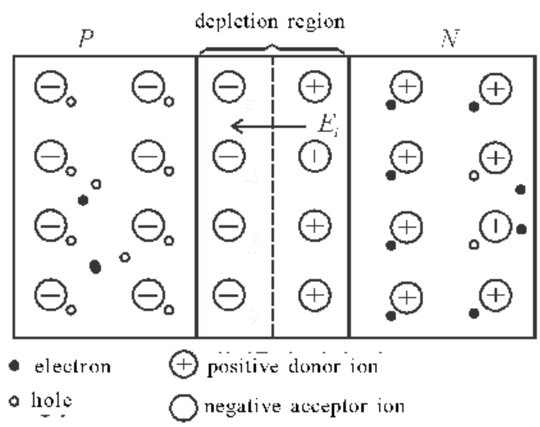


Fig. 16.13 P-N junction

Due to the ions present in the depletion layer, an electric field is generated in this layer directed from N-side to P-side. Any hole near the junction is pushed by this electric field into P-side. Likewise, any free electron near the junction is pushed by electric field into N-side. Thus diffusion current is opposed. However, this electric field supports the drift of minority electron son P-side into N-side and drift of minority holes from N-side to P-side. This electric field is called barrier electric field. Thus a drift current which is opposite in direction to the diffusion current starts. Initially, diffusion current is large and drift current is small. As the diffusion process continues, the space-charge regions on either side of the junction extend, thus increasing the electric field strength and hence drift current. The process continues until the diffusion current equals the drift current. In a P-N Junction under equilibrium there is no net current, but electric field still exists, this built in electric field causes a difference of potential across the junction of two regions. N-side is at a higher potential than P. Since, this potential tends to prevent the movement of electrons from the N-region into the P-region, it is often called a barrier potential. In general for silicon P-N Junction it is approximately 0.7 volt and for germanium PN-junction it is 0.3 Volt.

The width of the depletion layer (d) depends on the amount of impurities present in the, P and N sides. If the amount is more the width is small. This width is of the order of one micron (10^{-6} meter). Potential barrier V_B and barrier field E_i are related by the following formula.

$$E_i = \frac{V_B}{d}$$

If the width of the depletion layer of silicon P-N junction is assumed to be 1 micrometer;

$$E_i = \frac{0.7}{1 \times 10^{-6}} = 7 \times 10^5 \quad \text{V/m}$$

Any P-N junction can be connected in two ways in a circuit.

(i) When P-side is at a higher potential than N-side.

(ii) When N-side is at a higher potential than P-side.

The above situations are called forward biasing and reverse biasing respectively. In both these situations the electrical behaviour of P-N junction in the circuits is very different. We will study this in detail.

16.5.1 Forwad Biasing

When an external voltage V is applied across a semiconductor diode such that P-side is connected to the positive terminal of the battery and N-side to the negative terminal (figure 16.14); then P side is at higher potential than N-side. This is called forward biasing. In this situation, an external electric field, E is established at the junction which is directed from P to N. It is opposite to the internal electric field, E_i in the junction. In forward bias as barrier potential at the junction reduces it allow more diffusion to take place, while drift current remains unchaged. As a result, more number of holes and electrons reach the depletion layer and its width decreases. In this situation, diffusion current is more than drift current.

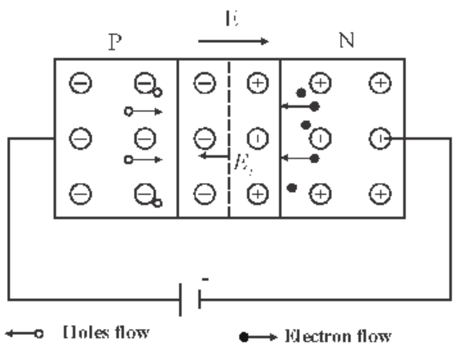


Fig. 16.14 : Forward biased P-N juction

In the P-region near the positive terminal of the battery if an electons reach this place by breaking a covalent bond, as a result of this process a hole is generated which moves towards the junction region while the electon moves through the connecting wires to reach the positive terminal of the battery. At the same time an electron is released form the negative terminal of the battery and enters N-region to move towards the junction. In this way, there is flow of current in the junction. In P-region it is due to holes and in N regions due to electrons which are the majority-charge carriers in these regions respectively. Therefore, this current is mainly due to the diffusion of majority charge carriers. It is obvious that such a bias helps in current build up and is called forward bias. The current in external circuit is due to electrons.

If the voltage applied by the battery is increased the potential barrier will reduce, more majority charge carriers will diffuse in the junction regions and the value of electric current will increase. Since, the flow of current is easy through the junction, hence, the resistance of the junction will be very small in this case.

16.5.2 Reverse Biasing

When the P-terminal of a P-N Junction is connected to the negative and N-terminal is connected to the positive terminal of the battary; then this is called reverse biasing. (figure 16.15). In this situation the external electric field E and internal electric field E_i both are in the same direction (form N side to P side). As a result, the barrier potential at the junction increases. Due to this the diffusion of majority charge-carriers from the junction is not possible as the hole in the P-region and the electrons in the N-region both move away from the junction; this increases the width of the depletion layer.

In this situation a very small current flows through the junction due to drift of minority-charge-carriers. For germanium P-N junction this current is microampere (10^{-6} A) order and for silicon P-N junction this is nanoampere (10^{-9}A) order. At normal applied voltages there is no change in the number of minority charge carriers, hence this current is constant for such potential differennce and is called reverse saturation current.

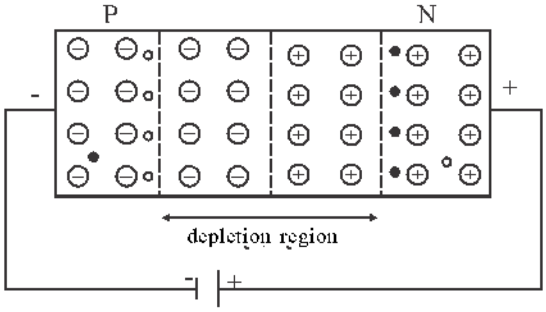


Fig. 16.15 : Reverse biased P-N junction

As a very small current exists in reverse bias condition, the resistance of PN junction is very large. If the temperature of the junction is increased then more covalent bonds break and the number of minority-charge-carriers increases. As a result reverse bias saturation current depends on temperature and increases with the increase in temperature of the junction. If the value of potential difference increases more than a limiting value then the current increase rapidly. This process is called breakdown which is discussed later.

16.6 Junction Diode and Its voltage current Characteristics

To connect a P-N junction in the circuit metallic electrodes are made at the P and N ends of the device. Hence, the device is called diode. Diode word is formed by di-electrode which means two electrodes. P-N junction diode is also called semiconductor diode.

The symbol used to represent diode in the circuit is shown in the figure (16.16).

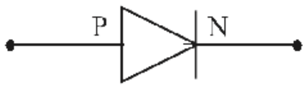


Fig. 16.16 : Circuit symbol for P-N junction diode

Here the arrow head represent P-region and bar represents N-region. The direction of arrow points from P to N representing direction of current flow under forward bias. The curves describing the variation in current through a diode with change in bias voltage are called V-I characteristics of diode. PN junction exhibits different behaviour under forward and reverse biased states. Therefore two types of characteristic curves are drawn :

(i) forward bias characteristics.

(ii) Reverse bias characteristics.

16.6.1 Forward Bias Characteristics

For obtaining the V-I characteristics of a forward biased P-N junction diode, the experimental circuit arrangement is shown in the figure (16.17).

Here the applied voltage. V on the diode is changed by the potential divider arrangement, and can be easily read by the voltmeter connected in parallel to the diode. The current *I* flowing in the diode corresponding to different applied voltages are noted by a milliammeter. The curve obtained for different values of *V* and *I* is shown in the figure (16.18). It is also called as the P-N junction diode forward characteristic curve.

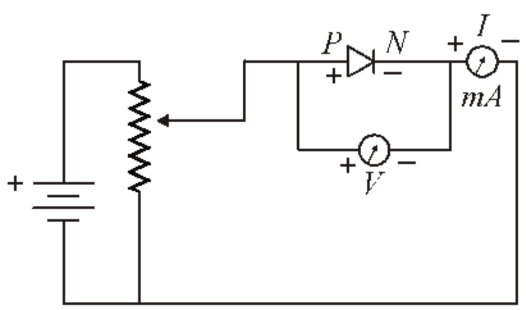


Fig. 16.17 : Experimental setup for characterstics of P-N diode in forward bias

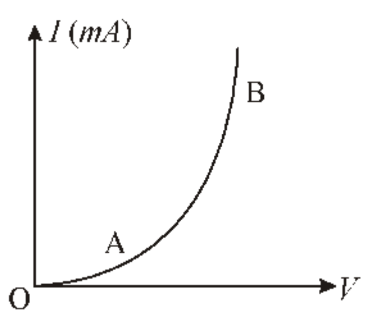


Fig. 16.18 : forward bias characterstics for P-N Junction

In forward bias, for very small values of voltage. (for Ge is 0.3 V and for Si is 0.7 V) the forward current is very small. The reason for this is that the value of applied voltage is less than the potential barrier due to which the current is very small. This behaviour of the diode is represented by the OA part of the curve. When the value of applied potential is increased the current in the diode increases exponentially. This behaviour is shown by part AB. The potential at which the current value increases rapidly is called the diode's knee voltage or cut in voltage. For germanium diode its value is approximately 0.3 V and for silicon diode its value is 0.7V.

16.6.2 Reverse Bias Characteristics

For obtaining the V-I characteristics of a reverse biased P-N junction diode the experimental circuit arrangement is shown in the figure 16.19 (a). Here with the help of a potential divider arrangement the P and N terminals of the diode are connected to the negative and positive terminals of the battery. Since reverse current is very small a miroammeter is used in place of milliammeter in this circuit. At different reverse voltages the corresponding reverse current values are noted and a curve is drawn as shown in the figure 16.19 (b).

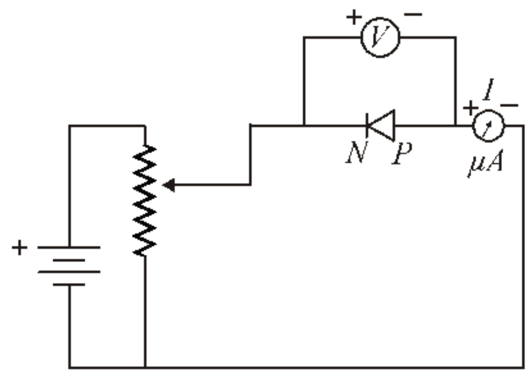


Fig. 16.19 (A) Experimental setup for characterstics of P-N diode in reverse bias

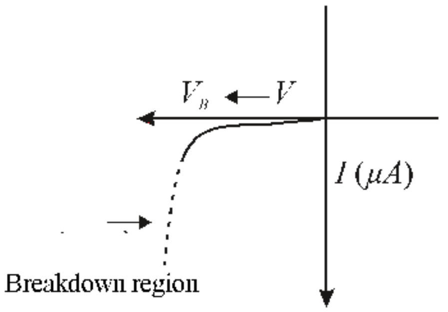


Fig. 16.19 (B) Reverse bias characterstics for P-N junction

As explained earlier in reverse bais, current is very small, as it is due to minority-charge-carriers, and remains constant till the reverse potential reaches the breakdown voltage (V_B) there after the reverse current increases very rapidly even for a slight increase in the breakdown voltage.

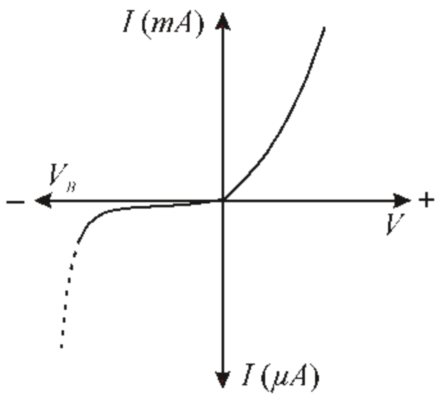


Fig. 16.19(C) : characteristic curve for P-N juction

for a semiconductor diode the complete forward and reverse bias characteristic curve is shown in the figure 16.19.(c). If we do not consider the break down regions of this then in forward bias large current (~ mA) flows while in reverse bias small current (μA or nA) flows. As a result P-N junction diode is a unidirectional device. The characteristic curve of the diode also shows that this device does not follow Ohm's law and hence is a non-linear device.

For a pN junction diode the variation of current with voltage is given by -

$$I = I_s\left[\exp\frac{qV}{kT} - 1\right]$$

Where $q = 1.6\times10^{-19}$ C,k is Boltzman constant, T is the temperature of the junction and V is the potential difference across the junction, I_s is reverse saturation current

In forward biasing as V is positive so when;

$$\exp\frac{qV}{kT} >> 1$$

and $$I = I_s \exp\frac{qV}{kT}$$

and current increase exponentially with the voltage. In reverse biasing where V is negative then;

$$\exp\frac{qV}{kT} << 1$$

$$\therefore \quad I = I_s$$

thus under reverse bias current is almost constant.

The above equations are quite valid to expalin I-V characteristics of Ge-PN diode. The diode made of silicon follows this equation only qualitatively. Also for both the types of diodes, the diode equation is not true for reverse voltages higher than breakdown voltage.

16.6.3 Diode resistance

Diode is a nonlinear device and it does not follow the Ohm's law thus the resistance of a diode is not constant. For such types of devices dynamic resistance is a more useful quantity rather than static or d.c. resistance.

(i) Forward dynamic resistance : By definition, forward dynamic resistance is given as;

$$r_f = \frac{\text{small change in the forward bias potnetial}}{\text{Corresponding change in forward current}} = \frac{\Delta V}{\Delta I}$$

In figure 16.20 for the calculation of forward resistance, ΔV and ΔI are shown near point C If ΔV and ΔI are very small then dynamic resistance at point C would be inversely proportional to the slope of the tangent drawn at this point on the curve. Forward dynamic resistance is generally very small $(1-100\Omega)$.

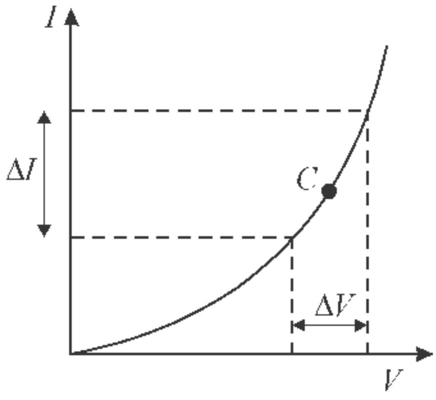


Fig. 16.20 : Forward dynemic resistant

(ii) Reverse Resistance : By definition, reverse resistance is;

$$r_r = \frac{\text{Small change in reverse voltage}}{\text{Corresponding change in reverse current}}$$

Since under reverse bias the current is very small, hence, reverse resistance is very high. This is of the order of mega ohm $(10^6\,\Omega)$.

16.6.4 Reverse Breakdown

As explained earlier if the reverse bias voltage across a PN diode is increased beyond a certain limit V_B the reverse current increases rapidly to a large value. This phenomenon is called reverse breakdown and the potential V_B at which it starts is called breakdown voltage. This breakdown can be due to one or both the mechanisms given blow-

(i) Avalanche Breakdown

(ii) Zener Breakdown

(i) Avalanche Breakdown : At large reverse voltages the minority charge carriers of P and N region get accelerated by high electric field and cross the junction. If these minority carriers acquire enough kinetic energy, from their collision with the covalent bonds new electron-hole-pair are generated. These charge carriers also get accelerated and produce new electron hole pairs. This process is cumulative due to which the number of minority charge carriers and hence reverse current increase rapily. This process is called avalanche breakdown. If the current is not controlled then due to the heat generated the diode may get damaged. If the potential is decreased before the diode gets damaged then diode current reduces and the diode again shows its normal reverse behaviour.

(ii) Zener Breakdown : This process occurs in a diode which is highly doped due to which the depletion layer is very thin. In this situation even for a small potential, the electric field at the junction is very strong. It exerts a force on the electrons of the covalent bonds, close to the junction and these bonds break. As a result minority charge carriers increase in number and the value of current increase rapidly. This process is called Zener breakdown.

The breakdown can be by due to any process but it is customary to called a diode meant to operate in break down region as Zener diode. We will study the use of zener diodes later.

Example 16.3 : For a P-N junction diode forward bias is increased form 2.0V to 2.5 V so forward current is changed from 16.5 mA to 26.5 mA. For same diode reverse bias is increased form 5V to 10 V so the reverse current changes from 20 microampere to 30 microampere. Calculate the dynamic resistance for this diode in both the situations.

Solution : According to question in, forward biasing ;

$$\Delta V_f = 2.5 - 2.0 = 0.5\ V$$

and change in forward current

$$\Delta I_f = 26.5 - 16.5 = 10mA$$

Therefore, forward dynamic resistance

$$r_f = \frac{\Delta V_f}{\Delta I_f} = \frac{0.5V}{10mA}$$

$$= \frac{0.5V}{10 \times 10^{-3} A} = 0.5 \times 10^2 = 50\Omega$$

(ii) For reverse biasing

$$\Delta V_r = 10 - 5 = 5V$$

and change in current is ;

$$\Delta I_r = 30 - 20 = 10\mu A$$

Therefore, reverse dynamic resistance.

$$r_r = \frac{\Delta V_r}{\Delta I_r} = \frac{5V}{10\mu A}$$

$$= 0.5 \times 10^6 \Omega = 0.5 M\Omega$$

Example 16.4 : For a P-N junction diode the dynamic resistance under forward bias is 25Ω. How much change should be made in forward bias potential so that there is 1 mA change in forward current?

Solution : Under forward bies

$$r_f = \frac{\Delta V_f}{\Delta I_f}$$

$$\Delta V_f = r_f \Delta I_f$$

$$= 25 \times (1 \times 10^{-3})$$

$$= 25 \times 10^{-3} mV$$

$$= 25 mV$$

Hence, there should be a change of 25 mV.

16.7 Use of a P-N Diode as Rectifier

We know that the generation and transmission of alternating current is more simple and less expensive than the generation and transmission of direct current. Due to this generally alternating current is used. But in many electronic instruments direct current or potential is also used. The direct current is obtained form cell or battery but in practice alternating current is converted into direct current and used in electric devices.

Rectification is the process by which alternating current is converted to direct current. The device used for this purpose is called rectifier. As stated earlier, a P-N junction diode allows current in forward bias and no current (negligible current) flows in reverse biasing. This unidirectional property of diode is used for rectification.

16.7.1 Half-Wave Rectifier

Figure (16.21) Shows the circuit of a half wave rectifier. In this an input alternating voltage source is connected to the primary coil of a transformer. The secondary coil of the transformer, is connected to a diode and load resistor R_L in series between point A and B. From the secondary coil of the transformer a desirable alternating voltage is obtained between point A and B. This alternating voltage can be more or less than the input alternating voltage. In case of a step up transfer V_S is more than in put V_{in} while for step down transformer V_S is less than V_{in} step up nature then V_S.

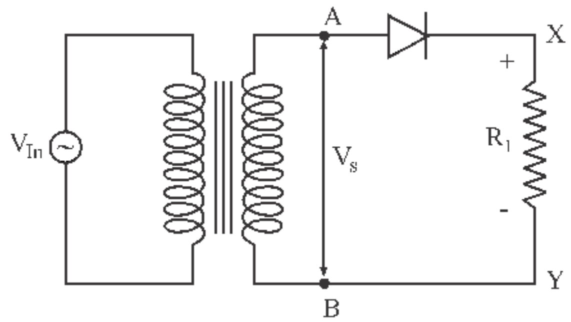


Fig. 16.21 : Half wave rectifier circuit

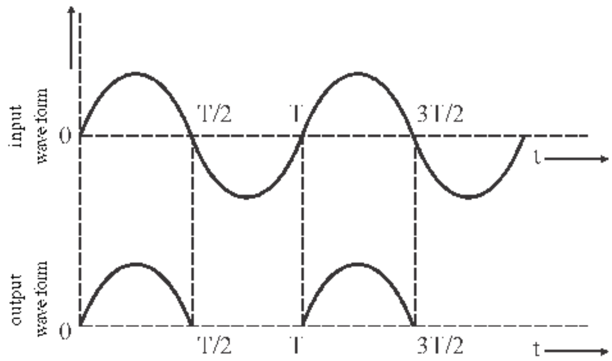


Fig. 16.22 : Wave form of output & input half wave rectifier

For positive half cycles of alternating potential V_i (or V_s) the end A of the secondary coil is at positive potenitial and end B is at to negative potential due to which the P terminal of the diode is at a higher potential than N terminal. Thus this diode will be conduction and allow current. In this case there will be a potential drop across the load resistor R_L. In the negative half cycle of input ac end A is at negative and end B is at positive potential, due to which diode is reverse biased. In reverse biasing the current is negligible hence diode is in non conducting state there will be no current in the circuit. In load resistor R_L the current flows only in one direction that is from A to B hence, potential drop at the ends of R_L is of dc nature.

However this potential drop across R_L is pulsating. It changes in magnitude but not in direction. In the above precess the negative half cycle of input ac is supressed by diode, and only the positive half cycle is obtained as dc. Hence, this circuit is called half wave rectifier. The above process is repeated for consecutive cycles of alternative voltage. Input ac voltage and output voltage waveform from the rectifier are shown in the figure (16.22).

In half wave rectifier only half cycle of input a.c if used remaining half cycle remains unused. In full wave rectifier the full cycle of the input ac is utilised. Hence, due to this the efficiency of half wave rectifier is half of the full wave rectifier. Therefore generally half wave rectifier is not used. Here, it should also be noted that in half wave rectifier the use of transformer is optional. The input voltage can be directly applied to diode load resistor combination.

16.7.2 Full Wave Rectifier

In a full wave rectifier circuit the output voltage is obtained for both the half cycles of input alternating voltage. Here two junction diodes are used such that one diode allows the positive half cycles of input ac and the other allows the negative half cycles. Figures (16.23) shows a circuit of full wave rectifier.

Input ac is applied ascross the terminals of the primary coil of a centre taped transformer.

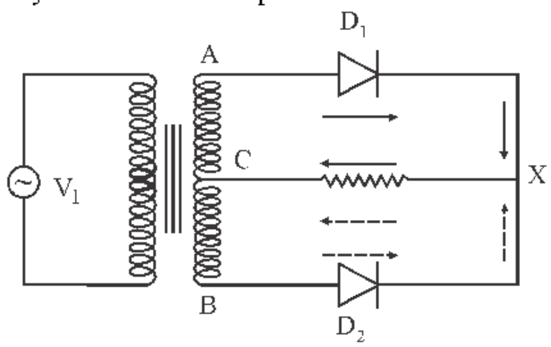


Fig. 16.23 : Full wave rectifier circuit

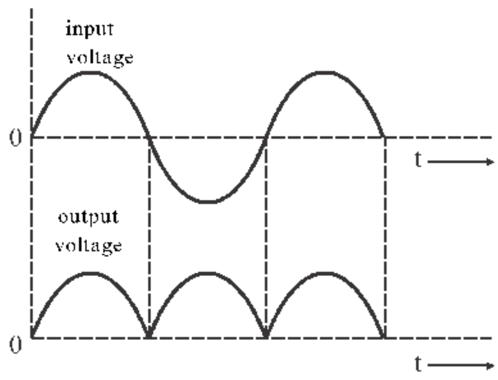


Fig. 16.24 : Wave form for input & output voltage for full wave rectifier

The P-terminals of the diodes is connected to the terminals A and B of the secondary of transformer. The N-side of the diodes are connected together and the output is taken across load register R_L connected between this common point of diode and the centre tap C of the secondary of the transforemer. The point C is at reference zero potential. Thus if a given instant if one terminal of secondary is positive with respect to point C, then other terminal will be negative. Let for the positive half of input ac terminal A is positive and B is negative. So diode D_1 gets forward biased and conducts (while D_2 being reverse biased is not conducting). Hence, during the positive half cycle we get an output current (and a output voltage across the load resistor R_L). During negative half cycle the voltage at A becomes negative with respect to centre tap, the voltage at B would be positive. In this part of the cycle diode D_1 would not conduct but diode D_2 would give an output current and output voltage (across R_L) ac. Note that for both halves of input, current in R_L is directed from X to C i.e. it is unidirectional. Thus, we get output voltage during both the positive as well as negative half of the cycle. Obviously, this is more efficient circuit for getting rectified voltage or current than the half wave rectifier current in R_L is from X to C.

For a full wave rectifies a centre tap transformer is necessary. Here an ordinary transformer cannot be used. If the current is to be obtained at high voltage then this transformer should be step up transforms and if the current is to be obtained at low voltage then the transformer should be step down transformer.

16.7.3 Full Wave Bridge Rectifier

In such type of full wave rectifier a centre tap transformer is not necessary but 4 (four) diodes are used instead of two diodes as shown in the figure (16.25) in the form of a bridge in the circuit. The input alternating voltage is obtained from the secondary coil of the transformer.

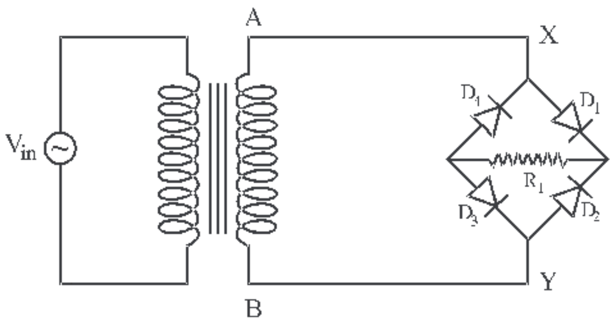


Fig. 16.25 : Bridge rectifier circuit

In the positive half cycle of the input voltage the terminal A of the secondary coil is positive and B is negative. Hence, diodes D_1 and D_3 are forward biased and diodes D_2 and D_4 are reverse biased. Current flows in AD_1XYD_3BA direction. There is no conduction is diodes D_2 and D_4.

In the negative half cycle of input voltage terminal A of the secondary coil is negative and B is positive. then diodes D_2 and D_4 are forward biased and diodes D_1 and D_3 are reverse biased. Now the current flows in directions BD_2XYD_4AB/D_2 and D_2 do no conduct.

In this way, in bridge rectifier at any instant only two junction diodes help in the flow of current and remaining two diodes do no conduct. But for every cycle the current flows from X to Y in load resister. Hence, this is unidirectional and the out voltage on R_L is rectified. As described earlier in place of centre tap transformer an ordinary transformer is used hence in comparison to full wave rectifier the bridge rectifier is less expensive which is an advantage.

The diodes used in rectifier circuit should be such that when they are not conducting i.e. when they are reverse biased then there should be no breakdown of diodes otherwise they would also be conducting and the process of rectification will not be possible. Hence, the rated potential of diodes should be higher than that of the maximum value of alternating voltage of secondary coil. These diodes are also called power diodes.

By the analysis of the waveform of output voltage (current) obtained from full wave rectifier it is clear that this potential is unidirectional but not constant, it is called fluctuating rectified potential whereas by definition pure dc voltage is constant in magnitude. From mathematical analysis of waveform of full wave rectifier, it is known that this type of changing potential is superposition of various harmonic frequencies and a pure dc component.

Using some special circuits, which are called smoothing filters; pure dc compoment is obtained by isolating the rectifying potential from alternating components. In such type of filters capacitors, inductors or their combination are used are because their behaviour is different for direct (rectified) potential and alternating potential. One such filter is shown in figure (16.26).

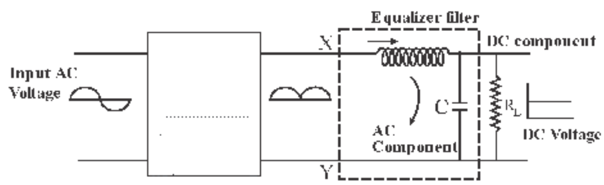


Fig. 16.26 : Full wave rectifier using equalizer filter

When rectifier's output voltage is applied between the terminals X and Y of the filter circuit then the due to the reactance offered by inductor L ac component of rectified output is opposed while dc component is not affeated ac component is further by passed by capacitors C connected in parallel with load resistor R_L. It is so because capacitor afters low reactance to ac components compared to the resitance offered by R also capacitor blacks dc so, it will not pass through C. Thus nearly pure dc potential will be obtained across R_L.

16.8 Special purpose diodes

Apart from rectification, P-N Junction diodes have other uses also. The diodes used for some specific purposes are different from the diodes used for some other specific purpose, in properties, like semiconductor, used used, amount of impurities and construction, etc. Now we will study about some special purpose diodes.

16.8.1 Zener Diode

We have seen that for a PN diode at reverse breakdown voltage the current rises Sharply however the potential difference across the diode remains practically constant at its breakdown voltage. The breakdown voltage for a diode depends on amount of doping. Thus depending upon the doping levels, diodes having specified breakdown voltage can be fabricated. Such diodes are called Zener diodes. Nowadays diodes having breakdown voltage from 1 V to several hundred volts are available. The circuit symbol for a Zener diode is shown in fig. (16.27).

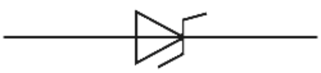


Fig. 16.27 : Symbol of zener Diode

As the voltage across a diode remains constant at its breakdown while current increases. This property makes it usefull for voltage stabilization and voltage regulation. We know that in an ordinary dc power supply consisting of rectifier and filters, ac is changed to dc. In such type of power supplies when there is a change in load current the value of dc voltage also changes. Therefore, in such situations where current is to be obtained at constant dc voltage such a power supply cannot be used. With the help of a zener diode the output voltage of a power supply can be kept constant.

Figure (16.28) shows a power supply whose output voltage is assumed to be V_i and a resistance R_s and a zener diode are connected across its output terminals. The load resistor R_L is parallel to the zener diode. The zener diode is so selected whose breakdown voltage V_Z is equal to the constant value of that dc voltage to be obtained at load resistor R_L. Zener diode is connected in the circuit in reverse biased condition. If the value of V_i is more than V_Z then the diode comes in breakdown region and the potential across its terminals remains V_Z. Since R_L is in parallel to the diode, hence potential across is also V_z. Even if the value of R_L is changed potential difference across it is still V_Z. The resistance R_S is so selected that the diode is not damaged by the thermal energy due to the increase in the zener current. This value of current is known from the data sheet given by the manufacturing company.

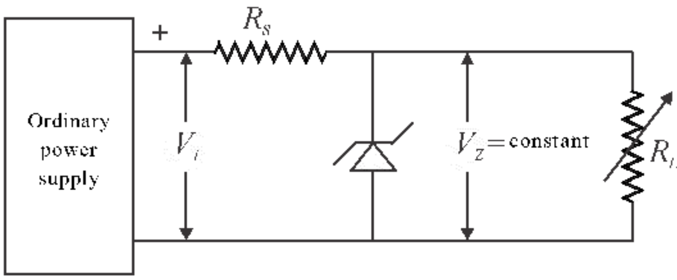


Fig. 16.28 : Voltage regulation by zener diode

16.8.2 Photo Diode

When light (electromagnetic radiation) of suitable frequency is incident on a semiconductor, it can be absorbed by semiconductor. The energy so recieved may cause transition of an electron from the valence band to conduction band, resulting in generation of an-electron hole pair. These electron-hole pairs are in addition to thermally generated electron-hole pairs. Obviously in the process conductivity of semiconductor increases. This increase in conductivity is called photo conductivity and phenomenon is called photo conductive effect. If the frequency of incident photos is v and their energy h_v is greater than or equal to the band gap energy Eg (i.e. $h_v > E_g$) then photo conductivity can be observed.

Photo diode is a PN junction whose function depends on the phenomenon of photo conductive effect. In such diodes one of the regions either P or N is made relatively thin so that the light photons incident on it are able to reach junction region.

Normally photo diodes are operated under reverse bias. The circuit symbol for photo diode is shown in Fig. (16.29). In the absence of light current through a reverse biased photo diode is small ($\sim N\mu A$). When light is incident photons absorbed near the junction create new-electron-hole pairs. These carries get separated due to electric field present in deplection region and cross the junction. Thus reverse current increases. The increase in reverse current is much larger than current under dark (no light) condition. While keeping the frequency constant if the intensity of light is increased reverse current increases more. Thus current is circuit is controlled by the intensity of the incident light. The effect of intensity level (ϕ) on reverse current is shown in fig. 16.31.Current in presence of light may be upto few hundred micro ampere. It is necessary to keep reverse voltage well below the breakdown voltage. Measuring the change in reverse current due to change in intensity of light, can be use to measure the intensity of light.

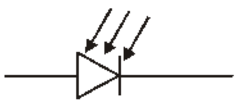


Fig. 16.29 : Circuit symbol for photo diode

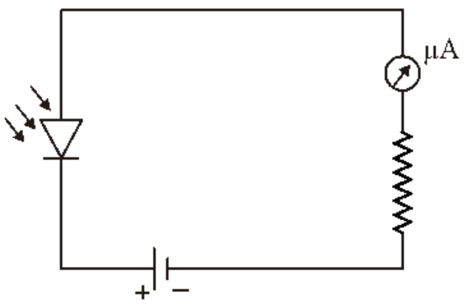


Fig. 16.30 : Photo diode for light detection

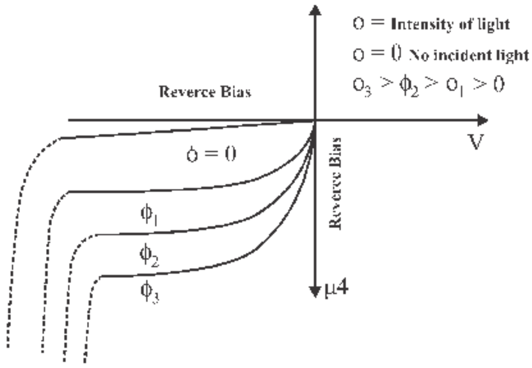


Fig. 16.31 : Effect of light intensity on reverse current voltage of a photo diode

Photo diode is used in following devices :

(i) Light detection.

(ii) Light operated switch.

(iii) Reproduction of sound in films.

(iv) For reading computer tapes and computer cards.

16.8.3 Light Emitting Diode (LED)

Light emitting diodes are those P-N junction diodes, which under forward bias, emit light. When an electron present in the conduction band of a semiconductor make transition to the hole present in the valence band, recombination of electron hole take place. In this process energy is released. Generally the electrons are at the minimum energy level of the conduction band (E_C) and the holes are at the maximum energy level of the valence band (E_V). Hence, energy relased in the process is E_C $E_V - E_g$ equal to the forbidden energy gap. In many semiconductor like silicon and germinaum, this energy is released in the form of heat (thermal energy) but in gallium arsenide phosphide (GaAsP) and gallium phosphide (GaP) like semiconductors, this energy is released in the form of visible light.

To obtain visible light of good intenaity electron-hole recombination events must occur in large number. But for intrinsic and extrinsic both types of semiconductors such recombination events occur in limited number. This is due to the reason that in intrinsic semiconductors the number of electrons hole pairs is quite small and in extrinsic semiconductors one type of charge carrier are in majority and the other type of charge carriers are in minority.

In semiconductors like GaP if the P and N region of the junction are heavily doped, so the depletion region is very thin. If the junction is forward bias then the large number of holes of P region move towards N region and the large number of electrons of N region move towards P region. Hence, near the depletion layer there would be a large number of recombinations of electrons and holes. As a result the emitted light will be of high intensity. The figure (16.32) shows the symbol for a light emitting diode.

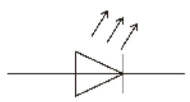


Fig. 16.32 : Symbol of LED

The semiconductor for LED is chosen as per the desired colour of light. At present the LED's of red, green, yellow, orange and blue colour are available.

The LED is used for following purposes :

1. In the form of light indicator.

2. In the form of seven segment display unit.

3. LED generating high intensity light are used in optical fibre communication.

4. High intensity light using LED bulbs.

16.9 Transistor

A transistor is a semiconduce device having three layers and two P-N junction with three terminals for connection to an external circuit. A transistor is capable of amplification of ac signals. The transistor was invented in the year 1948 in USA (United states of America) at Bell telephone laboretory by Bardeen, Brattain and Shockley. For this they were honoured by the nobel prize in the year 1956 in physics. The semiconductor devices like detectors and rectifiers were being used before 1948 but the invention of transistor has laid the foundation of present era of semiconductors electronics. Today many types of transistors for example, junction transistor, field effect transistor or MOSFET are available. Here we will study only about junction transistors. You will study about other transistors in higher classes.

16.9.1 Junction Transistor

A simple junction transistor is generally a single crystal of an extrinsic semiconductor (silicon or germanium) in which three regions of different conductivities are present. The width of the middle region is less then the other two regions and the type (N or P) of the semiconductor in this region is different than the other two regions. In this way we obtain two type of junction transistors which are called PNP and NPN transistors respectively. In a PNP transistor, there are two P-types semiconductors a thin layer of N type semiconductor is sandwiched between them. Similarly in an NPN transistor there are two N types semiconductor regions and a very thin layer P-type semiconductor is sandwiched between these two.

The middle region in both the types of transistors is called the base (B) and out of the other two regions one is called the emitter (E) and the other is called the collectors (C). [Figure (16.33)]. Connected to all the three regions there is a metallic electrode and a lead through which the transistor can be connected to external electric circuit. The leads are named as E lead, B lead and C lead.

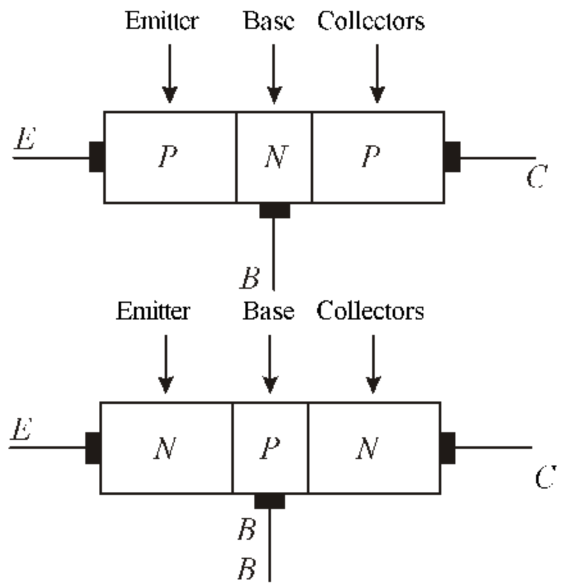


Fig. 16.33 : Construction of PNP & NPN transitor

The emitter and collector are made of the same type of semiconductor (P or N) but the amount of doping in them is different; also the collector region is bigger in size than the emitter. Due to differences in physical and electrical properties and because each region has its own specific purpose emitter and collector cannot be interchanged with each other while using a transitor.

Emitter is heavily doped because its purpose is to provide majority charge carrier in large numbers to the base. There is very little doping in base; also it is of very small width so that it does not provide opposite type of charge carrier in large number for recombination to the majority charge carriers coming from emitter. The purpose of the collector is to collect the majority charge carriers which pass through the base. Here the doping is less than the emitter but more than the base i.e it is moderatly doped.

The area of collector-base junction is larger than the that of emitter-base. Due to this the collection of the charge carriers take place very well and the transistor when used as an amplifier, this large area helps in quick dissipation of the generated heat. There are two P-N junctions in these transistors which are called E-B junction (emitter-base junction) and B-C junction (base-collector junction). Theoretically, both these junction can be considered to be connected back to back. (Figure 16.34).

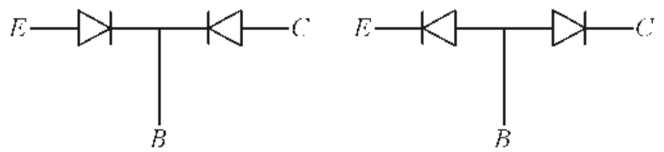


Fig. 16.34 : Assumption of transistor diode junction

However, it is worth noting that when two P-N junction are joined according to the figure (16.34) then a transitor is not obtained. In transistor, both these junctions are in present in the same crystal, whereas two diodes can not be joined to make a single crystal. The width of the base of the transistors is of micrometer order, hence both these junctions are very close.

When Transistor is used as an amplifier the emitter-base junction is always forward biased and base-collector junction is always reverse biased. Due to this the majority charge carriers always flow from the emitter towards the base, but the current flowing in this junction can be from emitter to base (E to B) or base to emitter (B to E) depending upon the nature of majority carriers. These two different directions of the electric current are used to differentiate between the symbols of PNP and NPN transistor. These symbols are shown in the figure (16.35).

The line segment having an arrow sign represents the emitter, the middle line segment represents the base and third line segment represents the collector. The arrow head shows the direction of current. Since in forward bias the emitter base junctionof the PNP transistor the majority holes from the P-type emitter move towards the N-type base hence, the current will flow in the junction from E to B. Therefore, in the symbol of PNP transistor the arrow head is pointing from E to B. Similarly, in NPN transistor in forward bias the majority electrons move form N-type emitter to P-type base hence the current will be in the direction from B to E. Therefore, in the symbol of NPN transistor the arrow head is pointing from B to E.

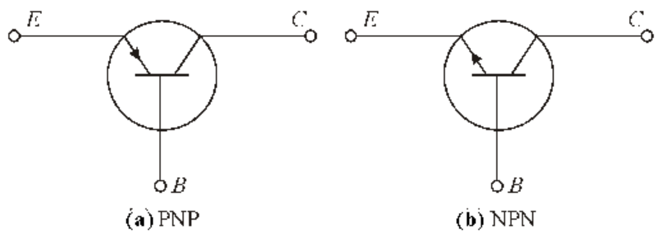


Fig. 16.35 : Symbol for junction diode

For biasing of two P-N junctions of a transistor following four possibilities are there:

1. When emitter-base junction is forward biased and base-collector junction is reverse biased; for such biasing, transistor operates in active region. Generally, transistor are biased in the this way.

2. When emitter-base junction and base-collector junction both are forward biased in such situation the transistors is in saturation region.

3. When emitter-base junction is reverse biased and base-collector junction is forward biased then transistor is in cut off region.

4. When emitter base junction in reversed baised on a collector base junction is reverse bias, transistor is considered in inverted mode.

From the above biasing possibilities only first i.e active bias is used for the working of a transistor as an amplifier. Hence, we make use of only this possibility to understand the transistor action. The other possibilities are used in switching and other circuits, which are not studied here.

16.9.2 Operation of Transitor

For any transistor to work properly E-B junction is forward biased and B-C junction is reverse biased. In this case the transistor is said to be in active state. Firgure 16.36 (a) and (b) shows active bias of PNP and NPN transistors. In both the figures the depletion regions corresponding to both the junctions are shown.

Since, E-B junction in forward bias and the doping in emitter is high, E-B junction will be narrow. Whereas B-C junction is reverse biased hence it is broader. The forward bias voltage V_{EB} at E - B Junction is small (0.5–1V) and the reverse bias voltage V_{CB} is more for B – C junction (5 to 15 V).

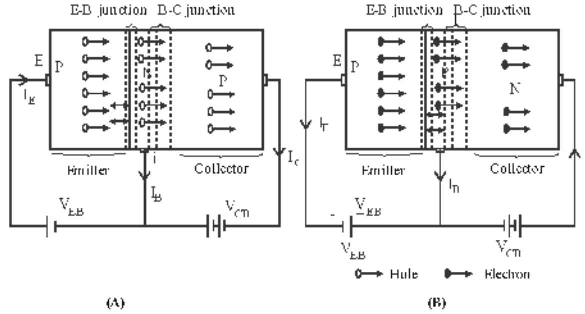


Fig. 16.36 Biasing of PNP and NPN transistor for its active operation

Now we consider the PNP transistor shown in the figure 16.36 (a). Since, E - B junction is forward biased hence majority charge carriers 'holes' from emitter will diffuse through this junction to base in large numbers. This is known as the injection of the holes from the emitter to the base. Similarly from the N-type base the electons cross the junction to reach emitter. The motion of both charge carriers are in opposite direction but the current related to them flows from E to B i.e from E to B. This current is called emitter current I_E which is due to both the holes and electrons. But due to less doping in base region of transistors this current is basically due to holes, for a PNP transitor.

The tendancy of the injected holes from the emitter to the base is of recombination with the electrons present in the base. But as the base is thin and lightly dopped very small (less than 5%) recombination will take place, and majority of holes pass base collector junction are able to reach the collector. Because the collector terminal is negative, these holes reach easily to this terminal to constitute collector current I_C.

Some holes injected form the emitter to the base, recombine with the electrons in the base and generate base current I_B. For every electron lost in the process of recombination an electron is supplied by negative terminal of battery V_{EB} connected to base. Therefore, here base current I_B flows outward from the base terminal B. For a PNP transistor the direction of currents I_E, I_B and I_C are show in the figure 16.36 (a).

If Kirchhoff's current law is used for transitor as a whole then, it is clear that emitter current will be equal to the sum of base current and collector current,i.e

$$I_E = I_B + I_C \quad ....(16.14)$$

Here $I_B << I_E$ and $I_B << I_C$

Since, E - B junction is forward biased hence its forward resistance, is very small, and B - C junction is reverse biased hence its reverse resistance is large. Due to this it is may appear that emitter junction current I_E would be much more than the current collector I_C. But from equation (16.14) as I_B is very small hence; I_C

$\approx I_E$. So from the view of operation, transistor is a device which transfers the current I_E from a very low resistance (forward biased EB junction) to the high resistance (reverse biased B_C junction) keeping the same value I_C ($I_C \approx I_E$). For this transfer process of current this device is named by the conjuction of two words transfer + resistor and abbreviated as transistor.

In other words, it can also be said that in for current conduction of transistor operation, the effect of voltage at B - E junction is very high on the collector current. If the value of V_{EB} is high then both emitter current and collector current are more.

The above description can also be used for the active operation of NPN transistor. In this transistor majority charge carriers are electron; which when injected form the emitter of N-type, diffuse through the P type base and reach the N type collector to produce collector current I_C. Figure 16.36 (b) shows the directions for I_E, I_B and I_C for an NPN transistor.

The emitter current in a NPN transistor is mainly due to electrons. Although both PNP and NPN transistors are used but due to the high mobility of electrons. **NPN transistors are more effective** in high frequency circuits.

16.10. Transistor Circuit Configurations :

Generally, the electronic circuits are four terminal networks in which two terminals are used for input signals and remaining two terminals are used for output signals. In a junction transistor there are only three terminals emitter (E), base (B) and collector (C). Hence, in such type of circuits, the transistor is so connected that out of the terminals E, B and C one terminal is common for both input and output. In this way to connect the transistor in the circuit following there configurations, are used.

1. Common Base (CB) Configuration.
2. Common Emitter (CE) Configuration.
3. Common Collector (CC) Configuration.

In any transistor circuit, all voltage can be taken relative to the earthed common terminal.

Figure (16.37) shows circuit diagrams for these configurations of transistor. In each circuit, the emitter base junction is forward biased and base collector junction is reverse biased.

16.10.1. Transistor Characteristic Curves

The graph representing the change in the current in the input and output circuits of the transistor with the applied voltages are called the characteristic curves of the transistor. When only direct current flows in the circuit and there is no load resistance connected between the output terminals then such curves are called static characteristics curves. When there are alternating currents in the circuit and there is load resistance connected between the output terminals then such curves are called dynamic characteristic curves. With the help of characteristic curves, the construction of various circuits of transistors is possible. Here we will study only about static characteristic curves. Generally, two types of

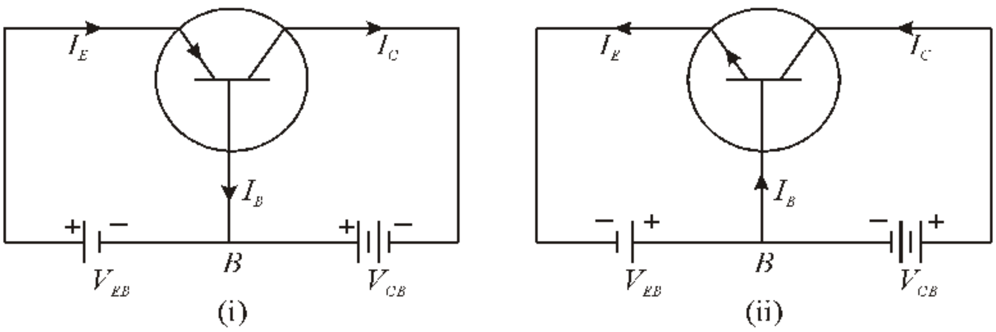
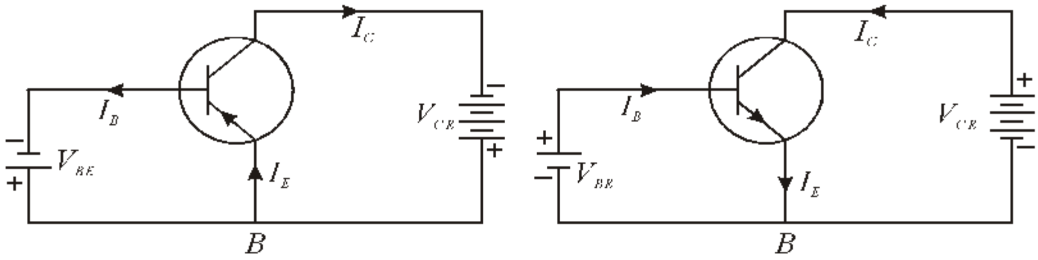


Fig. 16.37 (a) : CB circuit (i) PNP (ii) NPN



(i) (ii)

Fig. 16.37 (b) : CE circuit (i) PNP (ii) NPN

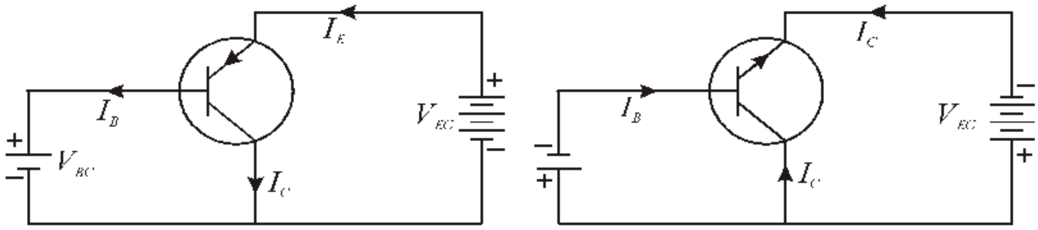


Fig. 16.37 (c) : CC circuit (i) PNP (ii) NPN

characteristic curves are useful.

Input Characteristic : By keeping the output voltage of a transistor constant, a graph drawn between input current and input potential difference is called input characteristic curve. For various constant input voltages many such curves are drawn. A family of these curves is called characteristics.

Output Characteristic : For constant input current a graph drawn between output voltage and output current is called the output characteristics curve. For various constant input currents a family of such curves is obtained called output characteristics.

In all the three configurations of the transistor there is difference between the characteristics and operation of a transistor. Hence, they are studied separately. Here, we will study only about common emitter and common base configurations.

16.10.2 Common Base Configuration

In this configuration (figure 16.37(a)) the base terminal of the transistor is common for input and output. The potential difference between the emiter and base is called the input voltage. Where as the potential difference between collector and base is called output voltage. Emitter current I_E is called input current and collector current I_C is called the output current. The arrangement shown in the figure (16.37 (a)) is the basic arrangement of this configuration. Fig (16.38) shows the circuit used for obtaining the characteristic curves of the transistor experimentally. In the figure shown a PNP transistor is used. By making a proper change in biasing arrangement a similar circuit can be made for obtaining NPN transistor characteristic curves.

In the figure (16.38) B - E junction is forward biased by battery V_{BB} and B - C junction is reverse biased by battery V_{CC}. Since, we have to see the effect on I_E and I_C by changing the applied voltage V_{EB} and V_{CB}. Hence, the potential divider R_1 and R_2 are employed with V_{BB} and V_{CC}.

Voltmeters and milli ammeters are used for the measurement of V_{EB} and V_{CB} and I_E and I_C respectively in the circuit. With the help of this circuit the input and output characteristic curves are obtained as follows.

Input Characteristics : Here the output voltage V_{CB} is kept constant and change in input current I_E is measured relative to the input voltage V_{EB}. For this with the help of R_2; V_{CB} is kept constant at some value. After this with the help of R_1, V_{EB} is varied starting from zero in descrete step (for example in the range of 0-5 volt). The corresponding values of I_E are measured with the

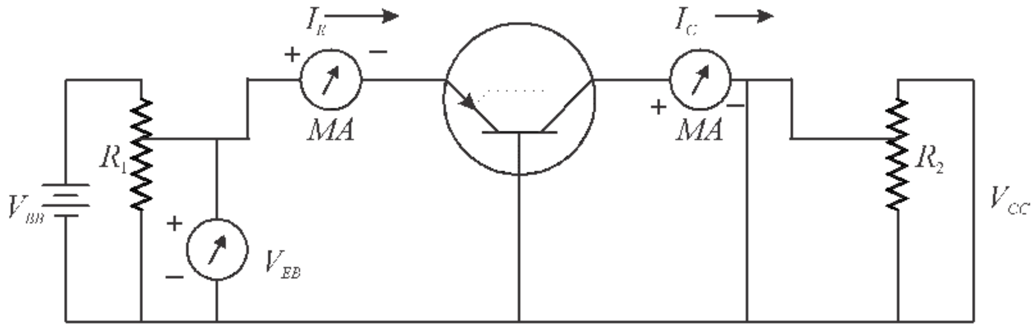


Fig. 16.38 : Circuit of obtaining transitor characterstics of a common base PNP transitor

help of a milliammeter. In this way at a constant value of V_{CB} a graphs is drawn for the various value of V_{EB} and corresponding I_E which is shown in the figure (16.39). The same process is repeated for other constant values of V_{CB}. It should be noticed that due to reverse biasing the value of V_{CB} is negative.

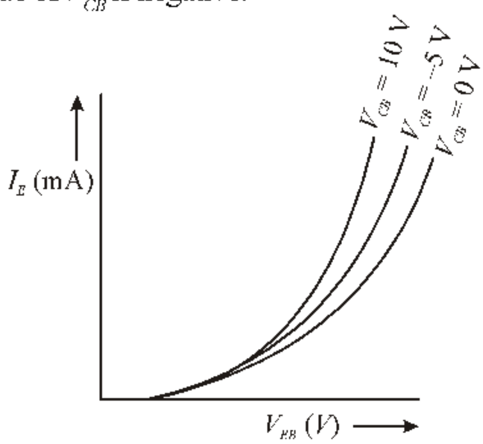


Fig. 16.39 : Input characteristic curve for CB transitor

The form of a input characteristic curve is similar to the graphs of forward characteristic of a P-N junction diode. Since, E - B junction is forward biased hence this is expected. Initially when V_{EB} is zero the current is also zero; hence the curve starts from origin. When the value of V_{EB} is initially increased then increases in I_E is negligible; but after a certain value of V_{EB} called the threshold voltage of the transistor; the current increases rapidly. The reason for this is also as explained for PN junction. According to the nature of the transistor its value is between 0.1 V to 0.5 V.

The dynamic input resistance for common base configuration of the transistor is given by -

$$R_{ib} = \left|\frac{\Delta V_{EB}}{\Delta I_E}\right|_{V_{CB}=\text{constant}}$$

The subscribt V_{CB} = constant indicates the curve for which the value of R_{ib} has been calculated.

The value of R_{ib} is between 50-100Ω.

Output Characteristics : Here input current I_E is kept constant and the changes in output current I_C is measured for various output voltage V_{CB}.

Initially, with the help of R_1, I_E is kept at a desired value. Now, with the help of R_2, V_{CB} is changed starting from zero in discrete steps, corresponding values of I_C are measured with the help of a milliammeter.

The graph drawn between V_{CB} and I_C is the output characteristic curve. The same process is repeated for other constant values of I_E and group of curves so obtained is called the output characteristic. Due to the reverse biasing of C-B junction for these graphs V_{CB} and I_C both are negative. By the study of these graphs the following facts are noticed :

(i) For $I_E = 0$, $V_{CB} = 0$, then I_C is also zero. But for $I_E = 0$ and non zero values of V_{CB}; a small I_C value is obtained. This is due to the reason that for $V_{CB} \neq 0$ in B-E junction is reversed biased and a negligible reverse current (~ μA) flows. The graph obtained for $I_E = 0$ is similar to the graph for reverse biased diode.

(ii) For other value of I_E ($I_E \neq 0$) at $V_{CB} = 0$ value of I_C is not zero. When the value of V_{CB} is increased from zero; for very small values of V_{CB}, I_C first increases then becomes almost saturated. In this situation graph is parallel to V_{CB} axis.

(iii) When the increases I_C increase of I_E. In saturated state the value of I_C is little less than the of I_E.

(iv) At non-zero values of I_E to make I_C zero, it is necessary that the polarity of V_{CC} is changed for V_{CB} and C-B junction is made forward biased in place of reverse biased. In such a case V_{CB} is positive.

When V_{CB} is made positive from zero; then at very low value of V_{CB}, I_C will be come zero. For various values of I_E in this forward biased state of collector-base junction the transitor is said to be working in saturated region. Figure (16.40) shows this.

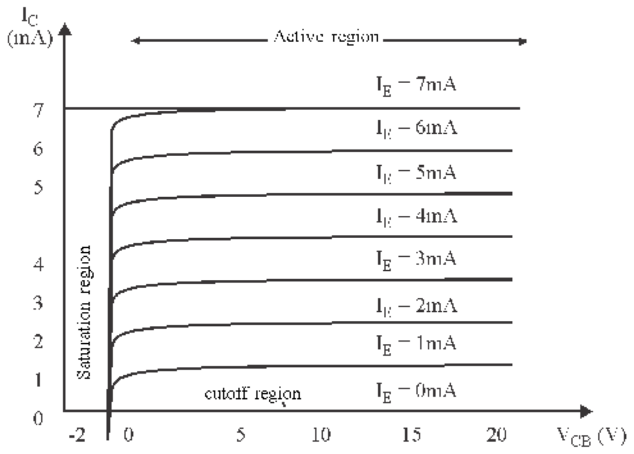


Fig. 16.40 : Output characterstics of CB transitor

The dynamic output resistance R_{ab} for common-base configuration is defined as follows:

$$R_{ab} = \left|\frac{\Delta V_{CB}}{\Delta I_C}\right|_{I_E=\text{constant}}$$

It is clear from output characteristic curve; that for a large change in V_{CB} there is small change in I_C. Due to this dynamic output resistance is high of the order of $10^6 \Omega$.

Inverse of output resistance is called output conductance.

16.10.3 Current Gain or Current Amplification Factor for Transistor in CB Configuration

In Common base configuration, at constant collector base voltage, the ratio of collector current I_C (output current) to the emitter current I_E (input current) is called as static current gain or static current amplification factor. It is denoted by α_{dc}.

$$\alpha_{ac} = \left.\frac{I_C}{I_E}\right|_{V_{CB}=\text{constant}} \quad ....(16.15)$$

At constant V_{CB}, if a small change ΔI_E in input current I_E and a small change ΔI_C in output current I_C, then dynamic current amplification factor is represented by α_{ac} or α

$$\alpha_{dc} = \left.\frac{\Delta I_C}{\Delta I_E}\right|_{V_{CB}=\text{constant}} \quad ....(16.16)$$

Since, $I_C < I_E$

and $\Delta I_C < \Delta I_E$

So, α_{dc} and α_{ac} are nearby 1 but less than 1.

Usually, $0.9 \le \alpha_{ac} \le 0.99$

16.10.4 Common Emitter Configuration

In this configuration emitter (E) is common in both input and output circuits. The potential between base and emitter is called potential difference input voltage whereas potential difference between collector and emitter is called output voltage. The basic arrangement for this configuration is shown in the figure 16.37 (b). Here base current I_B is input current and collector current I_C is output current.

For a PNP transistor the circuit for is common emitter characteristic curves shown in the figure (16.41). With the help of battery V_{BB} and the potential devider arrangement R_1 forward bias V_{BE} is provided to the base-emitter junction. the measurement of V_{BE} is done by the voltmeter connected across base and emitter terminals.

Here, the value of base current I_B is very low (~ μ A) hence it is measured by micro-ammeter. The potential difference V_{CE} between the collector and emitter is provided through the battery V_{CC} and potential divider R_2 arrangement. It is measured by the voltmeter and collector current I_C is measured by milliammeter.

Here, it is natural to ask that how does the base-collector junction is reverse biased, since there is no battery connected between them? To answer this question see figure (16.41) carefully. Here, both base and collector are connected to the negative terminals of batteries of repective circuits (V_{BB} and V_{CC}). If the magnitude of V_{CE} is higher then V_{BE} then base (N-type) will be at negative potential compared to collector (P-type). In other words, N-type base is at a higher potential than P-type collector, clearly this junction will be reverse biased.

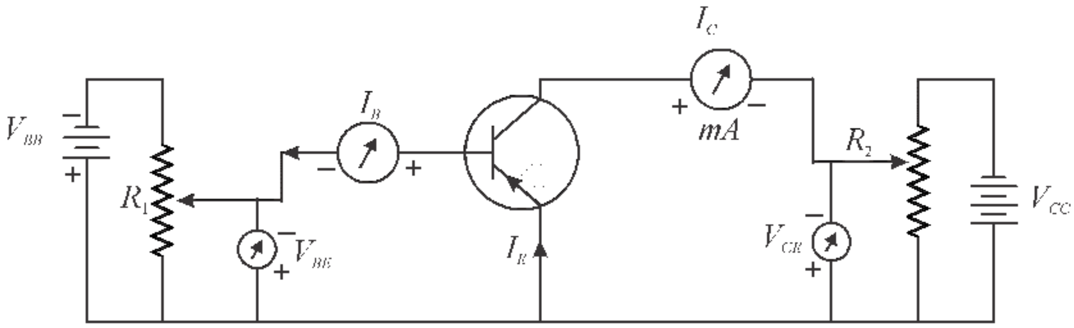


Fig. 16.41 : Circuit of obtaining transitor characterstics of a common emitter PNP transitor

Input Characteristics: Here output voltage V_{CE} is kept constant and the changes in input current I_B are studied related to the change in input voltage V_{BE}. For this with the help of R_2, V_{CE} is kept at a constant value. Now with the help of R_1, the value of V_{BE} is increased from zero in discrete steps and corresponding values of I_B are noted. A graph is drawn between I_B and V_{BE}, the same process is repeated for other values of V_{CE}. The graphs or curves so obtained are called the input characteristics of common emitter configuration. Figure (16.42) shows the characteristic curves for three constant values of V_{CE}.

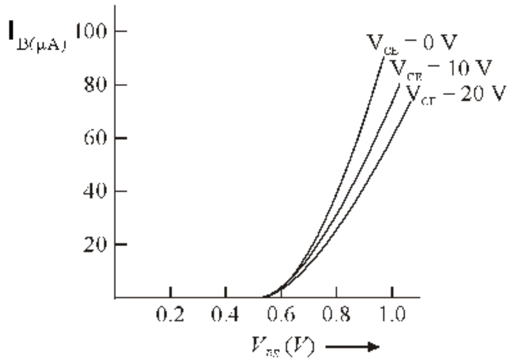


Fig. 16.42 : Input characterstics of CE transitor

Dynamic Input Resistance :

The dynamic input resistance for common emitter configuration of transistor is given by following relation :

$$R_{ie} = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE}=\text{Constant}}$$

The value of this is approximately of the order of 100Ω. If this value is compared with common base circuit input resistance R_{ib} then, $R_{ie} > R_{ib}$.

Output Characteristics : Here input current I_B is kept constant and the change in output current I_C corresponding to output voltage V_{CE} is studied. The value of V_{CE} is changed with the help of R_2 and corresponding value of I_C is measured. A graph is drawn between V_{CE} and I_C. This process is repeated for other values of I_B. The group of curves so obtained are called output characteristics. These are shown in the figure (16.43).

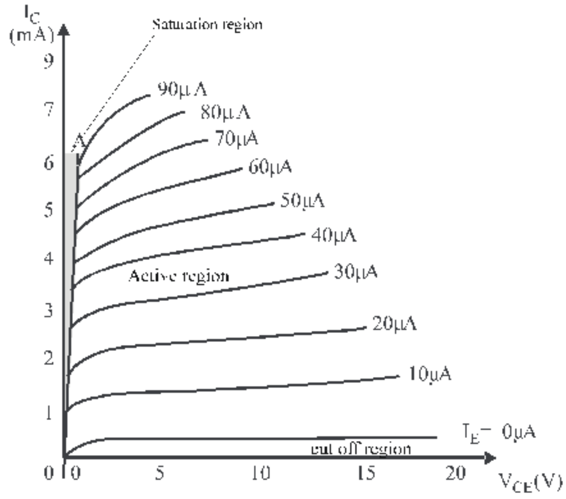


Fig. 16.43 : Output characterstics of CE transitor

At any constant value of I_B, when V_{CE} is increased from zero, I_C gets almost sturated rapily. After this the value of I_C increases slowly. The line OA in figure (1643) is called saturat line and the area between I_C axis and this line is called saturated region. When $I_B = 0$ but I_C is not zero, meaning when input current is zero still some output current flows. The region between I_B = 0 and V_{CE} axis is called cut off region.

The remaining region (apart from the cut of region and saturated region) is called the active region. Here, the emitter-base-junction is forward biased and base collector junction is reverse biased.

In active region I_C ia almost independent of V_{CE}.

The dynamic output resistance R_{oe} for this configuration is defined as follows :

$$R_{oe} = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{V_B=\text{Constant}}$$

Since, I_C does not change much with V_{CE} (leaving the initial values of V_{CE}) therefore, R_{oe} is charge of the order of 50-100kΩ.

Current Amplification Factor (β)

This is defined as the ratio of the change in collector current to the change in base current at a constant collector emitter voltage (V_{CE}) when the transistor is in active state.

The ratio of change in I_C (ΔI_C) to change in I_B (ΔI_B) at constant V_{CB} is called dynamic ampilication factor . It is denoted by β_{dc} ; $\beta_{dc} = \frac{I_c}{I_B}$

The ratio of change in I_C (ΔI_C) and change in I_B (ΔI_B) at constant V_{CB} is called dynamic amplificator factor β_{ac} or β

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$$

Since, $I_C >> I_B$ and $\Delta I_C >> \Delta I_B$. Therefore, β_{dc} and β both are grater than 1 i.e β >> 1.

Relation between α and β

For any configuration of a transistor the emitter current I_E is equal to the sum of base current I_B and the collector current I_C.

∴ $\quad I_E = I_B + I_C$ ....(16.18)

∴ $\quad$ for small changes in currents

$$\Delta I_E = \Delta I_B + \Delta I_C \quad \text{.........(16.19)}$$

Hence, it can be written;

or $$\frac{\Delta I_E}{\Delta I_C} = \frac{\Delta I_B}{\Delta I_C} + 1 \quad \text{.........(16.20)}$$

But $$\frac{\Delta I_C}{\Delta I_E} = \alpha \text{ and } \frac{\Delta I_C}{\Delta I_B} = \beta$$

Hence, putting the values in equation (16.20)

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

Or $$\frac{1}{\alpha} = \frac{\beta + 1}{\beta}$$

Or $$\alpha = \frac{\beta}{1+\beta} \quad \text{.........(16.22(a))}$$

From equation (16.21);

$$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad \text{.........(16.22(b))}$$

Since, the value of α is slightly less then 1 hence the value of β is very high in comparison to α.

16.11 Transistor Amplifier

Amplifier is an active electronic device in which the amplitude of output signal obtained is more than the amplitude of the input signal applied. Input signal is generally alternating current or voltage (ac). The process of increase in the amplitude of the signal is called amplification. In this there is no change in the shape and frequency of the signal. The necessary energy required for the increase in the amplitude of the signal is obtained from the dc power supply used in the amplifier circuit. Figure (16.44) shows the block diagram of an amplifier.

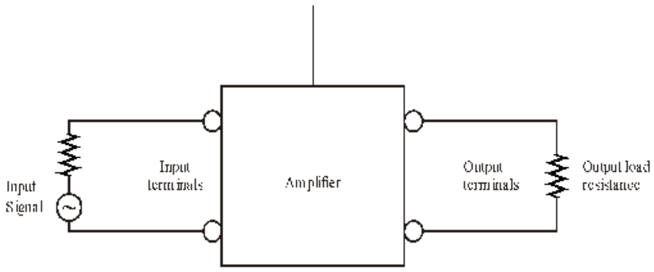


Fig. 16.44 : Block diagram of an amplifier

One type of amplifier, which is used in our daily life and which is known to us is the audio frequency amplifier. With the help of a microphone, the sound signals are converted into electrical signals of same frequency and given as input to the amplifier and output of high amplitude electrical signals are obtained which are again changed into sound signals by the loudspeaker. Hence, we hear a high intensity sound.

The ratio between output and input signals for an amplifier is known as amplification factor or gain. If the input signal voltage is V_i and output signal voltage is V_0 then;

Voltage amplification factor or voltage gain

$$A_i = \frac{\text{Output signal voltage}}{\text{Input signal voltage}} = \frac{V_o}{V_i}$$

Similarly, current amplification factor or current gain;

$$A_i = \frac{\text{Output signal current}}{\text{Input signal current}} = \frac{I_o}{I_i}$$

and Power amplification factor or power gain

$$A_p = \frac{\text{Output signal power}}{\text{Input signal power}} = \frac{P_o}{P_i}$$

$\because \quad P_o = V_o \times I_o$

and $\quad P_i = V_i \times I_i$

$\because \quad A_p = \frac{V_o I_o}{V_i I_i}$

$$A_p = A_v A_i$$

Hence, for amplifier all the three amplification factors are correlated.

When a transistor is used as an amplifier its one terminal is common the both of input and output circuits. The signal whose amplification is to be done is used in input circuit, and output circuit is taken across a load resistor R_L connected in circuit.The amplification by a transistor is possible for both the common base and common emitter configurations. But the voltage, current and power gain for common emitter amplifier is more than the common-base amplifier. Hence, here we will study only about common-emitter amplifier.

16.11.1 Common-Emiter Amplifier

Figure 16.45 shows a common-emitter amplifier circuit for PNP transistor. Here, emitter is common for both the input and output circuits. With the help of battery

V_{BB} the emitter-base junction is forward is provided. Whereas with the help of battery $V_{CC}(>V_{BB})$ reverse bias is provided to the collector-emitter junction. V_i is signal voltage whose amplification is to be done (input signal).

A load resistor R_L is connected to output circuit across which amplified voltage V_O is obtained.

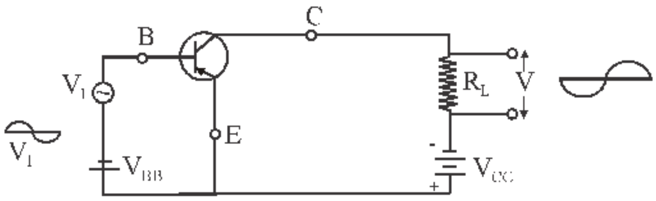


Fig. 16.45 : Common emitter amplifier

If alternative signal V_i is absent then potential difference V_{BE} across base-emitter junction is constant. Due to this the input current I_B and the output current I_C related to it are constants. In the presence of V_i, the potential difference across base-emitter junction changes. If for a small V_i changes in base and collector currents are i_b and i_c then, by definition :-

$$A_{ie} = \frac{\text{Output signal current}}{\text{Input signal current}}$$

$$= \frac{i_c}{i_b} = \beta$$

Since, $\beta >> 1$, therefore output symbol current is more than input current. Hence, in common emitter configuration the current is amplified. [In sign A_{ie} the subscript e represents common emitter configuration.]

Voltage Amplification :

By definition voltage amplification or voltage gain;

$$A_{ie} = \frac{\text{Output signal voltage}}{\text{Input signal voltage}} = \frac{V_o}{V_i}$$

Here, V_o =alternating voltage generated across load resistor R_L.

$$= i_c R_L$$

and $\quad V_i = i_b R_{ie}$

where R_{ie} is the input resistance in common-emitter configuration.

$$\therefore \quad A_{ve} = \frac{i_c R_L}{i_b R_{ie}} = \beta \times \frac{R_L}{R_{ie}}$$

$\frac{R_L}{R_{ie}}$ is called the resistance gain for common-emitter configuration.

As, $\beta >> 1$, and if R_L and R_{ie} are comparable;

$A_{ve} \approx \beta >> 1$ then voltage amplification is possible. If $R_L > R_{ie}$ then $A_{ve} > \beta$ (>>1) more voltage amplification is obtained.

Power Amplification

By definition power amplification

$$A_{pe} - A_{ie} A_{ve}$$

$$= \beta\beta \frac{R_L}{R_{ie}} = \beta^2 \frac{R_L}{R_{ie}}$$

As $\beta >> 1$ then $\beta^2 >> 1$ hence $A_{pe} >> 1$ means that in this configuration high power amplification is possible.

Phase Relationship

In common emitter amplifier there is 180° phase difference (opposite phase) between input signal and output signal.

Example 16.5 : The current gain is 0.99 in common-base configuration of a transistor. What will be the current gain for the same transistor in common-emitter configuration?

Solution : The current gains for common-base configuration and common-emitter configuration are α and β respectively.

Since, $\quad \beta = \frac{\alpha}{1-\alpha}$

$\because \quad \alpha = 0.99$

$$\therefore \quad \beta = \frac{0.99}{1-0.99} = \frac{0.99}{0.01} = 99$$

Example 16.6 : In a common base circuit collector resistance is 2.0 kΩ and the potential difference across it ends is 2.0 V. For the transistor α = 0.95. Calculate base current I_B.

Solution : The collector resistance is 2.0 kΩ and potential difference across it is 2 V.

$$\therefore \quad I_C = \frac{2V}{2k\Omega} = 1mA$$

$$\therefore \quad I_E = \frac{I_C}{\alpha} = \frac{1}{0.95} = 1.05mA$$

Therefore , $I_B - I_E \quad I_C = 1.05 - 1.0 = 0.5$ mA

Example 16.7 : In a junction transistor when its collector voltage V_{CB} is kept constant and emitter voltage V_{EB} is changed by 5 mV then its emitter current value changes by 0.15 mA. Calculate the input resistance of the transistor.

Solution : According to the question the transistor is in common-base configuration where its input resistance is :

$$R_{ib} = \frac{\Delta V_{EB}}{\Delta I_E}$$

According to the question;

$\Delta V_{EB} = 5$ mV

$\Delta I_{EB} = 0.15$ mV

$$\therefore \quad R_{ib} = \frac{5\times10^{-3}}{0.15\times10^{-3}} = 33.33\Omega$$

Example 16.8 : In an amplifier circuit a transistor is used in common-emitter configuration. A 20 μA change in base current brings 1 mA changes in collector current, and base-emitter voltage change by 0.04 V. Calculate :

(i) Input resistance,

(ii) Current amplification factor,

If a load resistor of 6 kΩ is used in the collector circuit also calculate the voltage gain of the amplifier.

Solution : In common-emitter configuration :

(i) Input resistance

$$R_{ie} = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.04}{20A} = \frac{0.04V}{20\times10^{-6}A}$$

$$= 2000\Omega = 2k\Omega$$

(ii) Current amplification factor

$$B = \frac{\Delta I_C}{\Delta I_B} = \frac{1\text{mA}}{20\text{A}} = \frac{1\times10^{-3}}{20\times10^{-6}}\frac{\text{A}}{\text{A}}$$

$$= 50$$

Voltage gain;

$$A_{ve} = \beta\frac{R_L}{R_{ie}} = 50\times\frac{6k\Omega}{6k\Omega} = 150$$

16.12 Digital Electronics

In electronics, we use basically two types of signals;

(i) Analog

(ii) Digital

When we consider a current (or voltage) as analog, we mean having currents or voltages varying continueously with time (Fig. 16.46). In such a signal current (voltage) has continuous values in some range. Rectifier and amplifier circuits considered in this chapter are called analog circuits as input and output signals are analog in nature.

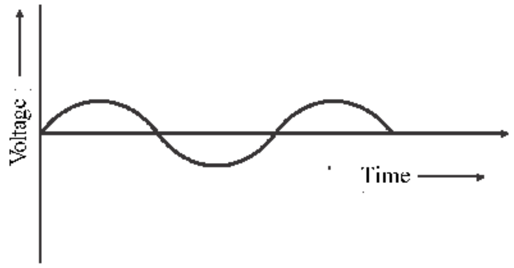


Fig. 16.46 : Analog signal

A digital signal is a signal in which the voltage (current) does not change continuously with time, here the voltage has a discrete value at each sampling point. In other words, the signals are obtained in the form of a pulse. For example, if we take a bulb connected to a switch, then in this bulb for voltage only two states are possible 0 when switch is off and maximum value 220 V when switch is ON. In this state a bulb is a device which can be assumed to be based on binary variable. If zero voltage is represented by 0 and maximum voltage 220 V is represented by 1. Then if switch is ON and OFF continuously then the voltage present on the bulb can be represented as in figure (16.47). Hence, it is an example of digital signal.

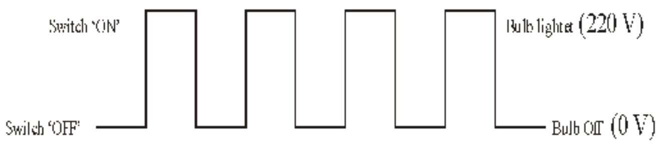


Fig. 16.47 : Digital Signal

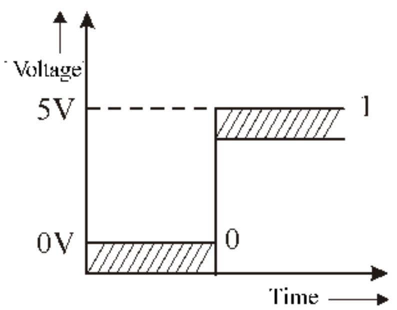


Fig. 16.48 : Representing digital signal

From the above example, it is clear that the digital signals can be represented by binary numbers 0 and 1 (bits). In reality 0 and 1 do not represent 0 V and 1 V. Bit 0 represents absence of signal and 1 represents presence of signal. Digital watches, modern computers, etc. are all examples of digital devices.

In digital circuits generally the high voltage level is V_{high} (5 ± 0.5) V which is represented by 1 and low voltage level V_{low} (0 ± 0.5) V which is represented by 0. Knowledge of exaxct values of V_{high} and V_{low} is not necessary, because the interval between these levels. A representative digital signal is shown in figure (16.48)

16.13 Logic Gates

A logic gate is an elementary building block of a digital circuit. Most logic gates have two inputs and one output. A logic gate is a digital circuit that follows certain logical relationship between the input and output voltages.The output signal is available only when some conditions are satisfied at input i.e. a logical relation exists between input signals. The five common logic gates used are OR gate, AND gate, NOT gate, NOR gate and NAND gate.

Each logic gate is indicated by a symbol and its function is defined by a truth table that shows all the possible input logic level combination with their respective output logic levels. Truth tables help to understand the behaviour of logic gates. These logic gates can be realised using semiconductor devices.

16.13.1 OR Gate

An OR gate has two or more inputs with one output. The output Y is 1 when either input A or input B or both are 1s, that is; if any of the input is high, the output is high.

Since logic gates are based on such variables for which only two values 0 and 1 are possible, the algebra for such variables is different than ordinary algebra. This type of algebra is known as Boolean algebra and the equations representing these relationships between variables are called Boolean expressions.

For a two input OR gate if inputs are given as A and B respectively and output is Y then boolean expression is

$$Y = A + B$$

Here the + sign between A and B represents OR operation. Y = A + B means Y is A or B.

where A and B can have values O or 1 A = 0, 1 B = 0, 1.

The truth table for this operation is as follows;

A	B	Y = A + B
0	0	0
1	0	1
0	1	1
1	1	1

Thus when either A or B or both are 1 then outputY is also 1. When Both A and B are zero then output Y is also zero.

OR operate can be explained by electrical switches connected in parallel. As shown in figure (16.49) the switches A and B are connected in parallel and are connected with a battery and a bulb Y.

If closed state (ON state) of A and B is represented by 1 and open state (off state) is represented by 0 and the illuminated and non illuminated situation of the bulb is indicated by 1 and 0 then for this circuit there are four possibilities.

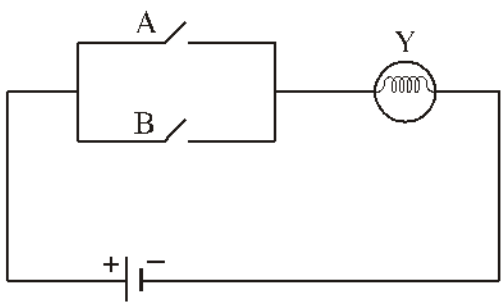


Fig. 16.49 : Representing OR operation

(i) If A and B both are open then there is no current in Y, hence bulb is not illuminated. In mathematical form A = 0, B = 0 then Y = A + B = 0.

(ii) If switch A is closed (A = 1) and B is open (B = 0) then the current will flow through switch A and bulb Y, illuminated hence Y = 1 : ∴ A= 1, B = 0 then Y = A + B = 1.

(iii) If A is open and B is closed then also bulb will be illuminated. ∴ A = 0, B = 1, Y = A + B = 1.

(iv) If A and B both are closed (A = B = 1) then bulb will be illuminated (Y = 1). ∴ A = B = 1, Y = A + B =1.

All the above four possibilities are according to the truth table. Figure (16.50) shows the symbol for OR gate.

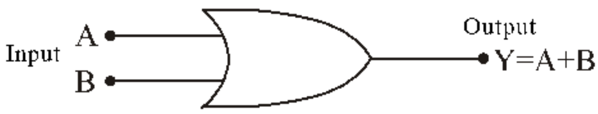


Fig. 16.50 : Symbol for two input OR gate

In practice an OR gate is constructed by circuit formed by two diodes D_1 and D_2 as shown in Fig. (16.51).

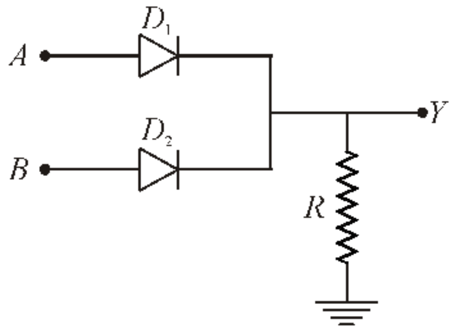


Fig. 16.51 : OR gate based on diode

The value of 0 V or 5 V is provided to the inputs A and B. 0 V is represented by 0 and 5 V represented by 1. R is the resistance at output Y and the other end of resistance R is earthed.

(v) When A = B = 0 meaning there is no signal then no diode will be operated and there will be no potential drop at R or Y = 0. This is equivalent to the first row of the truth table.

If input A is 5 V (1) or B = 0 V (0) then diode D_1 will be forward biased and behave as closed switch hence is in on state. If D_1 is ideal diode then there is no potential drop across it and a potential drop of 5 V (with respect to ground) will be available across R or will get Y = 1 condition. This will be similar to second row of truth table.

(vi) If A = 0 and B = 5 V (1) then in place of D_1, D_2 will be in on state. Still Y = 1. This is similar to third row of truth table.

(vii) If A and B both the diodes are (5 V) then both diodes will be in operation again Y = 1. This will be similar to fourth row of truth table.

16.13.2 AND Gate

An AND gate has two or more inputs and one output. The output Y of AND gate is 1 only when input A and input B are both 1. The boolean expression for the AND operation is represented as; Y = A•B

Here dot (•) represents AND operation

The above boolean equation means, Y = A and B. The truth table for this;

A	B	Y = A•B
0	0	0
1	0	0
0	1	0
1	1	1

Hence, in AND gate when both inputs are is 1 then only output is 1.

The AND gate can be explained by electrical switches; in series (Figure 16.52). When both switches A and B are open (A = B = 0) then there is no current in circuit. Hence, bulb Y is not illuminated. This is equivalent to the first line of the truth table. If any of the two switches is open then also Y = 0. When both switches are closed (A = B = 1), the Y = 1, bulb is illuminated. Figure (16.53) shows symbol for a two input AND gate.

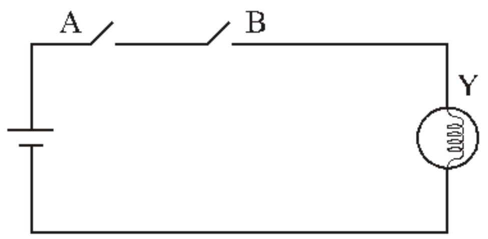


Fig. 16.52 : Representing AND gate

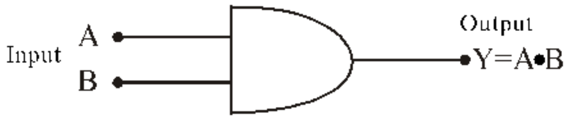


Fig. 16.53 : Symbol for AND gate

AND gate is also obtained with the help of diodes. Figure (16.54) shows a two input AND gate formed by diodes.

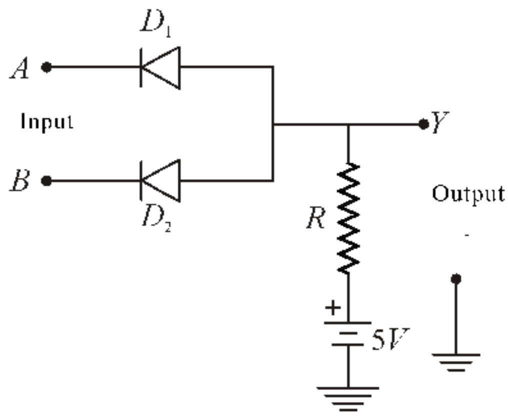


Fig. 16.54 : AND gate using diode

Here resistance R is connected to the positive terminal of a battery of potential difference 5 V and the negative terminal is earthed. Input terminals A and B are given 0 or 5 V level.

If A and B both are at zero potential (A = B = 0) then diodes D_1 and D_2 both are forward biased. As diodes are ideal there will not be any potential drop across any of the diodes and 5 V potential drop is available across R its end Y will be at zero potential with respect to ground (Y = 0). This represents the first line of truth table.

If A and B any of the two has value 5 V and the other has 0 V, then diode A is in operation and B is in non-operation mode. If diode A is ideal the there will be no potential drop across it. Again 5 V potential drop is there across R and Y = 0 the same is true A = 0 and B = 1. This represents the third line. If A = B = 5 V, then (A=B=1) then both diodes are non conducting and there is no current in R so its upper end is at same potential as its lower end i.e at + 5 V with respect to earth so Y=1 hence this represents the fourth line of truth table.

16.13.3 NOT Gate

This is a logic gate, with one input and one output. It produces '1' output if the input is '0' and vice-versa. That is it produces an inverted version of the input at its output. This is why it is also known as an inverter.

The Boolean expression is given by;

$Y = \bar{A}$.

$\bar{A}$ means NOT A meaning which is not A. ($\therefore$ A=0, Y = $\bar{0}$, =1) and (A = 1, Y = $\bar{1}$=0). The truth Table is givien as follows :

A	Y
0	1
1	0

Not operation can be understood by electrical circuit shown in Fig. 16.55. When switch is on (1) then Y (bulb) is off (0). When switch is off (0) then Y is illuminated (1). Figure (16.56) shows NOT gate symbol.

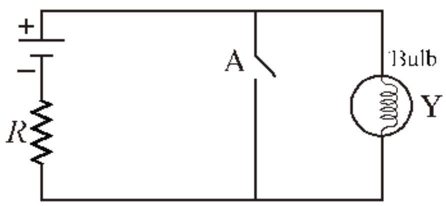


Fig. 16.55 : Circuit representing NOT gate

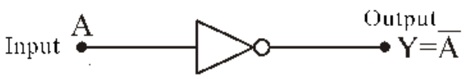


Fig. 16.56 : Symbol for NOT gate

For NOT gate transistor is used. A simple circuit is shown in the figure (16.57), in which an NPN transistor is used. The base B of the transistor is connected with the terminal A through a resistor R_B. Emitter E is earthed and collector C is connected to the positive end of dc supply V_{CC} = (5 V) through a resistor R_C. Negative terminal of the supply is grounded. Y is the potential difference of collector C relative to ground.

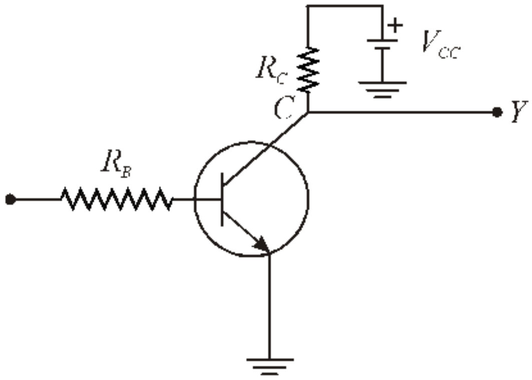


Fig. 16.57 : Transitor NOT gate

When, input terminal A is earthed then $V_A = 0$ base is also earthed. So there is no bias at EB junction by BC junction. In this the base current is zero, emitter current is zero the collector current is also zero, and the transistor is in cut off state, so collector is at + 5 V potential with respect to ground i.e at high potential and Y=1.

Now If relative to earth + 5 V voltage is used at base (meaning A = 1) then base-emitter junction is forward biased. Now base current, emitter current and collector current all are present. If the values of R_B and R_C are chosen in such a way the collector current is high then transistor is in saturation state. In this state, the potential drop across R_C is + 5 V which is equal to voltage V_{CC} so voltage at C is zero and Y = 0.

16.13.4 NOR Gate

It has two or more inputs and one output. A NOT operation applied after OR gate gives a NOT-OR gate or simply NOR gate. Its output Y is 1 only when both inputs A and B are 0. This logic is shown in the figure (16.58).

Fig. 16.58 : Action of NOR gate

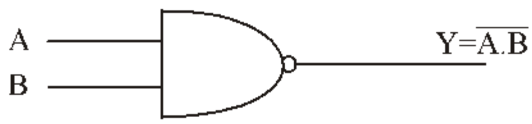


Fig. 16.59 : Symbol for NOR gate

The symbol is shown for NOR gets in figure (16.59), in which a bubble is placed at the OR gate output.

Boolean expression for this gate is given by

$$Y = \overline{A + B}$$

The truth table for this is;

A	B	A+B	$Y = \overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

It is clear from this table that when all the inputs are zero, then only output is 1.

For three inputs the NOR gate is shown in the figure 16.60.

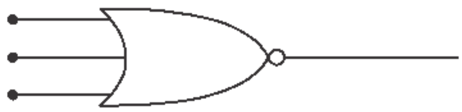


Fig. 16.60 : Symbol of NOR gate with three inputs

16.13.5 NAND Gate

This is an AND gate followed by a NOT gate as shown in Fig. 16.61. If inputs A and B are both 1 the output Y is 0. Its circuit symbol is shown in Fig. 16.62 NOT AND behaviour provides NAND gate.

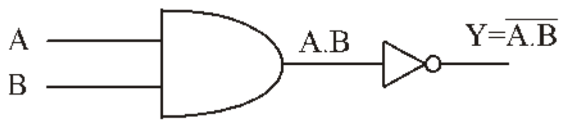


Fig. 16.61 : Action of NAND gate

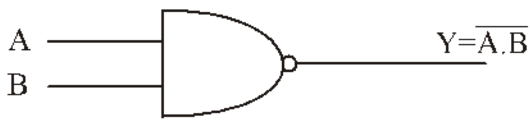


Fig. 16.62 : Symbol of two input NAND gate

The truth table for this gate is shown below.

A	B	A . B	$Y = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

16.13.6 XOR Gate

The Boolean expression for XOR operation is :

$$Y = A \oplus B$$

When any one of the input signals A or B has value 1 then only Y = 1. If both the two inputs are 0 or 1 then output is 0. Hence, truth table for this gate is :

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

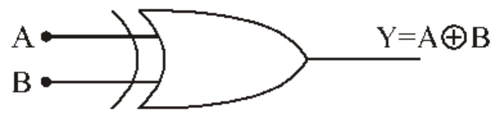


Fig. 16.63 : Symbol for XOR gate

The XOR gate symbol is shown in the figure 16.63. By making following truth table one can verify that -

$$A \oplus B = A\overline{B} + \overline{A}B$$

A	B	$\overline{A}$	$\overline{B}$	$A\overline{B}$	$\overline{A}B$	$A\overline{B} + \overline{A}B$
0	0	1	1	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
1	1	0	0	0	0	0

Hence, by using boolean expression $A\overline{B} + \overline{A}B$ XOR gate can be constructed. This is shown in the figure (16.64) in which we have used AND, OR and NOT gates.

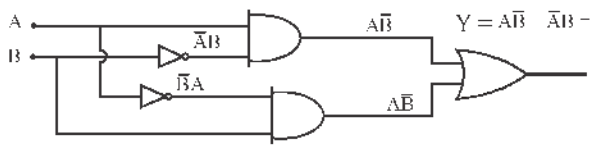


Fig. 16.64 : Boolian circuit for XOR gate

Special Notes

NAND gates are also called universal gates since by using these gates you can also realise other basic gates like OR, AND and NOT. Figure 16.65 shows how various gates are realised by NAND gate.

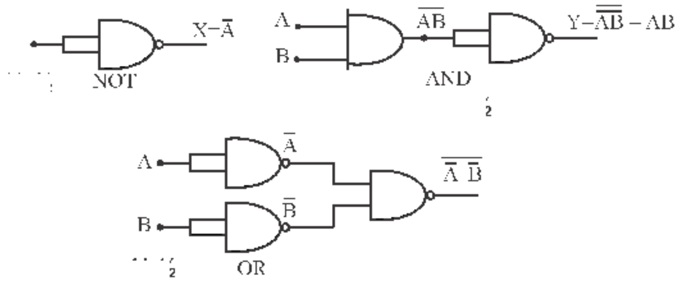


Fig. 16.65 : Different logic operations using NAND gate

In Fig. 16.55 (A) both the inputs are A so output from NAND operation is Y = $\overline{A}\,.\,\overline{A} = \overline{A}$ which is NOT operation. In fig. 16.55 (B) output of first NAND gate is $\overline{AB}$ which is input for second NAND gate, so output for second NANO gate is Y = $\overline{\overline{AB}}$ =AB which is AND operation.

In Fig. 16.55 (c) construction of OR gate is shown. Input A and B are inverted through separate NAND gates and then signals $\overline{A}$ and $\overline{B}$ have been applied to third NAND gate. The output so obtained is Y = $\overline{\overline{AB}}$. From the following truth table it can be seen that $\overline{\overline{AB}}$ = A + B.

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\overline{B}$	$\overline{\overline{AB}}$
0	0	1	1	1	0
1	0	0	1	0	1
0	1	1	0	0	1
1	1	0	0	0	1

NOR gates are also called universal gate since by using these gates we can also make other basic gates like OR, AND and Not gate Figure 16.66 shows how various gates are made by using NOR gates.

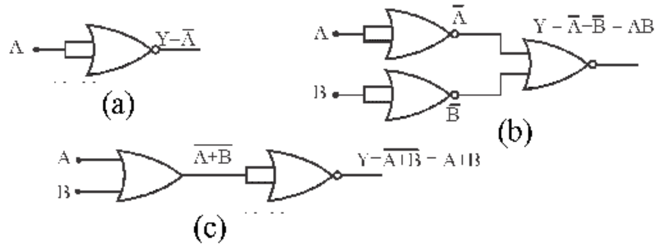


Fig. 16.66 : Different logic operations using NOR gate

In Fig. 16.66 (a) both the inputs for NOR gate are A so output X = $\overline{\overline{A}\overline{B}}$ = A + B which is NOT operation.

In fig. 16.66 (b) inputs A and B applied through two separate NOR gates respectively. The respective outputs $\overline{A}$ and $\overline{B}$ now act as input for another NOR gate the output for which us Y= $\overline{A} + \overline{B}$. Using a truth table it can be shown that Y = $\overline{A} + \overline{B}$ =AB so AND operation is achieved. Students are advised to verity the same.

In figure 16.66 (c) the output of $\overline{A+B}$ of first NOR gate is input for the second NOR gate. The output of second NOR gate is Y = $\overline{\overline{A+B}}$ = A + B so OR operation is achieved.

Important Points

1. In isolated atoms energy levels are discrete. In a solid because of interaction between atoms energy level of atoms splits and energy bands are formed in place of energy levels. Every electron in an atom stays in one of these pre-defined energy levels. the continuous energy groups are called energy bands.

2. These are forbidden energy gaps between the energy bands. No electron is relevant to these energy intervals in solids.

3. On the basis of electrical conductance and energy band structure substances are divided as (i) conductors (ii) insulators and (iii) semiconductors. Completely empty and fully filled bands do not participate in conduction.

4. In conductors, either the band related to the valence electrons is partially filled or it over laps with next band so the new band is partly filled and hence transition is possible. Partially filled bands help in conduction.

5. In insulators valence band is completely filled and higher band (conduction band) is fully empty. The energy gap between these bands is called forbidden energy gap (E_g). In insulators this forbidden energy gap is 3 to 6 eV.

6. In intrinsic semiconductors the forbidden energy gap is very small in comparison to insulators and is of 1 eV range. At absolute zero temperature their behaviour is like insulators. At room temperature some electons of valence band get thermal energy and reach conduction band. In valence band in place of these electrons holes are generated.

7. In intrinsic conductor both electrons and holes participate in conduction and their conductivity is in between the conductivity of conductors and insulators. When temperature increases the conductivity of intrinsic conductors increases.

8. In intrinsic semiconductor if the impurity of suitable kind is mixed in very small quantity their conductivity increases very rapidly. Such conductors are called extrinsic semiconductors.

9. Extrinsic semiconductors are of two types - (i) N-type (ii) P type.

10. Tetravalent intrinsic semiconductors like silicon and germanium when doped with pentavalent element like arsenic, phosphorous in very small quantity then N-type extrinsic semiconductors are obtained. The impurity is called donor impurity.

11. When a trivalent impurity (element) like aluminium, boron, indium is doped in very smallquantity with intrinsic semiconductor then P-type semiconductor are formed, such impurities are acceptor impurities.

12. In N-type semiconductors electrons are majority charge carriers and holes are minority charge carriers. In P-type semiconductors reverse is the case. Both types of semiconductors are electrically neutral.

13. When a P type semiconductor is connected to a N-types semiconductor at atomic level then their contact surface is called P-N junction. Almost all semiconductor devices use P-N junction. A depletion layer is formed close to the P-N junction in which there are bounded positive and negative ions and the number of free electrons and holes reduces (negligible).

14. The device based on P-N junction is called P-N diode or semiconductors diode. In forward biasing of junction, the P-terminal is at higher potential than N-terminal. In reverse biasing it is opposite.

15. In forward biasing there is conduction of current by the diode whereas in reverse biasing there is negligible

current flow hence diode is in non conducting state.

16. The P-N junction diode is basically used for rectification in which ac is changed to dc.

17. Semiconductor diodes are also used for some special purposes. Out of these Zener diode is operates in reverse biased state to help in voltage regulation. On the basis of optical properties of the semi conductors there are P-N devices like photo diode, light emitting diode, etc.

18. Junction transistor is a very important device in which there are two P-N junction. There are of two types (i) PNP (ii) NPN. Their middle part 'base' is very thin and very lightly dopped, and the other two parts are emitter and collector, emitter has higher doping than collector but the nature of impurity is same.

19. For active operation of junction transistor, base-emitter junction is forward biased and base-collector junction is reverse biased.

20. In circuit combination the transistor is so joined that out of base (B), collector (C) and emitter (E) any one is common for both input and output circuits. Hence, three types of circuit configurations are possible for a transistor but they are different in electrical properties.

21. the current amplification factor for common base and common emitter configurations are α and β respectively.

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } \alpha < 1 \text{ and } \beta >> 1$$

22. In common emitter transistor amplifier high voltage gain, high current gain and high power gain are obtained.

23. In digital electronics binary number system is used. In this we use only 0 and 1.

24. A logic gate is an elementary building block of a digital circuit. The gates are OR gate, AND gate, NOT gate, XOR gate, NAND gate and NOR gate. NAND and NOR gates are called universal gates; because with the help of these other gates can be obtained.

Questions For Practice

Multiple Choice Type Questions

1. At absolute zero temperature intrinsic Germanium and intrinsic Silicone; are:

   (a) Super conductor (b) Good semiconductors

   (c) Ideal insulator (d) Conductors

2. In insulators the forbidden energy gap between valence band and conduction band is of ......

   (a) 1 eV (b) 6 eV

   (c) 0.1 eV (d) 0.01 eV

3. At room temperature in intrinsic Silicon the number of charge carriers per unit volume is $1.6 \times 10^{16}/m^3$. If mobility of electons is $0.150\ m^2V^{-1}s^{-1}$ and mobility of holes is $0.05\ m^2V^{-1}s^{-1}$. Then conductivity of silicon is ($\Omega^{1}\ m^{1}$) is :

   (a) 1.28×10^{-4} (b) 3.84×10^{-4}

   (c) 5.12×10^{4} (d) 2.14×10^{4}

4. If a NPN transistor is used as an amplifier then;

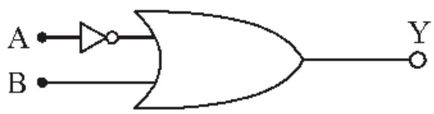
   (a) Electrons move from base to collector.

   (b) Holes move from emitter to base.

   (c) Holes move from base to emitter.

   (d) Electrons move from emitter to base.

5. the boolean equation for given circuit will be :



(a) $Y = A + \overline{B}$ (b) $Y = \overline{A + B}$

(c) $Y = \overline{A} + B$ (d) $Y = \overline{A}.B$

6. For some 'AND GATE' the three inputs are A, B and C then the output Y will be;

(a) Y = A.B + C (b) Y = A + B + C

(c) Y = A + B.C (d) Y = A.B.C.

7. The current amplification factor for common base circuit of a transistor is 0.95. When emitter current is 1mA then base current is :

(a) 0.1 mA (b) 0.2 mA

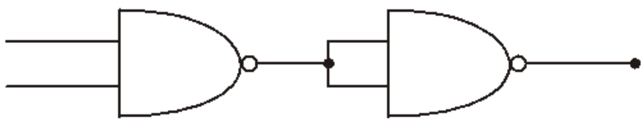
(c) 0.19 mA (d) 1.9 mA

8. the forbidden energy gap in Germanium is 0.7 eV. The wavelength at which its absorption is starts by Germanium is :

(a) 35000 Å (b) 17700 Å

(c) 25000 Å (d) 51600 Å

9. The logic gate obtained by two NAND gates shown in figure is :



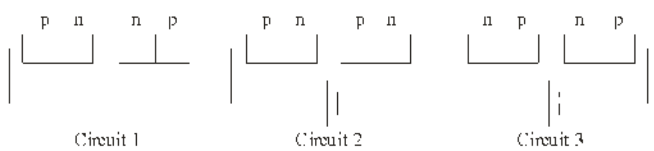
(a) AND gate (b) OR gate

(c) XOR gate (d) NOR gate

10. Two identical PN junction are joined in series with a battery (fig.). For which potential drop is same.

(a) Circuit 1 and 2 (b) Circuit 2 and 3

(c) Circuit 3 and 1 (d) Only circuit 1.



Answers (Multiple Choice Type Questions)

1. (c) 2. (b) 3. (c) 4. (d) 5. (c)

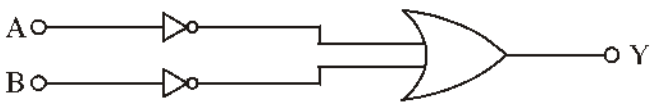
6. (d) 7. (a) 8. (b) 9. (a) 10. (b)

Very Short Answer type Questions

1. What is the direction of diffusion current in junction diode?
2. Write down the relation between α and β (current amplification factor) of the transistor.
3. Can the barrier potential of a forward biased pn junction be measured by a voltmeter connected at the ends of the device?
4. Make truth table for OR gate.
5. Write down the name of that logic gate in which the output is 1 when all the input are 1.
6. To use the transistor as an amplifier which junction is reverse biased?
7. What will be the value of α for a transistor for which β = 19?
8. The diode shown in the figure is in which biasing?

Short Answer Type Question

1. What is rectification? Draw the figure of bridge full wave rectifier.
2. In a transistor why is the base made thin in comparison to the emitter and collector?
3. Make complete I-V characteristic curve for ideal PN junction diode. Define dynamic resistance in forward biased state.
4. What do you understand by logic gate? Draw the symbol for XOR gate and also write the truth table.
5. Draw the transistor based circuit diagram for NOT gate and also give its truth table.
6. Write down the boolean expression for given logic circuit. And also write truth table for it.



7. Draw the circuit used for voltage regulation by zener diode and also explain process in short.

Essay Type Questions

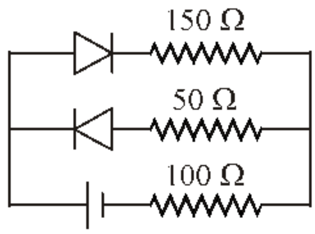
1. Differentiate between conductors, insulators and semiconductors on the basis of energy band theory. Explain the process of electrical conductance in intrinsic semiconductors.
2. What is PN junction? Explain the process occuring during its formation on the junction surface. If this junction is forward biased then explain the effect on depletion layer.
3. Draw the figure of full wave rectifier used to change ac to dc and explain its working.
4. Draw and explain the circuit arrangement for obtaining characteristic curve for forward and reverse biased PN-junction diode. Also draw the these curves.
5. What is junction transistor? Make the necessary diagram and explain the working of a PNP transistor.
6. Draw the circuit arrangement and also explain the characteristics for a transitor common emitter configuration. Also draw the curves and write the formula for voltage gain and current gain.
7. What do you understand by amplification? Make the diagram of a PNP transistor common amplifier and explain the process of amplification and calculate the formula for voltage gain.
8. Write down the names of some diodes used for special purposes and make their circuit diagrams. Write about their working and uses in short.
9. Draw circuit diagram for two input diode OR gate and AND gate. Also explain their working and give truth table.

Numerical Questions

1. Calculate the electic current generated in an intrinsic germanium plate at room temperature whose area is 2×10^{-4} m^2 and width is 1.2×10^{-3} m and a potential difference of 5 V is applied across its faces. Intrinsic charge carrier density is 1.6×10^{6}/m^3 for germanium at room temperature. the mobility of electrons and holes is 0.4 m^2 V^{-1}s^{-1} and 0.2 m^{2}v^{1}s^{1}.

   [**Ans :** 1.28×10^{-13} A]

2. As shown in the circuit below the forward resistance of both diodes is 50 Ω and reverse resistance in infinite. If emf of the battery is 6 V then calculate the current flowing through 100Ω.



   [**Ans :** 0.02 A]

3. The current amplification is 0.99 for amy transistor in common-base configuration. Calculate the change in collector current when there is 5.0 millampere change in emitter current. What will be the change in base current?

   [**Ans :** 0.9 mA, 0.05 mA]

4. For a PN junction the average value of the potential barrier is 0.1 V and the electric field 10^5 V/m is present at junction region. What will be the width of depletion layer for this junction?

   [**Ans :** 10^{6}m]

5. A transistor is connected in common emitter configuration. A power supply of 8V is there in the collector circuit and the potential drop of 0.5 V is on the resistance of 800Ω connected in series with the collector. If current amplification factor is $\alpha = 0.96$. Then calculate base current.

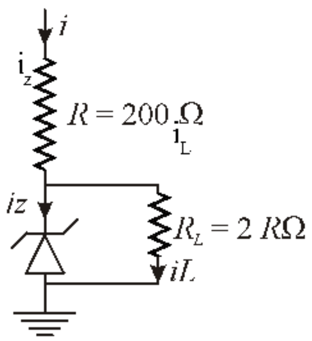
   [**Ans :** 26×10^{-3} Amp]

6. In a common emitter amplifier an increase of 50 μA in base current causes a 1.0 mA increase

in collector current. Calculate current gain β. What will be the change in emitter current? Calculate α with the help of β.

[**Ans :** $\beta = 20$, $\Delta I_E = 1050$ A, $\alpha = 0.95$]

7. Calculate the current and potential difference across ends of zener diode in the given circuit. If load resistor $R_L = 2\ k\Omega$ has a potential difference of 15V across its ends. The lowest working current of zener diode is 10mA.



[**Ans :** 17.5 mA, 15 V]

Chapter - 17

Electromagetic Waves, Communication and Contemporary Physics

Electromagnetic wave is present in our atmosphere in different forms, sometimes we are unable to detect their presence in the space. Light is also a form of electromagnetic wave. Infrared waves are also electromagnetic wave and keep the atmosphere heated-up. Microwaves are very useful in kitchen *i.e.*, in microwave oven and also in communication like radio, mobile telephony and RADAR.

In this chapter, we will study about the nature and properties of electromagnetic waves. Maxwell's equations are basis for electromagnetic waves. The most important prediction emerged from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) time-varying electric and magnetic fields that propagate in space. This led to the remarkable conclusion that light is an electromagnetic wave. Maxwell's work thus unified the domain of electricity, magnetism and light. We will discuss about qualitative aspect of these. Communication is possible due to electromagnetic waves, technological use of electromagnetic waves by Marconi and others led in due course, the revolution in communication that we are witnessing today.

Later we will discuss about nanotechnology. This is new branch of science emerged in three decades. This basically depends on Quantum mechanics. Nanotechnology will bring a revolution in science. For example you consider that in summer you are wearing such clothes which make you feel cool and in winter you are wearing such clothes which make you feel warm after wearing.

17.1. Displacement current

We have discussed in Chapter-7 that an electric current produces a magnetic field around it. Maxwell showed that for logical consistency, a changing electric field must also produce a magnetic field.

To see how a changing electric field gives rise to a magnetic field, let us consider the process of charging of a capacitor and apply Ampere's circuital law given by

$$\oint \vec{B}.\vec{dl} = \mu_0 I \qquad ....(17.1)$$

Where line integral is along a closed path which encloses conduction current and I is given by $I = dq/dt$

To find magnetic field at a point outside the capacitor : Figure 17.1, shows a parallel plate capacitor C which is a part of circuit through which a time dependent current I flows. Let us find the magnetic field at a point p in a region outside the parallel plate capacitor. For this, we consider a plane circular loop of radius r whose plane is perpendicular to the direction of the current-carrying wire, and which is centred symmetrically with respect to the wire. (fig. 17.2 (a)). From symmetry, the magnetic field is directed along the circumference of the circular loop and is the same in magnitude at all points, the left side of equation (17.1) is B (2πr).

So we have $B.(2\pi r)= \mu_0 I$ . . . (17.1 (a))

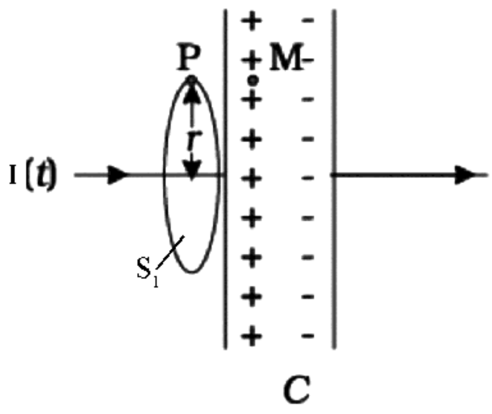
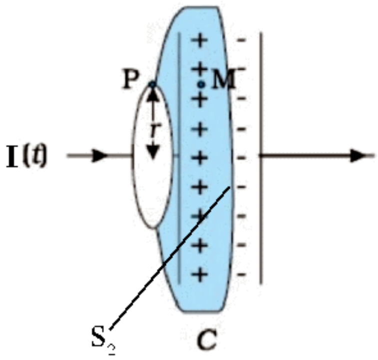
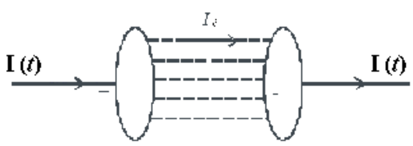


Fig. 17.1 : Time dependent current I (*t*) through capacitor & surfaces S_1 under considration

Now, consider a different surface, which has the same boundary. This is a pot like surface. (Figure 17.2 (a)) which nowhere touches the current, but has its bottom between the capacitor plates; its mouth is the circular loop mentioned above. Another such surface is shaped like a tiffin-box (without the lid) figure 17.3, On applying ampere's circuital law to such surfaces with the same perimeter, we find that the left hand side of equation (17.1) has not changed but the right hand side is zero and not $\mu_0 i$, since no current passes through the surface of figure 17.2(b). So we have a contradiction; calculated one way, there is a magnetic field at a point P; calculated another way, the magnetic field at P is zero.



(a)



(b)

Fig. 17.2 : (a) Electric field between charging plates
(b) Displacement current between the plates

Since the contradiction arises from our use of Ampere's circuital law, this law must be missing something. The missing term must be such that one gets the same magnetic field at point P, no matter what surface is used.

We can actually guess the missing term by looking carefully in figure 17.3. Is there anything passing through the surface S between the plates of the capacitor? Yes, of course, the electric field. If the plates of the capacitor have an area A, and a total charge Q, the magnitude of the electric field E between the plates is $\frac{Q}{A\varepsilon_0}$. The field is perpendicular to the surface S of figure 17.2 (b). It has the same magnitude over the area A of the capacitor plates and vanishes outside it. So what is the electric flux ϕ_E through the surface S? Using Gauss's law, it is

$$\phi_E = |\vec{E}|\, A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0} \qquad ......[17.1\text{ (c)}]$$

Now if the charge Q on the capacitor plates changes with time, there is a current $I = (dQ/dt)$, so that using equation (17.1 (c)) we have

$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

This implies that for consistency,

$$\varepsilon_0\left(\frac{d\phi_E}{dt}\right) = I \qquad ......(17.2)$$

This is the missing terms in Ampere's circuital law. If we generalise this law by adding to the total current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the total has the same value of current I for all surfaces. If this is done, there is no contradiction in the value of B obtained anywhere using the generalised ampere's law. B at the Point P is non-zero no matter which surface is used for calculating it. B at a point P outside the plates [figure 17.1(a)]is the same as at a point M just inside, as it should be. The current carried by conductors due to flow of charges is called conduction current. The current given by equation (17.2) is a new term, and is due to changing electric field (or electric displacement). It is therefore, called displacement current or Maxwell's displacement current. Figure. 17.3 shows the electric and magnetic fields inside the parallel plate capacitor discussed above.

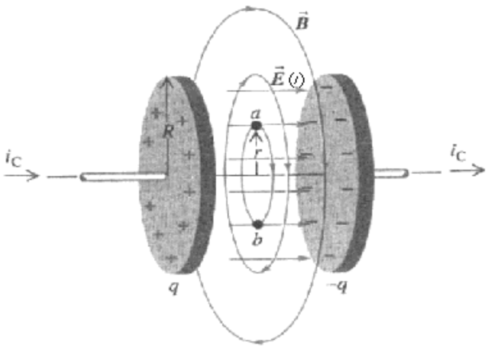


Fig. 17.3 : Magnetic field between the plates due to displacement current

The generalisation made by Maxwell then is the following. The source of a magnetic field is not just the conduction current due to flowing charges but also the time rate of change of electric field. More precisely, the total current I is the sum of the conduction current denoted by I_C, and the displacement current denoted by ($I_d = \varepsilon_0 d\phi_E / dt$), so, we have

$$I = I_c + I_d$$

$$= I_C + \varepsilon_0 \frac{d\phi_E}{dt}$$

In explicit terms, this means that outside the capacitor plates, we have only conduction current $I_C = I$, and no displacement current *i.e.*, $I_d = 0$. On the other hand, inside the capacitor, there is no conduction current *i.e.*, $I_C = 0$, and there is only displacement current, so that $I_d = I$.

The generalized and correct law : Ampere's circuital law has the same form as equation (17.1), with one difference, the total current passing through any surface of which the closed loop is the perimeter' is the sum of the conduction current and the displacement current. The generalized law is

$$\oint \vec{B}.\vec{dl} = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

and is known as Ampere-Maxwell law.

In all respects, the displacement current has the same physical effects as the conduction current. In some cases, for example, steady electic fields in a conducting wire, the displacement current may be zero since the electric field E does not change with time. In other cases, for example, the charging capacitor above, both conduction and displacement currents may be present in different regions of space. In most of the cases they both may be present in the same region of space, as there exist no perfectly conducting or perfectly insulating medium. Most interestingly, there may be large regions of space where there is no conduction current, but there is only a displacement current due to time varying electric fields. In such a region, we expect a magnetic field, though there is no (conduction) current sources nearby! The prediction of such a displacement current can be verified experimentally. For example, a magnetic field (say at point M) between the plates of the capacitor in figure 17.1 (a) can be measured and is seen to be the same as that just outside (at P),

We can understand it by as follows

$$\oint \vec{B}.\vec{dl} = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Using the relation for the surface inside the capacitor plates Fig. 17.2(b). We have

$B(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}$ ($\phi_E = EA$ and Ic = 0 inside the plates)

or $$B = \left(\frac{\mu_0 \varepsilon_0 \pi R^2}{2\pi r}\right)\frac{dE}{dt}$$

$$B = \left(\frac{\mu_0 \varepsilon_0 R^2}{2r}\right)\frac{dE}{dt}$$

This shown that B is produced due to dE/dt.

Example 17.1 : In charging a parallel plate capacitor of capacity 10μF, it takes 0.5 second to reach potential difference of 50 V. If plate area of the capacitor is 10×10^{-12}m², then find :

(i) Average conduction current at that time.

(ii) Average displacement current at that time.

(iii) Rate of change of electric field at that time.

Solution : (i) Average value of conduction current.

$$I = \frac{\Delta q}{\Delta t} = \frac{q_2 - q_1}{t_2 - t_1} = \frac{CV - 0}{0.5}$$

$$= \frac{10\times10^{-6}\times50}{0.5} = 10^{-3} A$$

(ii) Average displacement current in capacitor = Conduction current

$$I_d = I = 10^{-3} A$$

(iii) $\therefore I_d = \varepsilon_0 A \frac{dE}{dt}$

$$\therefore \frac{dE}{dt} = \frac{Id}{\varepsilon_0 A} = \frac{10^{-3}}{8.85\times10^{-12}(10\times10^{-12})}$$

$$= 1.1\times10^{9} V/ms$$

17.2. Maxwell's Equation, Qualitative Discussion:

Maxwell's equations represent the four basic laws of electricity and magnetism. These four laws are : Gauss's law in electrostatics, Gauss's law in magnetism, Faraday's law of electromagnetic induction and Maxwell-Ampere's circuital law. These equations are.

(i) $\oint \vec{E}.\vec{dA} = \frac{q}{\varepsilon_0}$ .... (17.3)

(ii) $\oint \vec{B}.\vec{dA} = 0$ .... (17.4)

(iii) $\oint \vec{E}.\vec{dl} = \frac{-d\phi_B}{dt}$ .... (17.5)

(iv) $\oint \vec{B}.\vec{dl} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$ .... (17.6)

Equation (17.3) represents Gauss's law in electrostatics. It states that the total electric flux through any closed surface S is always equal to $\frac{1}{\varepsilon_0}$ times the net charge inside the surface. This equation is called Maxwell's first equation. It is time independent steady state equation. It is true for both stationary and moving charges. We can infer from this equation that the electric lines of force do not constitute continuous closed path.

Equation (17.4) represents Gauss's law in magnetism. It state that the total magnetic flux through a closed surface is zero. This equation is called Maxwell's second equation. It is time independent equation. It expresses the fact that isolated magnetic poles do not exist in nature. We can infer from this equation that the magnetic field lines constitute continuous closed path.

Equation (17.5) represents Faraday's law of electromagnetic induction. this states that the line integral of electic field along a closed path is equal to the time-rate of change of magnetic flux through the surface bounded by that closed path. This equation is called Maxwell's third equation. It is time-dependent equation. We can infer from this equation that the time variation of magnetic field generates an electric field. This equatin is a relation between space integration of $\vec{E}$ and the time variation of $\vec{B}$. In this case electric field lines are closed loops.

Equation (17.6) represents Maxwell's Ampere's circuital law. It states that the line intergral of magnetic field along a closed path is equal to μ_0 times the sum of conduction and displacement currents. This equation is called Maxwell's fourth equation. It is a time dependent equation. This equation tells us that the time variation of electric field generates magnetic field.

If the electric field and magnetic field are present at any place, then the force on a charged particle is $\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$ and with the help of Maxwell equation all the events related to electro magnetism are explained easily.

17.3 Electromagnetic Wave and Their Characteristics

How are electromagnetic waves produced? Neither stationary charges nor charges in uniform motion (steady currents) can be sources of electromegnetic waves? The former produces only electrostatic fields, while the later produces magnetic fields that, however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge). This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. the oscillating electric and magnetic fields thus regenerate each other, so to speak, as the waves propagates through the space. The frequency of the electomagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source which accelerate the charge.

From the preceding discussion, it might appear easy to test the prediction that light is an electromagnetic wave. We might think that all we needed to do was to set up an ac circuit in which the current oscillate at the frequency of visible light, say yellow light. But, also, that is not possible. The frequency of yellow light is about 6×10^{14} Hz, while the frequency that we get even with modern electronic circuits is hardly about 10^{11} Hz. This is why the experimental demonstration of electromagnetic wave had to came in the low frequency region (the radiowave region), as in the Hertz's experiment (1887).

Hertz's successful experiment test the Maxwell's theory created a sensation and sparked off other important works in this field, two important achievements in this connection deserve mention. Seven years after Hertz, Jagdish Chandra Bose, working at Calcutta (now Kolkata) succeeded in producing and observing electromagnetic waves of much shorter wavelength (25 mm to 5 mm). His experiment, like that of Hertz's was confined to the laboratery. At around the same time, Guglielmo Marconi in Italy followed Hertz's work and succeeded in transmitting electromagnetic waves over distances of many kilometers. Marconi's experiment marks the begining of the field of communication using electromagnetic waves.

(i) Nature of electromagnetic waves and Propagation :

It can be shown from Maxwell's equations that electric and magnetic fields in an electromagnetic wave are prependicular to each other *and* to the direction of propagation. It appears reasonable, say from our discussion of the displacement current B and E are perpendicular to each other. In figure 17.4, we show a typical example of a plane electromagnetic wave propagating along the X-direction. The fields are shown as a function of x co-ordinate at given time t. The electric field E_y to along y - axis law varies simusoidally with x at given time t. The electric and magnetic fields E_x and B_z are perpendicular to each other, and to the direction x of propagation. We can write E_y and B_z as follows :

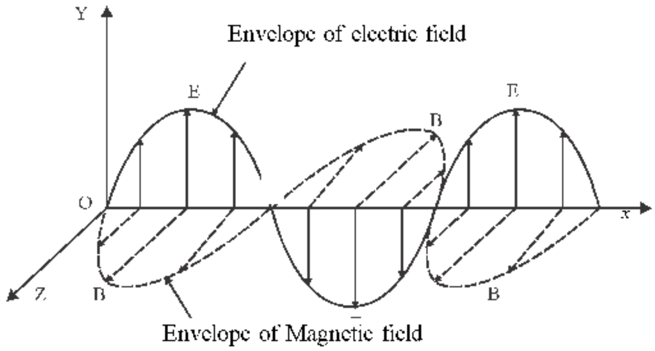


Fig. 17.4 : A linearly polarised EM wave propagating in the x-direction with the oscillating electric and magnetic field along Y and Z axis

The magnetic field B_z is along the Z axis and again varies sinusoidaly with x.

$$E_y = E_{oy}\sin(kx - \omega t) \qquad ....(17.7)$$

$$B_z = B_{oz}\sin(kx - \omega t) \qquad ....(17.8)$$

Here k is related to the wavelength λ of the wave by the usual equation $k = 2\pi/\lambda$ and ω is is the angular frequency, k is the magnitude of the wave vector (or propagation vector) $\vec{k}$ and its direction describes the direction of propagation of the wave. The speed of propagation of the wave is (ω/k) using equation (17.7) and (17.8) for E_y and B_z and Maxwell's equation, one finds that

$$c = \frac{\omega}{k} \text{ where, } c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad ....(17.9)$$

The relation ω = ck is the standard one for waves. This relation is often written in terms of frequency ($v = \omega/2\pi$) and wavelength ($\lambda = 2\pi/k$) as

$$2\pi v = c\left(\frac{2\pi}{\lambda}\right)$$

Or $\quad v\lambda = c$

It is also seen from Maxwell's equation that the magnitude of the electric and magnetic fields in an electromagnetic wave are related as

$$B_0 = E_0 / c \qquad ....(17.10, a)$$

and $\quad E_o = cB_o \qquad ....(17.10, b)$

If electromagnetic wave propagate in any medium other than vaccum, then speed of light is given by :

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \qquad ....(17.11)$$

Where ε is electric permittivity of the medium and μ is the magnetic permeability of the medium. ε_r is relative permittivity and μ_r is the relative permeability of the medium. This equation can be written as

$$\upsilon = \frac{c}{n} \qquad ....(17.12)$$

where, $n = \sqrt{\mu_r \varepsilon_r}$ is refractive index of the medium.

(ii) Energy Transmission by Electromagnetic Waves :

Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields. Energy density of an e.m. wave is the energy in unit volume of the space through which the wave travels.

We know that energy is stored in space wherever electric and magnetic fields are present.

In free space, the energy denisty of a static field E is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Again, in free space, the energy density of a static magnetic field is

$$u_B = \frac{B^2}{2\mu_0}$$

$\therefore B = E/c$ and $c = 1/\sqrt{\mu_0 \varepsilon_0}$, then

$$u_B = \frac{B^2}{2\mu_0} = \frac{E^2}{2c^2\mu_0} = \frac{E^2}{2\mu_0}(\mu_0 \varepsilon_0)$$

$$= \frac{1}{2}\varepsilon_0 E^2 = u_E$$

∴ Total instantaneously energy density

$$u = u_B + u_E = 2u_E = 2u_B$$

$$= \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \qquad ......(17.13)$$

If $E = E_y = E_m \sin(kx - \omega t)$, then for full cycle, average value of

$$\sin^2(kx - \omega t), \; <\sin^2\theta> = <\cos^2\theta> = \frac{1}{2}$$

∴ Total mean energy density

$$u_{av} = \varepsilon_0 <E^2>_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0} \qquad ......(17.14)$$

Rate of flow of energy through unit area is called Poynting vector, $\vec{S}$.

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \qquad ......(17.15)$$

and its magnitude, $S = \frac{EB}{\mu_0}$ ......(17.16)

For sinusoidal waves, Intensity I of plane electromagnetic wave is average of the poynting vector.

$$\therefore I = S_{av} = \frac{E_m B_m}{2\mu_0} = \frac{E_m^2}{2\mu_0 c} = cU_{av} \qquad ......(17.17)$$

(iii) Momentum and pressure Associated with Electromagnetic Waves :

An electromagnetic wave transports linear momentum as it travels through space. If an elecromagnetic wave transfers a total energy U to a surface in time t, then total linear momentum delivered to the surface is

$$p = \frac{U}{c} \qquad ......(17.18)$$

[Only for complete absorption of energy U]. If the wave is totally reflected, the momentum delivered to the surface will be $2U/c$.

$$p = \frac{2U}{c} \qquad ......(17.19)$$

If surface is totally absorbing then the radiation pressure is

$$p = \frac{I}{c} \qquad ......(17.20)$$

and for complete reflecting surface,

$$p = \frac{2I}{c} \qquad ......(17.21)$$

Where I is intensity of the wave.

(iv) Electromagnetic waves obey the principle of superposition. They show the properties of reflection, refraction, interference, diffraction and polorisation. Doppler effect is also observed for them.

Example 17.2 : An electric bulb is emitting uniform spherical wave in all directions. Assuming electromagnetic emission to be 50 W, then calculate (a) intensity (b) radiation pressure (c) magnitude of electric and magnetic fields at a point 3m away from the bulb.

Solution : At *r* distance from bulb, the energy will uniformly spread on area $4\pi r^2$.

(a) $$\text{Intensity} = \frac{\text{Power}}{\text{Area}} = \frac{50}{4\pi r^2}$$

$$= \frac{50}{4 \times 3.14 \times (3)^2} = 0.44 \text{ W/m}^2$$

(b) Radiation pressure,

$$p = \frac{I}{c} = \frac{0.44}{3 \times 10^8} = 1.47 \times 10^{-9} \text{ N/m}^2$$

(c) $$I = \frac{E_0^2}{2m_0 c}$$

$$E_0 = \sqrt{2\mu_0 Ic}$$

$$= \sqrt{2(2\pi \times 10^{-7} m/A) \times 0.44(W/m^2) \times 3 \times 10^8 m/s}$$

$$= 18.2 \text{ V/m}$$

and, $$B_0 = \frac{E_0}{c} = \frac{18.2}{3 \times 10^8} = 6.08 \times 10^{8}\, T$$

17.4. Electromagnetic Spectrum

We have seen that all EM waves propagate with the speed of light c in vacuum. These waves transport momantum and energy. At the time of maxwell we knew only about visible light (some what about infrared). In 1888 Hertz generated radio waves. At present we are familier with all EM waves from radiowaves to γ - rays. In all the waves there is role of accelarated / decelarated charges. All these waves differ only in wavelength / frequency energy and related effects. Set of these waves expressed in a certain order of wavelength is called EM spectrum. This classification does not have well difined sharp boundries, their boundries overlap each other. For example wave of wavelength 0.1 Å may be called x-ray or γ-ray. They differ only in their origin (γ-ray are produced from within the nucleus where as x-rays are produced by bombarding high energy electrons on a heavy matel, and are of atomic origin).

Now we will discuss these different waves in the order of their decreasing wavelength.

Radiowaves : These waves are produced by oscillating electrons in a dipole antenna or conducting wires. LC circuits and electronic devices are used for their generation. They are used in Radio, Television and other communication systems. Their range is from 500 kH_z to 1000 MH_z. For AM modulated band the range is from 530 kH_z to 1710 kH_z. For FM band the range is between 88 MH_z to 108 MH_z, and approximately same for TV broadcast ie 54MH_z to 890 MH_z. UHF frequency ranging from 840 MH_z to 935 MH_z is used in cellural phone.

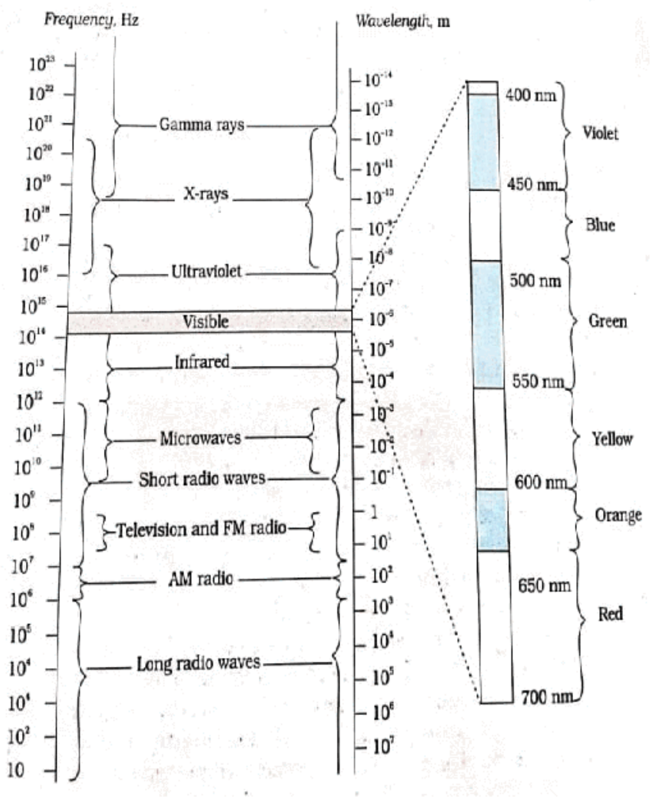


Fig. 17.5 : The electro magnetic spectrum with common names or various part of it.

Detection of radiowave is done by dipole antenna or magnetic a loop antenna. For first type electric field of the wave generate electric current while in loop antenna emf is induced and detected.

Microwaves : Range of these types of waves are from λ = 0.001 m to 0.1 m. They are also called mm waves. Their frequency is of the order of GHz. They are produced by electronic devices like magnetron, klystron and Gun diodes etc. They are used in RADAR system and navigation and speed guns. They are also used to investigate the properties of atoms and molecules. An interesting domestic use is in microwave oven. Water containing food can be cooked in microwave oven. The natural frequency of water molecule is 3GHz. When food is exposed to microwave radiation the water in it absorbs the microwave of this frequency by resonance and heated up and cooks the food. Utencils of porcelene or glass which has larger molucules and very low natural frequency are used. Hence food is cooked without heating the utencils thus saving energy. Metalic utencils are not used to avoid shock due to induced charges.

Infra red rayes : These rays are produced by oscillations of atoms and molecules in a hot body. Their wave length is from 1m to 700nm. These rays are used in physiotherapy, infrared photography and in remote control devices of electronic gadgets. The warmth that we feel in sun light is due to these rays.

Visible light : Human eyes are sensitive for these rays, of wavelength ranging from 380 nm to 788 nm corresponding to violet to red light. Sensitivity of our eyes differs for different wavelengths and is maximum for yellow green wavelength of 550 nm. These waves provide energy for photosynthesis in plants. These wave are produced by heating effect (bulb), transition of electrons from higher energy level to low energy level in excited atoms (flourosecent lamp) and direct conversion of bond energy to light energy in LED.

Ultraviolet rays : This part of EM spectrum ranges from 1 nm to 380 nm. Sun is the major source of these rays. They are harmful to our skin and may cause skin cancer. Fortunately our atmosphere provides a natural shield in the form of Ozone layer which absorbe these rays and protect us. But human activity using CFC in aerosole and refegerators are responsible for depletion of ozone layer. Arc lamp and welding arc produced UV rays in large amount. The weldor uses special protection mask to save his eyes from UV rays which may damage ratina. They are used to detect fake currency notes. The numbers on genuine currency notes gives fluorescence in UV rays.

X-rays : They are produced by bombarding high energy electron beam on a heavy metal like tungston. The wavelength range is in between 1 nm to 10^{-3} nm. They are used in medical science and is crystallography. Over and unnecessory exposure should be avoided as it may cause cancer and other damages to tissues.

γ-rays : These are the shortest wavelength and most energetic waves known. Thery are produced from within a radioactive nuclied. After emmission of α or β rays. The nucleus comes in exeited state, which stablizes after emitting γ-rays. They are used to destroy cancer cells. To sterlize the medical equipment their uncontrolled exposure on human body may cause cancer too. The sun is the a natural source of almost entire EM spectrum.

17.5 Propagation of EM waves

EM waves are transmitted by an antenna in a communication system. As they propagate their intensity go on decreasing due to various reasons. Noise is also introcued during propagation. There are three modes of transmission of EM waves from transmitter to receiver (a) ground wave or surface wave propagation, (b) Space wave propogation, (c) Sky wave propagation.

17.5.1 Ground or Surface wave propogation

In this mode, both the transmitter antenna and reciever antenna are very near to Earth surface. Hence the wave propagates just gliding to the surface. The electric field of the wave induces charge on the surface and the wave get short circuited and get vertically polarized. The intensity of the wave decreases with distance. More over this mode is suitable only for low frequency EM waves, frequency less then 1 MHz. Since high frequency waves are more absorbed by earth. This mode is used for a limited distance of (500 km.) propagation.

17.5.2 Space wave propagation

In this mode of propagation the modulated carrier waves propagates direct from transmitter antenna to the receive antenna through troposphere. Slight bending towards earth is helpfull due to different refractive index of layers of troposphere (actually ionosphere). Due to curvature of earth the long distance communication is not possible and it limited to line fo sight distance. The height of antenna is an important factor in the range of communication system.

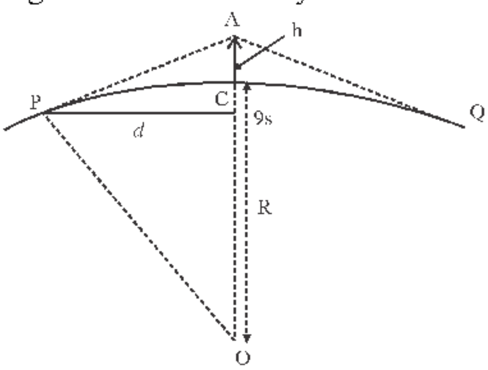


Fig. 17.6 : LOS in space wave propagation

In fig. 17.6, an antenna of height h is at a point C on earth surface. The EM waves from the top of antenna reach up to point P and Q on the surface. Hence the distance for the transmission is CP = CQ = d for the receiver of zero antenna height. If the radius of the earth is R and O is the center of the Earth from right angle tringle OPA,

$$OA^2 = OP^2 + PA^2 \quad (h << R, PA = PC)$$

or $$(R+h)^2 = R^2 + d^2 \, (PA \simeq PC = d)$$

or $$R^2 + 2hR + h^2 = R^2 + d^2$$

or $$h(2R+h) = d^2$$

or $$h(2R) = d^2$$

($h << 2R$, so h^2 is negligible)

hence $$d = \sqrt{2Rh} \qquad \ldots (17.22)$$

In this mode they waves between the frequency range 100 MHz to 200 MHz are used as carrier waves.

17.5.3 Sky waves propagation

Long range transmission is possible in this mode. The ionosphere plays a role of reflecting the waves back to the ground to receiver. The density of ions in the different layers is different and the reflective index of medium (ionosphere) decreases gradually with height. This reflection is not as reflection from a mirror but the wave gradualy bends back to the earth by the phenomenon of i.e. total internal reflection. This type of reflection is affected by atmospheric disturbances, frequency used and day night effect. During night the reflection is better due to merger of some layers of ionosphere. The maximum frequency that can be used is 30 MHz. Frequency greater than this is not reflected and so escapes.

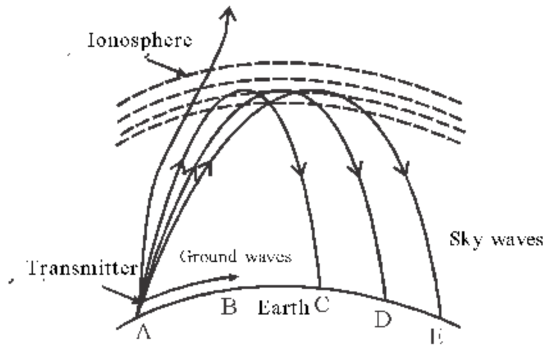


Fig. 17.7

Communication with Satellite

A communication satellite is at the height of ≈ 36000 km abvoe the earth and revolve round the earth with a time period of 24 hours. Hence it is stationery with respect to earth. The waves of frequency > 30 MHz which escape the ionosphere, reaches the satellite. The satellite retransmits the wave with another frequency called down link frequency. The frequency used in this mode is in UHF (Ultra high frequency) region. This mode is used in cellular telephony.

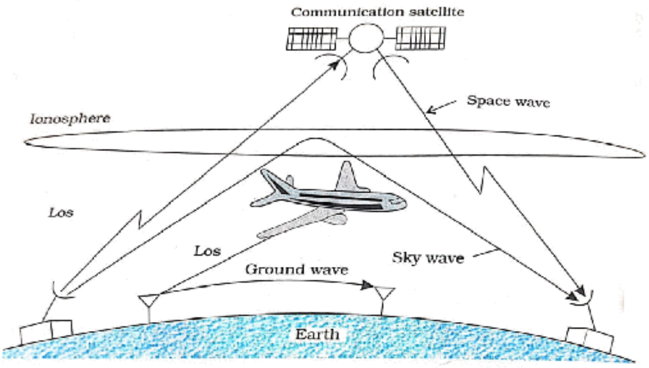


Fig. 17.7 : Various propagation modes for EM waves

17.6 Communication System

The exchange of information between two persons or system in the form of video, audio or data is called communication. Art of communication is as old as the human race. The art and the mechanism changed with time. In modern time there is revolation in this field. Mobile, cellular telephony and use of computer in fox, e-mail and other modes made the life easy. The basic components of a communication system are (i). Transmitter (ii). Communication channel (iii). Receiver Fig 17.8 gives the bolock diagram of the system. The information may be is any form such as video, audio, text and data which are non electrical in nature. First a device called a transducr is used to convert information in electrical signal which is suitable for transmission. This is message signal. This message signal is sent to receiver through chennel. This chennel could be a wire, coaxial cable, optical fiber or EM wave (Which carries the information after modulating HF carrier wave with information signal). In EM mode the channel is space, the wave travel from transmitter to receiver through space with speed of light. The antenna at receiver end defect the signal and change it into usable form. Unwanted signal is introduced as a noise during propagation. The information signal may be a analog or digital.

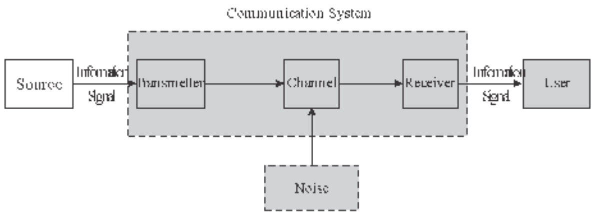


Fig. 17.8 : Communication system

17.7 Modulation

Normally the information signals are of low frequency, so even after converting into electrical form are not suitable for long distance transmission. A high frequency wave called carrier wave (Since it carries the signal) is used for transmission. The information signal is superimposed or loaded with carrier wave. The process of superimposing the signal with carrier wave is called moderation. The carrier wave may be mathematically represented by $c(t) = A_c \sin(\omega_c t + \phi)$

Here $c(t)$ is the instantanious value of voltage or current, the wave has three characters. Namely Amplitude A_c , frequency ω_c and ϕ (is initial phase angle). In the process of modulation any one character of the carrier wave is modified as per the information signal. Hence the modulation is of three types :

(i) Amplitude modulation, called AM.

(ii) Frequency modulation, called F.M

(iii) Phase modulation called, PM.

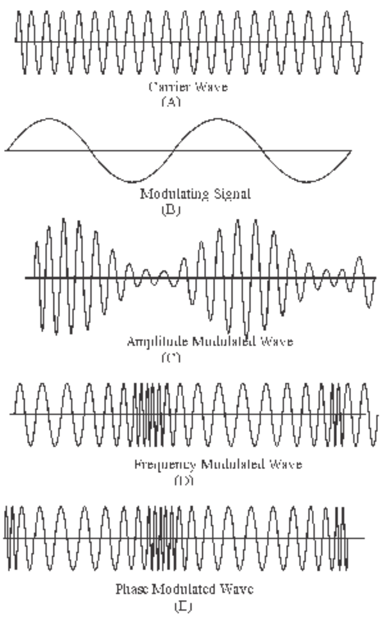


Fig. 17.9 : Analog modulation

(i) Amplitude modulation:-

The figure (A) Showes a carrier wave of deferent high frequency. Figure (B) shows modulating signal. The figure (c) shows an amplitude modulated wave modulated carrier wave. Note that the frequency and phase of the wave remain unchanged, but the amplitude changes with change in signal. The modulated wave carries the information as the change in amplitude which can be sensed (detected) at the receiver end.

(ii) Frequency Modulation :-

Frequency modulated wave is represented by fig (D). Shown is the diagram that amplitude and phase of the modulated wave remains unchanged, while the frequency of the carrier wave is changed as per the amplitude of the signal. ie when signal amplitude is high the frequency is high and vice-versa. Hence the information is carried by the wave in the form of change in frequency.

(iii) Phase modulation:-

In this mode, the phase of the carrier wave is changed as per the instantaneous values of signal voltage. It is to be noticed in the diagram (D) and (E) that both FM and PM are similar. In both, change of frequency is there, but is PM the phase of the wave either leads or lags as per signal voltage.

17.7.1 Need of Modulation :-

As it is clear that low frequency information signal is not suitable for long range transmission even if it is converted into electrical form. We need a carrier wave to carry the signal.

(i) Need of Antenna :

In electronic circuits the modulated wave is electrical in nature and it has to be converted into EM wave, which is done by a dipole antenn. It needs very high antenna (at least of the order of $\lambda / 4$) so that it carries all the information of the signal (or the time variation of signal). For a wave of 20 kHz , λ is 15km. Clearly such long antenna is not possible. To reduce the high of antenna we should load the information on the high frequency carrier wave.

(ii) Effective Power Radiated by Antenna :-

The effective power radiated by an antenna is related to (i) antenna length l and (ii) the radiated frequency/ wave length λ by $P \propto (l/\lambda)^2$. It is clear from the given relation that for same antenna length, the wave length of the radiated wave should be very small for more power to be radiated. That is the reason for loading the signal on a high frequency/low wavelength carrier wave.

(iii) Mixing of the information:-

When so many persons are talking at a time, the listener is unable to make any sense of that. Similarly when so many transmitter are radiating information, the receiver can't make any sense. That`s why each transmitter is allotted one fixed carrier frequency (which carries the information) and the receiver is tuned to this frequency only, so others are eliminated.

17.7.2 Amplitude Modulation :

In amplitude modulation, the amplitude of the carrier wave is changed as per change in signal amplitude.Let $c(t) = A_c \sin \omega_c t$ and $m(t) = A_m \sin \omega_m t$ represent carrier wave and singal respectively, where $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$ are freqeuncy of carrier wave and signal.

After modulation the carrier wave is -

$$c_m(t) = (A_c + A_m \sin \omega_m) \sin \omega_c t$$

$$= A_c \left(1 + \frac{A_m}{A_c} \sin \omega_m t\right) \sin \omega_c t$$

Here the singal is present in the amplitude of the above wave. We can write it as -

$$c_m(t) = A_c \sin \omega_c t + mA_c \sin \omega_m t \sin \omega_c t \qquad \ldots(1)$$

where $m = \frac{A_m}{A_c}$ is modulation index, and $m \le 1$.

Using - $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

We can write eqn (1) for $c_m(t)$ as -

$$c_m(t) = A_c \sin \omega_c t + \frac{mA_c}{2} \cos(\omega_c - \omega_m)t - \frac{mA_c}{2} \cos(\omega_c + \omega_m)t$$

here $\omega_c - \omega_m$ and $\omega_c + \omega_m$ are called lower and upper sideband frequencies these, hence the carrier wave also contains these upper and lower sideband frequencies along with ω_c.

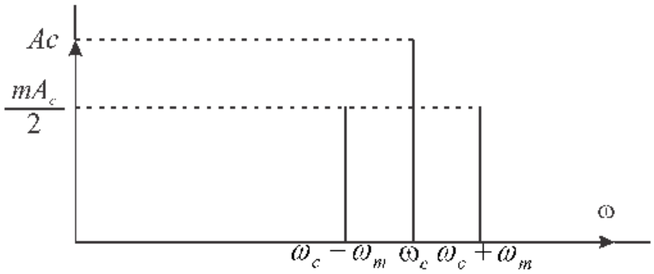


Fig. 17.10 Graph between amplitude and frequency in amplitude modulation

Note : In amplitude modulation, the amplitude of signal voltage is added with the amplitude of carrier wave changes between A_{max} and A_{min} this change represents up to what extent the carrier wave is modulated. Amplitude modulation index is -

$$i.e., \mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

If m = 0; there is modulation

$m \geq 1,$ over modulation

$0 < m < 1,$ normal modulation

17.7 .3 Transmission and Reception of Amplitude modulated waves

The above mentioned modulated wave can not be transmitted as such. First its power is increased using a power amplifier, then it is transmitted by an appropriate antenna. The process is shown is block diagram -

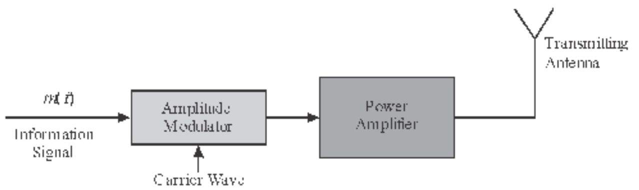


Fig. 17.11 Block diagram of a transmitter

The modulated transmitted wave get weakned (attenenated) in transmission channel, hence the signal that we recieve by the recieving antenna is very weak. It can not be used as such. We amplify this signal in two steps. (1) Radio freqeuncy amplification. (2) I F or intermediate frequency amplification. Now the information signal (which is mixed carrier wave) is obtained by separating it from carrier wave using a detector, this process is called demodulation. The information is signal is again amplified to get it in a usable form.

Example 17.4 : A carrier wave of peak value 12V is used for transmission. What will be the peak value of signal to obtain a modulation index of m = 0.75.

Solution : Amplitude of carrier wave $A_c = 12V$

Modulation index $m = 0.75$

$\because \quad A_m = mA_c = 0.75 \times 12 = 9\text{V}$

Example 17.5 : After modulation the amplifide of carrier wave varies between 5V and 2V. What is depth of modulation ?

Solution : Here $A_m = 5V$ and $A_{min} = 2V$

Modulation index $m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{5-2}{5+2} = \frac{3}{7}$

or $\quad m = \frac{3}{7} \times 100\% = 42.8\%$

17.8 Nanotechnology :

Nano science is the branch of technology in which study of bodies of size less than 100 nm is taken

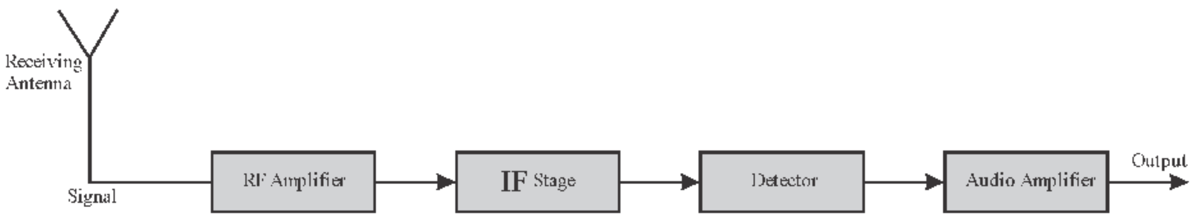


Fig. 17.12 : Block diagram of a receiver

up. Thickness of human hair is of the order 60000 to 80000 nm. From this we can experience the smallness of nanotechnology. Scienstists have discovered serveral nano particles and thin films whose properties are different than that of their ordinary size. There are enormous possibilities of fabricating better structures, devices and materials using nano particles and nano films. There are three specialities in nanotechnologies -

1. Size less then 100 nm.
2. Unique properties due to small size.
3. Control on structure and properties up to nano scale.

There are many examples of nano structures in nature. For example, some catalysts, filter particles and some minerals which have different properties at nanoscale. In previous decade the fabrication of functional enginnered materials and devices were made possible after understanding and controlling the nanostructures.

During last 40 years a techniques of nano and microlithography have been developed, which revolutionised the micro electronics. With the help of this technique new microprocessors were developed which have small size and many fold efficiency. In previous years micro mechanic and micro optical devices were developed.

Another branch of nanotechnology is moleculer and chemical technique by which new consumer products are being obtained by controlling the chemical properties of the molecules.

17.8.1 Nanostructures in Nature

Fine observation of plants and animals around us reveals many specific features at nano level. Few esamples are -

1. Insect eyes have many small bulges of hexagonal shape of size few hundres nanometre in size. These are smaller than the wave length of vissible light (380-780nm), hence they have less reflectivity and more absorption of light which make them able to see in a better way as compared to human beings in dim light conditions. On this basis scientists have fabricated such nano strutures which absorb more infrared light these nano structures are used to increase efficiency of thermo volt aic cell.
2. Wings of butterfly has multilayered nano patterns, which filters the white light and reflects a particular color. Thses nano structurs are of the size of wavelength of light, the interfreance of ligth at the wings are used in critical analysis of colours.
3. Edlewiess is an alpine plant, which is found in hight mountain ranges. whave level of UV rays is high. The flower of this plant is covered by hollow nanofibers which absorbs UV rays and refflects white light. So the flower appears white. These nano structures protact plant from hight energy radiations. On the basis scientist are working on device which can protect us from high energy radiations.

17.8 .2 Observations of nano strucutre

The ordinary microscopes are not suitable for observing nano structure. Some complex devices are used for this purpose.

Few are-

(i) **Optical Microscope -** It can be used to oberserve a nano structure of size 250 nm which is longer than the 1actual nano partides. This also the limit of such microscopes.

(ii) **Electron Microscope -** In this microscope electron beam is used in place of visible light due to which particles of few nano meter size can be studied. Due to its high resolution structure of a few non can be observed.

(iii) **Scanning Probe Microscope -** The nano particles of size 1nm can be observed by such microscope. Nano technology is the most active field of research in solid state physics, chemistry, electical enginnering bio-chemistry and approximately all branches of science, which is going to have a deep impact on our economy and life style.

Important Points

1. **Displacement Current :** it is that current which comes into existence (in addition of conduction current) whenever the electric field and hence the electric flux changes with time. It is given by

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

2. **Maxwell's Equations :**

(i) $\oint \vec{E}.\vec{dA} = \frac{q}{\varepsilon_0}$

(ii) $\oint \vec{B}.\vec{dA} = 0$

(iii) $\oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt} = -\frac{d}{dt}[\oint \vec{B}.\vec{dA}]$

(iv) $\oint \vec{B}.\vec{dl} = \mu_0 \left[I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right]$

3. Basic Properties of E.M. Waves :

(i) The oscillation of $\vec{E}$ and $\vec{B}$ fields are perpendicular to each other as well as to the direction of propagation of the wave.

The direction of wave is in the direction of $(\vec{E} \times \vec{B})$

(ii) In free space they travels with speed $c = 3 \times 10^8$ m/s. their velocity in a medium of refractive indes n is $\upsilon - c/n$ where $n = \sqrt{(\mu_r \varepsilon_r)}$.

(iii) The electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields. The average energy density of an electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\left[\varepsilon_0 E_0^2 + \frac{B_0^2}{\mu_0} \right] \text{ in free space.}$$

(iv) Momentum of the electromagnetic wave $p - U/c$ (U = total transferred energy) and pressure on totally absorbing surface due to this momentum, pressure = I/C For Totally reflecting surface $p = 2I/C$

5. the main part of an electomagnetic spectrum in the order of increasing wavelength from 10^{-2}Å or 10^{-12} m to 10^6m are γ-rays, X-rays, UV rays, visible light, infrared rays, microwaves and radiowaves.

6. **Propagation of Electromagnetic Waves :**

These are propagated through three modes :

(i) Ground wave propagation

(ii) Space wave propagation

(iii) Sky wave propagation

7. **Communication Systems :** The setup used to transmit information from one point to another is called communication system. A communication system mainly consists of

(i) Transmitter

(ii) Communication channel

(iii) Receiver

8. **Modulation :** Modulation is the process of changing some characteristics (amplitude, frequency or phase) of high frequency carreir wave in accordance with the instantaneous value of the low frequency audio signal called the modulating signal. We have three types of modulation :

(i) Amplitude modulation

(ii) Frequency modulation

(iii) Phase modulation

9. **Need for Modulation :**

(i) Size of the antenna

(ii) Radiated power by the antenna

(iii) Mixing of different signals

10. Amplitude modulated wave consist of carrier frequency ω_c in addition to $\omega_c - \omega_m$ and $\omega_c + \omega_m$ called side bands

Modulation index $\mu = \frac{A_m}{A_c}$

11. Nanotechnology is the engineering of tailoring of functional systems at the molecular or atomic scale.

Questions for Practice

Multiple Choice type Questions :

1. Average energy density radiated in electromagnetic wave is related to :

(a) Only electric field

(b) Only magnetic Field

(c) Both electric and magnetic field

(d) Average energy density is zero

2. Waves related to telecommunication are

(a) Infrared (b) Visible light

(c) Microwaves (d) Ultraviolet ray

3. electromagnetic waves does not transport :

(a) Energy (b) Charge

(c) Momentum (d) Information

4. If $\vec{E}$ and $\vec{B}$ are electric and magnetic field vectors of electromagnetic waves, then the propagation of electromagnetic wave is along :

(a) $\vec{E}$ (b) $\vec{B}$

(c) $\vec{E} \times \vec{B}$ (d) $\vec{E}.\vec{B}$

5. Which radiation has least wavelength?

(a) X - ray (b) γ-ray

(c) β-ray (d) α-ray

6. Mark the wrong option related to characteristic of electromagnetic waves :

(a) Both electric field and magnetic field vector occupy maximum and minimum value at same time and position.

(b) In electromagnetic waves, the energy is equally distributed among electric and magnetic field vectors.

(c) Electric and magnetic field vectors are parallel to each and they are perpendicular to the direction of propogation of the wave.

(d) They do not require any medium for their propagation.

7. For whom the ground waves are possible ?

(a) Low radio frequency at low range

(b) High Radio frequency at low range

(c) Low radio frequency at high range

(d) Low radio frequency at low range

8. The height of TV tower is h meter. If radius of the Earth is R, then during TV transmission, the area coverage is (if h < R) :

(a) πR^2 (b) πh^2

(c) $2\pi Rh$ (d) πRh

9. For propagation of radiowaves the mode used is :

(a) Ground wave propagation

(b) Sky wave propagation

(c) Space wave propagation

(d) All of the above

10. In a amplitude modulated wave the maximum amplitude is 10 V and minimum amplitude is 2V. The modulation factor m is :

(a) 2/3 (b) 1/3

(c) 3/4 (d) 1/5

11. Modulation Factor of Over modulated wave is

(a) 1 (b) Zero

(c) < 1 (d) > 1

Answer (Multiple Choice Question)

1. (c) 2. (c) 3. (a) 4. (c)
5. (b) 6. (c) 7. (a) 8. (c)
9. (d) 10. (a) 11. (d)

Very Short Answer Type Questions :

1. What is the speed of the electromagnetic waves in vaccum?

2. For electromagnetic waves, what is the effect on refractive index of Ionosphere on increasing height from Earth's surface.

3. the vibration of $\vec{E}$ of an electromagnetic wave propagation in X-direction are parallel to Y-axis. Then in which axis vibration of $\vec{B}$ are parallel?

4. For long distance propagation, which method is used for propagation?

5. What are the frequency limits of sky waves in sending singnal to remote places?

6. Name the part of the communication system which converts signal to suitable form to send on communication channel to receiver.

7. Name the method used to superimpose signal on carrier wave?

8. Which types of matter are studied in Nanotechnology?

Short Answer Type Questions :

1. Name the components of electromagnetic spectrum in decreasing order of wavelength.

2. Write four main characteristics of electromagnetics waves.

3. Explain ground waves and sky waves.

4. What is communication system?

5. Name the part of the communication system.

6. Explain modulation.

7. Give the name of the instruments used to observe nanostructures.

Essay Type Questions :

1. What is the nature of electromagnetic wave? Explain the Hertz's experiment related to electromagnetic wave.

2. Describe various components of electromagnetic wave and explain their characteristics.

3. Explain the process of modulation and demodulation. How they are used in message signal propagation?

4. Explain with diagram amplitude modulation, frequency modulation and phase modulation.

5. Describe some examples of Nanotechnique observed in nature .

Numerical Questions :

1. A plane electromagnetic wave travels in free space along the X-direction. At a particular point in space the maximum value of $\vec{E}$ is 600 volt/meter. What is $\vec{B}$ at this point? ($c=3\times10^8$ m/s)
   [**Ans.** 2×10^{-6} wb/m^2]
2. A TV transmitting antenna is 75 m tall. How much maximum distance and area cover by it? radius of Earth = 6.4×10^6 m
   [**Ans.** 31 km, 3014 km^2]